Constraints on the nuclear Equation of State: From terrestrial experiments to neutron star observations

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Laboratory experiments constraining the Nuclear Equation of State (EoS) of symmetric/asymmetric nuclear matter

- **Heavy-Ion Collisions- HICs (RHIC, FAIR, NICA)**
  Probes high-density matter ($1–5\rho_0$) via flow, π/K production
- **Neutron Skin Measurements (PREX/CREX)**
  Parity-violating e^- scattering on $^{208}Pb/^{48}Ca$
  → symmetry energy (*J*, *L*)
- **Giant Resonances**
  Isovector IVGDR dipole R (Texas A&M, RCNP) → Sym. energy
  Isoscalar ISGMR monopole resonance → *Incompressibility*.
- **Accurate Masses and radii of Neutron-rich and Exotic Nuclei**
  Structure, stability, Surface energy and diffuseness (FRIB,RIKEN,ISOLDE) → symmetry energy at low densities
- **Electroweak Scattering (Jefferson Lab, GANIL)**
  Charge distributions, transfer reactions → isospin asymmetry effects

Neutron stars and astrophysical observable constraining the EoS

Observable	Probes EoS at	Missions/Events
Mass-Radius	$\sim 1\text{–}10\rho_0$	NICER, radio timing
Tidal Deformability	$\sim 1\text{–}3\rho_0$	LIGO/Virgo (GW170817)
Moment of Inertia	Core density (up to 10 ρ_0)	Pulsar binaries
Cooling	Core composition	X-ray telescopes (Chandra)
GW Mergers	High density ($>3\ \rho_0$)	LIGO, Einstein Telescope

- ✓ The particle fractions in npeμ matter of NS core at β-equilibrium.
- ✓ The core-crust transition density and pressure.
- ✓ The crustal fraction of the moment of inertia.
- ✓ The URCA process in NS (The NS cooling).

Nuclear Models and forces

- **The non-relativistic Hartree–Fock approach with**

  Semi-microscopic Density-dependent M3Y interaction

  Skyrme NN interaction

  Argonne AV18 interaction **Gogny force**

  Isospin- and momentum-dependent MDI interactions

- **RMF model with different parameterizations**

  NL1, NL2, NL3, NL-SH, FSU-Gold, DD …

- **Phenomenological EOSs** :

with a reasonable fit to empirical masses and radii of stable nuclei, or NS structure

EOS of ANM based on density dependent CDM3Y nucleon-nucleon interactions

JPG 38, 035102 (2011)

CDM3Y-Paris
CDM3Y-Reid
NN
Interactions

Non-relativistic
Hartee-Fock
scheme

SNM – ANM - PNM
Equations of State
(K_0=150-330 MeV)

$$v_{00(01)}^{D(Ex)}(\rho,r) = F_{0(1)}(\rho)\,v_{00(01)}^{D(Ex)}(r)$$

$$F_{0(1)}(\rho) = C_{0(1)}f(\rho) = C_{0(1)}\left(1 + \alpha_{0(1)}e^{-\beta_{0(1)}\rho} - \gamma_{0(1)}\rho\right),$$

CDM3Y-Paris

$$v_{00}^{D} = 11061.625\frac{e^{-4r}}{4r} - 2537.5\frac{e^{-2.5r}}{2.5r},$$

$$v_{00}^{\mathrm{Ex}} = -1524.25\frac{e^{-4r}}{4r} - 518.75\frac{e^{-2.5r}}{2.5r} - 7.8474\frac{e^{-0.7072r}}{0.7072r},$$

$$v_{01}^{D} = 313.625\frac{e^{-4r}}{4r} + 223.5\frac{e^{-2.5r}}{2.5r},$$

$$v_{01}^{\mathrm{Ex}} = -4118.0\frac{e^{-4r}}{4r} + 1054.75\frac{e^{-2.5r}}{2.5r} + 2.6157\frac{e^{-0.7072r}}{0.7072r}$$

Saturation and symmetry properties of isospin asymmetric nuclear matter

ANM energy density

$$E_A(\rho,I) = \frac{3\hbar^2 k_F^2\left[(1+I)^{5/3}+(1-I)^{5/3}\right]}{20m} + f(\rho)\frac{\rho}{2}\left\{C_0 \mathcal{J}^D_{00} + I^2 C_1 \mathcal{J}^D_{01} + \frac{1}{4}\int\left[C_0 v^{Ex}_{00} B_0^2 + C_1 v^{Ex}_{01} B_1^2\right] d\vec{r}\right\}.$$

$$I = (\rho_n - \rho_p)/\rho.$$

2D-expansion of E_A with respect to $\boldsymbol{\rho}$ and $\boldsymbol{I}$ (up to I^8 & ρ^8)
(**24 characteristic NM coefficients**)

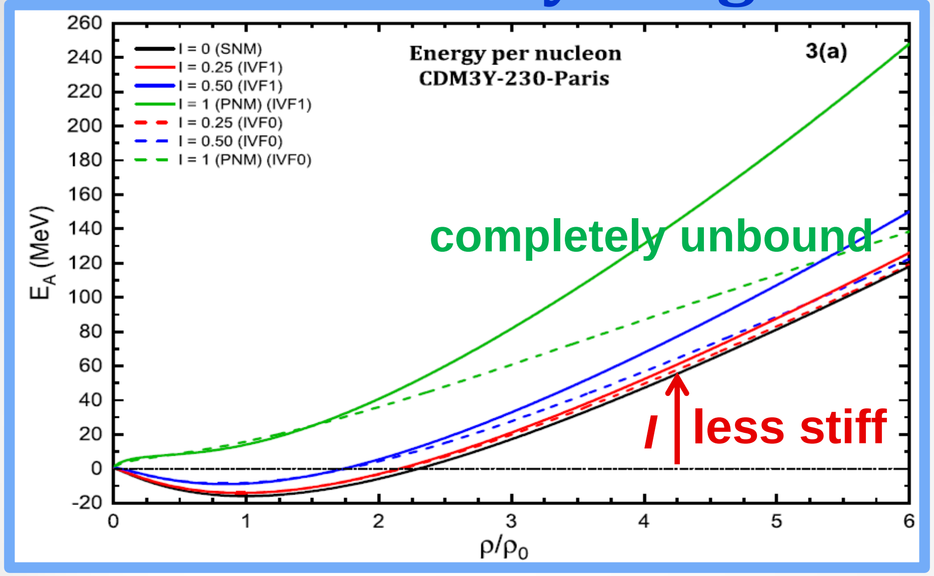
ANM saturation properties
$\boldsymbol{\rho_{0I}}$ **,** $\boldsymbol{E_A(\rho_{0I})}$ **,** $\boldsymbol{K_{0I}}$
(Analytical formulae)

$$E_A(\rho,I) = \sum_{n=0,1,2,\ldots} \frac{1}{(2n)!}\frac{\partial^{(2n)}}{\partial I^{(2n)}}\left[\sum_{m=0,1,2,\ldots}\frac{1}{m!}\frac{\partial^m E_A(\rho,I)}{\partial \rho^m}\bigg|_{\rho_0}(\rho-\rho_0)^m\right]\Bigg|_{I=0} I^{(2n)}.$$

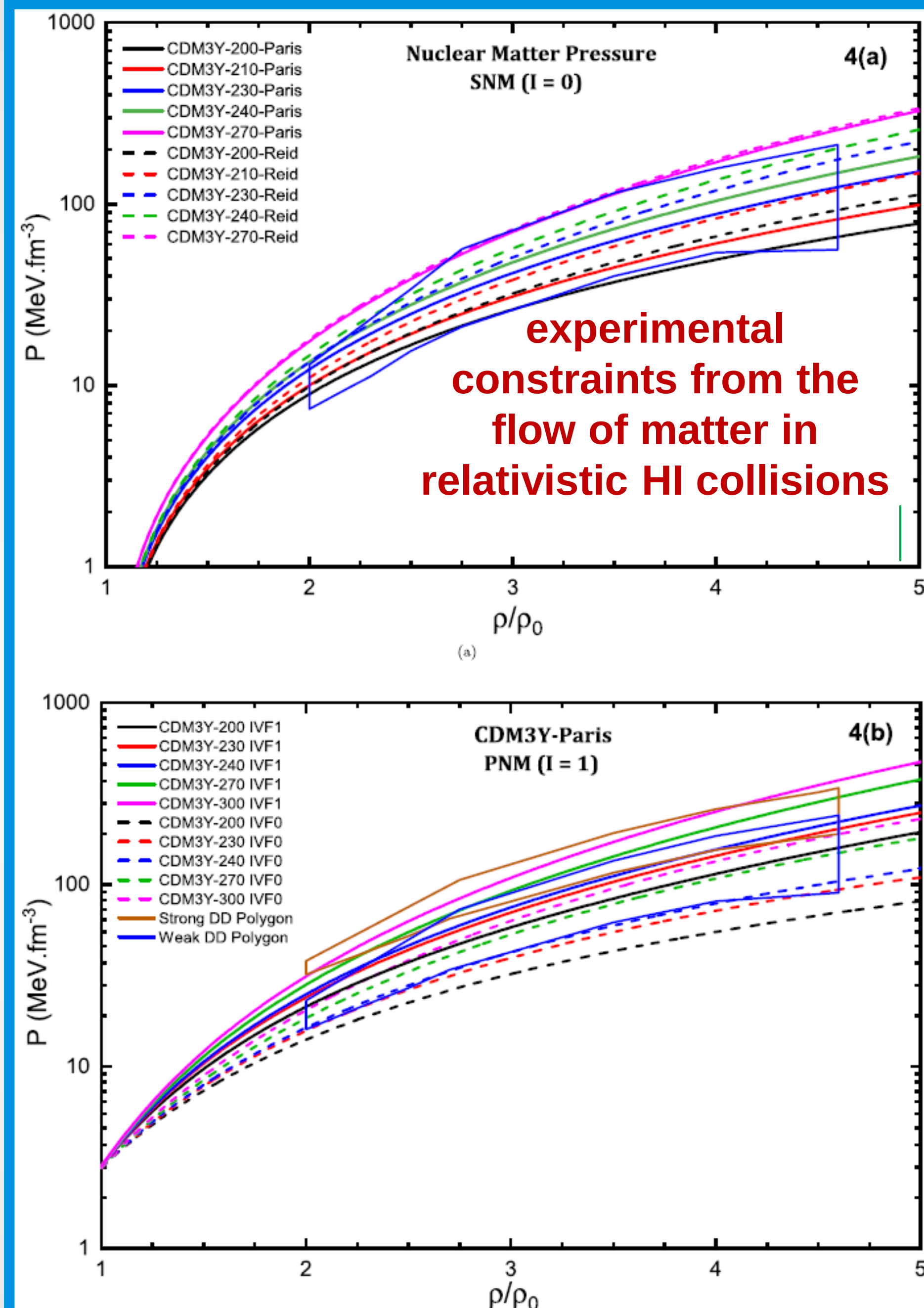
The **quadratic terms** are sufficient to describe ANM up to **2ρ₀**.
What about **supra-saturation densities**?

Nucl. Phys. A 878, 14(2012)

✓ The SNM is the Strongest bound over wider density range.



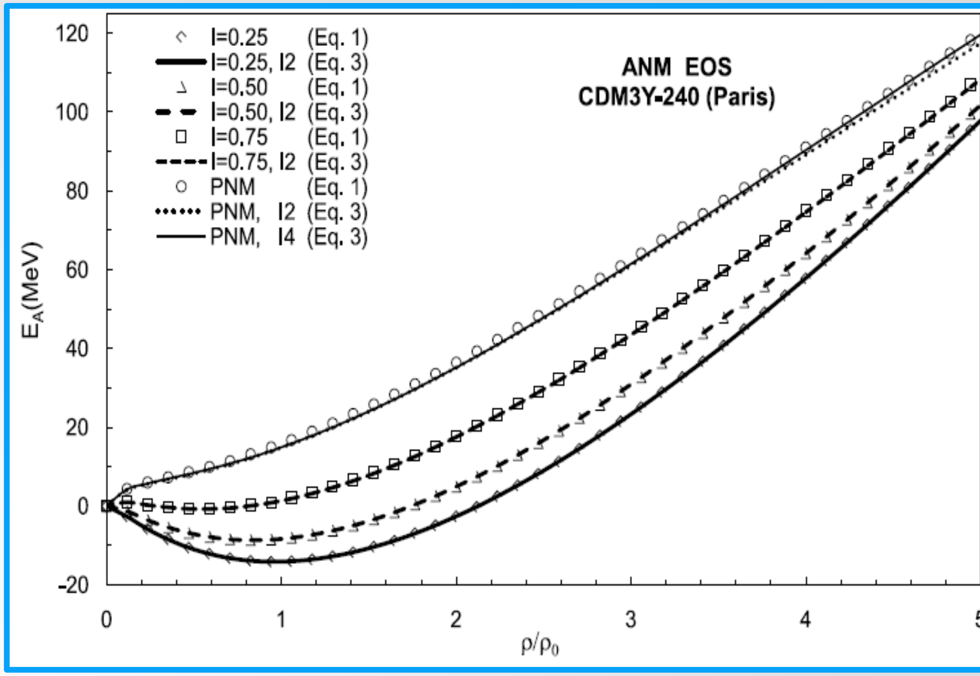
The fine ranges of Paris (K_0= 240 -250 MeV) and Reid (K_0 = 230 - 240 MeV) EOSs maintain simultaneous consistency with experimental/theoretical constraints on NM energy, symmetry energy, and pressure.

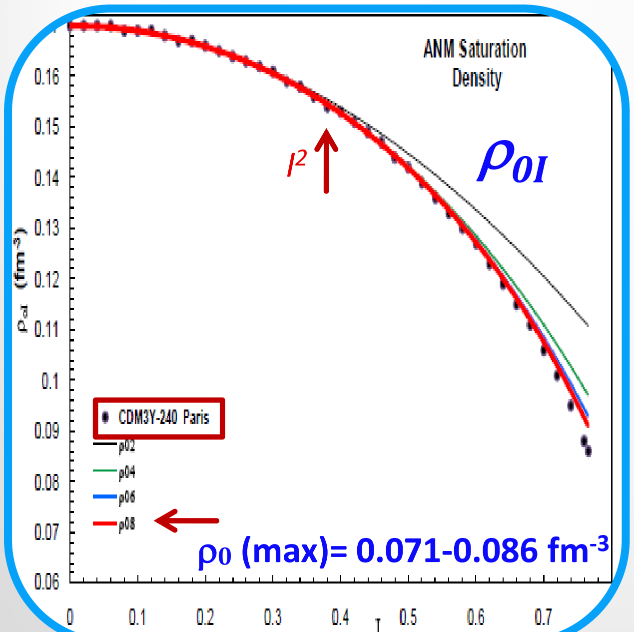


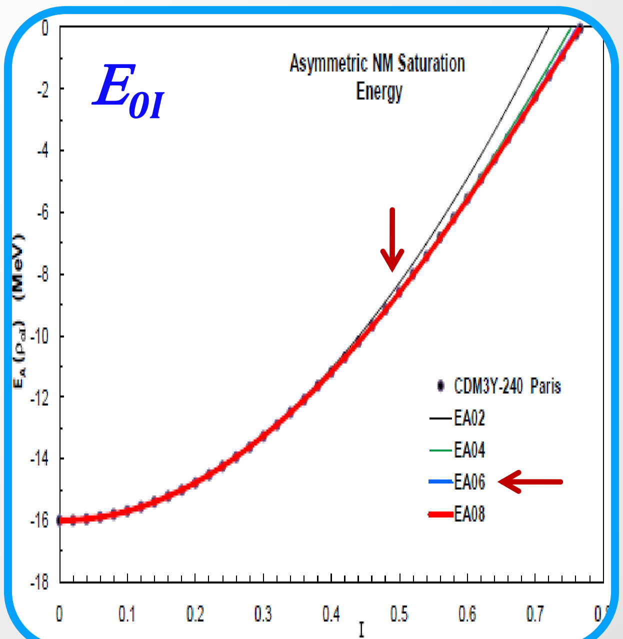
NPA 1035, 122668 (2023)

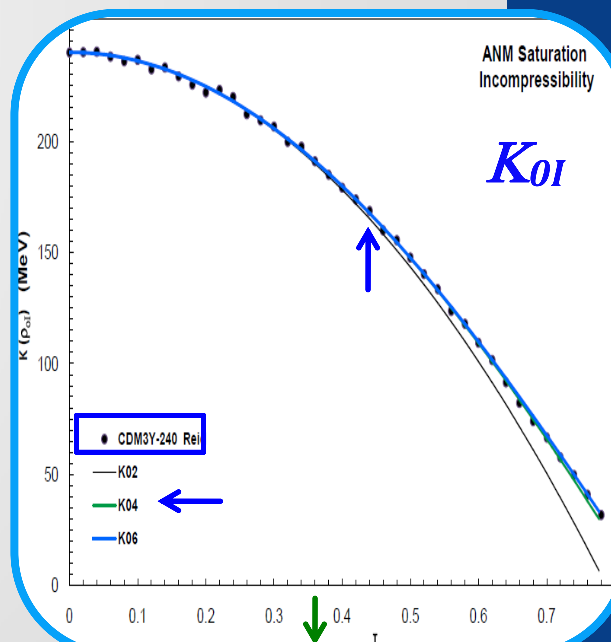
Isospin asymmetry dependence of ANM saturation properties

The **more neutron-rich ANM** is the **less bound** and **less stiff** one, over **narrower bound density range**.









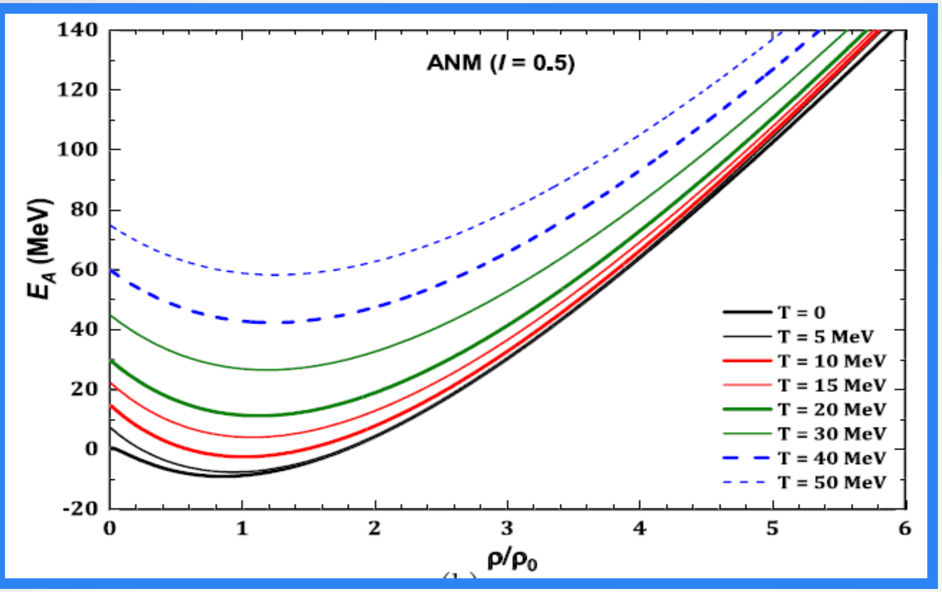
ρ_0 (I_{max})= 0.071-0.086 fm^{-3} ← I_{max} (bound-ANM)= 0.766 - 0.780 → K_{0I} (I_{max})= 30-49 MeV

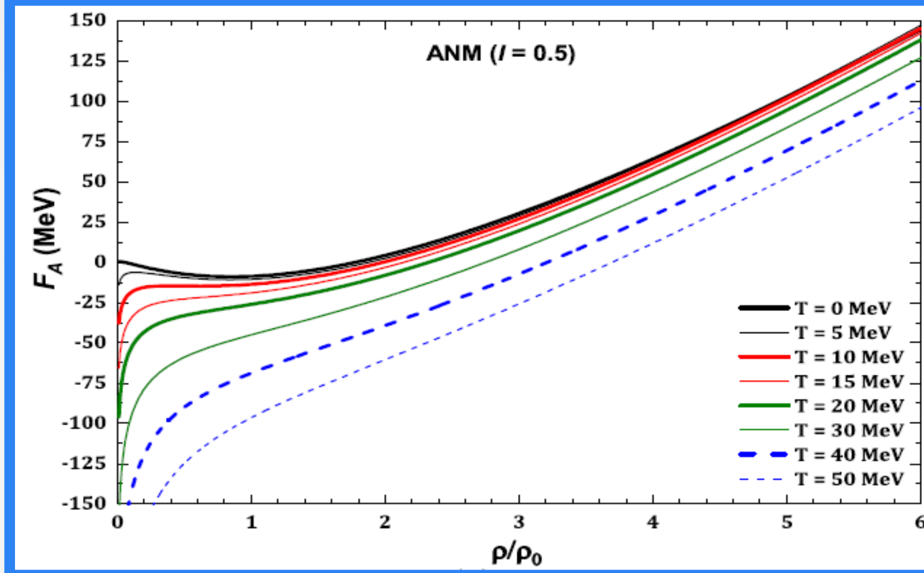
Saturation properties of hot ANM

NPA 1008, 122142 (2021)

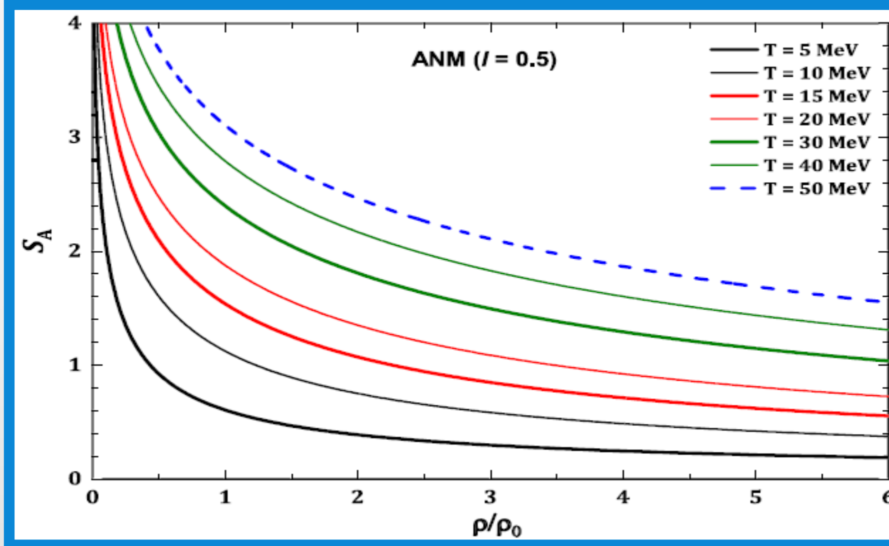
Helmholtz free energy per nucleon

$$F_A(T,\rho,I) = \frac{F(T,\rho,I)}{A} = E_A(T,\rho,I) - TS_A(T,\rho,I).$$





- Both the **repulsive internal energy** and the **negativity** of the **free energy increase** with ***T***, leading to less bound ANM, with a larger entropy.
- **Increasing the density increases** the **free energy**, and the **nucleons** become **less free in phase space,** with less **entropy**.



Conclusions I

- Based on the semi-realistic M3Y-Paris (Reid) *NN* interactions, the isospin asymmetry **terms up to I^8** (with only **18 characteristic quantities**) are needed to describe reasonably the different ANM saturation properties.
- Up to **$4\rho_0$**, some properties, such as ANM (PNM) **energy per nucleon,** are well expressed by their expansion up to only I^2. **NPA 878, 14(2012)**

- **Increasing the temperature ($<T_c$)** of **hot ANM**

[NPA1008, 122142(2021)]

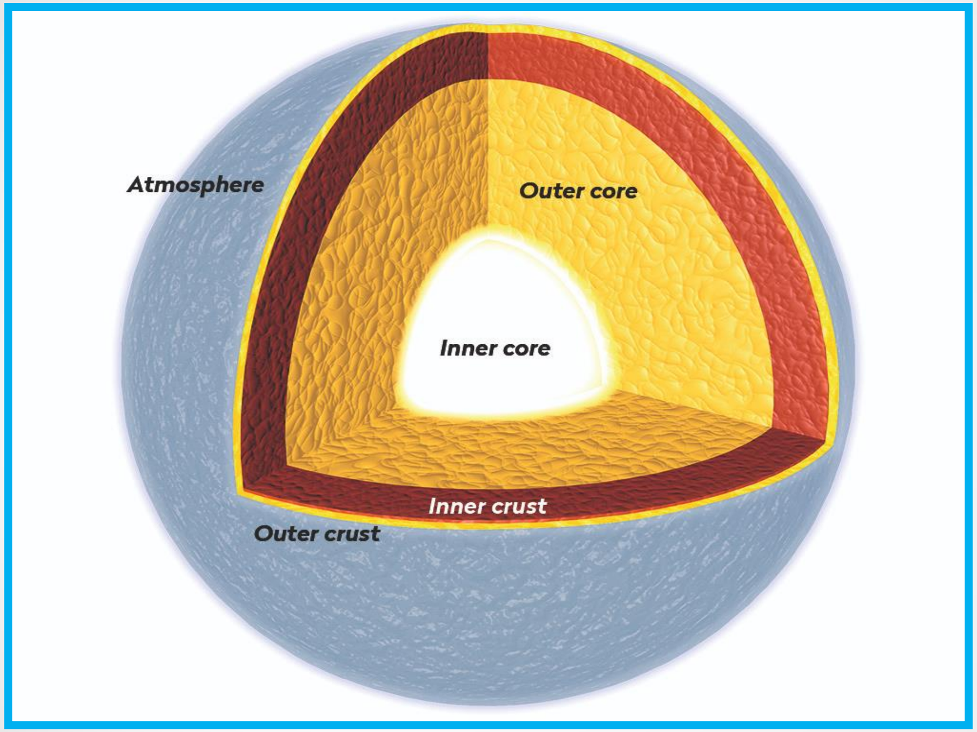
✓ **decreases** its **equilibrium binding energy**, shifting it to **higher equilibrium density**,

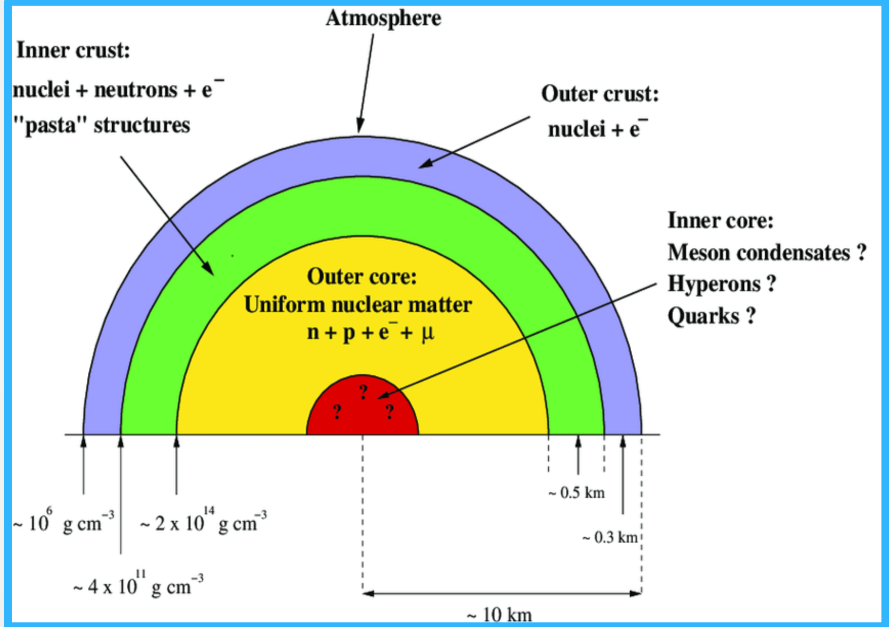
✓ **increases** its **attractive saturation free energy**, shifting it to **lower ρ**.

∴ The **hotter ANM** is **less bound**.

- **Increasing the isospin-asymmetry** (ANM) **decreases both the saturation binding- and free-energy,** and their corresponding **density**.
- The **saturation binding (free energy) disappears** at ***T*>(14-17) MeV**.
- **I_{max}=0.81±0.02** is indicated for a **semi-cold** bound ANM having **saturated free energy**.

Structure of Neutron Stars and massive pulsars





Observations: Electromagnetic emissions in all bands, (infrared, optical, ultraviolet, X-ray, gamma-ray), Neutrinos, and **Gravitational waves**.

From NN interaction to NS structure

CDM3Y-Paris (Reid)
Eff. *NN* interaction

HF

CDM3Y-K_0 EOSs

TOV

NS
Structure
properties

Energy density and pressure of npeμ matter

$$E_A(\rho, x_p) = \frac{3\hbar^2 k_F^2\left[(2-2x_p)^{5/3}+(2x_p)^{5/3}\right]}{20\,m} + \frac{\rho}{2}\left\{F_0(\rho)J_{00}^D + (1-2x_p)^2 F_1(\rho)J_{01}^D\right\}$$
$$+\frac{\rho}{8}\int\left[F_0(\rho)v_{00}^{Ex}(B_0(x_p, r))^2 + F_1(\rho)v_{01}^{Ex}(B_1(x_p, r))^2\right]d\vec{r},$$

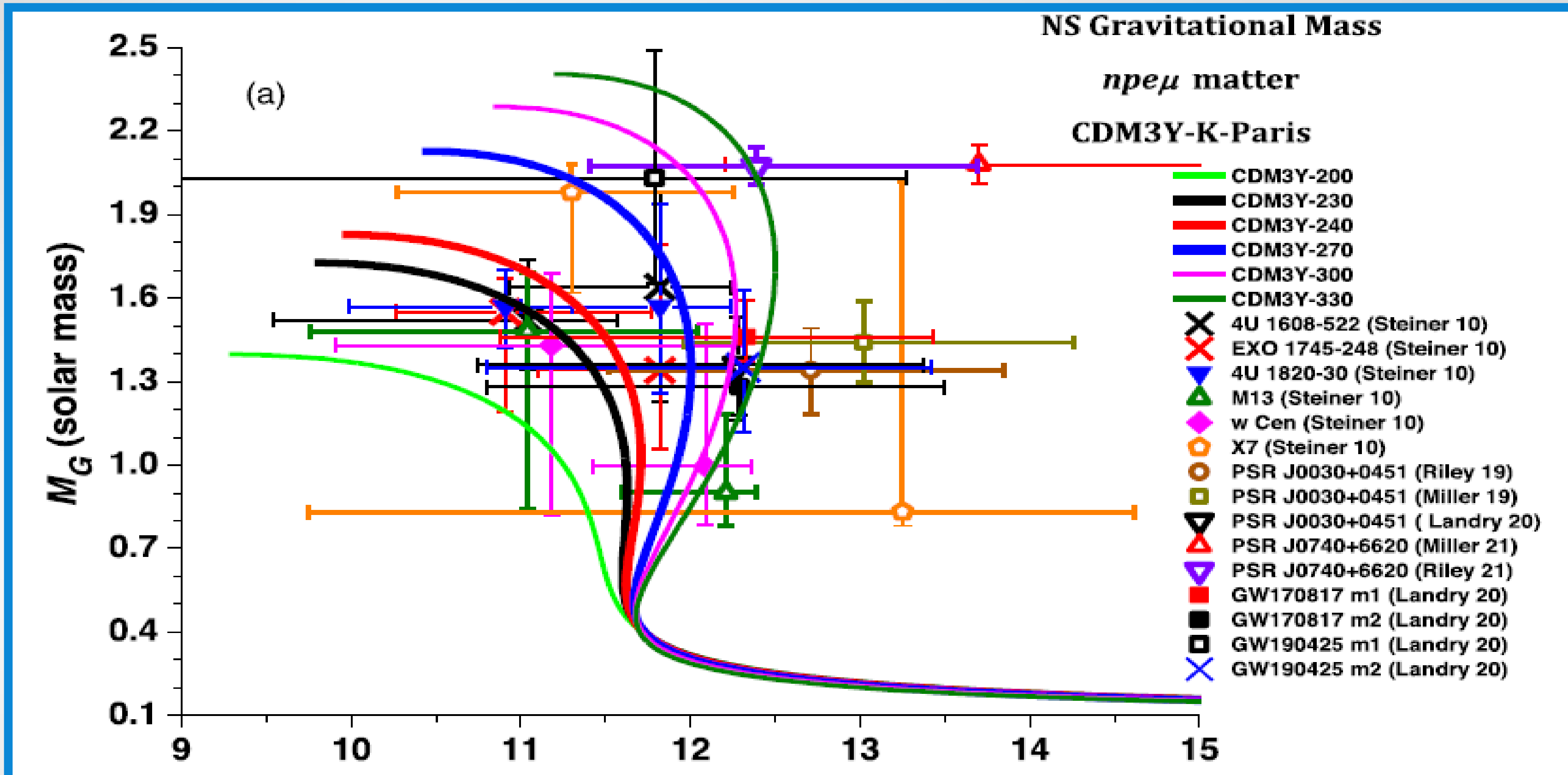
$$B_0(x_p, r) = (2-2\,x_p)\hat{j}_1(k_{Fn}r) + 2x_p\hat{j}_1(k_{Fp}r),$$
$$B_1(x_p, r) = (2-2\,x_p)\hat{j}_1(k_{Fn}r) - 2x_p\hat{j}_1(k_{Fp}r).$$

$$P(\rho, x_p, x_e, x_\mu) = P_b(\rho, x_p) + \sum_{l=e,\mu} P_l(\rho, x_l),$$
$$P_b(\rho, x_p) = \rho^2\frac{\partial E_A(\rho, x_p)}{\partial\rho},$$
$$P_l(\rho, x_l) = \mu_l\,\rho\,x_l - \varepsilon_l(\rho, x_l).$$

Tolman-Oppenheimer-Volkoff equations at hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\frac{G\,\varepsilon(r)\,M(r)}{c^2r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi\,r^3P(r)}{c^2M(r)}\right)\left(1-\frac{2\,GM(r)}{r\,c^2}\right)^{-1}$$
$$\frac{dM(r)}{dr} = \frac{4\pi\,r^2\varepsilon(r)}{c^2}.$$

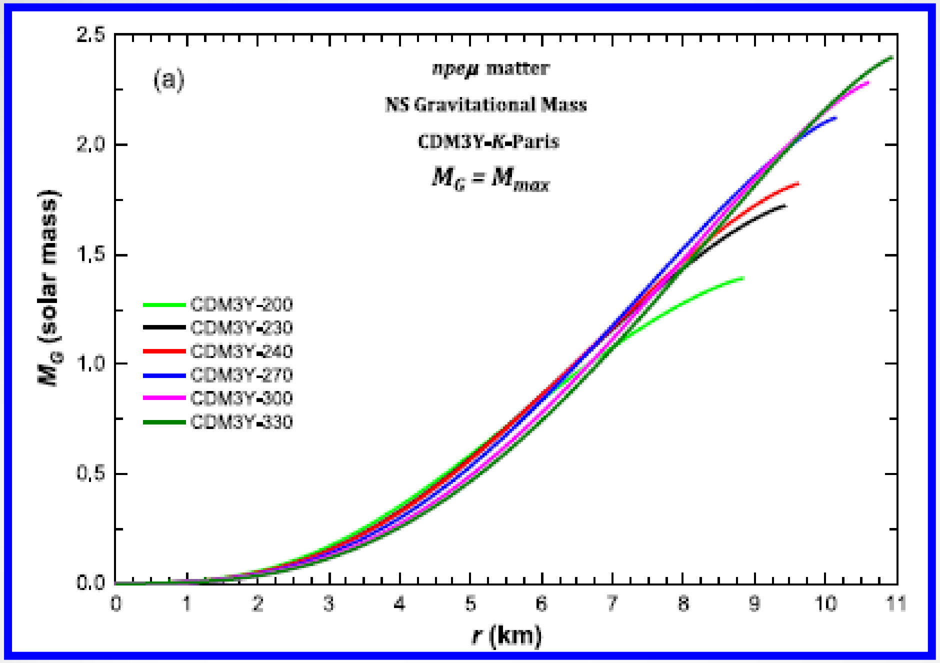
Mass-Radius relation of NS



JPG 51, 065203 (2024)

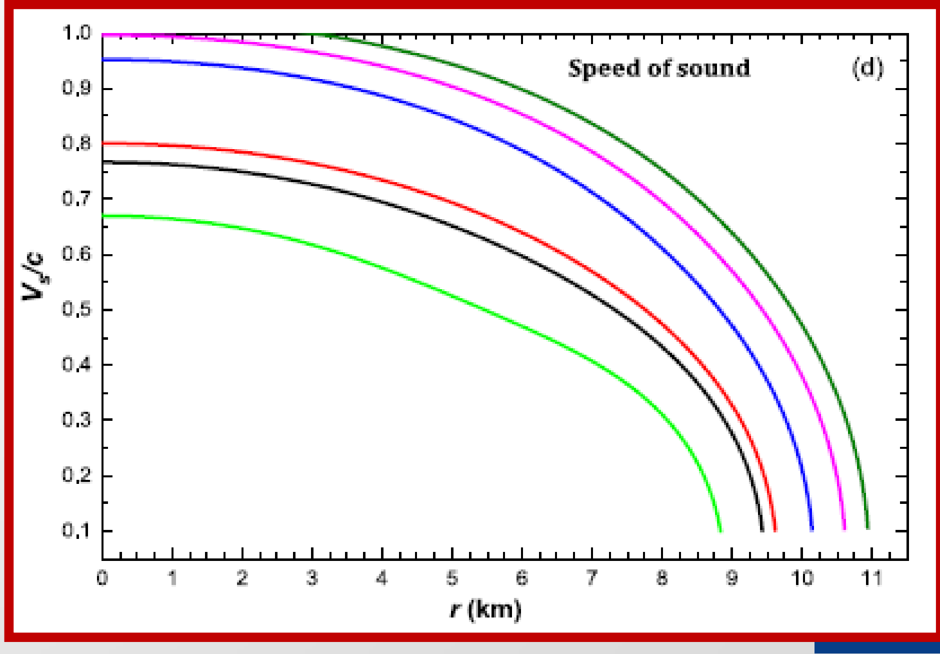
The CDM3Y-K EOSs (K_0 = 230–270 MeV) verify most of the investigated recent constraints on the masses and radii of NS, estimating radius of 11.67 ± 0.34 km for NS of 1.4$M_\odot$ mass, and M_{max}(NS) of 1.93 ± 0.21 $M_\odot$.

The **accumulated gravitational mass** and **speed of sound** inside NS (M_{max})

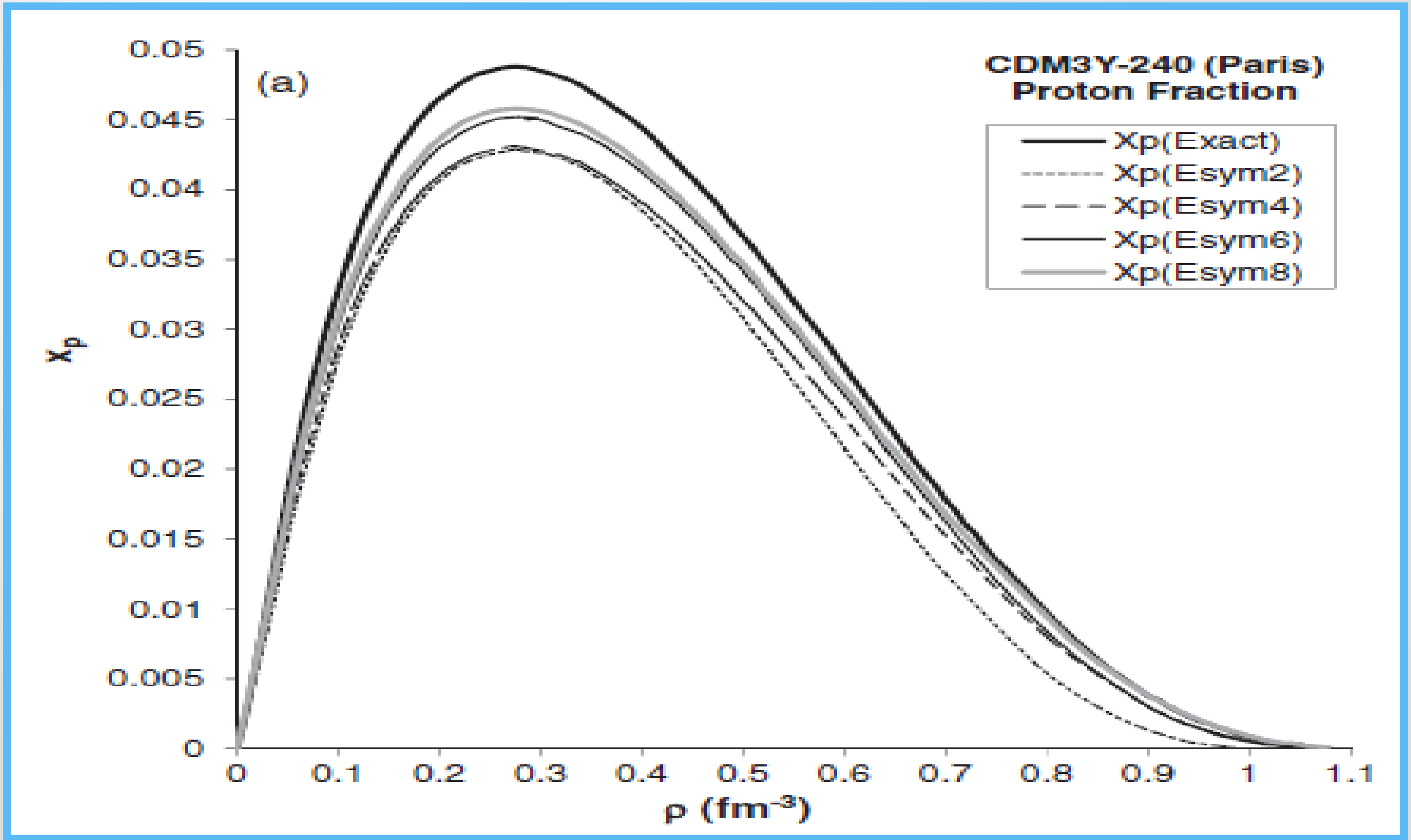


- ✓ **The softer EOS indicates less radius of vanishing pressure, which in turn indicates a less NS radius.**
- ✓ **The estimated maximum compactness (GM_{max}/Rc^2) of NS increases with increasing the stiffness of its NM.**

- ➢ **The central speed of sound approaches the speed of light for the EOSs of K_0 > 270 MeV.**
- ➢ **The density at which the speed of sound relative to c starts to break its conformal limit ($1/\sqrt{3}$) decreases with increasing the stiffness of the NM in NS.**



The symmetry-energy coefficients, up to its eighth order (E_{sym8}) contribute to the proton fraction in β-stable NS matter at different NM densities, and to the core-crust transition density and pressure.



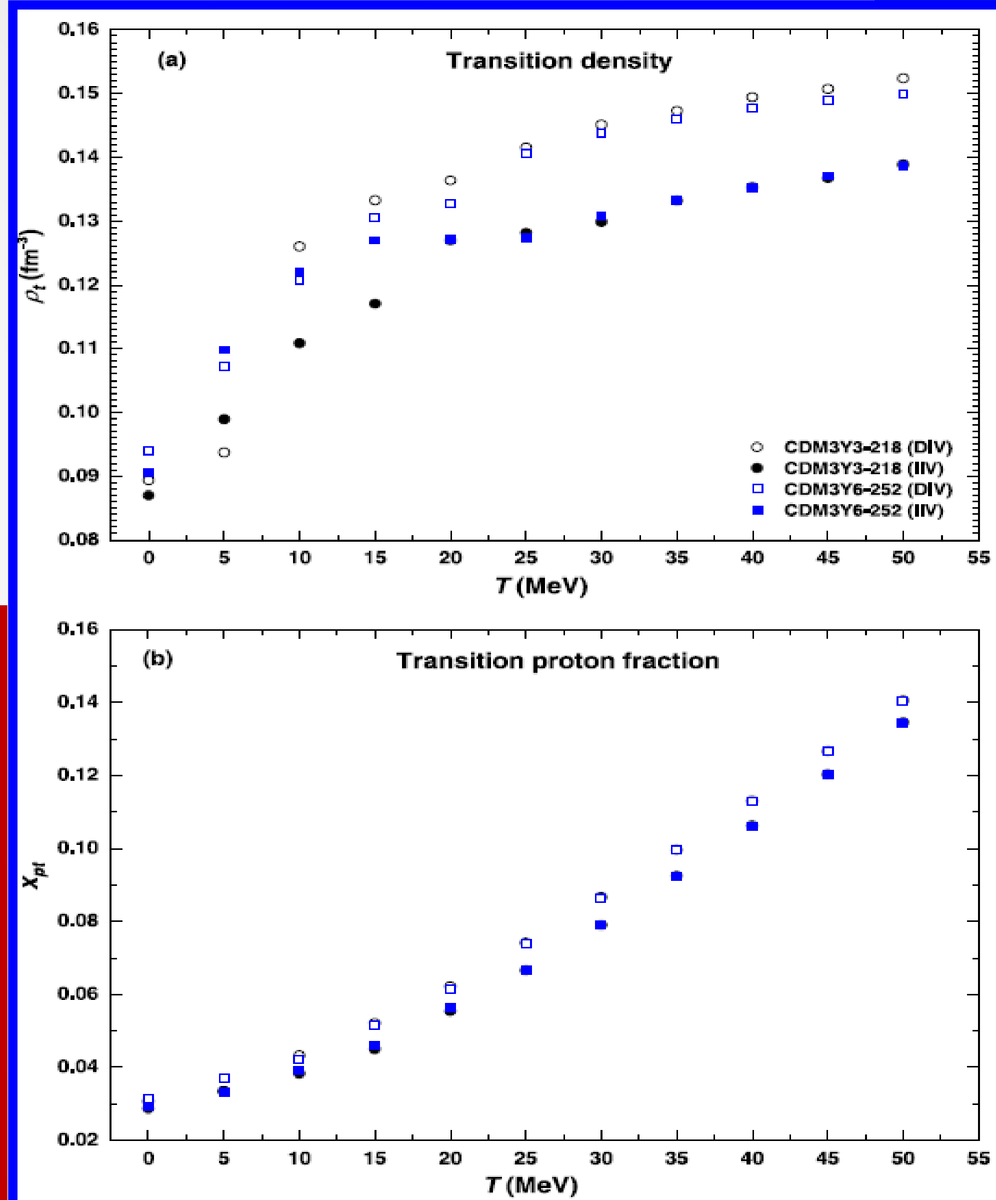
PRC 89, 028801 (2014)

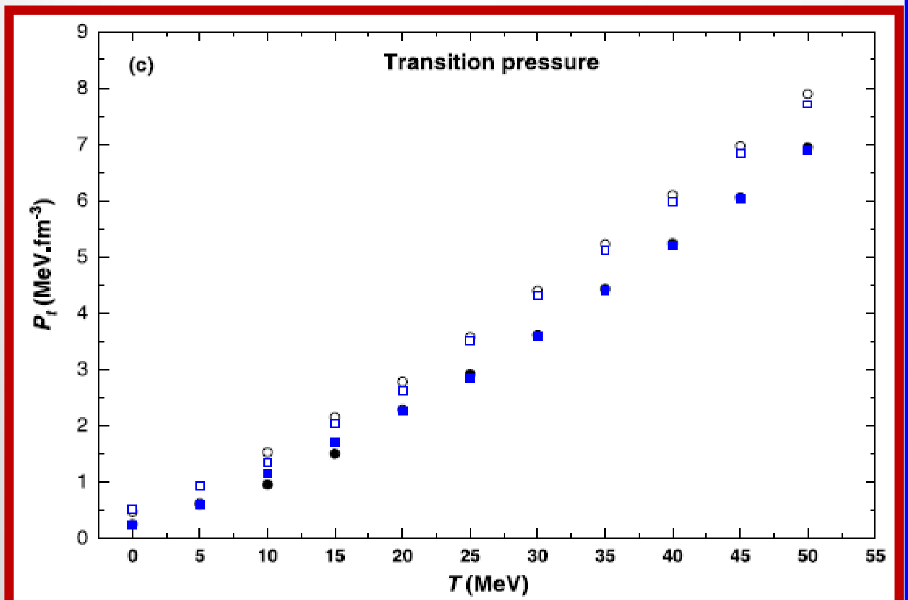
The liquid core – solid crust

- ✓ transition density,
- ✓ pressure, and
- ✓ proton fraction

PRC 106, 015801 (2022)

increase
with temperature.





➢Compactness

It measures how strongly spacetime is curved near the NS surface.

$$\mathcal{C} = \frac{\text{NS gravitational radius (Schwarzschild radius)}}{\text{actual } R\ (NS)} = \frac{R_g}{R} = \frac{G\ M}{c^2 R}.$$

✓**A more compact NS ⇒ a stronger gravitational field near its surface.**

✓**Extremely dense objects ⇒ $\mathcal{C}$=0.1 – 0.3.**

➢Tidal deformability

It quantifies how much NS is deformed by an external tidal gravitational field (such as that of its companion star in a binary system).

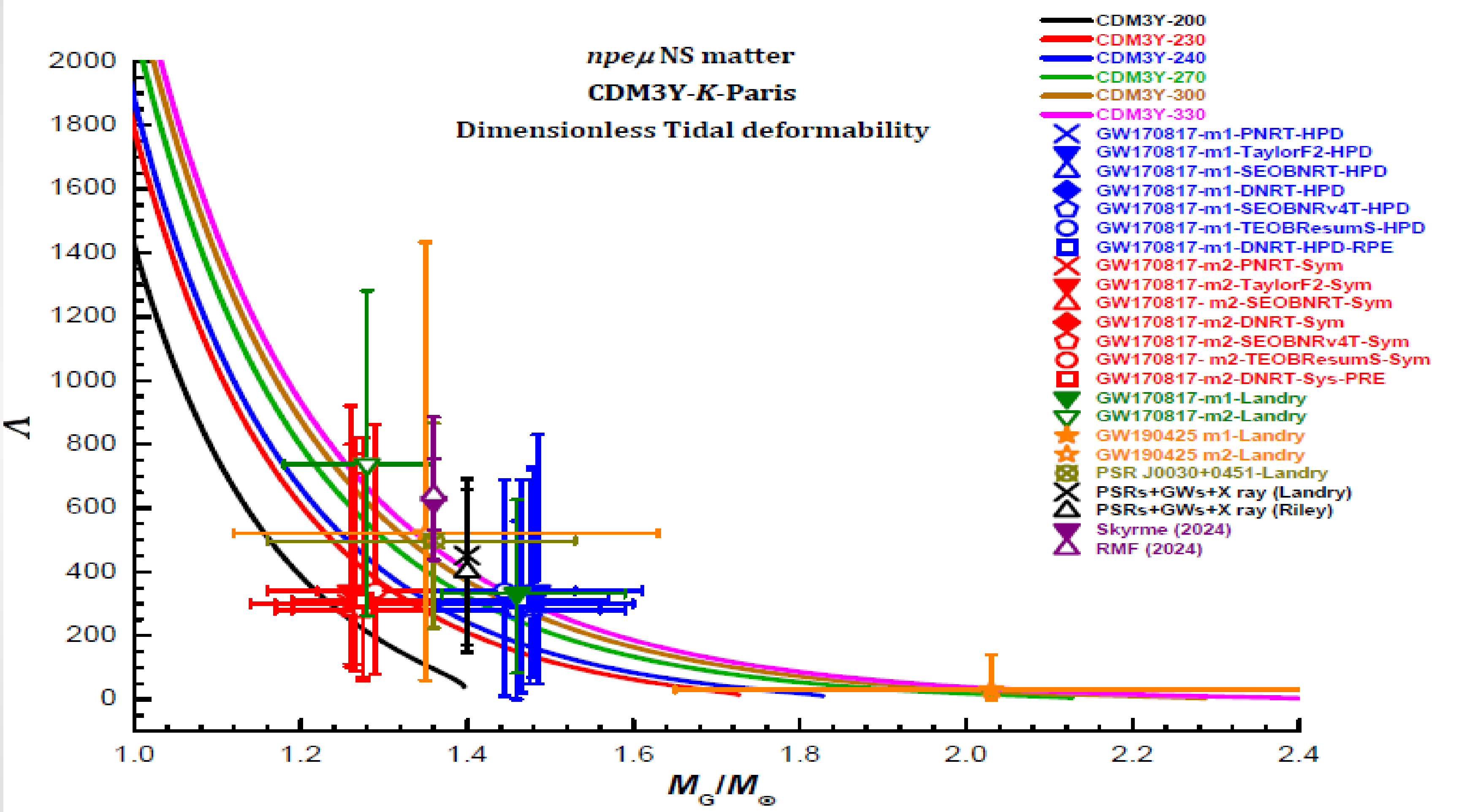
$$\Lambda = \frac{2}{3}\frac{\text{NS Love number } (k_2)}{(Compactness)^5} = \frac{2\ k_2}{3\ \mathcal{C}^5}.$$

✓ k_2 ≡ how the NS **internal structure** responds to **tidal force**. It is defined by the induced **quadrupole moment** (Q_{ij}) and the **external tidal field** E_{ij}, $\quad Q_{ij} = -k_2 R^5 E_{ij}/G$.

✓**A larger Λ means more easily deformed NS of less compactness.**

✓**It is measured in GW observations (like from binary NS mergers).**

Tidal deformability and compactness of neutron stars and massive pulsars



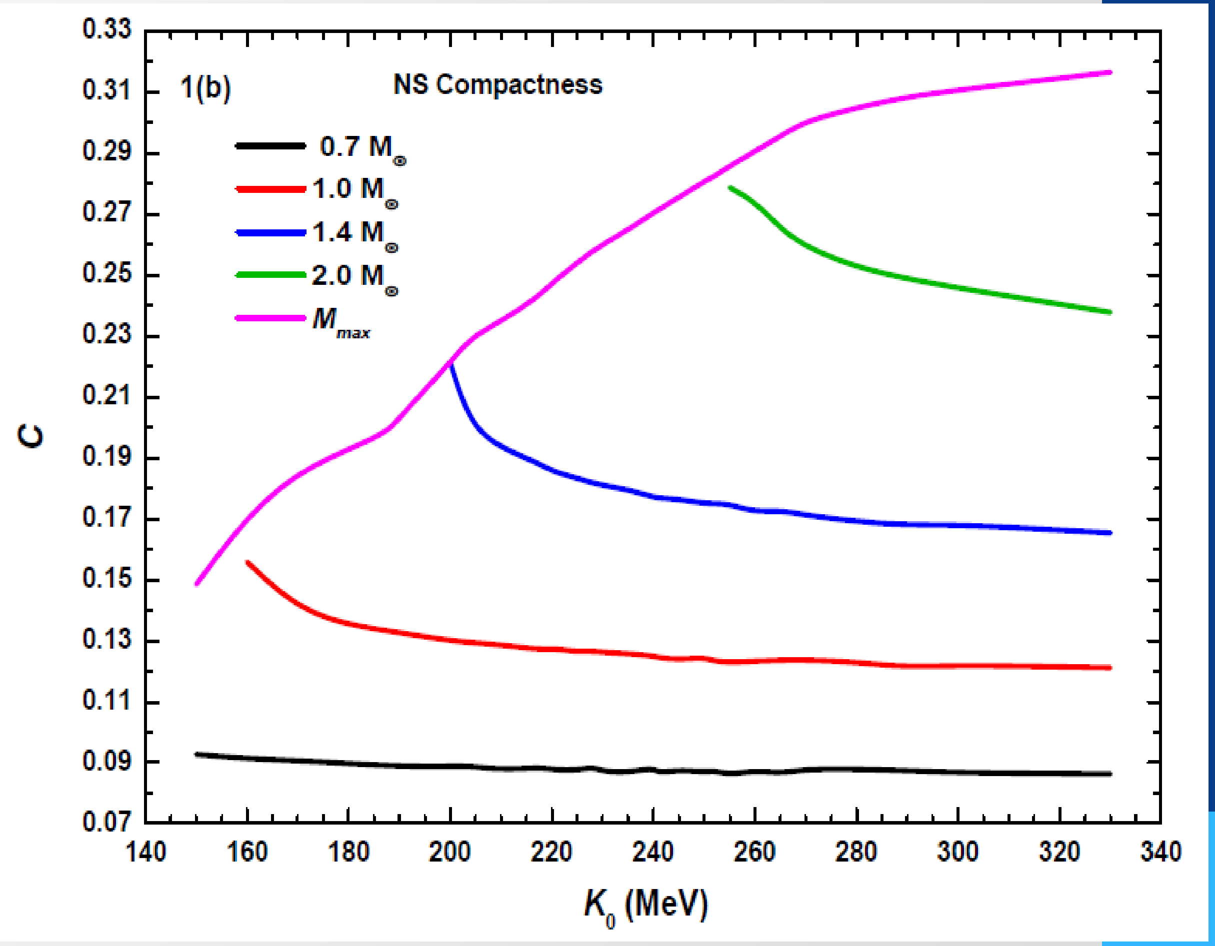
The EoSs from CDM3Y-230 to CDM3Y-330, together, yield more limited ranges of radii and tidal deformability for NS objects , than their empirically inferred ranges.

Compactness of NS

PRC 111, 035806 (2025)

The compactness brings further constraints on the mass–radius relation for NSs, determines their tidal deformability, and influences the propagation of their emissions.

- **The indicated compactness of light NS (< $M_{\odot}$) is almost constant with the incompressibility of the employed EOS.**
- **The sensitivity of the NS compactness increases with its mass, where stiffer NM anticipates less compactness.**

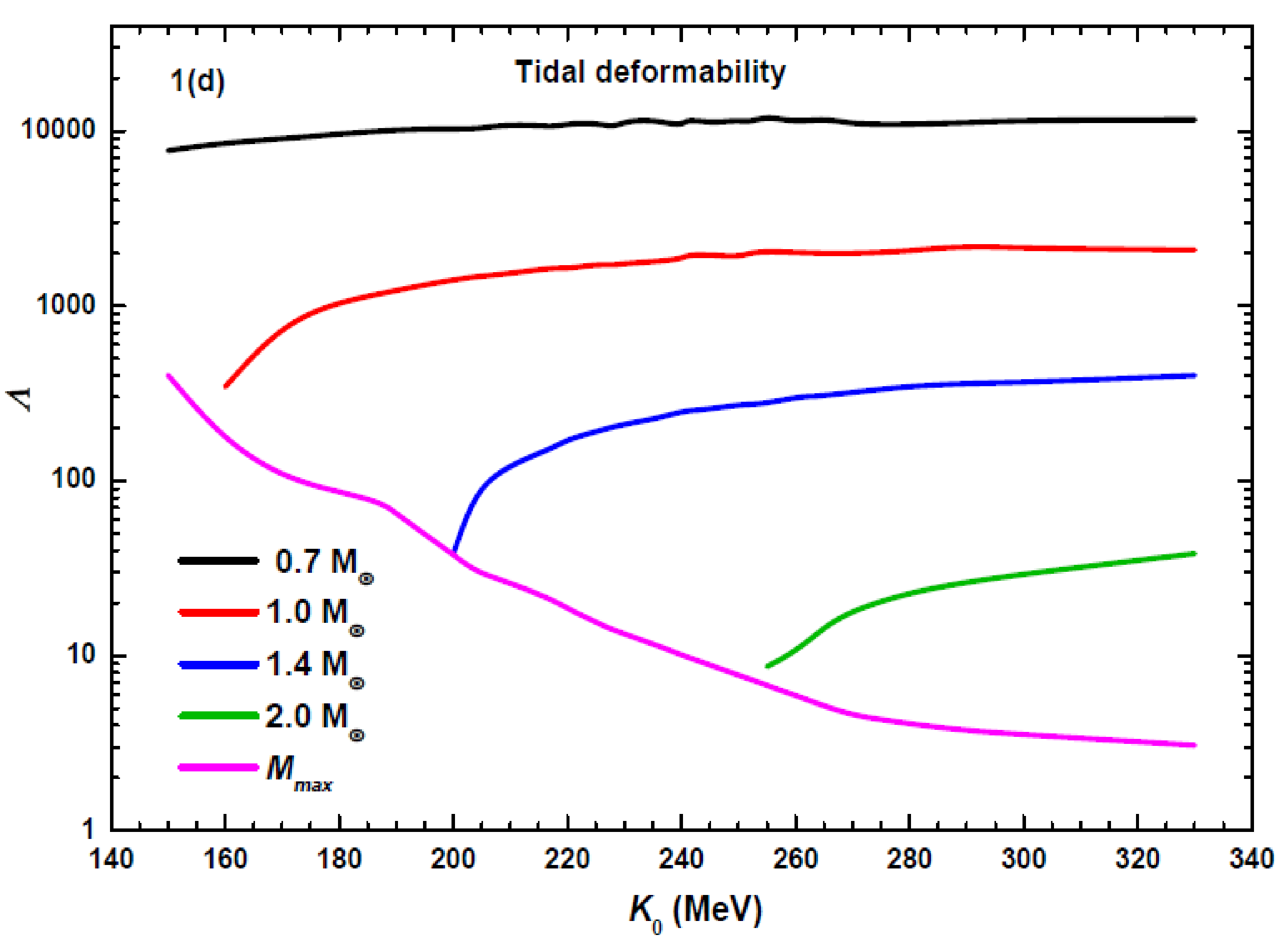


- **The largest compactness associated with the heaviest NS (M_{max}) for a given EOS increases with its incompressibility.**

Tidal Deformability

PRC 111, 035806 (2025)

- **The behavior of Λ with K_0 reverses the behaviour of C with it.**
- **$\Lambda(M<M_\odot)$ is almost independent of the stiffness of the EOS.**
- **Λ for the heavier NSs increases with K_0, keeping its order of magnitude unchanged.**



- **The order of magnitude of the indicated Λ decreases with increasing the NS mass.**
- **The minimum indicated tidal deformability reached for a NS of M_{max}, based on a certain EOS, decreases with K_0, spanning three orders of magnitude over the range from K_0=150 MeV to 330 MeV.**

Density curvature, skewness, and kurtosis of the nuclear symmetry energy and their impact on the compactness and tidal deformability of neutron stars

PRC 111, 035806 (2025)

$$E_A(\rho, I) = \sum_{n=0,2,4,6,\ldots} \left[E_{\mathrm{sym}n}(\rho_0) + L_n \mathbb{X} + \frac{K_n}{2!}\mathbb{X}^2 + \frac{Q_n}{3!}\mathbb{X}^3 + \frac{I_n}{4!}\mathbb{X}^4 + \frac{H_n}{5!}\mathbb{X}^5 + \frac{G_n}{6!}\mathbb{X}^6 + \cdots \right] I^n$$

$$\equiv E_A(\rho, 0) + E_{\mathrm{sym2}}(\rho) I^2 + E_{\mathrm{sym4}}(\rho) I^4 + E_{\mathrm{sym6}}(\rho) I^6 + \cdots ,$$

$$\boxed{\mathbb{X} = \left(\frac{\rho - \rho_0}{3\rho_0}\right)}$$

$$E_A(\rho, I=0) = E_A(\rho_0, I=0) + \frac{K_0}{2!}\mathbb{X}^2 + \frac{Q_0}{3!}\mathbb{X}^3 + \frac{I_0}{4!}\mathbb{X}^4 + \frac{H_0}{5!}\mathbb{X}^5 + \frac{G_0}{6!}\mathbb{X}^6 + \cdots ,$$

$$E_{\mathrm{sym2}}(\rho) \equiv \frac{1}{2!} \frac{\partial^2 E_A(\rho, I)}{\partial I^2}\bigg|_{I=0} = E_{\mathrm{sym}}(\rho_0) + L\mathbb{X} + \frac{K_2}{2!}\mathbb{X}^2 + \frac{Q_2}{3!}\mathbb{X}^3 + \frac{I_2}{4!}\mathbb{X}^4 + \frac{H_2}{5!}\mathbb{X}^5 + \frac{G_2}{6!}\mathbb{X}^6 + \cdots ,$$

$$E_{\mathrm{sym4}}(\rho) \equiv \frac{1}{4!} \frac{\partial^4 E_A(\rho, I)}{\partial I^4}\bigg|_{I=0} = E_{\mathrm{sym4}}(\rho_0) + L_4\mathbb{X} + \frac{K_4}{2!}\mathbb{X}^2 + \frac{Q_4}{3!}\mathbb{X}^3 + \frac{I_4}{4!}\mathbb{X}^4 + \frac{H_4}{5!}\mathbb{X}^5 + \frac{G_4}{6!}\mathbb{X}^6 + \cdots ,$$

$$L_n = 3\rho_0 \frac{dE_{\mathrm{sym}n}}{d\rho}\bigg|_{\rho_0}, \quad \text{with} \quad L = L_2, \quad K_{\mathrm{sym}n} = (3\rho_0)^2 \frac{d^2 E_{\mathrm{sym}n}}{d\rho^2}\bigg|_{\rho_0},$$

$$\text{with } K_0 = K_{\mathrm{sym}0}, \; Q_n = (3\rho_0)^3 \frac{d^3 E_{\mathrm{sym}n}}{d\rho^3}\bigg|_{\rho_0}, \; I_n = (3\rho_0)^4 \frac{d^4 E_{\mathrm{sym}n}}{d\rho^4}\bigg|_{\rho_0},$$

$$\boxed{K_{\mathrm{sym}} = K_{\mathrm{sym2}} = K_2 \text{ and } K_{\mathrm{sym}n} = K_n}$$

	NS mass			The causality condition	Threshold proton fraction for DU	SNM-PNM pressure, energy, and symmetry energy	Combined constraints of NM and NS	Other studies from nuclear structure and reaction, NM, and NS
	$M_\odot$	$1.4\,M_\odot$	$2\,M_\odot$					
K_0 (MeV)	$\geqslant 160$	$\geqslant 200$	$\geqslant 255$	$\leqslant 300$	$\leqslant 280$ [40]	$240 \leqslant K_0 \leqslant 250$ [41]	260 ± 20	$K_0^{\mathrm{exp/theo}} = 230 \pm 20$ [86]
								283 ± 32 [79]
								258^{+2}_{-24} [87]
								238 ± 22 [88]
K_2 (MeV)	$\leqslant -10$	$\leqslant -17$	$\leqslant -28$	$\geqslant -36$	$\geqslant -32$	$-27 \leqslant K_2 \leqslant -24$	-28 ± 4	$K_2^{\mathrm{exp/theo}} = -81 \pm 47$ [88]
								-128 ± 103 [92]
								-107 ± 88 [93]
								-50^{+90}_{-180} [23]
								-100 ± 100 [86]
								-120^{+80}_{-100} [87]
								$-200 \leqslant K_2^{\mathrm{theo}} \leqslant 50$ [81]
K_4 (MeV)	$\geqslant -1.03$	$\geqslant -0.84$	$\geqslant -0.60$	$\leqslant -0.38$	$\leqslant -0.48$	$-0.65 \leqslant K_4 \leqslant -0.62$	-0.57 ± 0.09	$-5.8 \leqslant K_4^{theo} \leqslant 10.77$ [97]

PRC 111, 035806 (2025)

	NS mass			The causality condition	Threshold proton fraction for DU	SNM-PNM pressure, energy, and symmetry energy	Combined constraints of NM and NS	Other studies from nuclear structure and reaction, NM, and NS
	$M_\odot$	$1.4\,M_\odot$	$2\,M_\odot$					
$K_{\tau 2}$ (MeV)	$\leqslant -43$	$\leqslant -139$	$\leqslant -242$	$\geqslant -307$	$\geqslant -281$	$-233 \leqslant K_{\tau 2} \leqslant -215$	-248 ± 33	-239 ± 56 [89]
								-370 ± 120 [97]
								$-517 \leqslant K_{\tau 2}^{\rm theo} \leqslant -220$ [94]
								$-742 \leqslant K_{\tau 2}^{\rm theo} \leqslant -228$ [96]
								-355 ± 30 [95]
								-400 ± 100 [86]
								-595 ± 245 [79]
								-550 ± 100 [80]
								-500^{+125}_{-100} [83]
$K_{\tau 4}$ (MeV)	$\geqslant -399$	$\geqslant -175$	$\geqslant -10$	$\leqslant 54$	$\leqslant 34$	$-46 \leqslant K_{\tau 4} \leqslant -21$	$-46 \leqslant K_{\tau 4} \leqslant 34$	$-32 \leqslant K_{\tau 4}^{\rm theo} \leqslant 347$ [97]
$K_{\tau 6}$ (MeV)	$\leqslant 478$	$\leqslant 74$	$\leqslant -42$	$\geqslant -47$	$\geqslant -46$	$-39 \leqslant K_{\tau 6} \leqslant -31$	-38 ± 6	

$$K_{0I} \equiv (3\rho_{0I})^2 \frac{d^2 E_A(\rho, I)}{d\rho^2}\bigg|_{\rho_{0I}} = K_0(\rho_0, I=0) + K_{\tau 2} I^2 + K_{\tau 4} I^4 + K_{\tau 6} I^6 + \cdots .$$

$$K_{\tau 2} = K_2 - 6L - \frac{L Q_0}{K_0}$$

$$K_{\tau 4} = K_4 - 6L_4 + \frac{9L^2 - Q_0 L_4 - Q_2 L}{K_0} + \frac{L^2 I_0 + 2Q_0 K_2 L + 6L^2 Q_0}{2K_0^2} - \frac{L^2 Q_0^2}{2K_0^3}$$

PRC 111, 035806 (2025)

Increasing K_0 and the fourth-order skewness coefficient Q_4, and decreasing the negativity of the isoscalar Q_0 and kurtosis H_0 and that of the isovector K_4, I_2, G_2 and $K_{\tau 4}$ coefficients

Increase the estimated NS radius
Tidal deformability
Tidal Love number of NS ($M \geq M_{\odot}$)
The central speed of sound (v_s) NS ($\leq 1.4\ M_{\odot}$)
The predicted M_{max}(NS) based on a given EOS
The corresponding maximum compactness
Surface redshift z_{surf} (M_{max}),

Indicate less compactness
z_{surf} of a given NS
less minimum tidal Love number
$k_{2,\mathrm{min}}(M_{max})$
Tidal deformability $\Lambda_{\mathrm{min}}(M_{max})$.

- **Increasing the I_0, G_0, Q_2, K_6, H_2 and $K_{\tau 6}$ (decreasing the negativity of the K_2 and $K_{\tau 2}$) coefficients <u>inverts</u> the behavior of the NS quantities with K_0 and with the aforementioned coefficients.**

Increasing the stiffness of ANM is found to

- **increase** the **core–crust transition density** and slightly, **the transition proton-fraction**,
- but to **decrease** the **abundance of the proton, muon, and electron over npeμ core matter** of NS, as well as the estimated **central density.**
- This in turn determines the possibility and **optimum conditions for** **direct URCA cooling process** in β-stable NS matter.

- **The EOS of the isospin asymmetric nuclear matter and its symmetry properties are still largely unknown.**

- **The results are model dependent !**

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Thank You