Physics of non-inertial frames (selected topics)

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Motivation

"Global Λ hyperon polarization in nuclear collisions"
STAR Collaboration, Nature 548, 62 (2017)

Quark-Gluon Plasma formed in nuclear collisions as a relativistic fluid at local thermodynamic equilibrium with acceleration and vorticity

Thus, in principle, describing observations in an accelerated frame belongs to heavy-ion physics

From pure theoretical viewpoint it is about an extention of the equivalence principle.

Indeed, according to the equivalence principle mechanical equation of motion in accelerated frame is the same as in external grav. field (Einstein's Elevator)

Plan of the talk

A. Extending “Einstein’s Elevator"to thermodynamics

B. Applications: Unruh effect without detectors .
A few words about lattice simulations

C. Gravitational Striction, or “Kronecker Anomaly”

Step back in time: Son-Surowka paper (2009)

Claim that chiral anomaly (quantum, or loop effect) has direct impact on hydrodynamics in external e-m field

Logic: hydrodynamics is built on conservation laws
Consider massless fermionic constituents. Naively axial current is conserved. There is an anomaly, however:

$$\partial_\alpha J_5^\alpha \;=\; e^2 C \epsilon^{\alpha\beta\gamma\delta} A_\beta \partial_\gamma A_\delta$$

where J_5^α is axial current,
A_α is e-m potential, e is electromagnetic coupling
Constant C depends on constituents (say, quarks)

Cnt'd

Utilize hydrodynamic "machinary"
(equation of motion..., positivity of entropy change)

Get a noval piece in the current:

$$J_5^\alpha = \mu^2 C \epsilon^{\alpha\beta\gamma\delta} u_\beta \partial_\gamma u_\delta$$

where u_α is 4-velocity of fluid, μ is chemical potential
Constant C is the same as in anomaly above

Generated macroscopic axial current (absent from Landau - Lifshitz) starting from quantum anomaly. Great excitement.

Alternative derivation of chiral effects

To find an equilibrium state, or state with maximum entropy, one introduces (fictitious) interaction:

$$\hat{H}_{eff} \;=\; \hat{H}_0 \;-\; \mu\hat{Q}$$

where $\hat{H}_0, \hat{Q}$ are Hamiltonian of the system and conserved charge, and μ is associated chemical potential

Crucial step: in language of Lagrangian density:

$$\delta L \;=\; \mu u_\alpha J^\alpha$$

where J^α is conserved (e-m) current, u_α is fluid 4-velocity

Substitute

$$eA_\alpha \;\to\; \mu u_\alpha \qquad (e_5 A^5_\alpha \;\to\; \mu_5 u_\alpha)$$

into anomaly and immediately get all chiral effects right

A few references

The substitution above was introduced by
Sadofyev, Shevchenko, VZ (2010)
and immediately put on heavy use by Dubna group
O. Rogachevsky, A. Sorin, O. Teryaev (2010)

It would be natural to generalize to gravitational case but this step turned very difficult and made in "Thermodynamic equilibrium with acceleration and the Unruh effect", F. Becattini, Phys.Rev.D 97 (2018) 8, 085013

$$\hat{H}_{eff} = \vec{\Omega} \cdot \hat{\vec{J}} + \vec{a}\hat{\vec{K}}$$

$\vec{\Omega}, \vec{a}$ are angular velocity and acceleration, $\hat{\vec{J}}, \hat{\vec{K}}$ are generators of rotations and of boosts.
To introduce this $\hat{H}_{eff}$ Becattini had to go beyond textbooks

Duality between statistics and field theory

$*$ Consider rotated and/or accelerated QGP in equilibrium. Statistically, effective, or macroscopic interaction

$$H_{eff} = \vec{\Omega} \cdot \vec{M} + \vec{a} \cdot \vec{K}$$

where $\vec{M}$ is angular momentm $\vec{K}$ is the boost
$**$ In field th. effect of rotation and acceleration can be described in terms of an external grav. field

$$\hat{H}_{fundamental} = \frac{1}{2} \hat{\Theta}^{\alpha\beta} h_{\alpha\beta}$$

where $\Theta^{\alpha\beta}$ is the energy momentum tensor, $h_{\alpha\beta}$ is the grav. potentials accomodating the same $\vec{\Omega}$, $\vec{a}$
$***$ Evaluate "external probes", $< \Theta^{\alpha\beta} >, < J_5^{\alpha} >$ for quantum particles. Results are expected to be the same. It

Further remarks on duality

Hydrodynamic perturbation theory does not exist beyond one loop

Moreover, one has to choose matter described by a conformal theory. As a result only very limited number of loops exists, all of them polynomial

The duality discussed was introduced in year 2020 and in fact is simplest (toy) example of duality between quasiclassical gravity in the bulk and conformal theory on the boundary

Conclusion to part A

Terms linear in acceleration are absorbed into Einstein's equivalence principle.

Higher powers of acceleration enter thermodynamical quantities both in accelerated frames and in external gravitational field - extended equivalence principle.

Unruh effect

Observer, accelerated with respect to the Minkowski vacuum, sees thermal bath of particles at temperature

$$T_{unruh} = \frac{a}{2\pi}$$

In the original paper, consideration of detectors is crucial

We consider a thermodynamic version of the effect: evaluate chracteristics of gas in accelerated frame without mentioning detectors ("medium as a detector")

We impose conditions: particles are described by a conformal theory (massless partciles) and energy is positive, i.e. medium is stable

Statistics-gravity duality at work

Evaluate energy density Θ_{00} of quantum massless spinors as function of independent a, T using the thermodynamical pertubative expansion

$$\rho_{vac} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2}$$

$$\rho_{vac}(T_{Unruh}) = 0$$

One-loop exact evaluation of the Unruh temperature since all higher order terms in a/T vanish because of use of conformal theory

Everything seems consistent,
although subtleties are not ruled out

A few words about lattice simulations

Results of lattice measurements are much more difficult to predict because the Hamiltonian of unaccelerated lattice, $\hat{H}_0$ is much more complicated than $\hat{H}_{non-interactinggas}$ we used

One thing seems safe to say: there are no linear in acceleration terms in thermodynamic quantities.

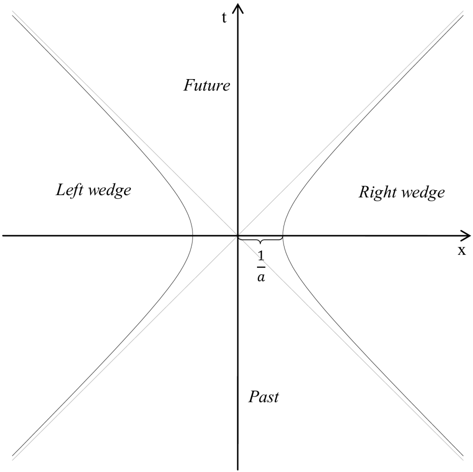
If one still gets linear terms, it seems better to average over $\pm a$.

Acceleration: Minkowski vs Euclid

If we describe observations of an observer at rest with respect to accelerated frame everything looks smooth. It is not so if we stay at a Minkowskian frame. Then measurements can be made in two regions ("wedges") which causally separated and physics might look very different. On the theoretical side, one should remember that we understand Minkowskian picture as analytical continuation of the Euclidean one.

Acceleration and temperature blow up to infinity near the origin of coordinates. Careful analysis of this region is crucial and apparently complicated. Goes back to "On the Duality Condition for Quantum Fields", J.J Bisognano , E.H. Wichmann (1976)

Rindler coordinates



Quantum vacuum state of non-inertial frames

Define energy-momentum tensor of Minkowski vacuum as vanishing: $< \theta_{\alpha\beta} >_{MInkowski} = 0$

Go to accelerated frame. This is a kind of gauge transformation in GR . Therefore,

$$< \theta_{\alpha\beta} >_{accelerated} >= 0$$

On the other hand, we are aware of the Unruh effect :

$$\epsilon_{Unruh} \sim T^4 \neq 0$$

Way out: kill it with negative energy of accelerated vacuum:

$$\epsilon_{Unruh} + \epsilon_{Casimir} = 0$$

In general, it is probably true that the Casimir energy is to be taken into account (fitted)

Kroneker Anomaly?? Conclusions

If we embed $\epsilon_{Casimir}$ into a cosmological constant, there is tension between $g_{ii} \sim (-1, -1, -1)$ for cosmological constant and $g_{ii} \sim (-1/3, -1/3, -1/3)$ for Unruh gas.

Similar to Kroneker Anomaly??
"Kronecker anomalies and gravitational striction" Alexander M. Polyakov , Fedor K. Popov e-Print: 2203.07101 [hep-th]

Conclusions (if time permits): comparison with literature