Description of the low-energy fragmentation reactions in the transport-statistical approach

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Outline

- Motivation.
- Description of heavy-ion collisions with modified transport (BNV)-statistical (SMM) approach . Comparison with

  experimental data for reactions with two different projectiles ^{18}O (35 MeV per nucleon) and ^{86}Kr (64 MeV per nucleon) on two targets : ^{181}Ta and ^{9}Be
- Comparison with other model calculations (Abrasion-Ablation, EPAX and Fracs)
- Explanation of target ratio dependence on mass number *A* from the point of view of BNV-SMM calculations will be given
- Conclusion

To model heavy-ion collision microscopically we use kinetic theory:

Transport theory: Boltzmann-Nordheim-Vlasov (BNV) approach

time evolution of the one-body phase space density: f(r,p;t)

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla} f - \vec{\nabla} U \vec{\nabla}_p f = I_{coll}[f, \sigma]$$

Physical input:

mean field potential U (equation of state) and in-medium elastic cross section

F. Bertsch, S. Das Gupta, Phys. Rep. **160** (1988) 189
V. Baran, M. Colonna, M. Di Toro, Phys. Rep., **410** (2005**)** 335

Density functional
U(ρ(r)) = Nuclear Mean Field + Symmetry terms + Coulomb

$$U(\rho) = A\left[\frac{\rho}{\rho_0}\right] + B\left[\frac{\rho}{\rho_0}\right]^d + C(-1)^k\left(\rho_n - \rho_p\right)/(\rho_n + \rho_p) + U_{\mathrm{coul}}$$

$$A = -356 MeV, B = 303 MeV, d = 7/6, k = 1(p), 2(n), C = 36 MeV$$

Solution of transport equation

Partial integro-differential equation for *f(r,p;t)* is solved by simulation with the test particle method:
N -finite element test particles (TP) per nucleon.
Each TP carries charge and isospin number.
A – number of nucleons in the system
g – the shape of the TP in space
ρ – the density

$$f(\vec{r},\vec{p},t)=\frac{1}{N}\sum_{i}^{NA} g(\vec{r}-\vec{r}_i(t))\,\bar{g}(\vec{p}-\vec{p}_i(t))$$

$$g=\mathrm{e}^{-(\vec{r}-\vec{r}_i(t))^2/L^2}\ \ldots;\ \bar{g}=\mathrm{e}^{-(\vec{p}-\vec{p}_i(t))^2/l^2}$$

$$\rho(r;t)=\int d\vec{p}\ f(\vec{r},\vec{p};t)$$

Equations of motion of TP

(Newton equation of motions:):

$$\frac{\partial\vec{p}_i(t)}{\partial t}=-\vec{\nabla}_r U(r_i,t)\qquad \frac{\partial\vec{r}_i(t)}{\partial t}=\frac{\vec{p}_i(t)}{m}$$

Velocity Verlet (or leapfrog) algorithm, accuracy $(dt)^2$:

$$\vec{p}_i(t+\frac{1}{2}\Delta t)=\vec{p}_i(t)-\frac{1}{2}\Delta t\vec{\nabla}_r U(r_{i(t)})$$

$$\vec{r}_i(t+\Delta t)=\vec{r}_i(t)+\Delta t\ \vec{p}_i(t+\frac{1}{2}\Delta t)/m$$

$$\vec{p}_i(t+\Delta t)=\vec{p}_i(t+\frac{1}{2}\Delta t)-\frac{1}{2}\Delta t\vec{\nabla}_r U(r_{i(t+\Delta t)})$$

Collision term

$$I_{coll}[f_1,\sigma] =$$

$$\frac{g}{h}\int d^3p_2\, d^3p_3\, d^3p_4\, \sigma(12,34)\,\delta(\vec{p}_1+\vec{p}_2-\vec{p}_3+\vec{p}_4)\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3+\varepsilon_4)\left[\bar{f}_1\bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3\bar{f}_4\right]$$

Pauli blocking factors for final state $(1-f(r,v_i;t)) \equiv (1-f_i) := \bar{f}_i$

g degeneracy

Collision term: treatment by stochastic simulation

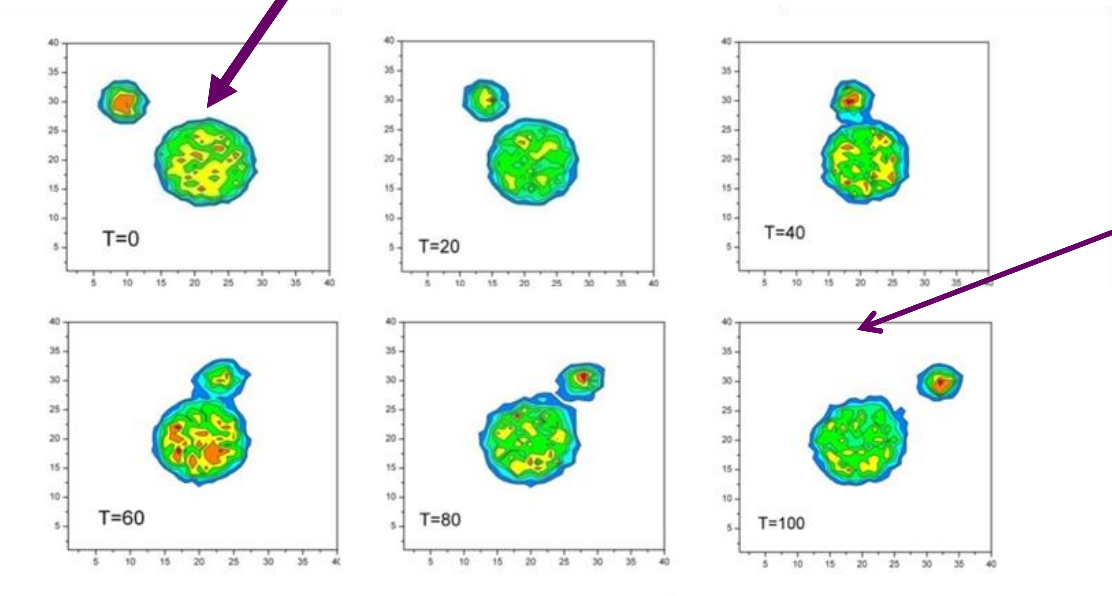
1.Select in each time step d*t* TPs with distance $d \le \sqrt{\sigma/\pi}$

2.Collide with probability $P=\sigma_{el}/\sigma_{max}$ with random scattering angle

3.Check Pauli blocking of final state in phase space

The computational code was developed on the basis of the code developed in LNS-INFN Catania in collaboration with M. Di Toro and H.H. Wolter

STEP 1: stochastic modeling of the system of two approaching ions. Minimization of energy in Woods-Saxon potential, taking into account Coulomb and Symmetry energy. The initial values of coordinates ***R*** and momenta ***P*** in the center of mass system are added



Density contour plots in the reaction
^{18}O(35 *A* MeV) + ^{181}Ta at *b* = 9 fm
(t=0, 20, 40, 60, 80, 100 fm / *c* (10 fm/*c*=3.3*10-23 c))

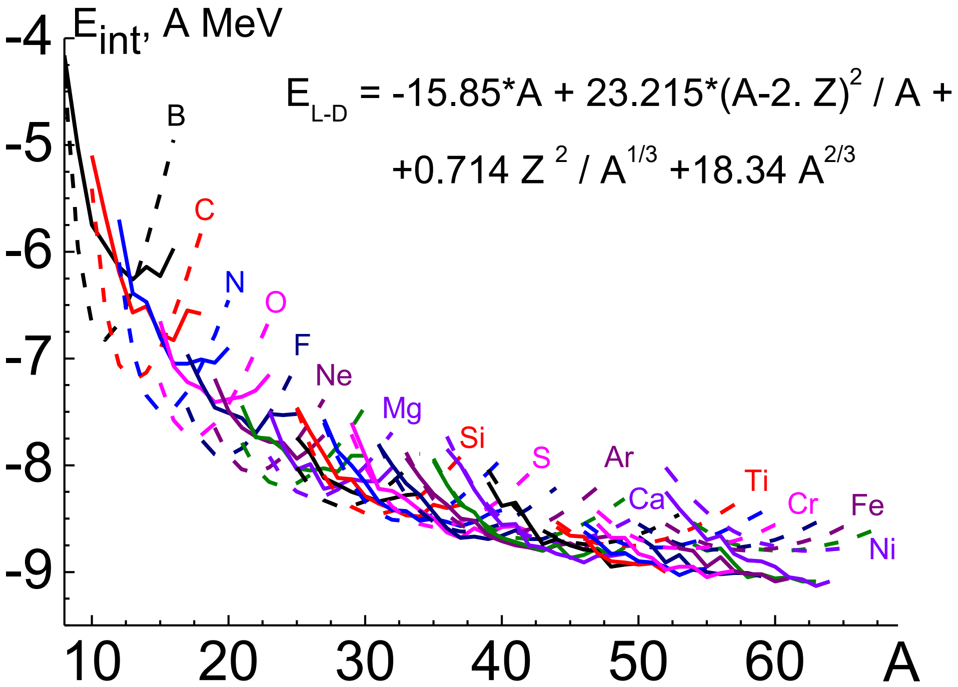
STEP 2: Evolution in time in the mean field until freeze-out time: only Coulomb potential, nuclear forces between fragments are negligible

Identify final fragments by coalescence method
Here:
Cut-off criterion in density

$$(\varrho(r;t_{\text{freeze-out}})<0.02)$$

Fragments characteristics:
mass number *A*, charge *Z*, intrinsic energy E_{int}, momentum ***P***, coordinates ***R***

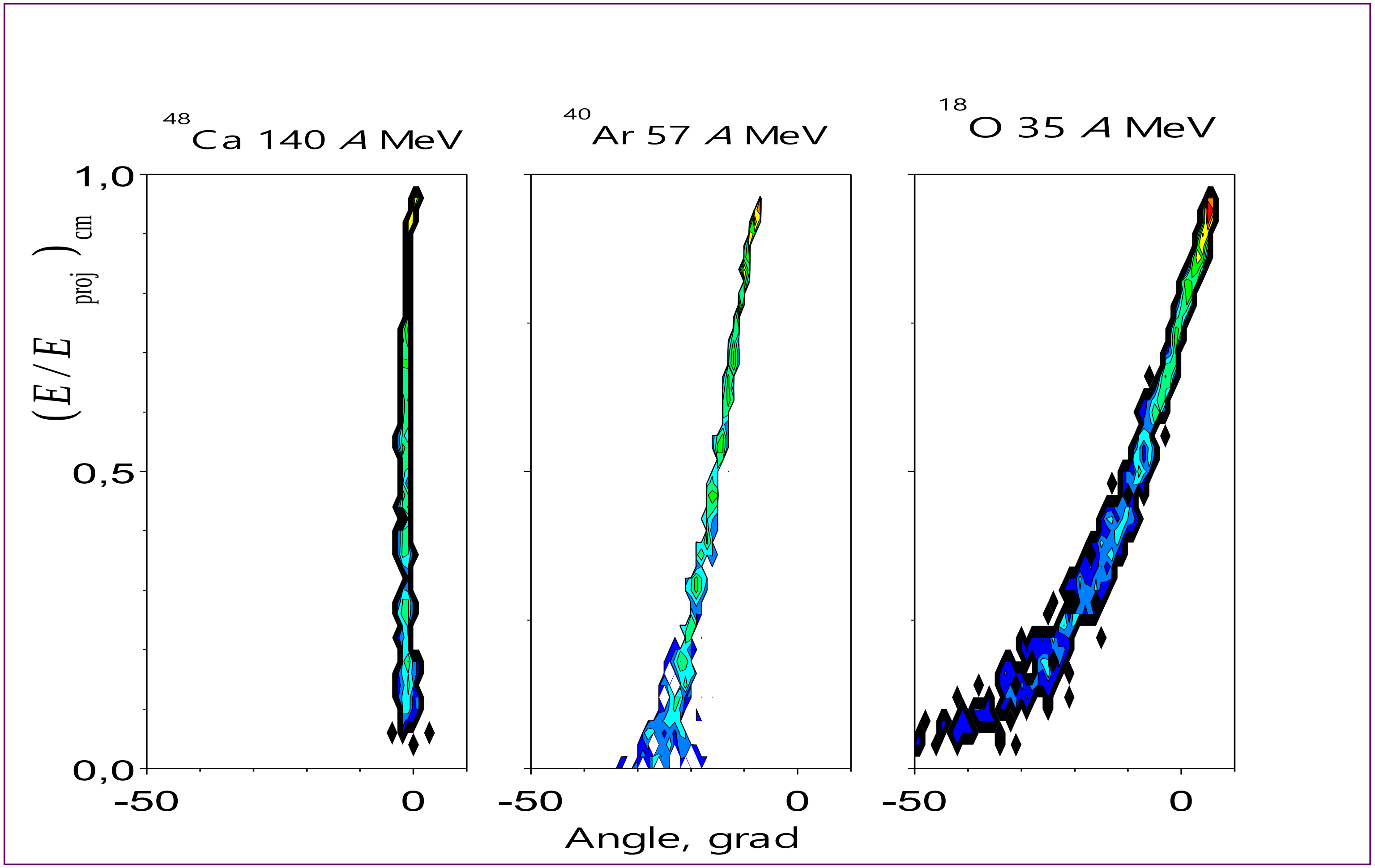
Calculation of the ground energy of a nucleus with the same density functional U(r) as used in the transport equation



G.S. binding energies for isotopic chains of the intialized nuclei (solid lines) with a liquid-drop formula (dashed lines).

--> reasonable agreement

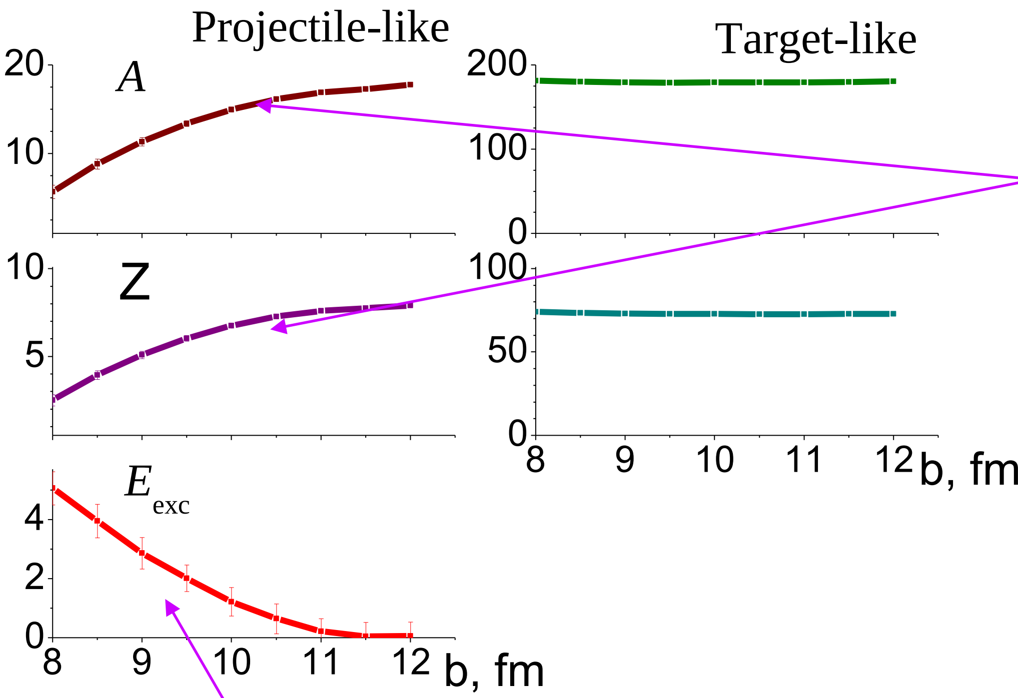
Calculation of Wilczynski diagrams for different projectiles and energies on ^{181}Ta in the transport equation



Calculations with 200 TP for nucleon

^{18}O(35 *A* Mev)+^{181}Ta

The results smoothly depends on the value of impact parameter *b*



Heavier ion stays practically the unchanged!

Excitation energy is calculated in self-consisted way with the same potentials as dynamical calculations are fulfilled

Fragments are excited!
Transport approach is semiclassical approach, it can’t describe quantum effects.

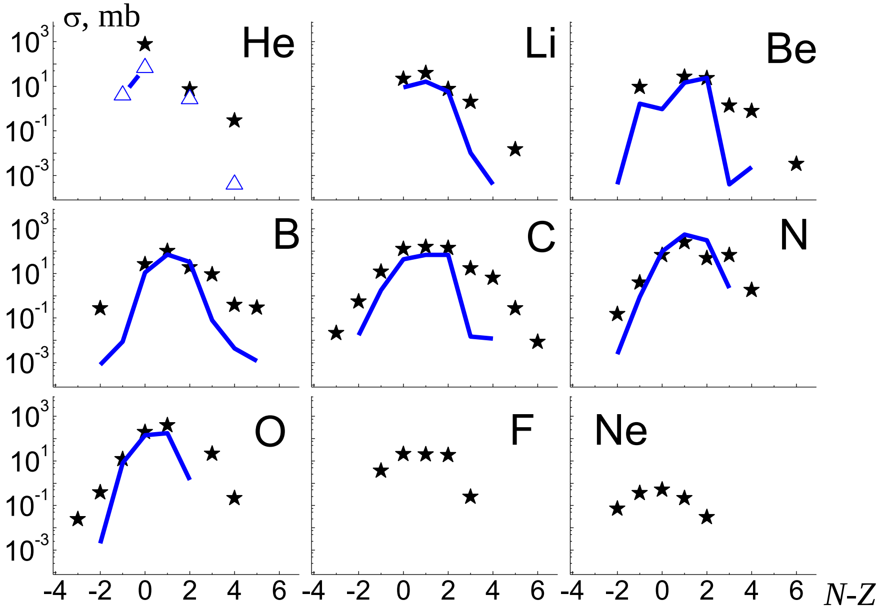
De-excite the fragments with the statistical code

STEP 3: **Calculating cold evaporation residues:**

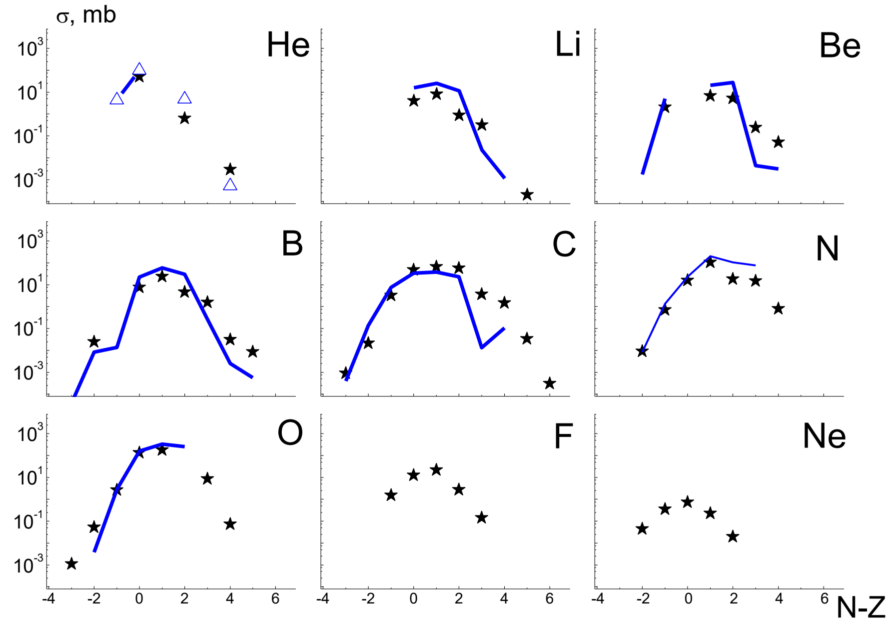
SMM code, P. Bondorf, et al.,Phys. Rep. 257, 133 (1995)

Input parameters: **A_{fr}, Z_{fr}, $\underline{E}_{exc}$ R, P** from **BNV** calculation

^{18}O(35 *A*Mev)+^{181}Ta



^{18}O(35 *A*Mev)+^{9}Be

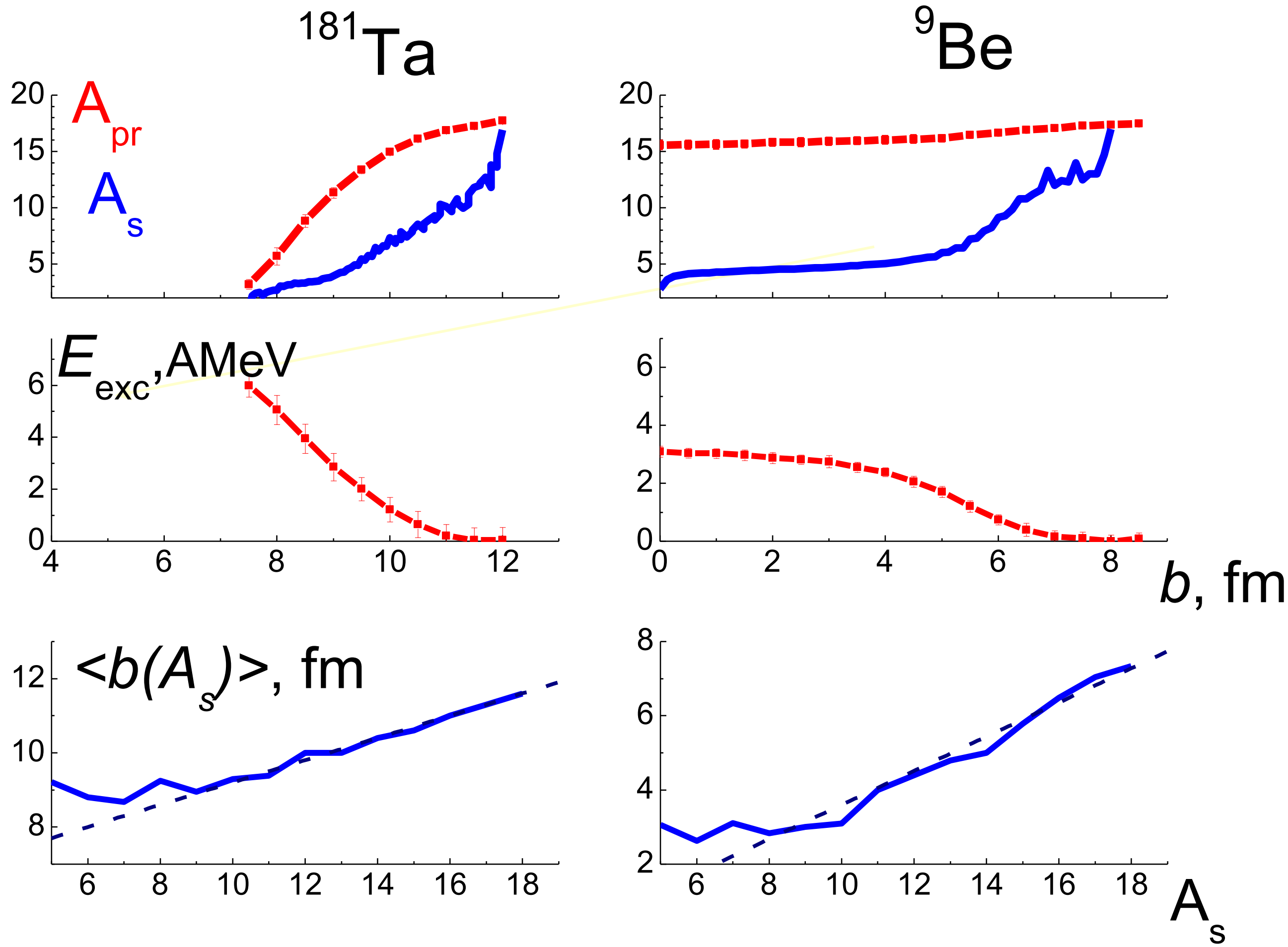


Experiment: Combas set-up, FLNR, JINR

Artukh, A.G, et al. Phys. Part. Nuclei Lett. 18, 19 (2021).

BNV-SMM calculations give reasonable agreement for *N*-Z close to zero nuclides

Comparison of the calculated results for two reactions with ^{18}O (35 *A* MeV) beam

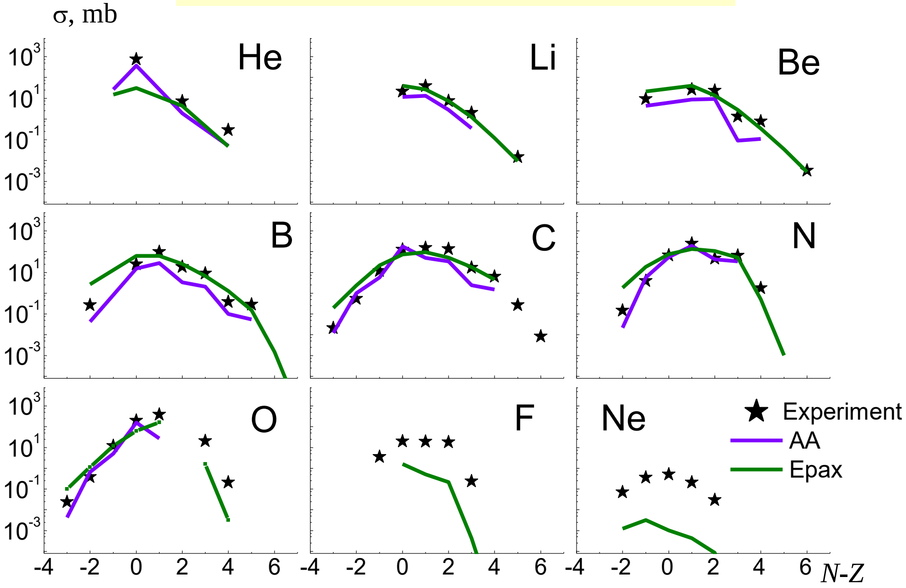


Different approaches can also be used to predict isotope distributions produced in nuclear collisions at Fermi energies

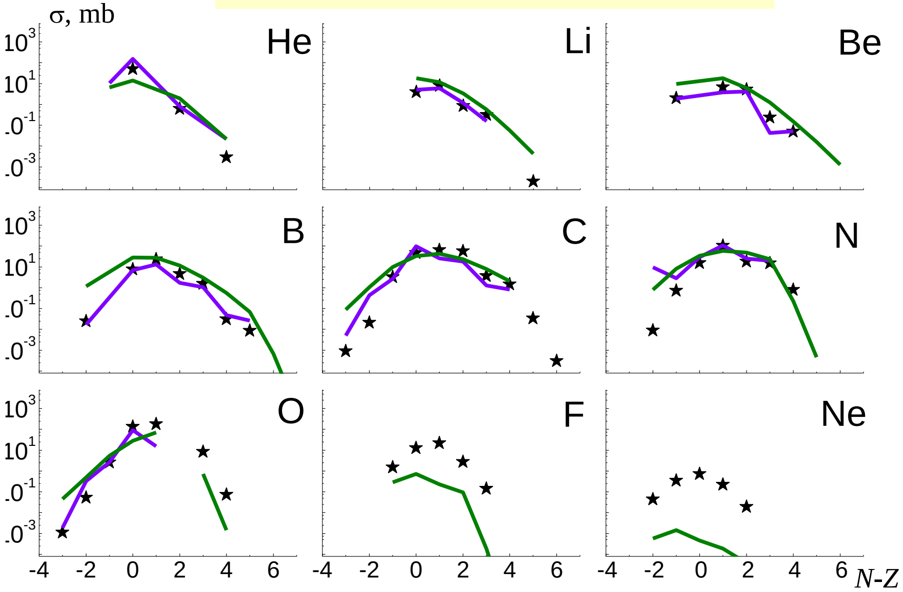
- **Transport approaches: QMD** (quantum molecular dynamics*) J. Aichelin, Phys. Rep.* ***202****, 233 (1991).***,**
  AMD(antisymmetrized/fermionic molecular dynamics)
  A. Ono and H. Horiuchi, Prog. Part. Nucl. Phys. ***53****, 501 (2004);*
- **EPAX (**an Empirical PArametrization of fragmentation CROSS sections)
  K. Summerer and B. Blank, Phys. Rev. C. 61, 034607 (2000).
- **Abrasion-Ablation model***, Bowman J.D., Swiatecki W.J., Tsang C.F. // LBL Report. 1973. LBL-2908.*
- **FRACS** (Mei B. Improved empirical parameterization for projectile fragmentation cross sections // Phys. Rev. C. 2017. V. 95. P. 034608.
- ***etc*****.**

Calculations of isotope distributions with EPAX and Abrasion-Ablation models

$^{18}O(35\ AMev)+^{181}Ta$



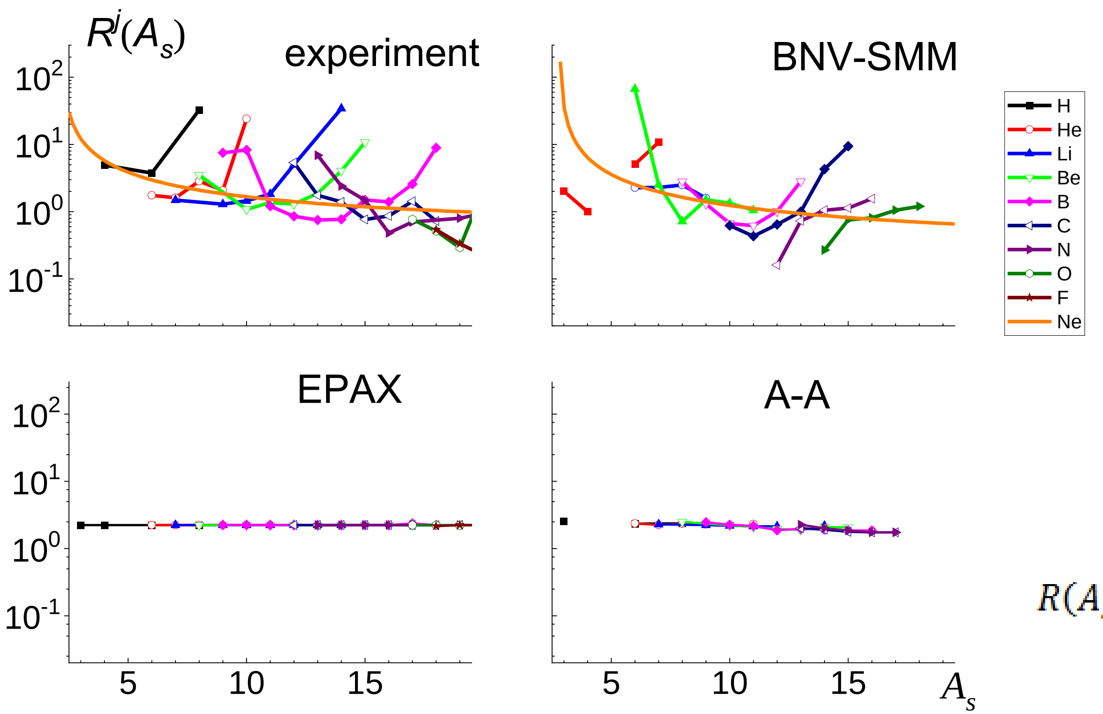
$^{18}O(35\ AMev)+^{9}Be$

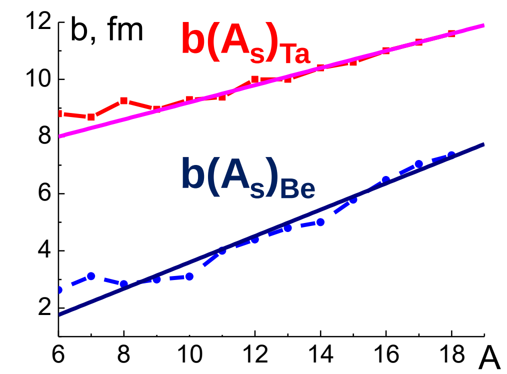


The best coincidence give EPAX calculations,
BNV-SMM calculations give reasonable agreement for *N*-Z close to zero nuclides

Experiment: Combas set-up, FLNR, JINR
Artukh, A.G, et al. Phys. Part. Nuclei Lett. 18, 19 (2021).

Target ratio $R_J(A_s) = \sigma_J(A_s)_{Ta} / \sigma_J(A_s)_{Be}$, **for projectile** ^{18}O, 35 A MeV



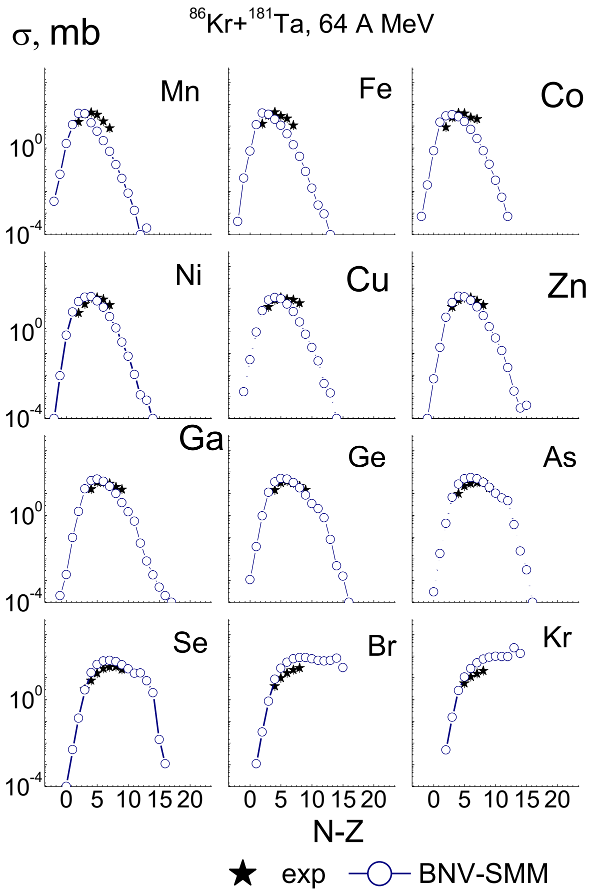


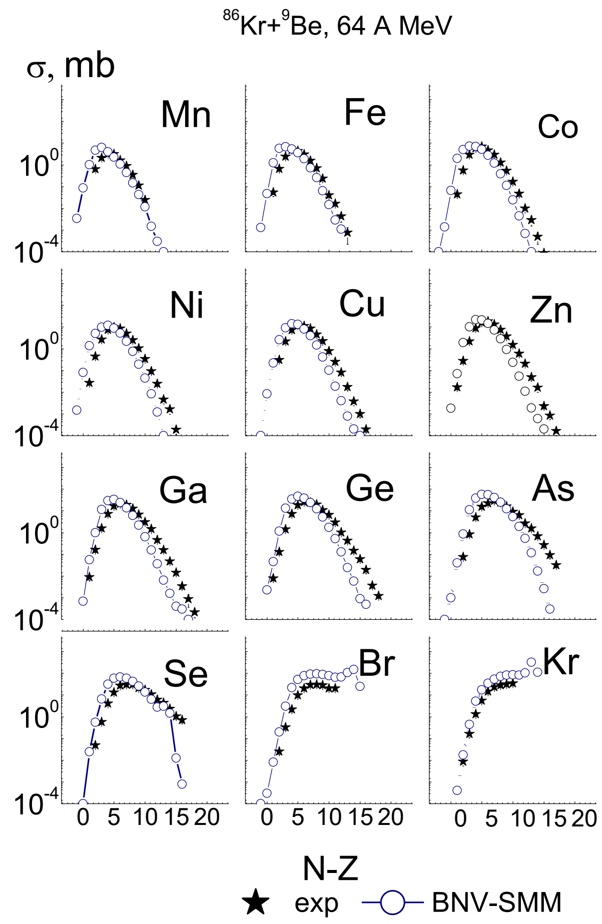
$$R(A_s) = b(A_s)_{Ta} \left(\frac{\partial b}{\partial As}\right)_{Ta} \Big/ b(A_s)_{Be} \left(\frac{\partial b}{\partial As}\right)_{Be}$$

Cold residues for reactions,
Experiment and BNV-SMM calculations:

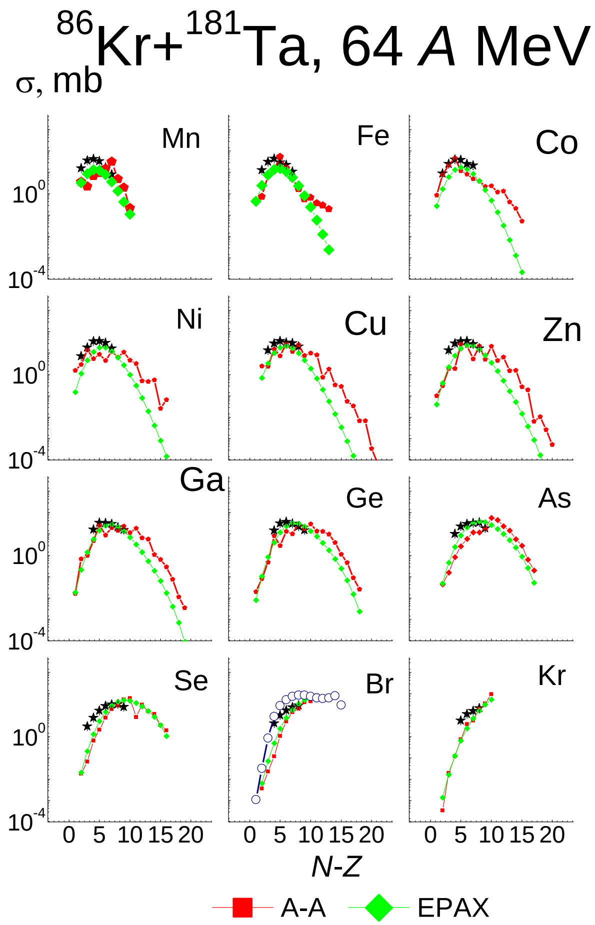
$^{86}Kr(64\ A Mev)+^{181}Ta$

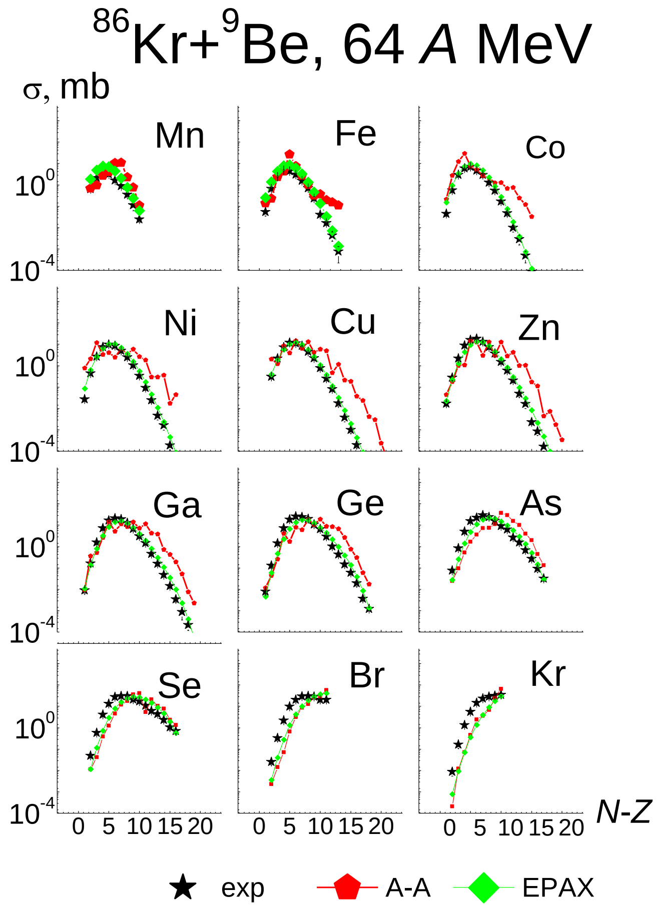
$^{86}Kr(64\ A Mev)+^{9}Be$





Cold residues for reactions, experiment, EPAX and Abrasion-Ablation models





Ratio σ(Kr+Ta) / σ(Kr+Be), 64 *A* MeV
M. Mocko et.al Phys. Rev. C76, 014609(2007)

σ(Kr+Ta) /σ(Kr+Be), 64 *A* MeV,
Experiment and model calculations

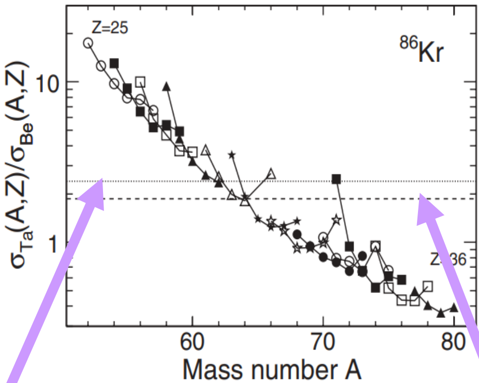
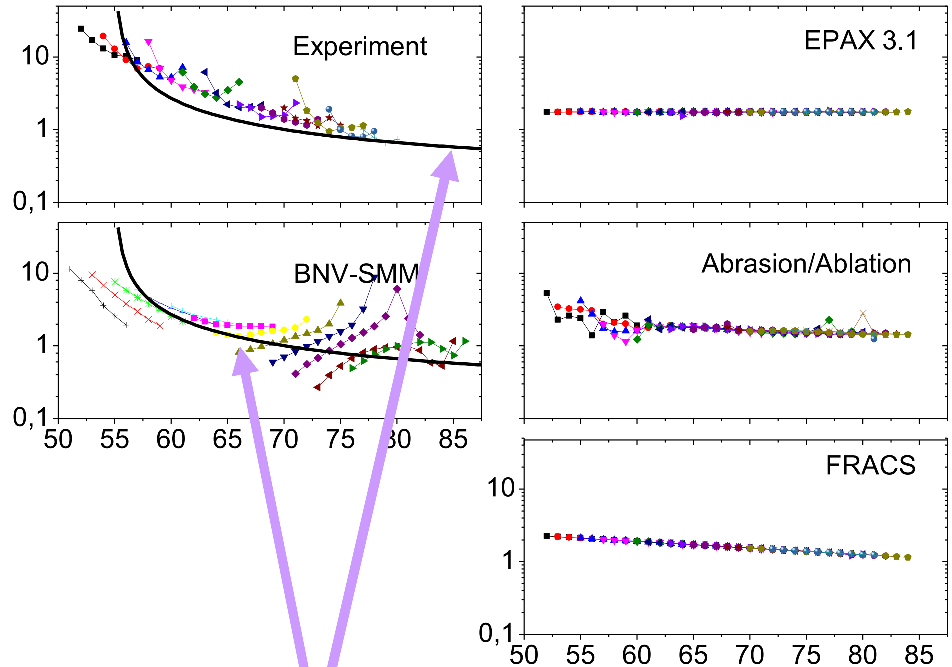


FIG. 10. Ratios of the fragmentation cross sections on Ta and Be targets, $\sigma_{Ta}(A,Z)/\sigma_{Be}(A,Z)$, for fragments with $25 \leqslant Z \leqslant 36$ for the ^{86}Kr beam. Only ratios with relative errors smaller than 25% are shown. Open and solid symbols represent odd and even elements starting with $Z = 25$. The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Eq. (4), respectively.

$$\frac{\sigma_{Ta}(A,Z)}{\sigma_{Be}(A,Z)} = \frac{(A_{Kr}^{1/3} + A_{Ta}^{1/3})^2}{(A_{Kr}^{1/3} + A_{Be}^{1/3})^2} = 2.4,$$

$$\frac{\sigma_{Ta}(A,Z)}{\sigma_{Be}(A,Z)} = \frac{(A_{Kr}^{1/3} + A_{Ta}^{1/3} - 2.38)}{(A_{Kr}^{1/3} + A_{Be}^{1/3} - 2.38)} = 1.9.$$



$$R(A_s) = b(A_s)_{Ta}\left(\frac{\partial b}{\partial As}\right)_{Ta} \Big/ b(A_s)_{Be}\left(\frac{\partial b}{\partial As}\right)_{Be}$$

Conclusions

We found that our calculations describe fragments close to *N*-*Z* = 0 rather well, but for neutron reach isotopes our calculations give smaller values than the experiment

The target ratios point out on the importance of taking into account two important characteristics of the reaction:

1) **target mass**
2) **impact parameter value**

The increase of the yields of the neutron-reach isotopes in the reactions on heavy targets in comparison with the light ones can be explained by the different character of the primary (excited) fragments formation

Thank you for attention