Relativistic r-mode instability windows in hyperonic stars

Kraav K. Gusakov M. Kantor E.
Ioffe Insitute

Dubna 2025

Outline of the talk

1. <u>Introduction</u>
   - What is an r-mode?
   - Why study r-modes?
   - The concept of the instability window
   - The LMXB paradox
   - Why relativistic r-modes in hyperonic stars?

2. <u>Hyperonic stars</u>
   - Model of a hyperonic star
   - Chemical reactions in hyperonic matter
   - Bulk viscosity and adiabatic index
   - Pairing effects

3. <u>Relativistic r-modes</u>
   - Hydrodynamic oscillation equations
   - R-mode eigenfunctions
   - Dissipation through bulk viscosity
   - Dissipation through shear viscosity
   - Amplification by CFS-mechanism

4. <u>Instability windows</u>
   - Calculation overview
   - Numerical results

5. <u>Conclusion</u>

Introduction

What is an r-mode?

Stellar oscillations can be classified according to
1) the dominant restoring force and **2) the geometry of the fluid motion**

R-modes – oscillations of rotating stars:

1. Restored by the **Coriolis force**
2. **Quasi-toroidal** motions:

Flow velocity:

$$\boldsymbol{v} = \boldsymbol{v}_0 + \delta\boldsymbol{v}$$

equilibrium velocity (rotation) | velocity perturbation (r-mode)

$$\delta\boldsymbol{v}(\boldsymbol{r},t) \approx [\boldsymbol{r}\times\boldsymbol{\nabla}]\mathrm{T}(\boldsymbol{r},t)$$

$$\mathrm{T}(\boldsymbol{r},t) = T_{lm}(r)P_l^m(\cos\theta)e^{im\varphi+i\sigma t}$$

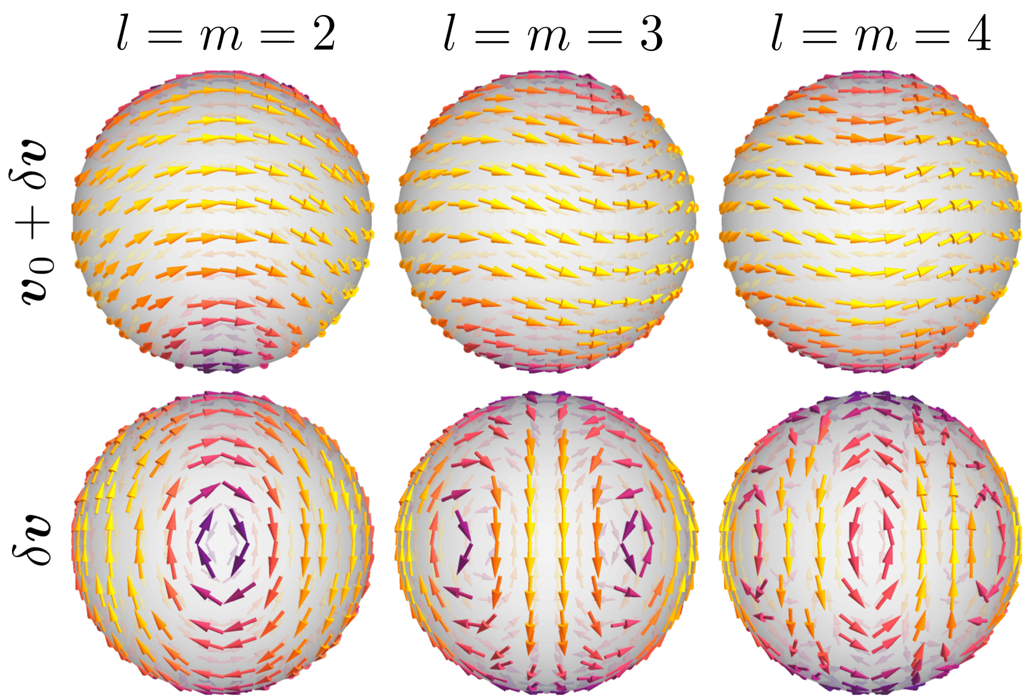
R-mode oscillation frequency:

$$\boxed{\sigma \approx \left[\frac{2m}{l(l+1)} - m\right]\Omega}$$

Notations and terminology:

Ω – angular velocity
T_{lm} – toroidal function
P_l^m – associated Legendre polynomials

Geometry of a purely toroidal motion:



Why study r-modes?

R-modes are the most susceptible
to the Chandrasekhar-Friedman-Schutz (CFS) instability
(with respect to emission of gravitational waves)
[e.g., Lindblom et al. Phys. Rev. Lett. 1998]

<u>CFS mechanism in a nutshell:</u>
[Chandrasekhar 1970; Friedman & Schutz 1978]

Oscillating star emits gravitational waves: $\dot{E}_{\rm GW}(\Omega) = \text{emitted energy/sec}$

$\dot{J}_{\rm GW}(\Omega) = \text{emitted angular momentum/sec}$

⇩

Change of the oscillation energy: $\dot{E}_{\rm CFS}(\Omega) = \dot{E}_{\rm GW}(\Omega) - \Omega \dot{J}_{\rm GW}(\Omega)$

⇩

Instability criterion: $\boxed{\dot{E}_{\rm GW}(\Omega) > \Omega \dot{J}_{\rm GW}(\Omega) \quad \Rightarrow \quad \dot{E}_{\rm CFS} > 0}$

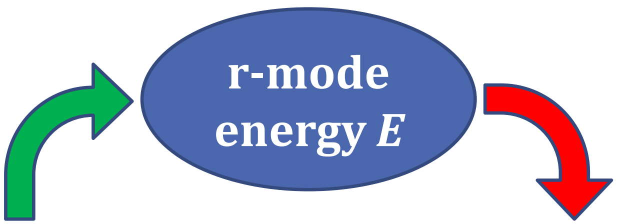
For r-modes the instability criterion is met at any Ω!!!

⇩

Under favorable conditions r-modes may become visible to GW detectors

The concept of the r-mode instability window

Evolution of r-modes



CFS mechanism:

$\dot{E}_{\rm CFS}(\Omega) > 0$

Energy dissipation:

$\dot{E}_{\rm diss}(\Omega, T^\infty) < 0$

T^∞ – stellar temperature (redshifted)

The most «popular» dissipative agents:

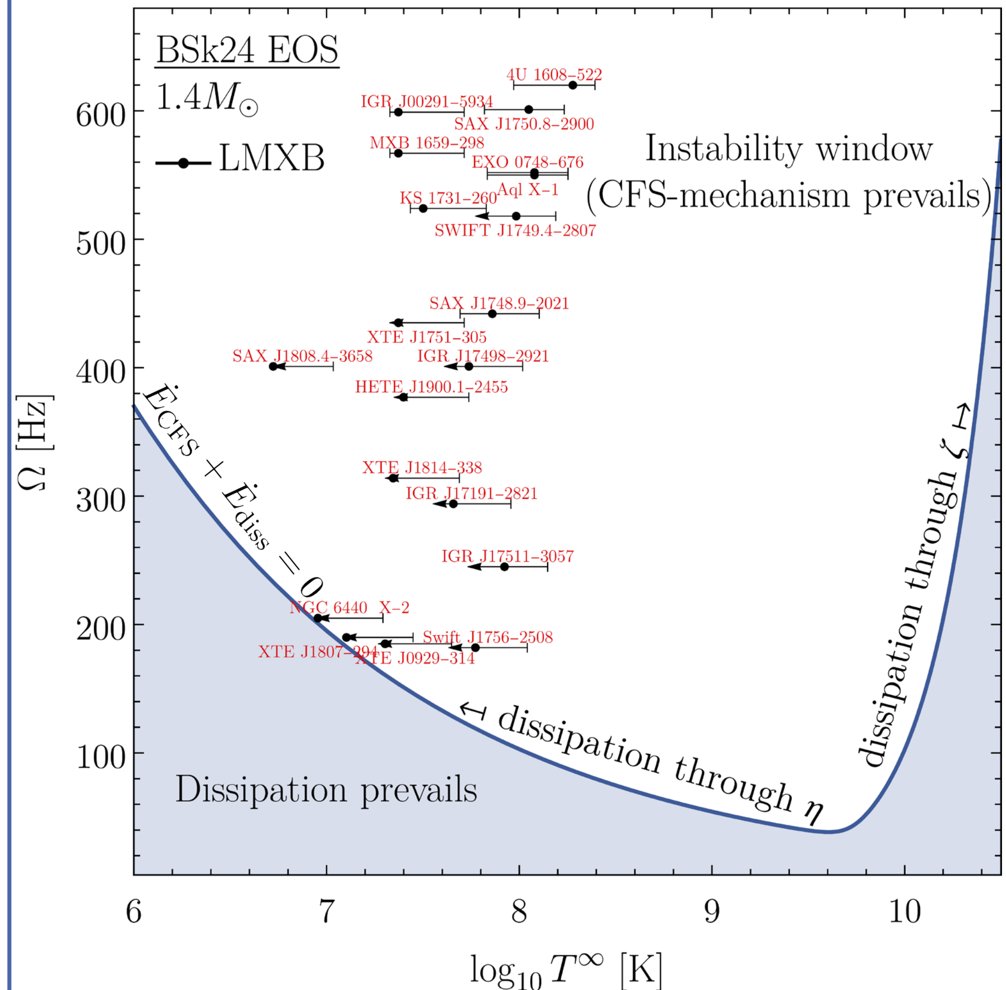
- **shear viscosity (η)**
  [friction of the fluid layers in relative motion]
- **bulk viscosity (ζ)**
  [heat production by out of equilibrium chemical reactions]

Instability window:

[e.g., Lindblom et al. 1998]

$$\dot{E} = \dot{E}_{\rm CFS}(\Omega) + \dot{E}_{\rm diss}(\Omega, T^\infty) > 0$$

An example: Newtonian nucleonic model



LMXB paradox: [e.g., Haskell 2015] the window likely should not contain observable sources!

The LMXB paradox

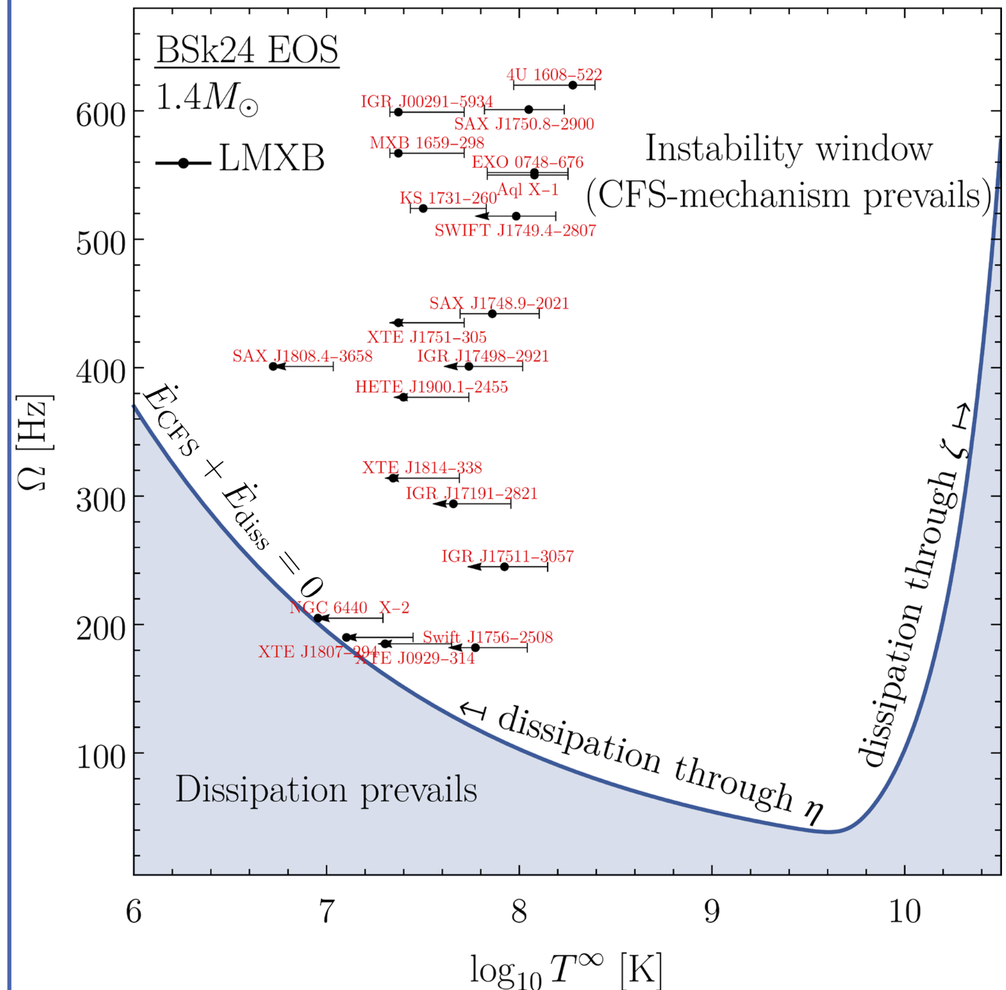
Attempts at stabilizing r-modes

- **particle diffusion**
  [K. Kraav, M. Gusakov, E. Kantor PRD 2024]
  [K. Kraav, M. Gusakov, E. Kantor MNRAS Lett. 2021]
- **resonant suppression by superfluid modes**
  [Kantor E., Gusakov M., Dommes V. PRD 2021]
  [Kantor E., Gusakov M., Dommes V. PRL 2020]
- **Ekman layer dissipation**
  [K. Glampedakis & N. Andersson PRD 2006]
  [Y. Levin & G. Ushomirsky MNRAS 2001]
  [L. Bildsten & G. Ushomirsky ApjL 2000]
- **mutual friction**
  [B. Haskell et al. MNRAS 2009]
- **amplified bulk viscosity in hyperonic matter**
  [D. Ofengeim et al. PRD 2019]
  [M. Nayyar & B. J. Owen PRD 2006]

<u>This study:</u>

Consider **relativistic** r-modes in **hyperonic** stars

An example: <u>Newtonian nucleonic</u> model

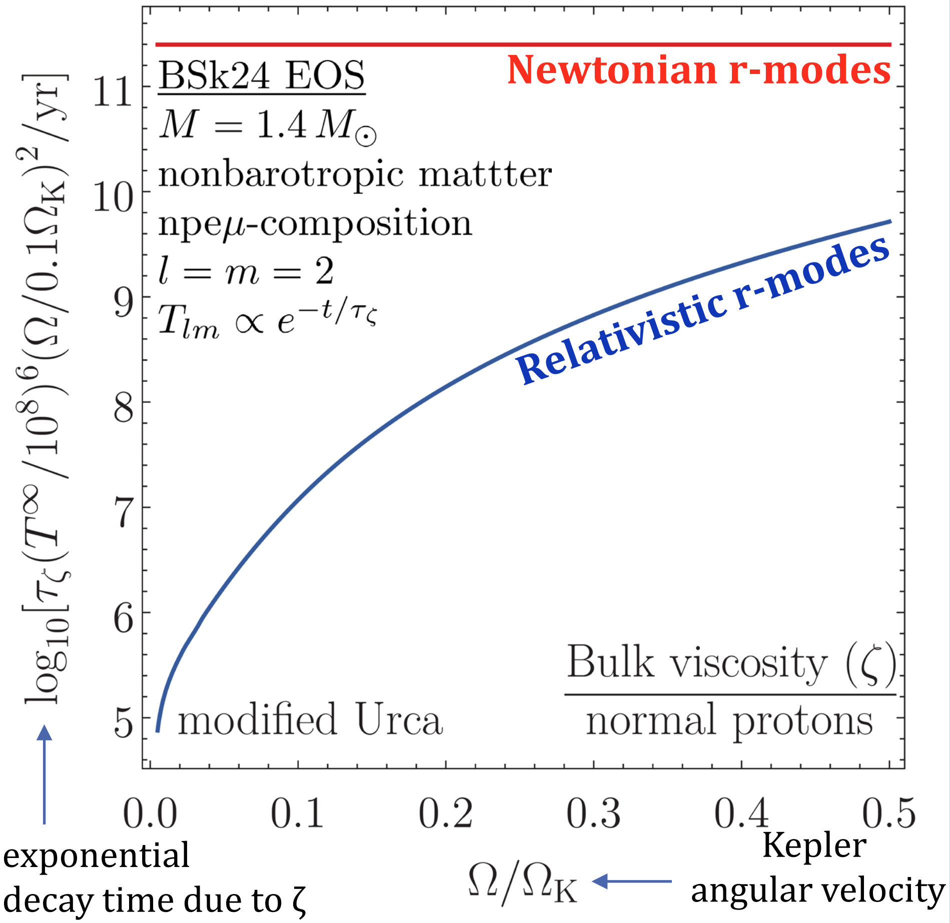


LMXB paradox: [e.g., Haskell 2015] the window likely should not contain observable sources!

Why relativistic r-modes in hyperonic stars?

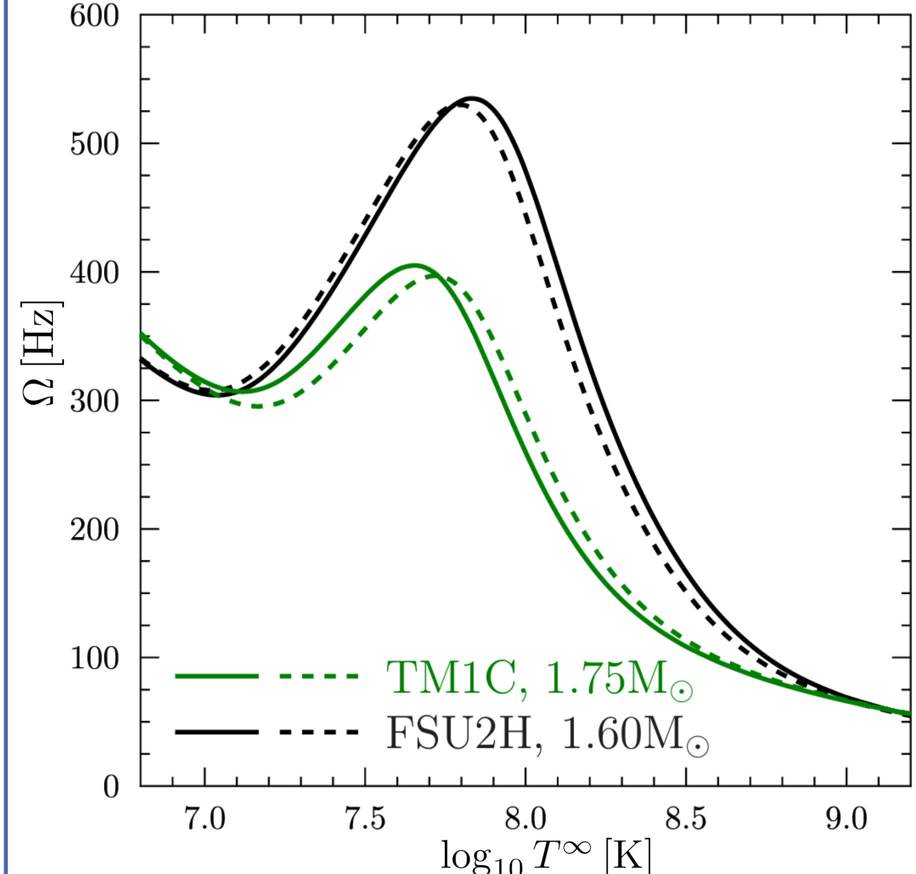
We expect extremely strong dissipation through bulk viscosity

Newtonian theory severely underestimates the r-mode damping by **bulk viscosity**



[from Kraav, Gusakov & Kantor PRD 2024]

Hyperons strongly enhance the **bulk viscosity** coefficient ζ



[from Ofengeim, Gusakov, Haensel & Fortin PRD 2019]

Hyperonic stars

Model of a hyperonic star

Hyperonic star – a star that contains hyperons in its interior

Hyperonic equations of state

- relativistic mean field
- compatible with massive ($\sim 2M_\odot$) stars
- Λ_{uds}- and Ξ^-_{dss}-hyperons appear first

$\lvert B \rvert$ [MeV]	E_{sym} [MeV]	L [MeV]	K [MeV]	$M_{\max}$ [$M_\odot$]
FSU2H (Providência et al. 2019)				
16.3	30.5	44.5	238	1.99
TM1C (Gusakov et al. 2014)				
16.3	36.9	111	281	2.05

B - binding energy
E

Simplified stellar model

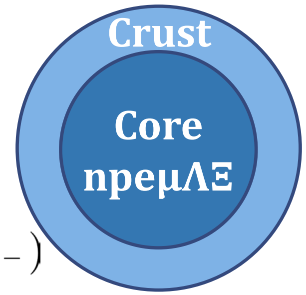
p – pressure
ε – energy density
$\rho = \varepsilon / c^2$ – density
n_k – number densities

Barotropic crust

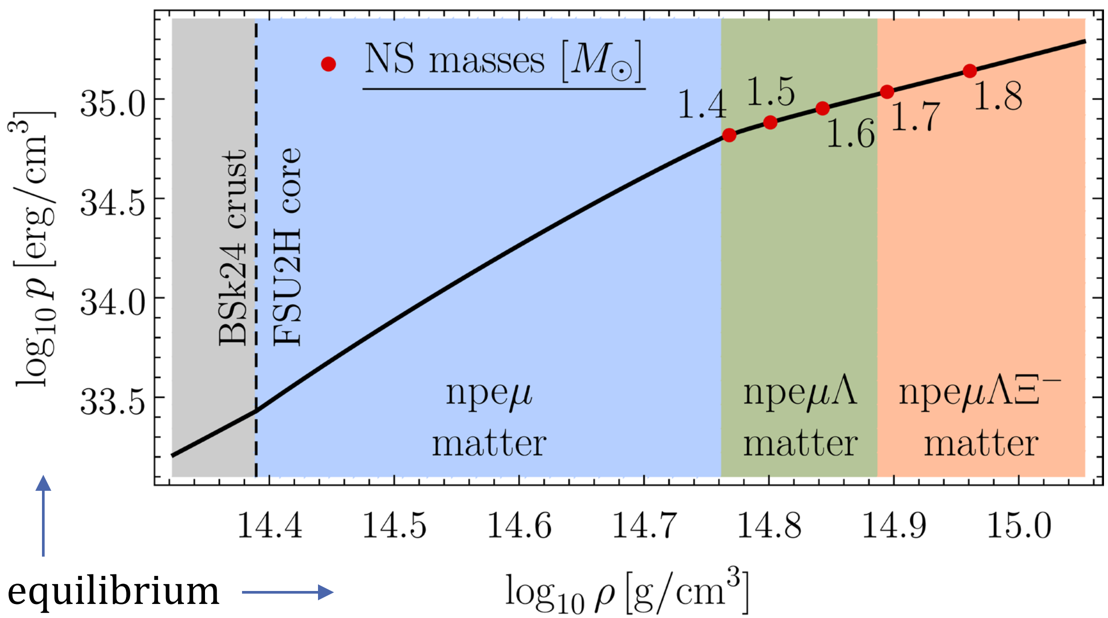
$$p = p(\varepsilon)$$

Nonbarotropic core

$$p = p(n_{\mathrm{n}}, n_{\mathrm{p}}, n_{\mathrm{e}}, n_\mu, n_\Lambda, n_{\Xi^-})$$



Representative core structure



Chemical reactions in hyperonic matter

Chemical reactions are classified by their reaction rates
(following closely Ofengeim, Gusakov, Haensel & Fortin PRD 2019)

Chemical reactions:
(for example)
direct: $1+2 \rightarrow 3+4$
inverse: $1+2 \leftarrow 3+4$

Chemical imbalance:
(deviation from chemical equilibrium)
$$\mu_{12\leftrightarrow 34} = \underbrace{\mu_1 + \mu_2 - \mu_3 - \mu_4}_{\text{chemical potentials}}$$

Reaction rates Γ and λ:
(reactions/time/volume)
$$\Gamma_{12\leftrightarrow 34} \equiv \Gamma_{12\rightarrow 34} - \Gamma_{12\leftarrow 34}$$
$$\Gamma_{12\leftrightarrow 34}(\mu_{12\leftrightarrow 34}) \approx \lambda_{12\leftrightarrow 34}\mu_{12\leftrightarrow 34}$$

extremely slow — **moderately fast** — **extremely fast**

Weak leptonic processes

modified Urca:
$$n + N \rightarrow p + N + l + \bar{\nu}_l$$
$$p + N + l \rightarrow n + N + \nu_l$$
$$\boldsymbol{\Gamma \propto (T/T_F)^6}$$

direct Urca:
$$n \rightarrow p + l + \bar{\nu}_l$$
$$p + l \rightarrow n + \nu_l$$
$$\boldsymbol{\Gamma \propto (T/T_F)^4}$$

l – leptons $\quad N$ – nucleons
ν_l – neutrinos $\quad \bar{\nu}_l$ – antineutrinos

Will be ignored as negligibly slow

Weak nonleptonic processes

$$n + n \leftrightarrow \Lambda + n$$
$$n + p \leftrightarrow \Lambda + p$$
$$n + \Lambda \leftrightarrow \Lambda + \Lambda$$
$$n + \Xi^- \leftrightarrow \Lambda + \Xi^-$$
$$n + \Lambda \leftrightarrow \Xi^- + p$$
$$\boldsymbol{\Gamma \propto (T/T_F)^2}$$

Total reaction rate:
$$\lambda_{\rm tot}(T) = \sum_{12\leftrightarrow 34} \lambda_{12\leftrightarrow 34}(T)$$

Will be treated as arbitrarily fast/slow

Strong process

$$\Lambda + \Lambda \leftrightarrow \Xi^- + p$$

chemical equilibration
at negligibly short timescales:
$$\tau_{\rm strong} \ll \tau_{\rm hydro}$$

$\Downarrow$

Will be treated as equilibrated:
$$2\mu_\Lambda - \mu_{\Xi^-} - \mu_{\rm p} = 0$$

Bulk viscosity and adiabatic index

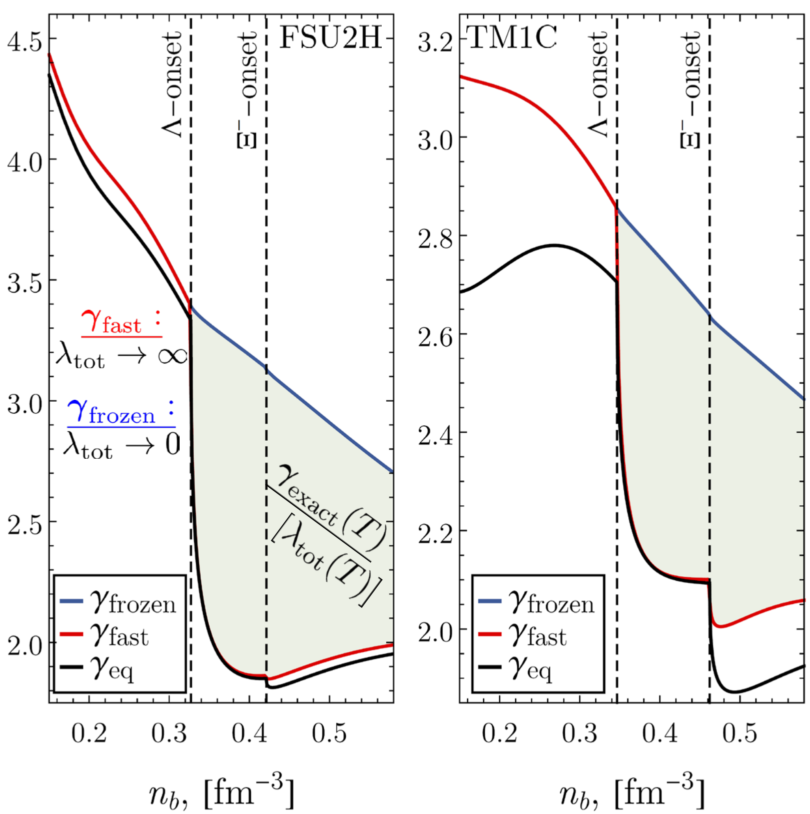
Adiabatic index (γ)

Stiffness of the matter under adiabatic compression

$$\gamma = \frac{w_0}{p_0}\left(\frac{\Delta p}{\Delta \varepsilon}\right)_{\rm ad} = \gamma_{\rm frozen}\left[1 - \beta\frac{(\lambda_{\rm tot}/\lambda_{\max})^2}{1+(\lambda_{\rm tot}/\lambda_{\max})^2}\right]$$

frozen reactions **effect of reactions**

$\gamma_{\rm frozen}$, β, and $\lambda_{\max}$: determined by the EOS



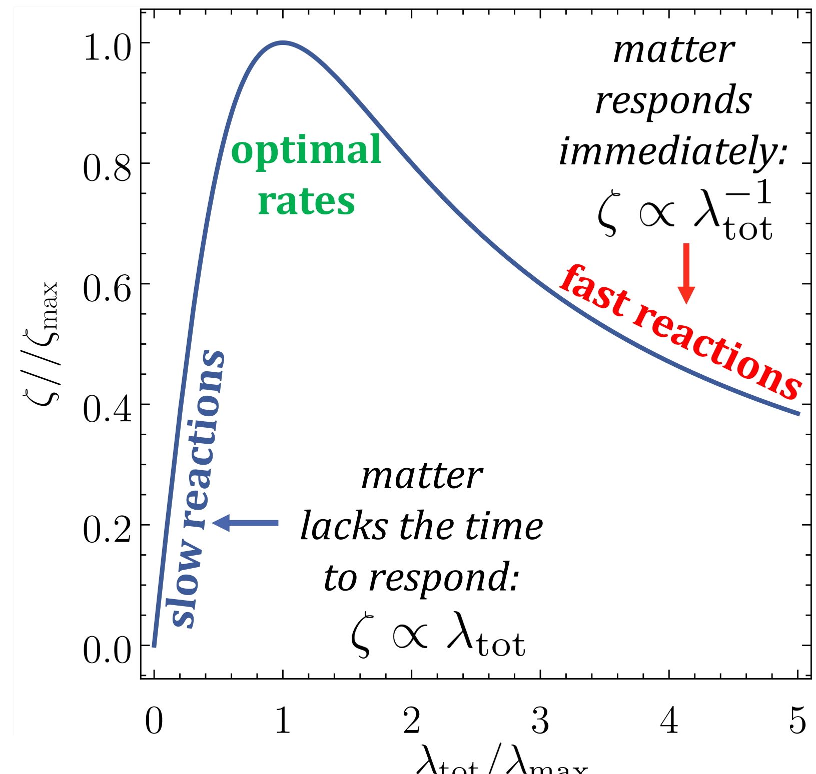
Bulk viscosity (ζ)

Heat production by out of equilibrium reactions
[following Ofengeim, Gusakov, Haensel & Fortin PRD 2019]

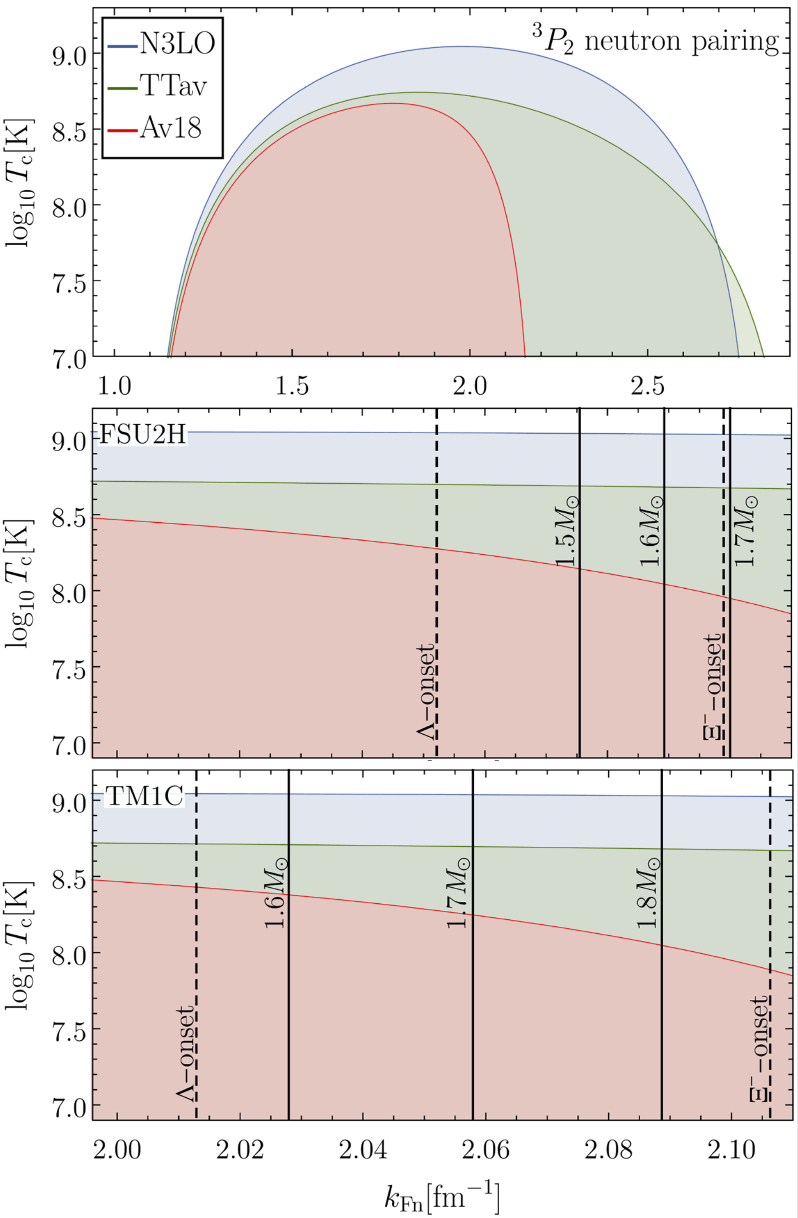
$\lambda_{\max}$, $\zeta_{\max}$: determined by the EOS

$$\Delta p - \Delta p_{\rm ad} = -\zeta\,{\rm div}\boldsymbol{v}$$

$$\zeta = \zeta_{\max}\frac{2(\lambda_{\rm tot}/\lambda_{\max})}{1+(\lambda_{\rm tot}/\lambda_{\max})^2}$$



NOTE: γ and ζ depend on T throuh $\lambda_{\rm tot}(T)$



[from Ding et al. PRC 2016; Ho et al. PRC 2015; Takatsuka & Tamagaki, Prog. Theor. Phys. 2004]

Chemical reactions and pairing effects

This study

Hyperons: normal ($T_{c\Lambda} \lesssim 10^7$ K) [Takahashi et al. PRL 2001; Wang & Shen PRC 2010; Takatsuka et al. Progr. Theor. Phys. 2006]

Protons: normal/strongly superconducting

Neutrons: normal/superfluid (N3LO, TTav, Av18)

1. Pairing affects the particle spectrum

$$\varepsilon_k(\boldsymbol{p}) - \mu_k = v_{Fk}(|\boldsymbol{p}| - p_{Fk})$$

$$\Downarrow\ T < T_{ck}$$

$$\varepsilon_k(\boldsymbol{p}) - \mu_k = \mathrm{sign}(|\boldsymbol{p}| - p_F)\sqrt{\delta_k^2 + v_{Fk}^2(|\boldsymbol{p}| - p_{Fk})^2}$$

T_{ck} – critical temperature $\quad$ δ_k – energy gap

p_{Fk}/v_{Fk} – Fermi momentum/velocity

2. The spectrum affects the momentum distribution

Fermi-Dirac distribution:

$$f_k(\boldsymbol{p}) = \frac{1}{1 + e^{[\varepsilon_k(\boldsymbol{p}) - \mu_k]/k_B T}}$$

3. The distribution affects reaction rates

$$\Gamma_{12\leftrightarrow 34} \equiv \Gamma_{12\to 34} - \Gamma_{12\leftarrow 34} \propto \mathcal{I}\left(\frac{\mu_{12\leftrightarrow 34}}{k_B T}\right) - \mathcal{I}\left(-\frac{\mu_{12\leftrightarrow 34}}{k_B T}\right)$$

$$\mathcal{I}(\xi) \equiv \int \delta\left(\sum_{m=1}^{4} z_m - \xi\right) \prod_{k=1}^{4} f_k dx_k \longleftarrow$$

summation over energy- conserving reactions

$$z_k = (\varepsilon_k - \mu_k)/k_B T \quad x_k = v_{Fk}(|\boldsymbol{p}| - p_{Fk})/k_B T$$

Chemical reactions and pairing effects

Neutron superfluidity

Reduction factors:

$\underline{|\mu_{12\leftrightarrow 34}| \ll k_\mathrm{B} T :}$

$$\boxed{\mathcal{R}_{12\leftrightarrow 34} = \frac{\lambda^{\mathrm{superfluid}}_{12\leftrightarrow 34}}{\lambda^{\mathrm{normal}}_{12\leftrightarrow 34}} = \frac{\Gamma^{\mathrm{superfluid}}_{12\leftrightarrow 34}}{\Gamma^{\mathrm{normal}}_{12\leftrightarrow 34}} = \frac{\mathcal{I}'^{\mathrm{superfluid}}_{12\leftrightarrow 34}(0)}{\mathcal{I}'^{\mathrm{normal}}_{12\leftrightarrow 34}(0)}}$$

Reactions involving one neutron:

[P. Haensel, K. Levenfish, D. Yakovlev A&A 2002]

$$\boxed{\mathcal{R}^{[1]}_{\mathrm{asympt}} = \frac{3}{2}\sqrt{\frac{\pi y}{2}}\, e^{-y}\, \frac{(y^2+y+\pi^2)}{\pi^2}}$$

$$\mathcal{R}^{[1]}_{\mathrm{fit}} = \frac{a_1^{5/4} + b_1^{1/2}}{2} \exp\left[0.5068 - \sqrt{y^2 + 0.5068^2}\right]$$

$$a_1 = 1 + 0.3118\, y^2 \qquad b_1 = 1 + 2.556\, y^2$$

Reactions involving three neutrons:

$$\boxed{\mathcal{R}^{[3]}_{\mathrm{asympt}} = \frac{3\sqrt{3}}{2\pi} e^{-2y}\, y\,(3\,y + 2)}$$

$$\mathcal{R}^{[3]}_{\mathrm{fit}} = \frac{3\sqrt{3}}{2\pi} e^{-2y} \left(a_3 - b_3 e^{-y}\right)$$

$$a_3 = 3\,y^2 + 2\,y + 15.68,$$

$$b_3 = 2.791\, y^3 + 7.732\, y^2 + 14.06\, y + 14.47$$

Strong proton superconductivity

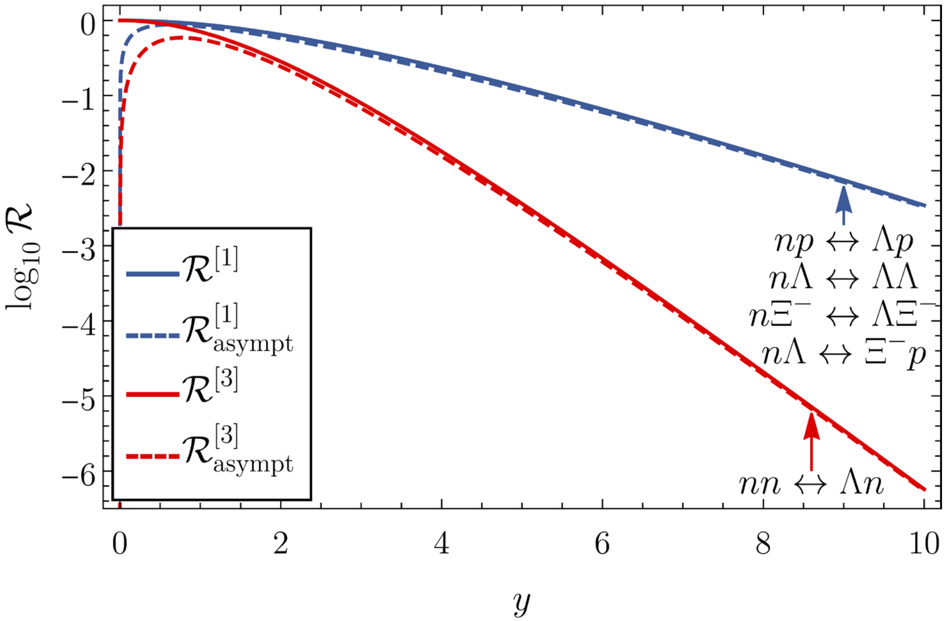
Suppresses weak reactions:

$\cancel{n + p \leftrightarrow \Lambda + p \qquad n + \Lambda \leftrightarrow \Xi^- + p}$ $\qquad \lambda_{12\leftrightarrow 34} \to 0$

Neutron energy gap

[e.g., Haensel et al. 2000]

$$\boxed{y = \frac{\delta(T)}{k_\mathrm{B} T}} \boxed{\frac{\delta(T)}{k_\mathrm{B} T} = \sqrt{1 - \frac{T}{T_\mathrm{c}}}\left[0.7893 + \frac{1.188}{T/T_\mathrm{c}}\right]}$$



Relativistic r-modes

Hydrodynamic oscillation equations

Eulerian perturbations:	Lagrangian displacement:	Lagrangian perturbations:
$f = f_0 + \delta f$ equilibrium	$\delta u^\mu = i(\sigma_{\rm loc}/c)\xi^\mu$ collective 4-velocity local oscillation frequency	$\Delta f = \delta f + \xi^\rho \nabla_\rho f_0$

Stellar equilibrium

1. Hydrostatic equilibrium:

Metric tensor:

[Hartle 1967]

$$ds^2 = -e^{2\nu(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + $$
$$+ r^2\{d\theta^2 + \sin^2\theta[d\varphi - \Omega\omega(r)dt]^2\}$$

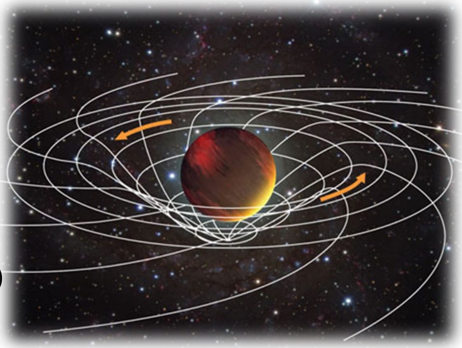
frame-dragging effect

Four-velocity:

$$u^\mu = \Lambda k^\mu$$
$$k^\mu = (1, 0, 0, \Omega/c)$$

Gravitational redshift:

$$\Lambda = (-k^\mu k_\mu)^{-1/2} \approx e^{-\nu(r)}$$



2. Thermal equilibrium: $T^\infty = T/\Lambda = \text{const}$

3. Chemical equilibrium:

$$\mu_{12\leftrightarrow34} = 0 \quad \Gamma_{12\rightarrow34} = \Gamma_{12\leftarrow34} \quad \Gamma_{12\leftrightarrow34} = 0$$

Hydrodynamic equations

1. Linearized Euler equation:

$$\delta\{w u^\mu \nabla_\mu u^\rho = -{}^\perp\nabla^\rho p + \perp^\rho_\lambda \nabla_\mu(\eta c \sigma^{\mu\lambda})\}$$

η – **shear viscosity (friction of fluid layers)**

u^μ – 4-velocity	w – enthalpy density
$g_{\mu\nu}$ – metric tensor	∇_μ – covariant derivative
$\perp^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$	${}^\perp\nabla^\mu \equiv \perp^{\mu\nu} \nabla_\nu$

2. Linearized continuity equations:

$$\delta\{\nabla_\mu(n_k u^\mu) = \Gamma_k/c\}$$

chemical reactions

$$\Delta\varepsilon + w_0 {}^\perp\nabla_\mu \xi^\mu = 0$$
$$\Delta p + \gamma p_0 {}^\perp\nabla_\mu \xi^\mu = -\zeta c {}^\perp\nabla_\mu u^\mu$$

3. EOS: $\varepsilon = \varepsilon(n_{\rm n}, n_{\rm p}, n_{\rm e}, n_\mu, n_\Lambda, n_{\Xi^-})$

4. Thermodynamics:

$$d\varepsilon = \mu_k dn_k \quad dp = n_k d\mu_k \quad w \equiv p + \varepsilon = \mu_k n_k$$

R-mode eigenfunctions

Looking for r-mode solutions

Velocity perturbation due to an r-mode

$$\delta \boldsymbol{v}(\boldsymbol{r},t) \approx [\boldsymbol{r} \times \boldsymbol{\nabla}]\mathrm{T}(\boldsymbol{r},t)$$

$$\mathrm{T}(\boldsymbol{r},t) = T_{lm}(r) P_l^m(\cos\theta) e^{im\varphi + i\sigma t}$$

$T_{lm}(r)$ – toroidal function

Relativistic r-mode peculiarities

Kraav K., Gusakov M., Kantor E.

[PRD 2022, Universe Lett. 2022, PRD 2024]

1. Sophisticated dependence of T_{lm} on Ω
2. Exponential suppression at $\Omega \to 0$

toy model: $$\Omega^2 \frac{d^2}{dr^2} T_{lm} - q_{lm}^2 T_{lm} = 0$$

$$T_{lm} \sim e^{(q_{lm}/\Omega) r}$$

$$dT_{lm}/dr \sim T_{lm}/\Omega$$

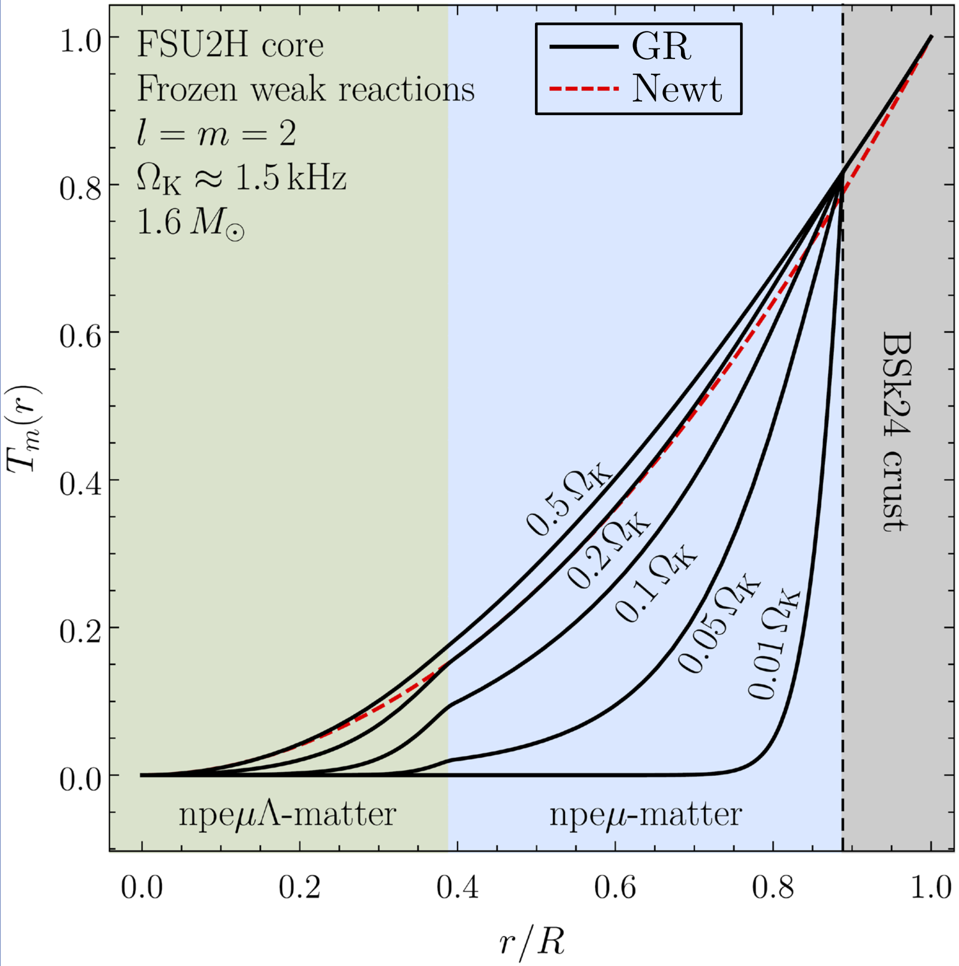
3. Peculiar "ordering"

$$\frac{\xi^r_{\mathrm{GR}}}{T_{lm\,\mathrm{GR}}} \sim \Omega \quad \text{vs} \quad \frac{\xi^r_{\mathrm{Newt}}}{T_{lm\,\mathrm{Newt}}} \propto \Omega^2$$

Toroidal function

Newt: Newtonian calculation (no dependence on Ω)

GR: Relativistic calculation (depends on Ω)



Dissipation through bulk viscosity

Energy loss due to ζ

Kraav, Gusakov & Kantor:
[PRD 2024]

General formula:

$$\dot{E}_\zeta = -\int \zeta(\nabla_\mu u^\mu)^2 c^2 \sqrt{-g}\, d^3x$$

R-modes:

$$\dot{E}_\zeta \propto -\Omega^6 \int \frac{\zeta(r)}{A^2(r)} \left[\frac{h'(r)}{\gamma(r)}\right]^2 F_\zeta(r)dr$$

$$h^{\mathrm{GR}}(r) = r^{-m} e^{(m-1)\nu(r)} T_m(r)$$

$$h^{\mathrm{Newt}}(r) = r^{-m} T_m(r)$$

does not depend on T_m

Schwarzschild discriminant:
(matter is nonbarotropic)

$$A(r) = \frac{w_0'(r)}{w_0(r)} - \frac{p_0'(r)}{w_0(r)}\left[1 + \frac{w_0(r)}{\gamma(r)p_0(r)}\right]$$

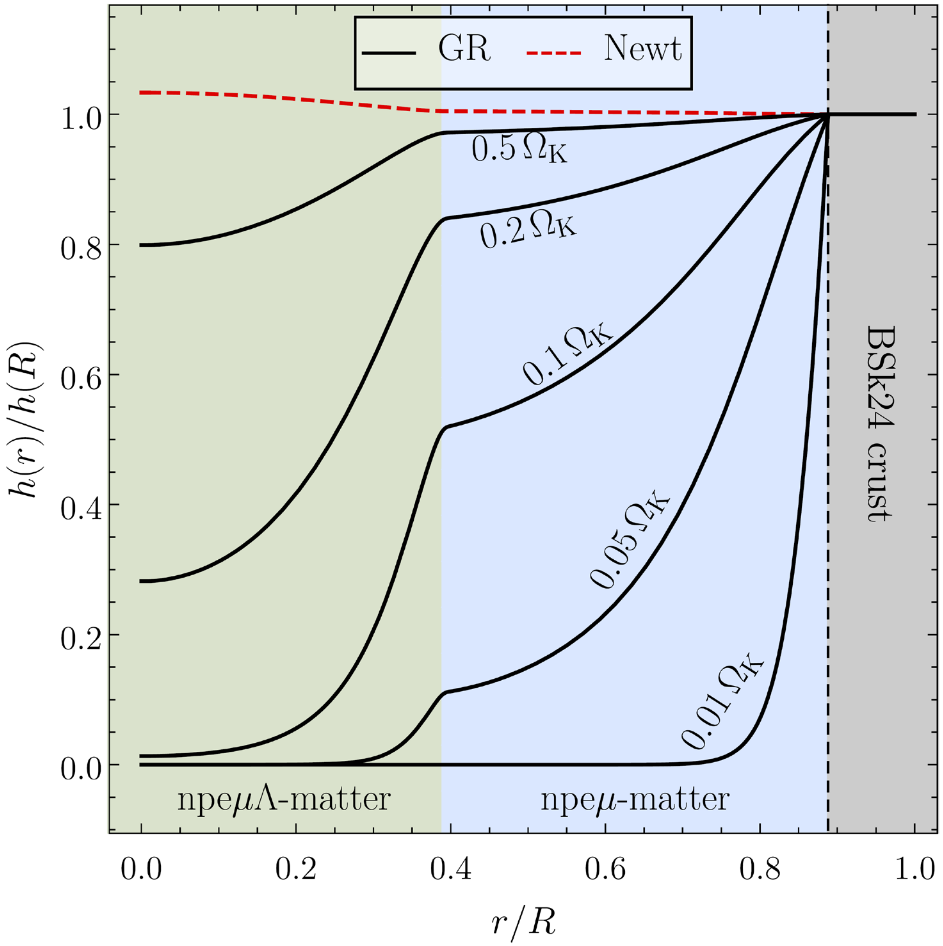
Amplification by relativistic effects

$$|h^{\mathrm{Newt}\prime}(r)| \ll |h^{\mathrm{GR}\prime}(r)| \quad \Rightarrow \quad |\dot{E}_\zeta^{\mathrm{GR}}| \gg |\dot{E}_\zeta^{\mathrm{Newt}}|$$

An example

$1.6\,M_\odot$ FSU2H model
Frozen weak reactions
$l = m = 2$ mode

Newt: any Ω
GR: depends on Ω

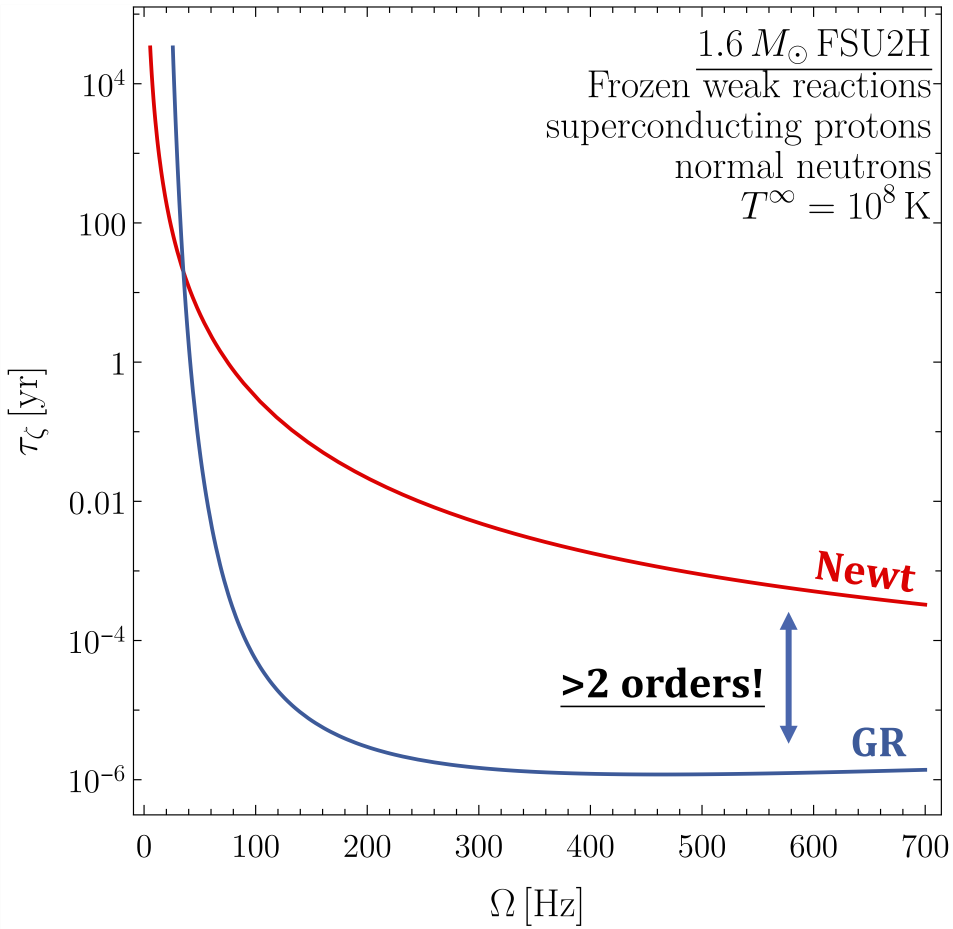


Dissipation through bulk viscosity

Damping time due to ζ

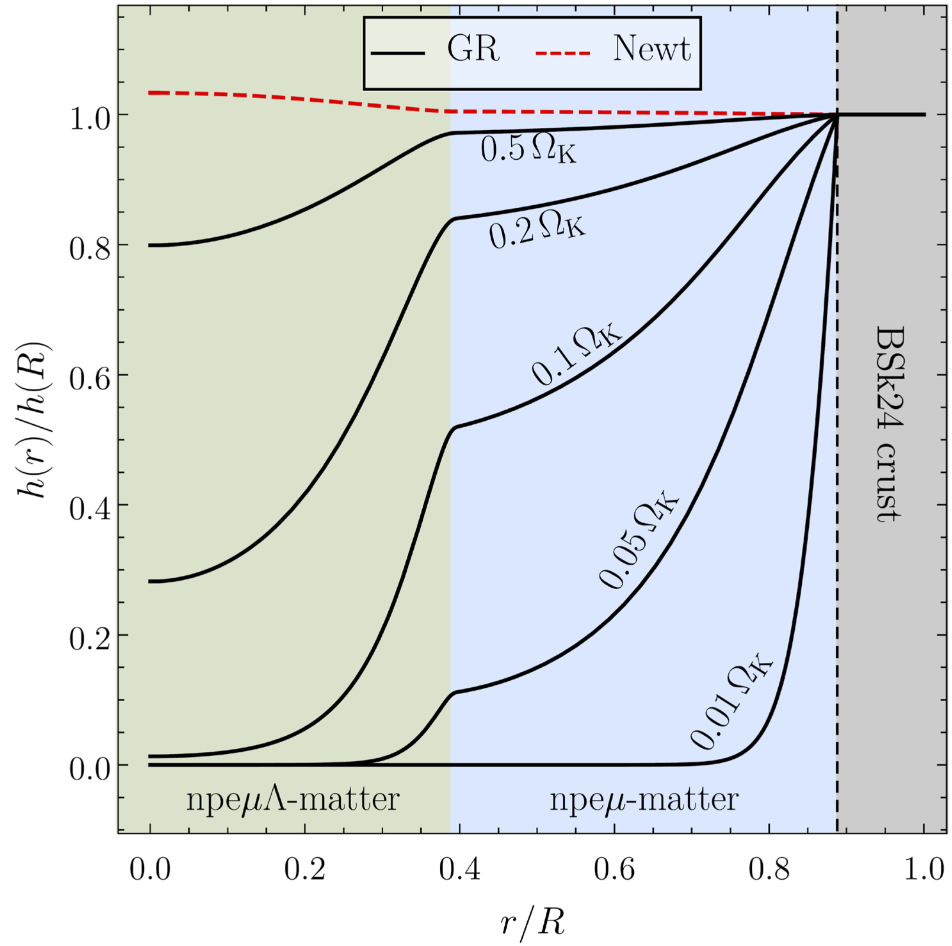
$$\tau_\zeta = \frac{2E}{|\dot{E}_\zeta|}$$

E – r-mode energy
$\dot{E}_\zeta$ – energy loss through ζ/time



Amplification mechanism

$$\dot{E}_\zeta \propto \Omega^6 \int \frac{\zeta(r)}{A^2(r)} \left[\frac{h'(r)}{\gamma(r)}\right]^2 F_\zeta(r)dr$$

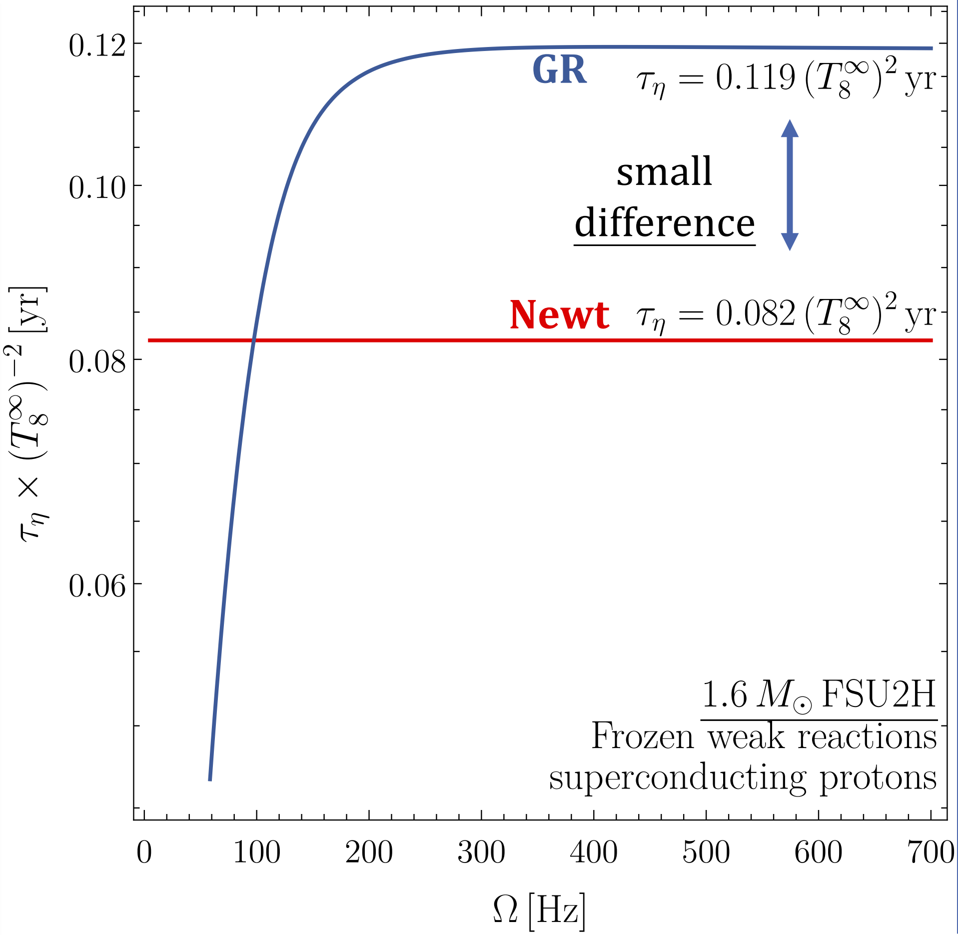


Dissipation through shear viscosity

Damping time due to η

$$\tau_\eta = \frac{2E}{|\dot{E}_\eta|}$$

E – r-mode energy
$\dot{E}_\eta$ – energy loss through η/time



Energy loss due to η

Kraav, Gusakov & Kantor:
[PRD 2024]

General formula:

$$\dot{E}_\eta = -\int \frac{\eta}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} c^2 \sqrt{-\mathrm{g}}\; d^3x$$

$$\sigma^{\mu\nu} \equiv {}^{\perp}\nabla^{\mu}u^{\nu} + {}^{\perp}\nabla^{\nu}u^{\mu} - \frac{2}{3}\perp^{\mu\nu} {}^{\perp}\nabla_{\lambda}u^{\lambda}$$

R-modes:

$$\dot{E}_\eta \propto -\Omega^2 \int \eta(r)\Big\{\big[T_m(r) - rT_m'(r)\big]^2 + (m-1)(m+2)e^{2\lambda(r)}T_m^2(r)\Big\}e^{-\lambda(r)}dr$$

The effect of r-mode peculiarities in GR

Enhanced dissipation
at sufficiently slow rotation rates

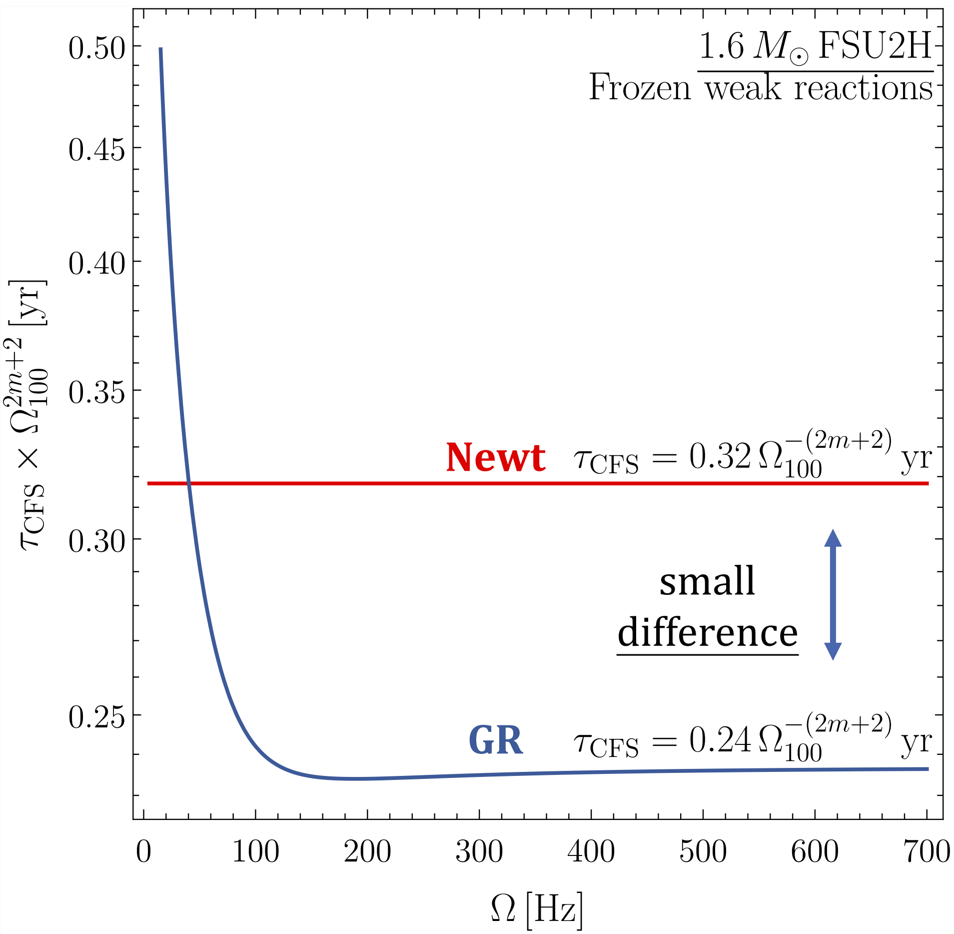
No significant differences
at rotation rates relevant for stars in LMXBs

Amplification by CFS-mechanism

Driving time due to CFS-mechanism

$$\tau_{\rm CFS} = \frac{2E}{\dot{E}_{\rm CFS}}$$

E – r-mode energy
$\dot{E}_{\rm CFS}$ – CFS energy supply/time



Energy pumping by CFS mechanism

Kraav, Gusakov & Kantor:
[PRD 2024]

Thorne 1980:

$$\dot{E}_{\rm CFS} = -\frac{\sigma_r}{\sigma}\frac{G}{c^3}\sum_{L=2}^{\infty}\sum_{M=-L}^{L}\left(\frac{\sigma}{c}\right)^{2L+2}\times$$
$$\times N_L\left[|\mathcal{I}_{LM}|^2 + |\mathcal{S}_{LM}|^2\right]$$

R-modes:

$$\dot{E}_{\rm CFS} \propto \Omega^{2m+4}\left[\int w_0(r)T_m(r)e^{2\lambda(r)}r^{m+2}dr\right]^2$$

The effect of r-mode peculiarities in GR

Suppressed CFS-amplification
at sufficiently slow rotation rates

No significant differences
at rotation rates relevant for stars in LMXBs

Instability windows

Calculation overview

Boundary of the instability window

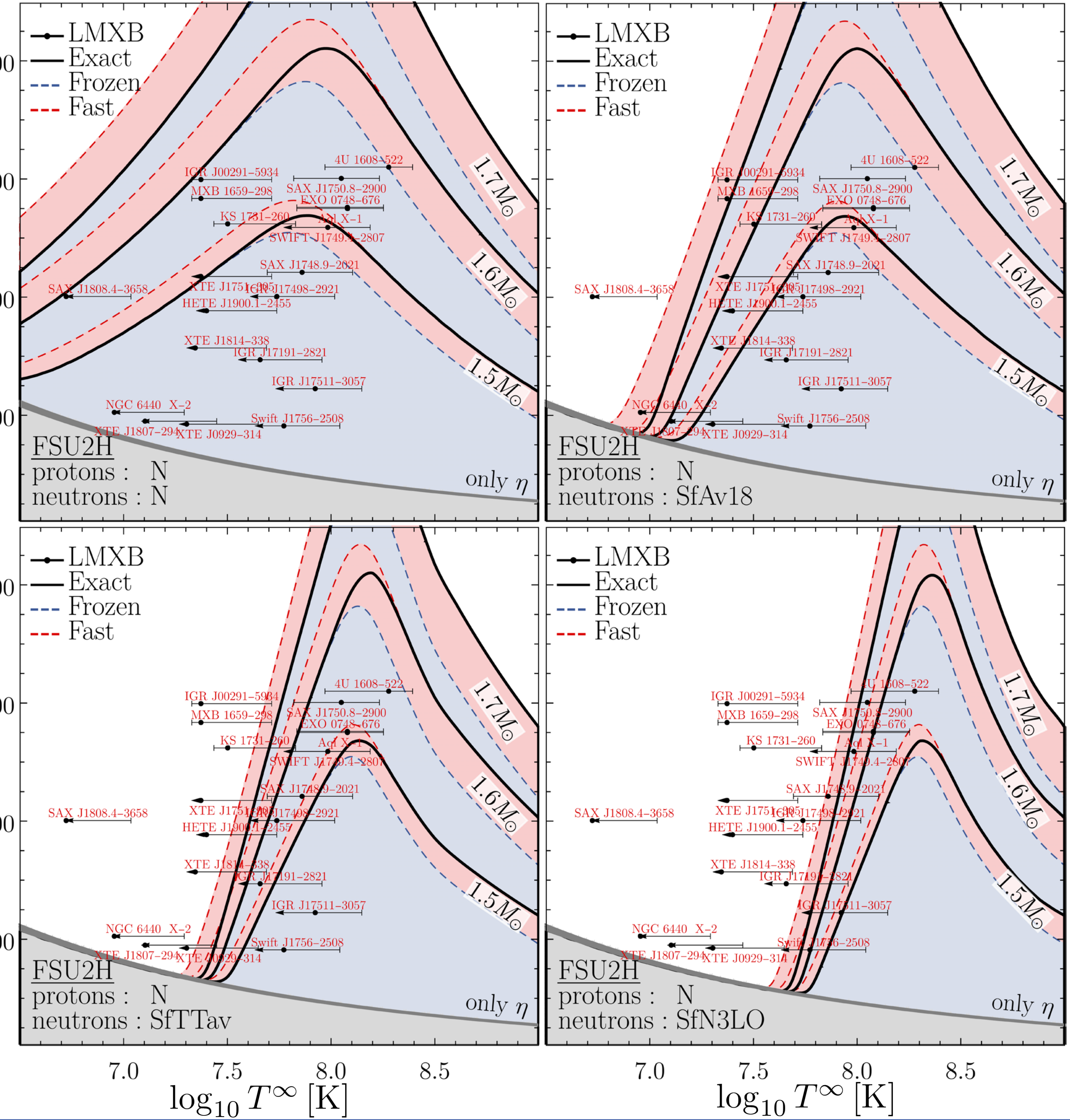
$$\dot{E}_{\rm CFS}(\Omega, T^\infty) + \dot{E}_\zeta(\Omega, T^\infty) + \dot{E}_\eta(\Omega, T^\infty) = 0$$

CFS-mechanism (GW-emission amplifies the r-mode)	$\dot{E}_{\rm CFS}(\Omega, T^\infty) \propto \Omega^{2m+4}\left[\int w_0(r) T_m(r,\Omega,T^\infty) e^{2\lambda(r)} r^{m+2} dr\right]^2$
Bulk viscosity (heat production in chemical reactions)	$\dot{E}_\zeta(\Omega, T^\infty) \propto -\Omega^6 \int \frac{\zeta(r,\Omega,T^\infty)}{A^2(r,\Omega,T^\infty)}\left[\frac{h'(r,\Omega,T^\infty)}{\gamma(r,\Omega,T^\infty)}\right]^2 F_\zeta(r) dr$
Shear viscosity (friction in the fluid)	$\dot{E}_\eta(\Omega, T^\infty) \propto -\Omega^2 \int \eta(r,T^\infty)\{[T_m(r,\Omega,T^\infty) - rT'_m(r,\Omega,T^\infty)]^2 + (m-1)(m+2)e^{2\lambda(r)} T_m^2(r,\Omega,T^\infty)\} e^{-\lambda(r)} dr$

NOTE: γ and A depend on T $\Rightarrow$ T_m and h depend on T^∞

Microphysical conditions

EOS:	**Protons**	**Neutrons**	**Adiabatic index**
1. FSU2H 2. TM1C	1. normal (N) 2. strongly superconducting (SSc)	1. normal (N) 2. superfluid (N3LO, TTav, Av18)	1. Frozen 2. Fast 3. Exact



Instability windows

Stellar models:
- FSU2H EOS
- M = 1.5/1.6/1.7 $M_\odot$

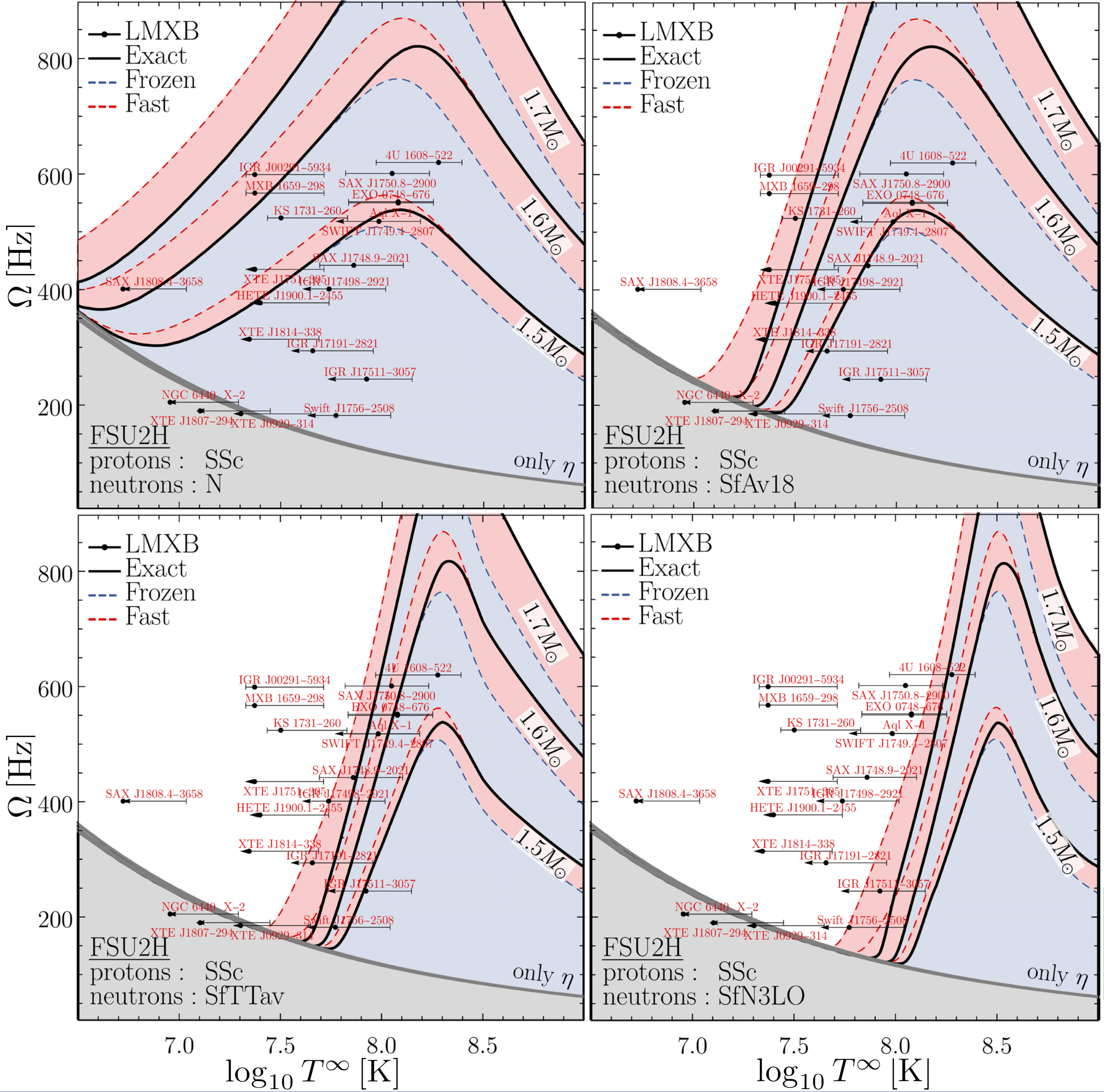
Protons:
normal (N)

LMXB data:
(see references in Kantor, Gusakov & Dommes PRD 2021)

Reaction rates:
Ofengeim et al. PRD 2019

Shear viscosity:
Schmitt & Shternin 2018

1. Very efficient ζ
2. Strong influence of neutron superfluidity
3. Broadening due to chemical reactions



Instability windows

Stellar models:

- FSU2H EOS
- $M = 1.5/1.6/1.7\ M_\odot$

Protons:

strongly superconducting (SSc)

LMXB data:

(see references in Kantor, Gusakov & Dommes PRD 2021

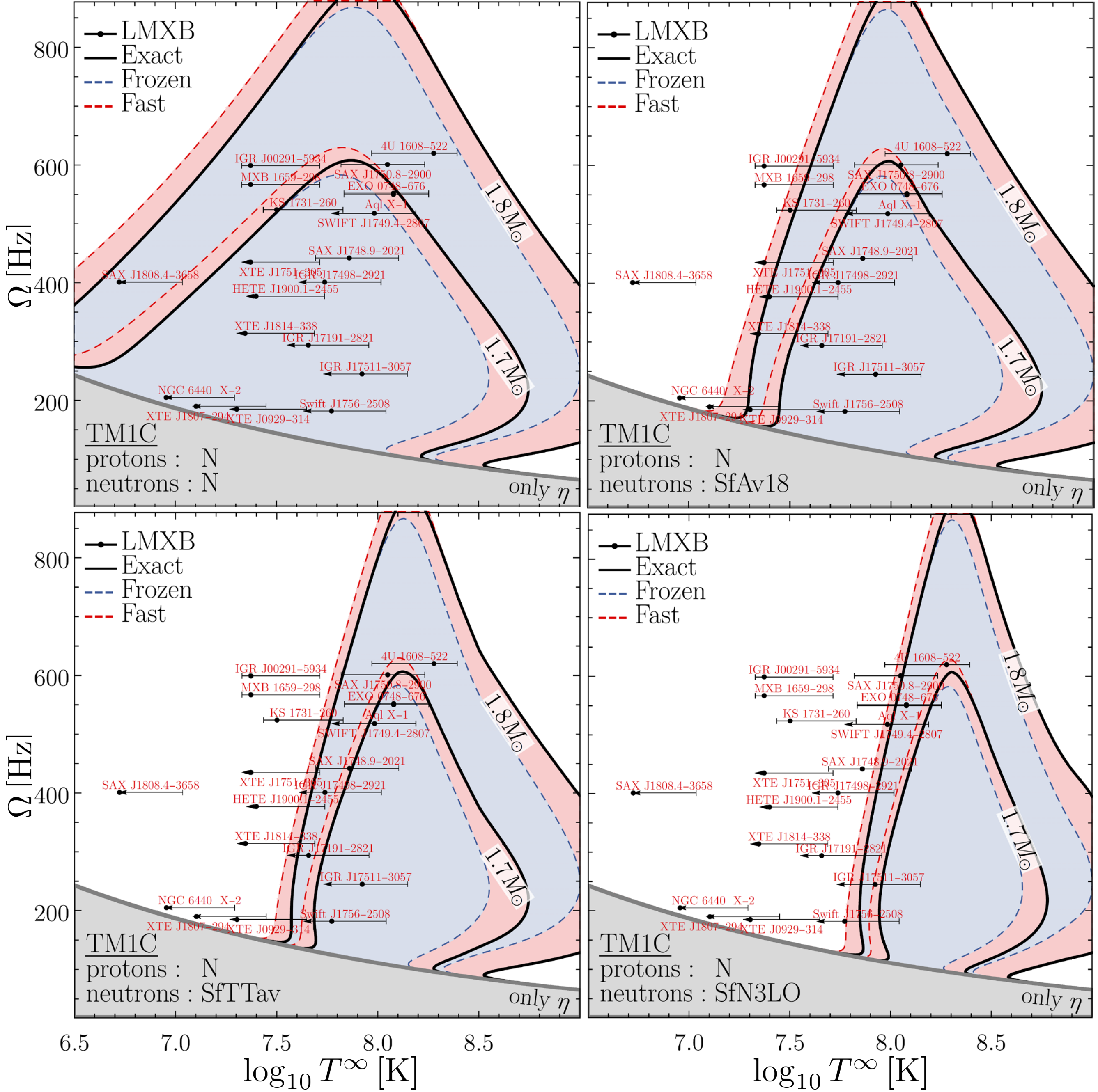
Reaction rates:

Ofengeim et al. PRD 2019

Shear viscosity:

Schmitt & Shternin 2018

Proton superconductivity has only little impact



Instability windows

Stellar models:

- TM1C EOS
- $M = 1.7/1.8\ M_\odot$

Protons:

normal (N)

LMXB data:

(see references in Kantor, Gusakov & Dommes PRD 2021)

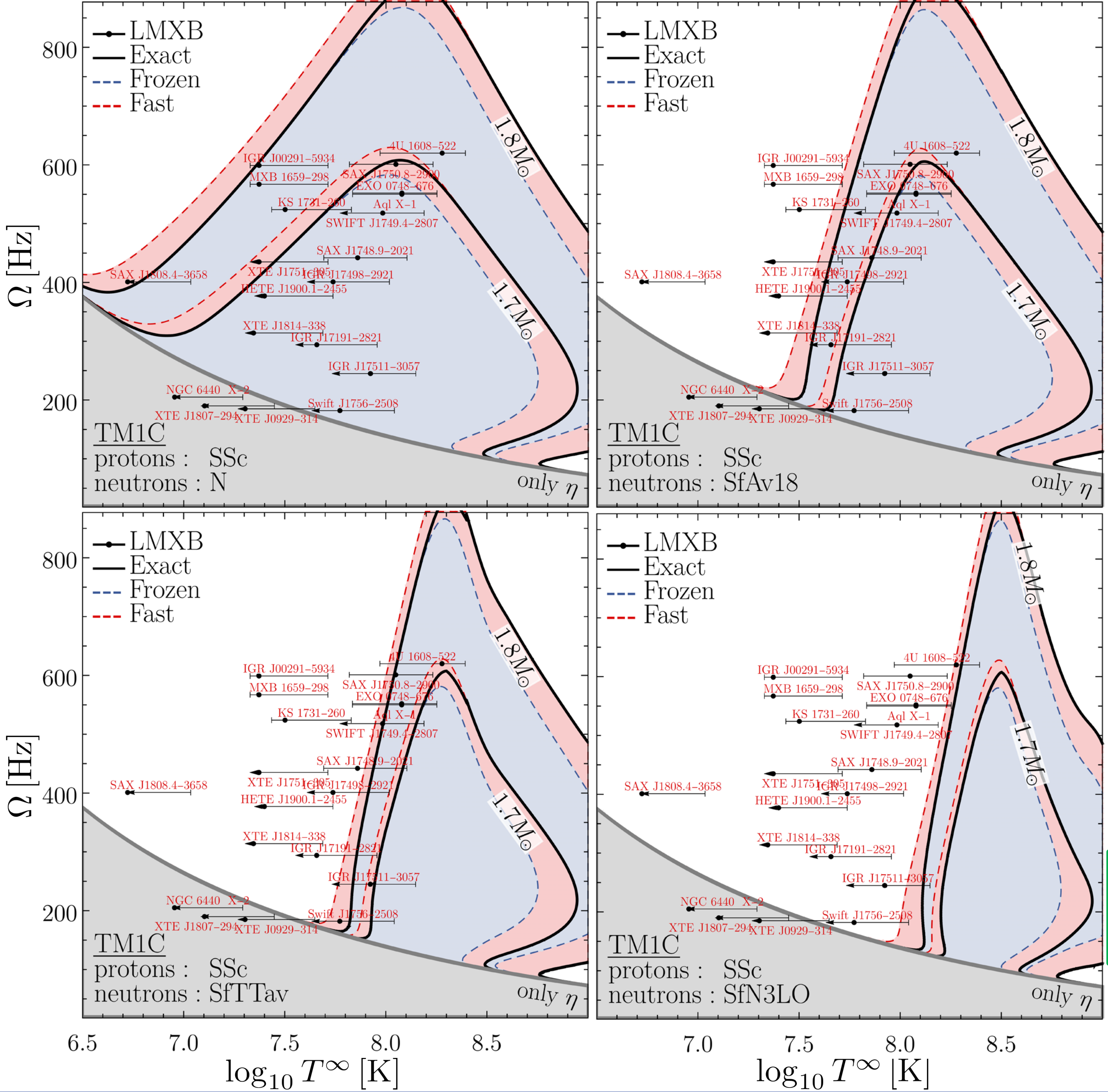
Reaction rates:

Ofengeim et al. PRD 2019

Shear viscosity:

Schmitt & Shternin 2018

1. TM1C requires higher masses than FSU2H
2. Peculiar shape



Instability windows

Stellar models:

- TM1C EOS
- $M = 1.7/1.8\ M_\odot$

Protons:

strongly superconducting (SSc)

LMXB data:

(see references in Kantor, Gusakov & Dommes PRD 2021

Reaction rates:

Ofengeim et al. PRD 2019

Shear viscosity:

Schmitt & Shternin 2018

Proton superconductivity has only little impact

Conclusions

Summary

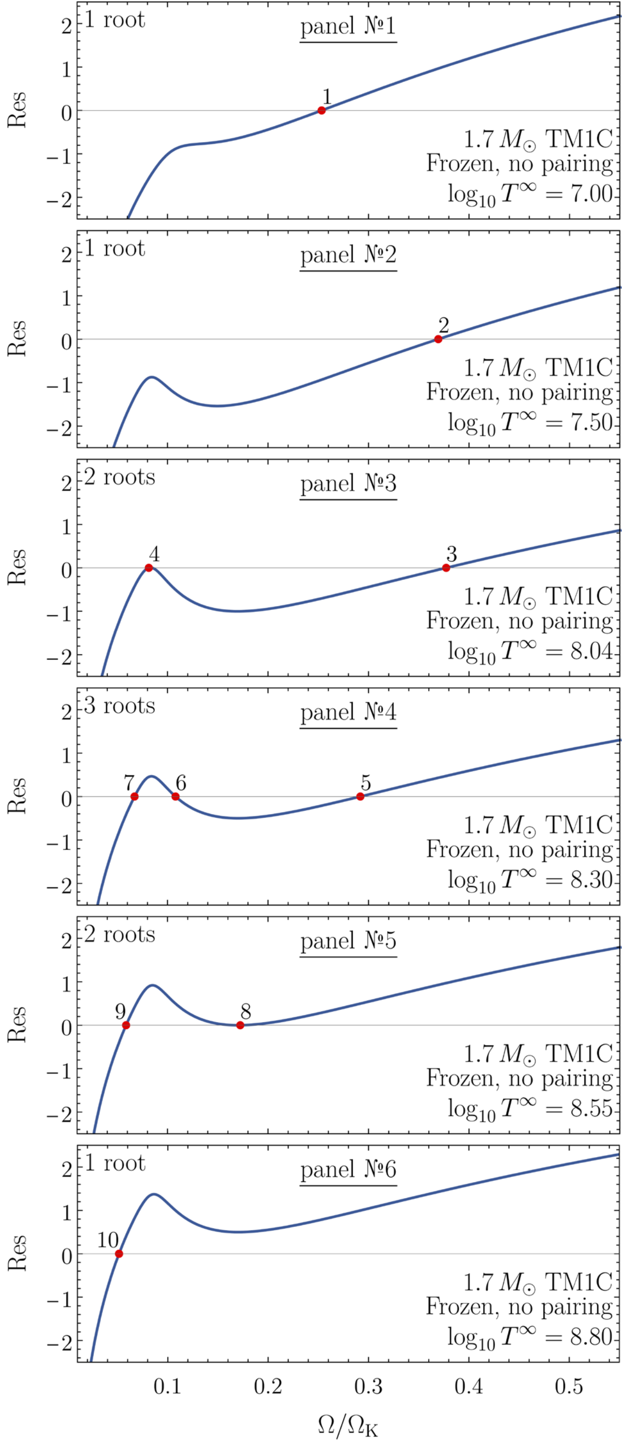
- We considered instability windows of relativistic r-modes in hyperonic stars
- We estimated the influence of nucleon pairing effects on the instability windows
- We investigated the importance of accounting for the effect of chemical reactions on the adiabatic index

Observations

- Proton superconductivity weakly affects instability windows
- Neutron superconductivity strongly suppresses dissipation through bulk viscosity at sufficiently low temperatures
- Accounting for the effect of chemical reactions on the adiabatic index slightly broadens the instability windows

Under reasonable physical conditions,
bulk viscosity may serve as an extremely efficient dissipative mechanism,
capable od stabilizing r-modes in LMXBs!

Supplementary materials

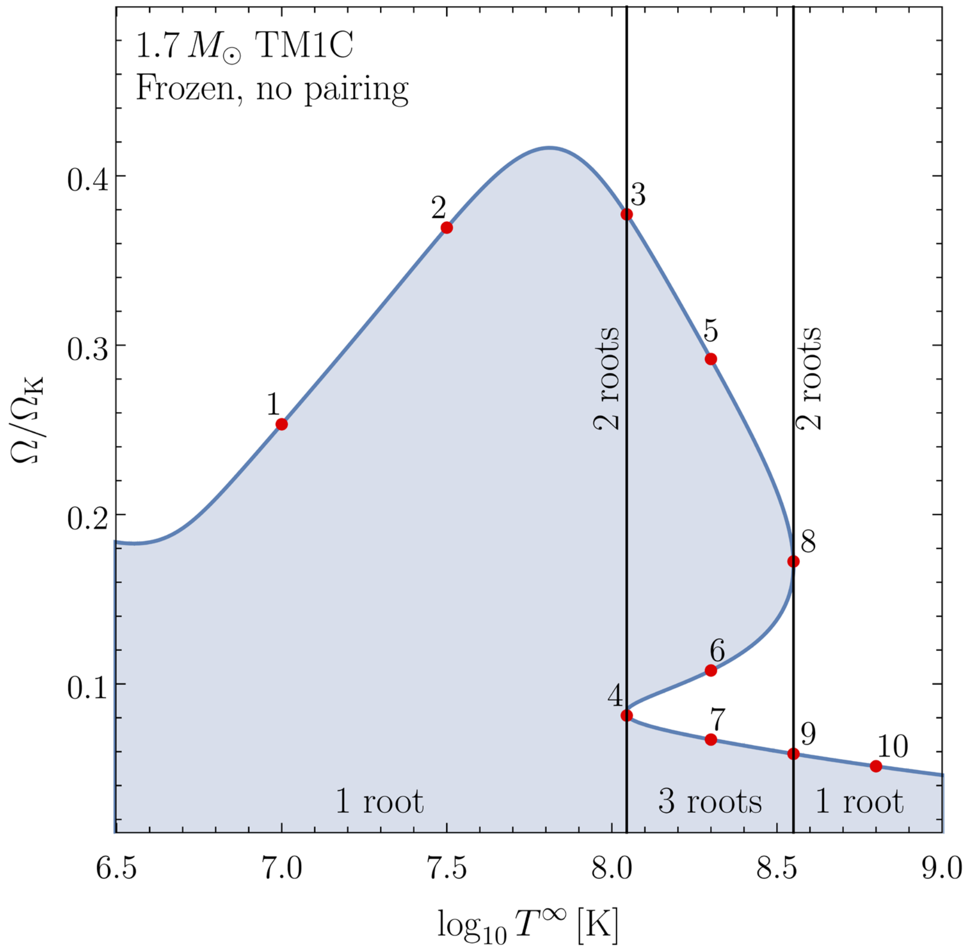


The shape of the TM1C instability windows

Residual function:

$$\mathrm{Res}(\Omega, T^\infty) \equiv \log_{10} \dot{E}_{\mathrm{CFS}} - \log_{10} |\dot{E}_{\mathrm{diss}}|$$

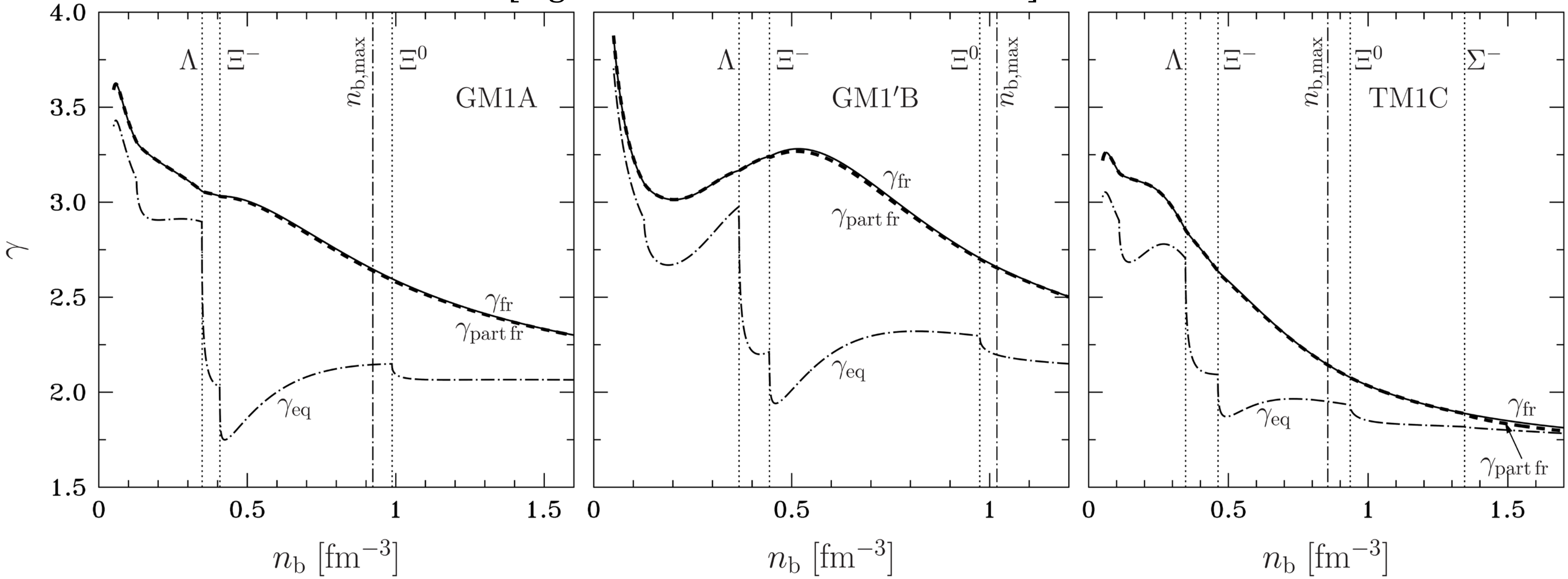
Instability window:



On accounting for strong reactions in adiabatic index

Strong chemical reactions weakly affect the adiabatic index

[Figure 3 from Gusakov et al. 2014]



Solid lines ($\gamma_{\rm fr}$): all chemical processes are frozen (including strong ones)

Dashed lines ($\gamma_{\rm part\ fr}$): frozen weak reactions and extremely fast strong reactions