# INDIVIDUAL TOROIDAL DIPOLE STATES IN $^{58}$Ni

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# Introduction

- Recently, the low-lying E1 individual toroidal states (ITS) were predicted within QRPA in strongly deformed $^{24}$Mg.

V. O. Nesterenko, A. Repko, J. Kvasil, and P.-G. Reinhard, Phys. Rev. C 120, 182501 (2018).

In (e,e’) DALINAC experiments states in spherical $^{58}$Ni with enhanced transversal form-factors were observed. M1 states?

W. Mettner, A. Richter, W. Stock, B. C. Metsch, and A.G.M. Van Hees, Nucl. Phys. A473, 160 (1987)

$(\gamma, \gamma')$ experiments:

F. Bauwens, et al., PRC 62, (2000)   T. Shizuma, et al., PRC 109, (2024)

(p,p’) experiment:

I. Brandherm, P. von Neumann-Cosel, PRC 110, (2024)

We analyze low-energy states in spherical $^{58}$Ni. The results of the work have been recently published in PRL 133, 232502 (2024).

PHYSICAL REVIEW LETTERS **133**, 232502 (2024)

Editors' Suggestion   Featured in Physics

**Candidate Toroidal Electric Dipole Mode in the Spherical Nucleus $^{58}$Ni**

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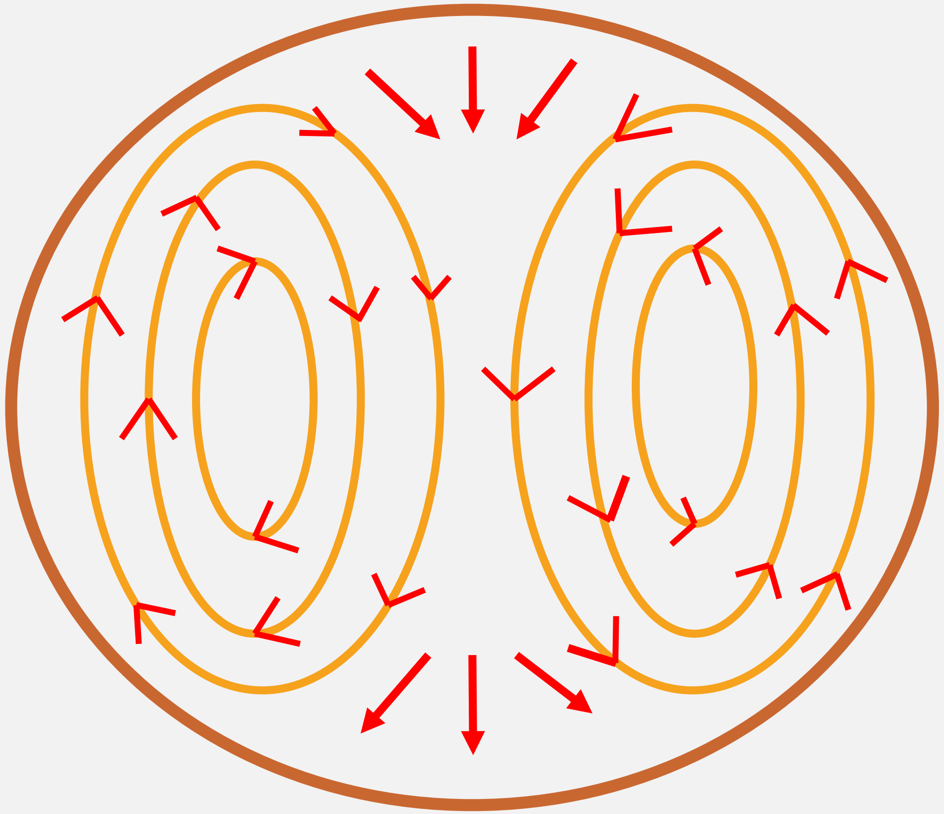
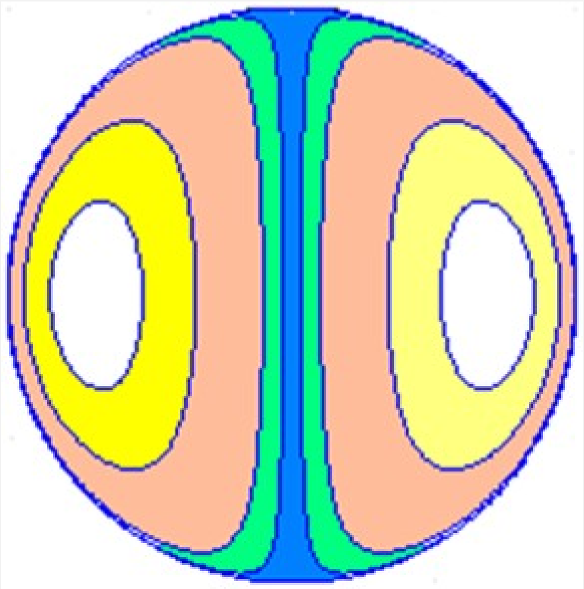
**toroidal currents**



V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).
S.F. Semenko, SJNP 34 356 (1981).

Hill vortex ring



Hill showed that the incompressible Euler equations has a steady solution as a spherical vortex ring. The spherical vortex ring propagates without change of either velocity or shape.

M. J. M. Hill, PTRS 185, 213 (1894).
PhD thesis, M. M. Scase (2003)

# Model

- The Skyrme QRPA code (Repko) for spherical nuclei.

A. Repko, J. Kvasil, V. O. Nesterenko, and P.-G. Reinhard, arXiv:1510.01248[nucl-th].

- Fully self-consistent QRPA (mean field and residual interaction are derived from the initial Skyrme functional, p-p and p-h channels, residual interaction takes into account all terms from the initial functional).
- Single-particle basis includes 190 proton and 226 neutron levels.
- For example, 2qp basis includes 143 proton and 249 neutron pairs for SV-bas.
- Spurious admixture are removed.

A. Repko, J. Kvasil, and V. O. Nesterenko, PRC 99, 044307 (2019).

- 5 Skyrme forces (SV-mas10, SV-bas, SV-mas08, SkM$^*$, SLy6)
- Volume (SkM$^*$ and SLy6) and surface (SV-mas10, SV-bas, SV-mas08) pairings.

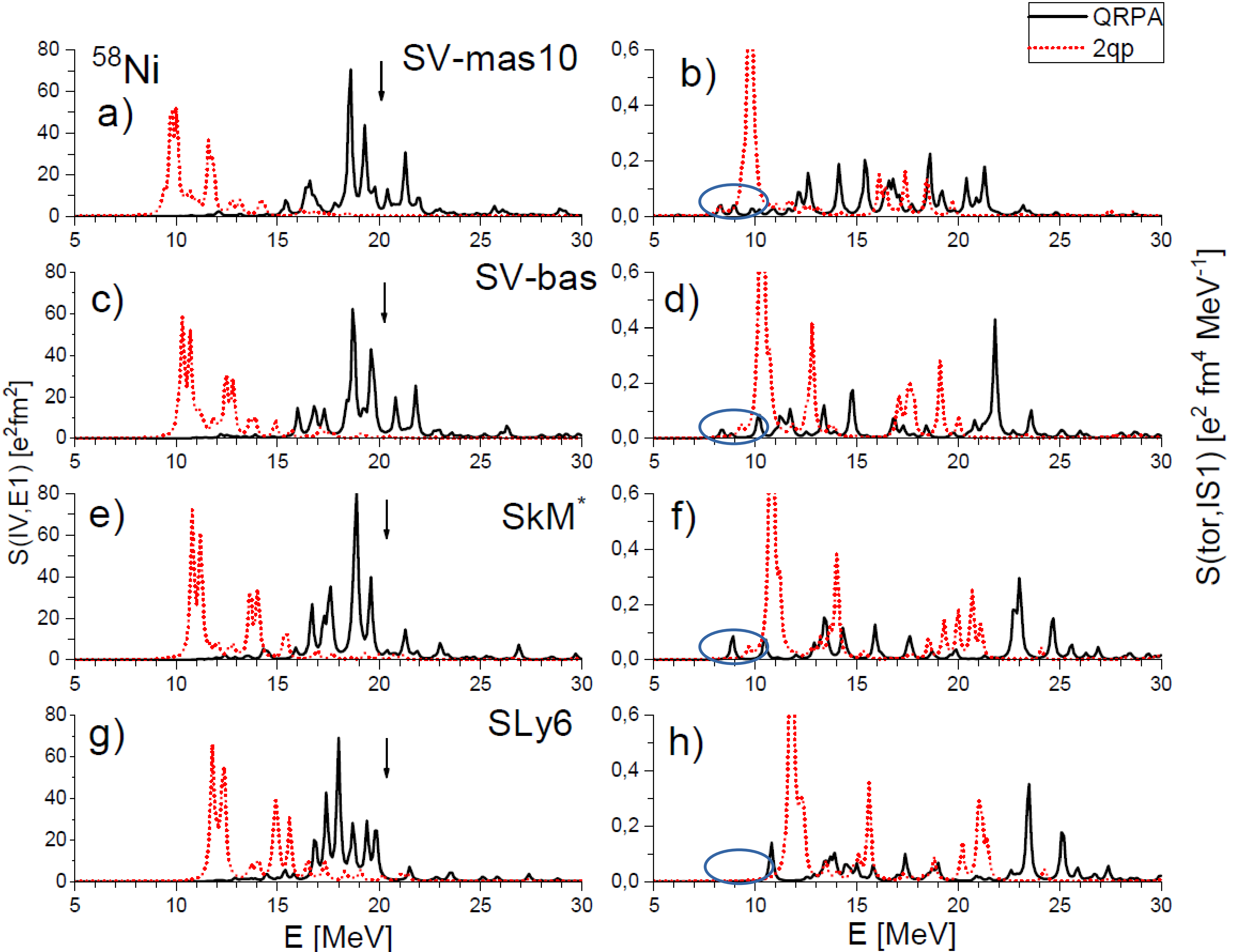
A. Repko, J. Kvasil, V. O. Nesterenko, and P.-G. Reinhard, EPJA 53, 221 (2017).

$$V^q_{\text{pair}}(\mathbf{r},\mathbf{r}') = G_q\left[1-\eta\left(\frac{\rho(\mathbf{r})}{\rho_{\text{pair}}}\right)\right]\delta(\mathbf{r}-\mathbf{r}')$$

| | $e_p$ | $e_n$ |
|---|---|---|
| IV | 0.517 | -0.483 |
| IS | 1 | 1 |

| | SV-mas10 | SV-bas | SV-mas08 | SkM$^*$ | SLy6 |
|---|---|---|---|---|---|
| m/m$^*$ | 1 | 0.9 | 0.8 | 0.79 | 0.69 |

# IV and tor IS1 strength functions



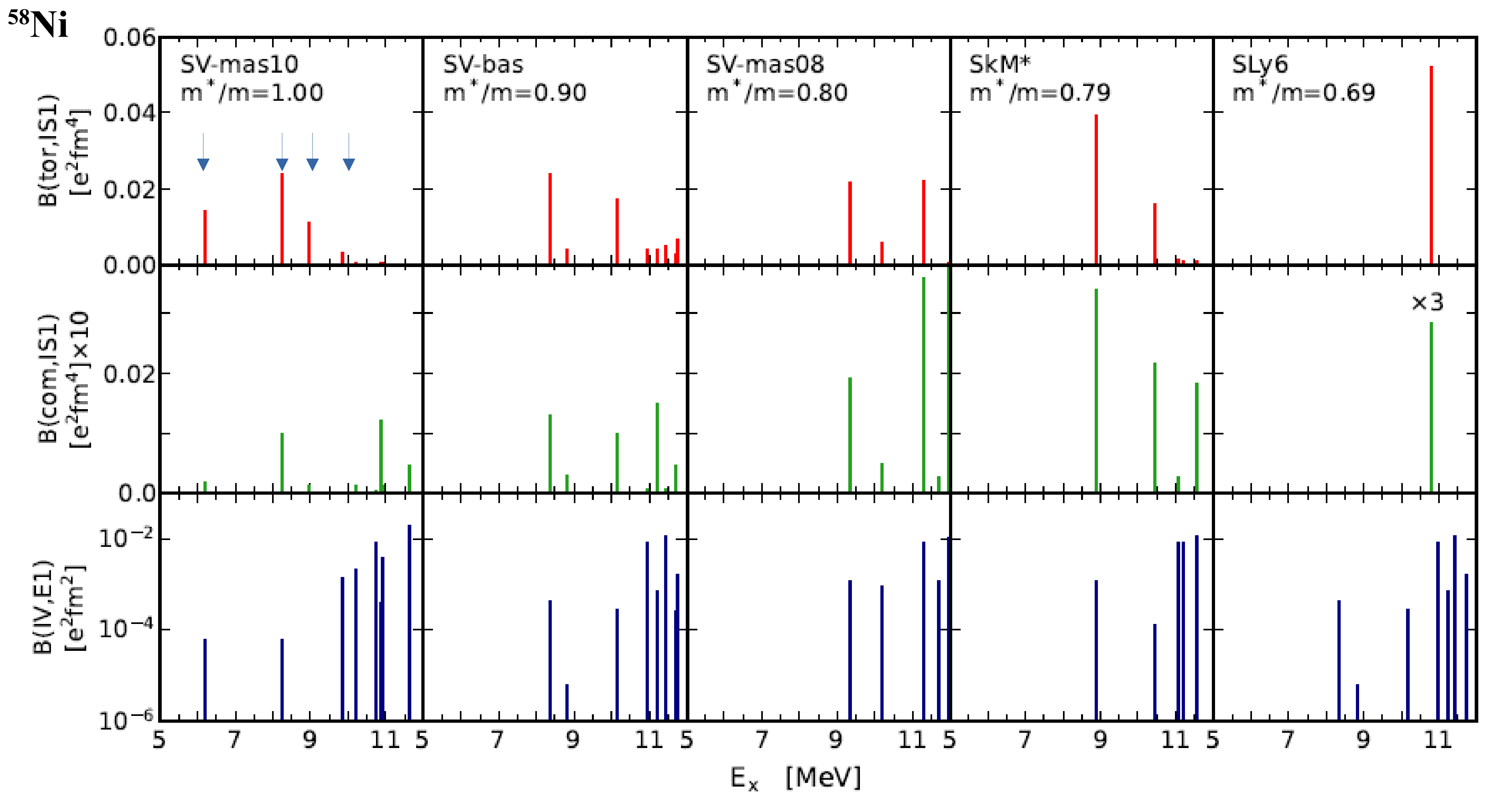

for IV:

- 2qp strength is located at 10-15 MeV;
- QRPA calculations correspond to IV GDR (16-22 MeV)

for tor IS1:

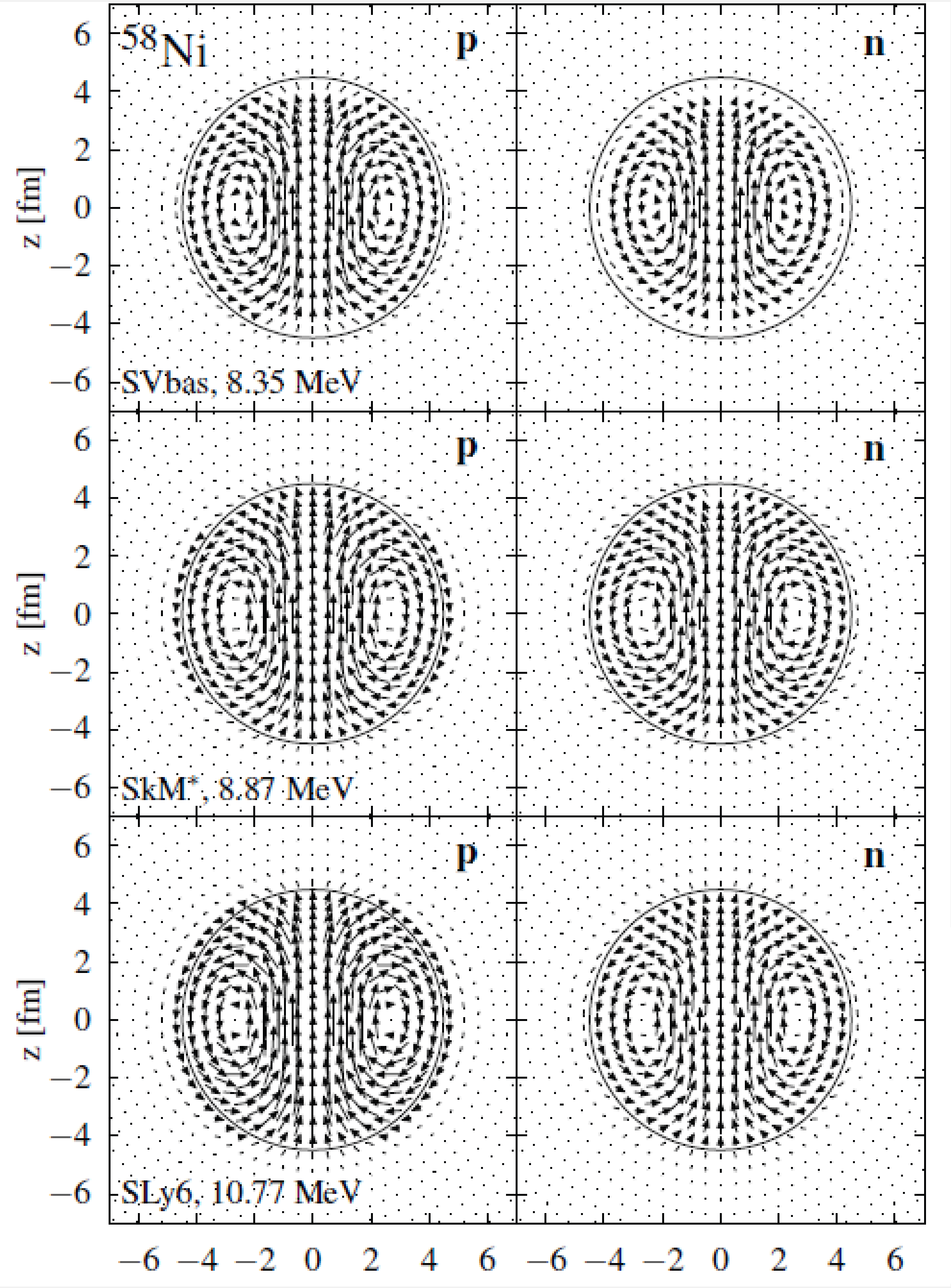
- two 2qp energy range;
- QRPA states at 6-11 MeV (our energy range!)

# tor IS1, com IS1, IV E1 strength

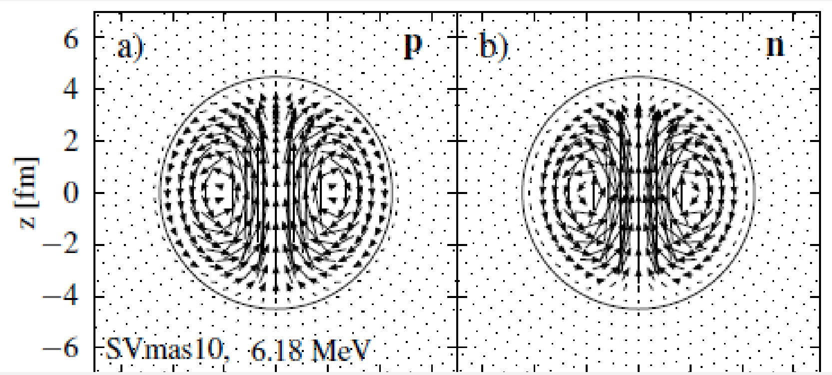



- The IS QRPA states have a toroidal character;
- Strength distribution depends on the effective mass.
- Good agreement for energies with SV-mas10

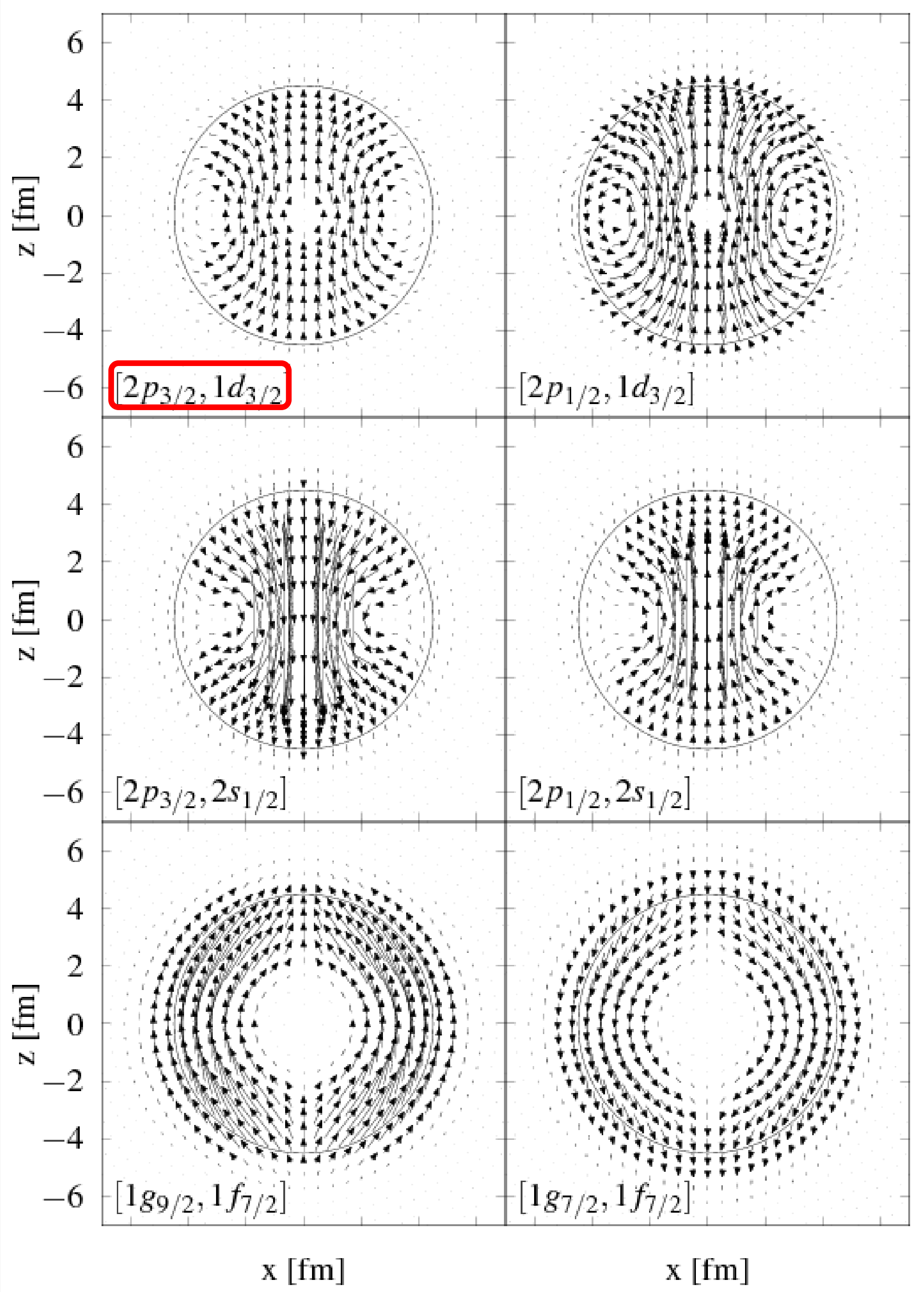
# Currents: lowest dipole states




| Force | E [MeV] | main 2qp components | % |
|---|---|---|---|
| SV-mas10 | 6.18 | pp $[2p_{3/2},1d_{3/2}]$<br>pp $[2p_{1/2},1s_{1/2}]$ | 42<br>11 |
| SV-bas | 8.35 | pp $[2p_{3/2},1d_{3/2}]$<br>nn $[2p_{3/2},1d_{3/2}]$ | 64<br>9 |
| SkM* | 8.87 | pp $[2p_{3/2},1d_{3/2}]$<br>pp $[1g_{9/2},1f_{7/2}]$ | 56<br>10 |
| SLy6 | 10.78 | pp $[2p_{3/2},1d_{3/2}]$<br>pp $[2p_{3/2},2s1_{/2}]$ | 29<br>16 |



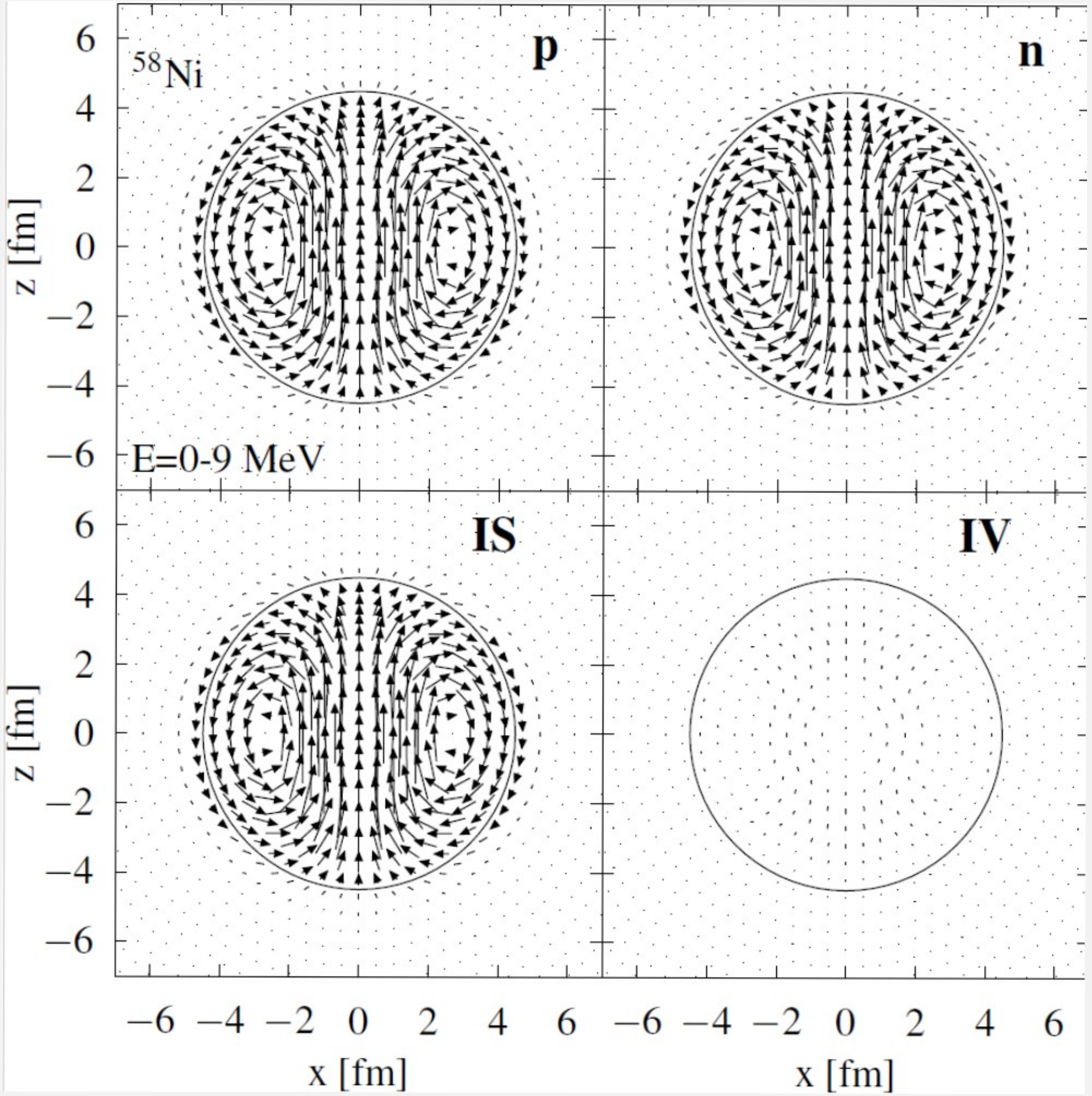

- the lowest E1 IS states have a toroidal character;
- the same basic 2qp states for all QRPA states

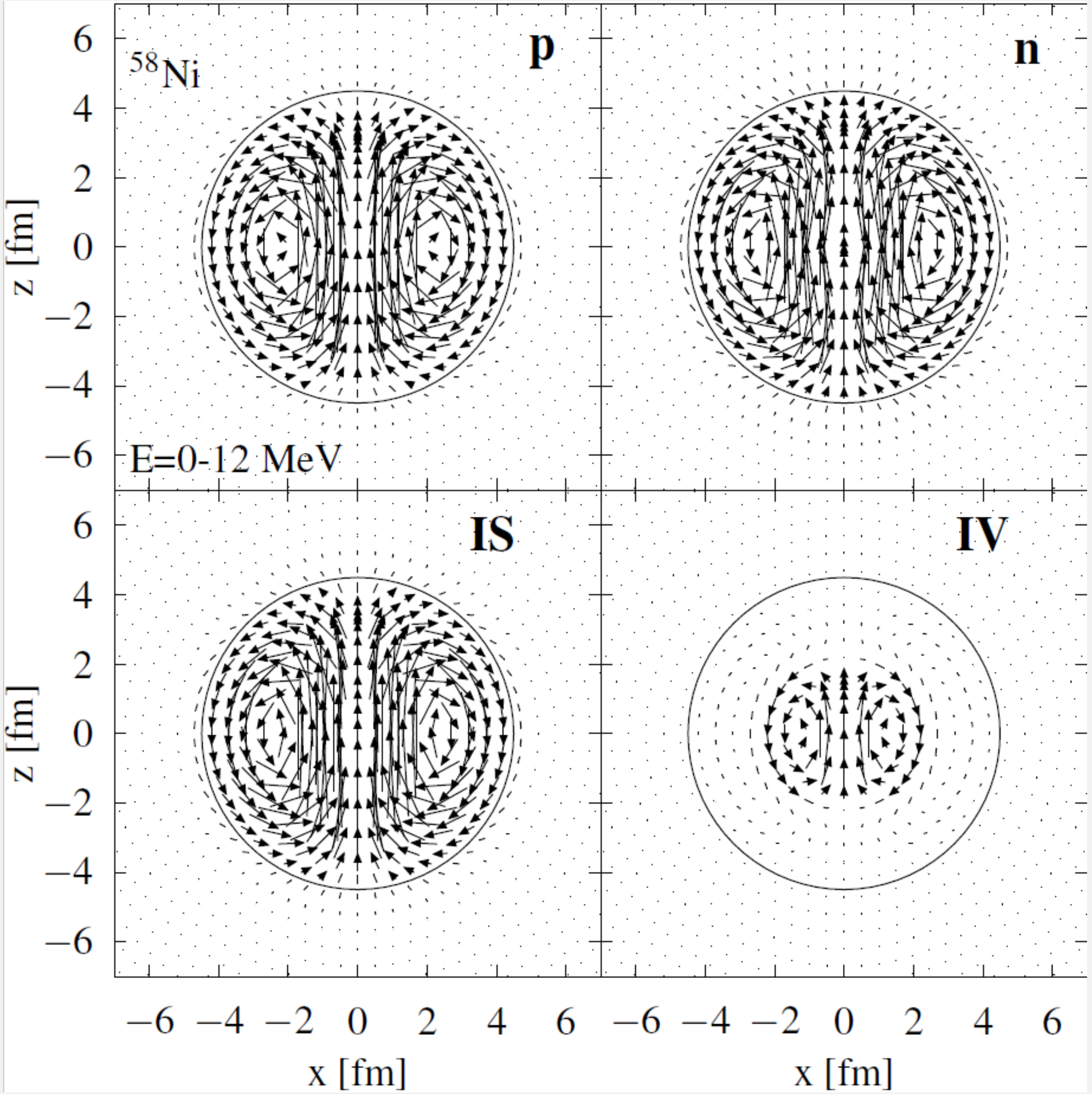



# Currents: 2qp proton configurations

- The currents of these 2qp configurations very remind the typical toroidal flow
- the vorticity of the 2qp states leads to the vorticity of the QRPA states.
- vortical flow has a mean-field origin

# SV-mas10: averaged currents



## Formalism

PWBA:

$$\frac{d\sigma}{d\Omega}(\theta, q_{\rm eff}, E_i) = 4\pi\sigma_{\rm Mott}(\theta, E_i) f_{\rm rec}(\theta, E_i) \times \left[ \left|F^C_{E\lambda}(q_{\rm eff})\right|^2 + \left(\frac{1}{2} + \tan^2(\frac{\theta}{2})\right) \left|F^T_{E\lambda}(q_{\rm eff})\right|^2 \right]$$

where:

$$\sigma_{\rm Mott}(\theta, E_i) = \left[\frac{e^2 Z}{8\pi E_i} \frac{\cos(\frac{\theta}{2})}{\sin^2(\frac{\theta}{2})}\right]^2$$

In our calculations:

$f_{\rm rec}=1$

Form factors:

$$F^C_{E\lambda}(q) = \sqrt{2\lambda+1}\int_0^\infty drr^2 \delta\rho^\nu_\lambda(r) j_\lambda(qr)$$

$$F^T_{E\lambda}(q) = \frac{1}{c}\int_0^\infty drr^2 [\sqrt{\lambda+1}\,\delta J^\nu_{\lambda,\lambda-1}(r) j_{\lambda-1}(qr) - \sqrt{\lambda}\,\delta J^\nu_{\lambda,\lambda+1}(r) j_{\lambda+1}(qr)]$$

where: $\delta\rho^\nu_k(\vec{r}) = \langle\nu|\hat{\rho}_k|0\rangle(\vec{r})$ is the transition density,

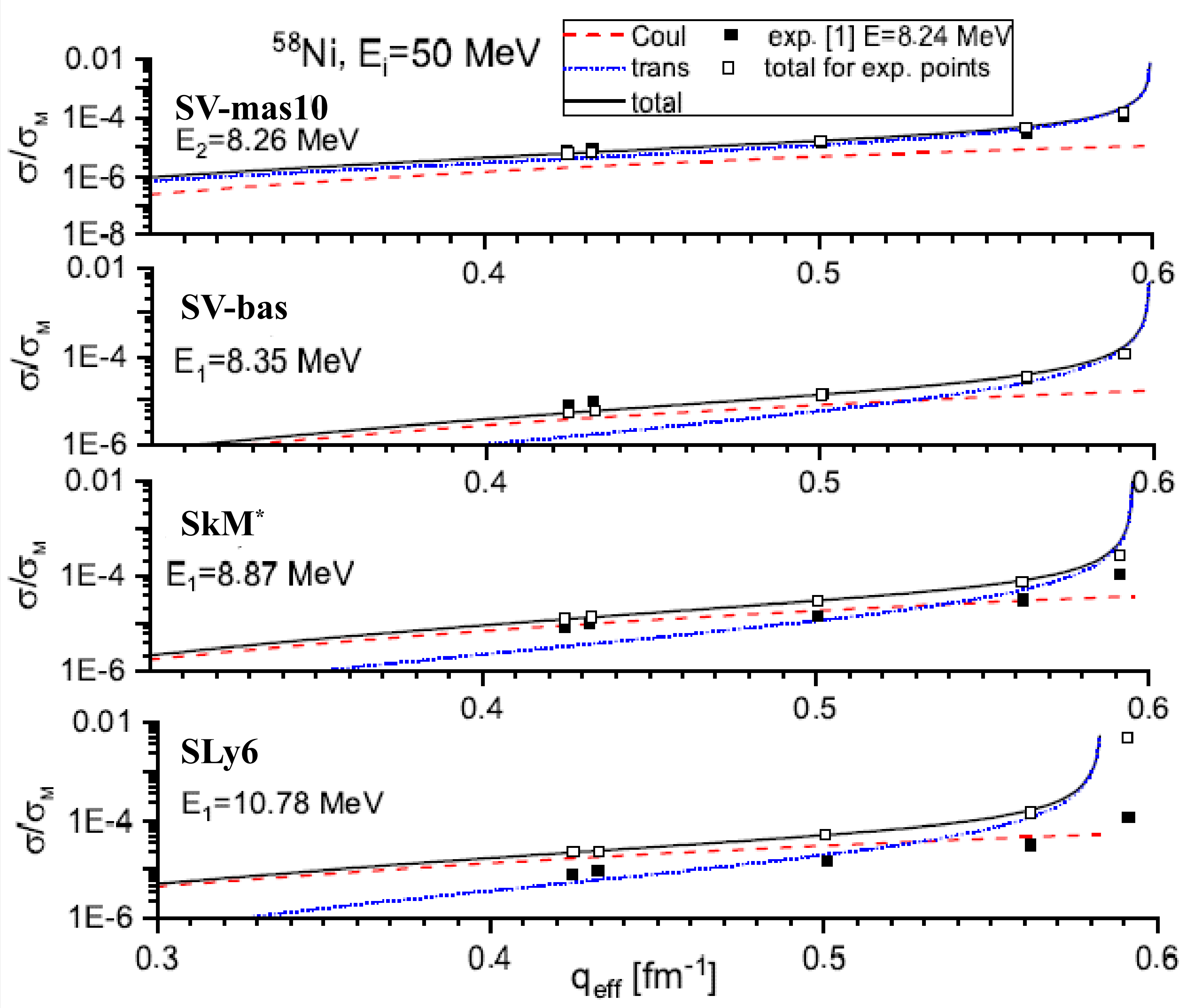
$\delta J^\nu_{\lambda,\lambda\pm1}(r)$ are radial components of CTD

Transfer momentum:

$$q = \frac{2}{\hbar c}\sqrt{E_i E_f}\sin\left(\frac{\theta}{2}\right)$$

Here, $E_f = E_i - E_\nu$ is the final electron energy, $E_\nu$ is the nuclear excitation energy

The effective transfer momentum: $q_{\rm eff} = q\left(1 + 1.5\frac{Z\alpha\hbar c}{E_i R}\right)$

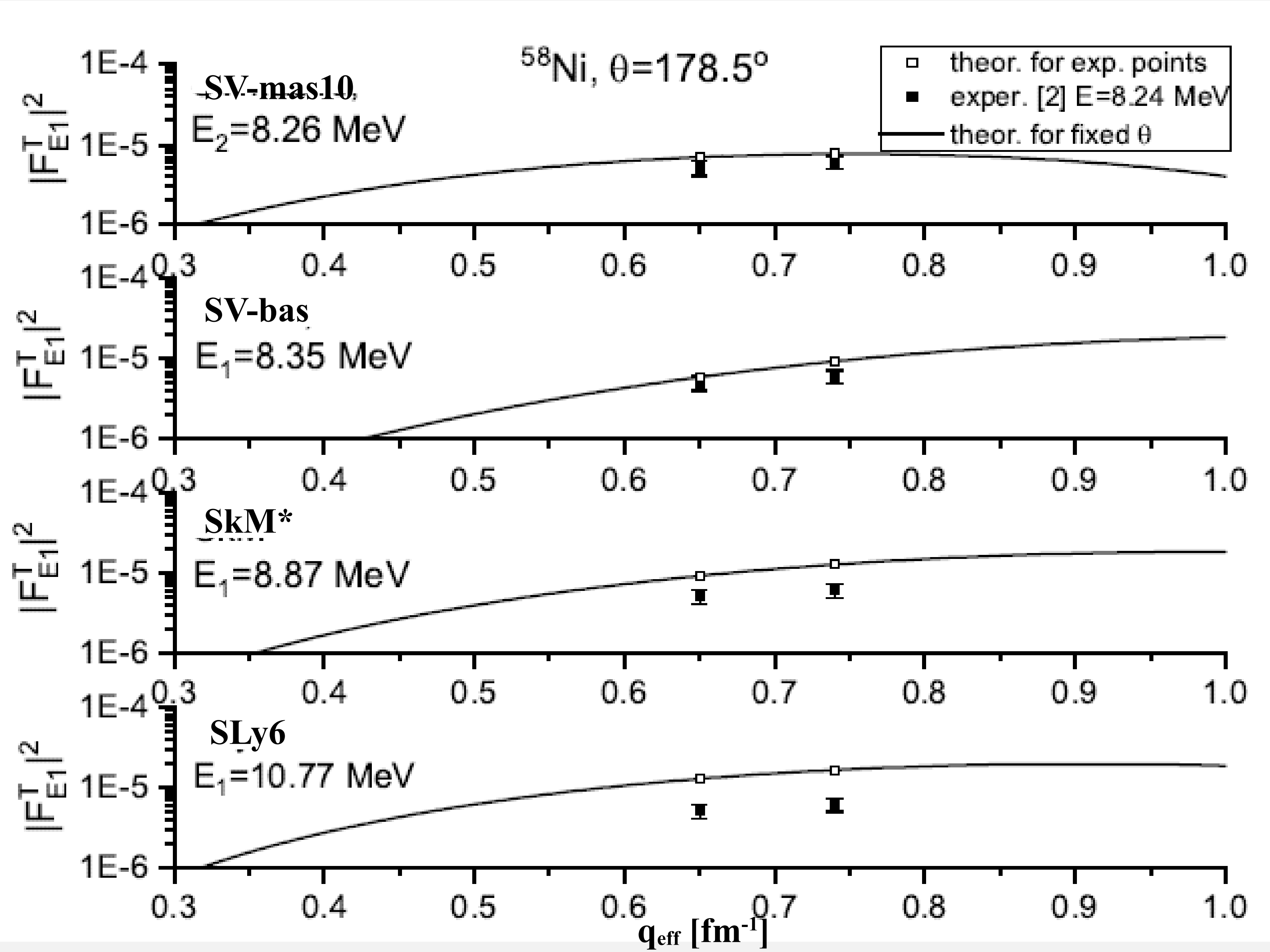



# Cross-sections

| E [MeV] | θ [deg] | q_eff [fm$^{-1}$] | $\sigma/\sigma_M$ |
|---|---|---|---|
| 49.4 | 92.9 | 0.4358 | 7.943E-06 |
| 50.4 | 92.9 | 0.4359 | 9.525E-06 |
| 49.9 | 116.9 | 0.5111 | 1.434E-05 |
| 50.4 | 140.9 | 0.5647 | 3.128E-05 |
| 50.4 | 164.9 | 0.5937 | 1.124E-04 |

W. Mettner, A. Richter et al, NPA 473, 160 (1987).

- QRPA well describes experimental data.
- The best agreement with experiment for SV-bas and SV-mas10 forces.



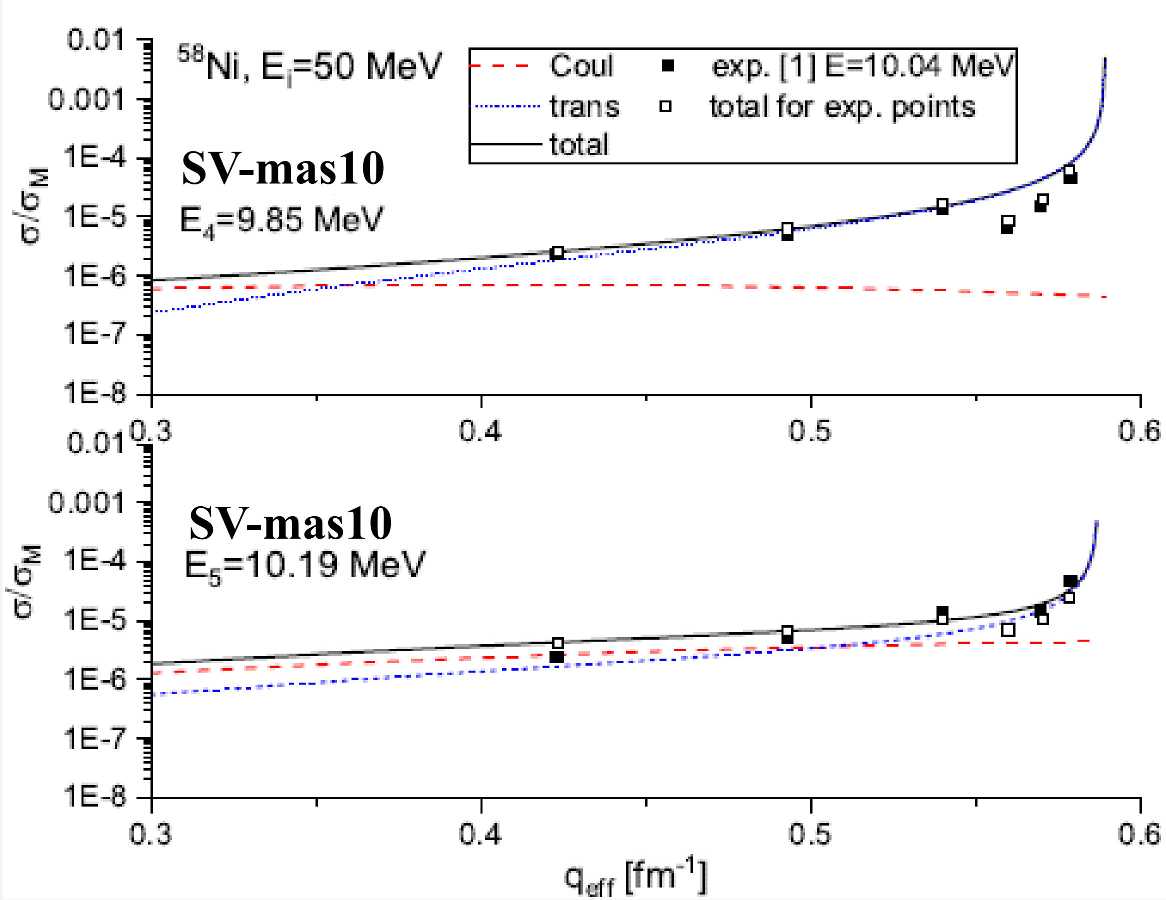

B. Reitz, (1986) private communication

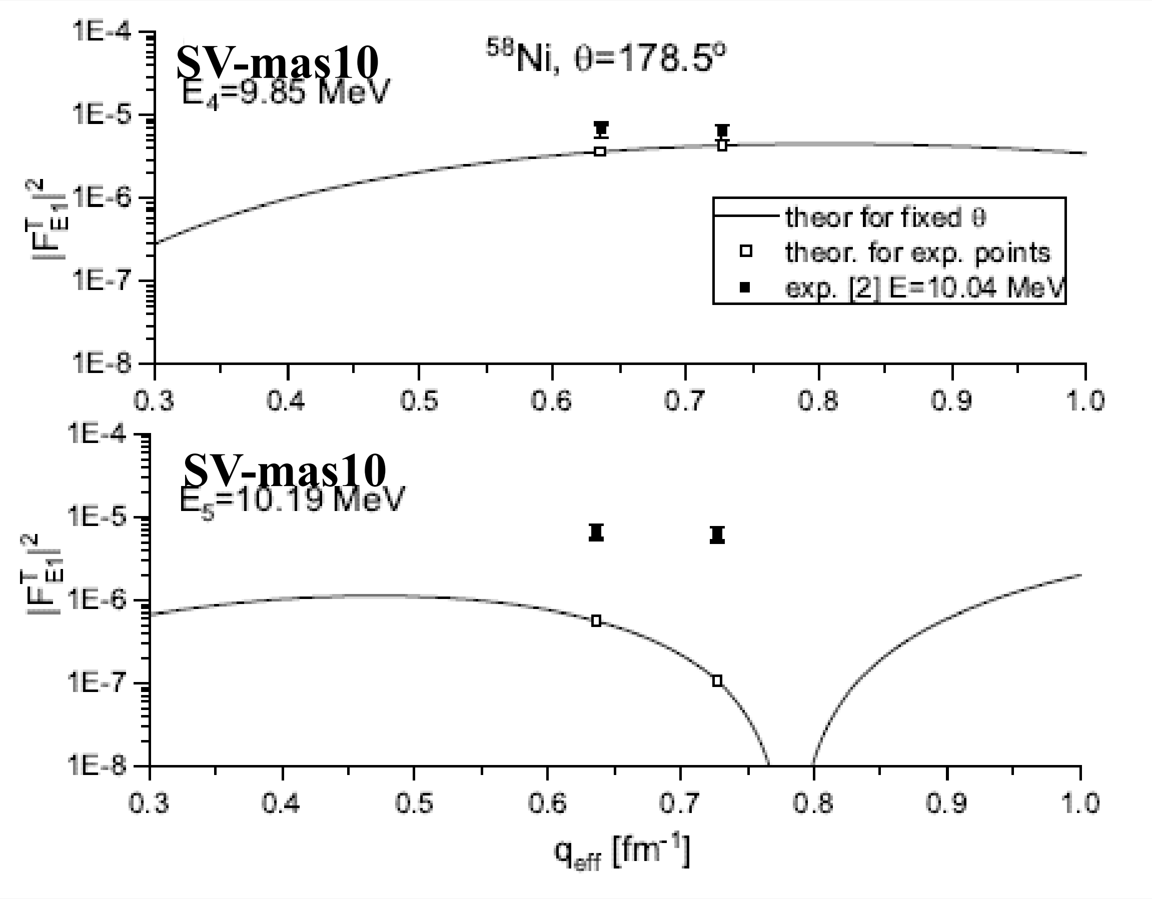
# θ=178.5

| E | θ | q_eff | $\lvert F_T \rvert^2$ |
|---|---|---|---|
| [MeV] | [deg] | [fm$^{-1}$] | |
| 56.6 | 178.5 | 0.65 | 5.13E-06 |
| 65.4 | 178.5 | 0.74 | 6.1E-06 |

- QRPA well describes the experimental data of squared form factors.
- The best agreement for the SV-mas10 force.

# 10.04 MeV



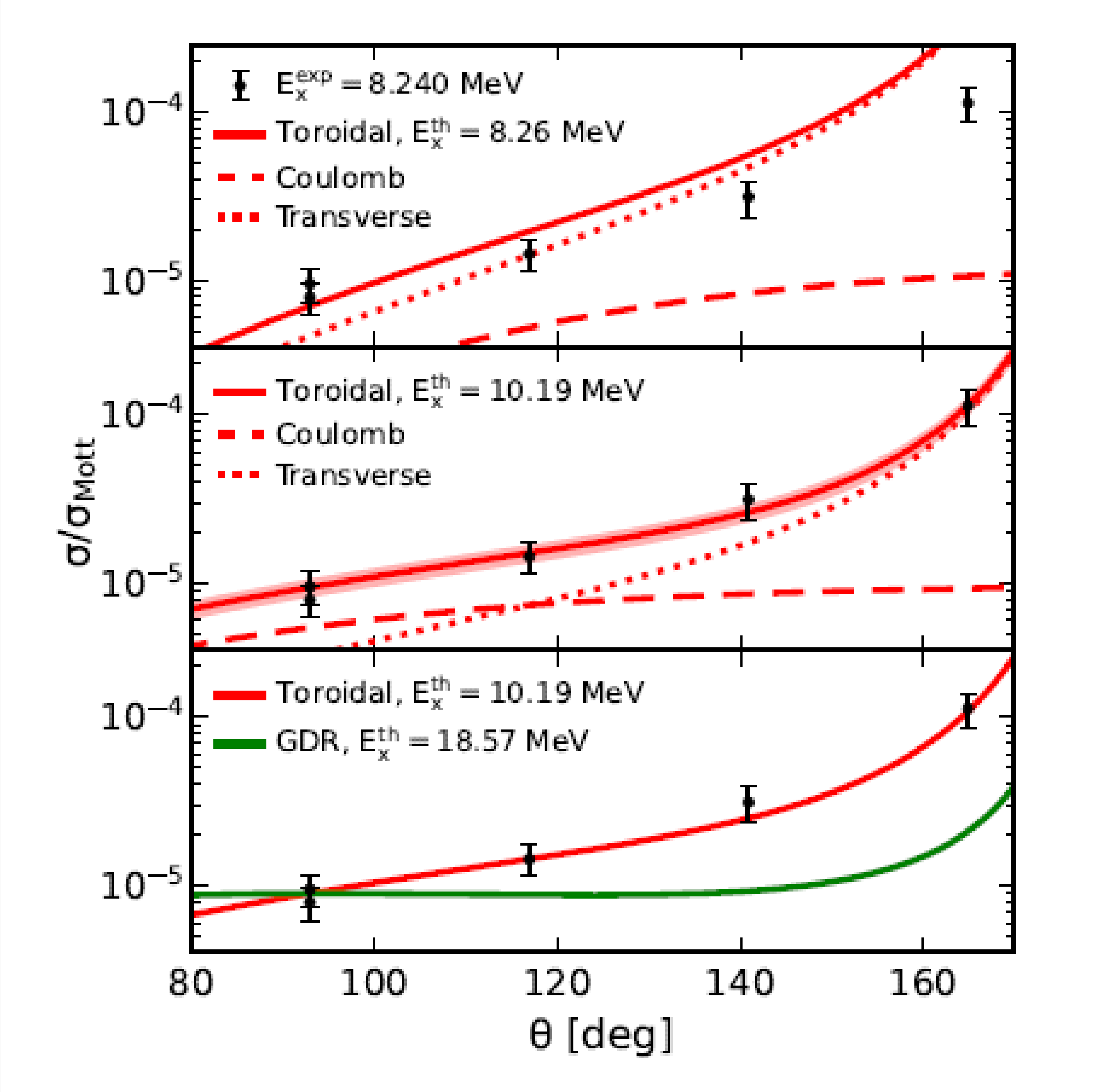




W. Mettner, A. Richter et al, NPA 473, 160 (1987).

B. Reitz, (1986) private communication

- It is reasonable to associate 10.04 MeV state with 9.85 and 10.19 MeV states obtained in calculations with SV-mas10.

# Cross section for SV-mas10



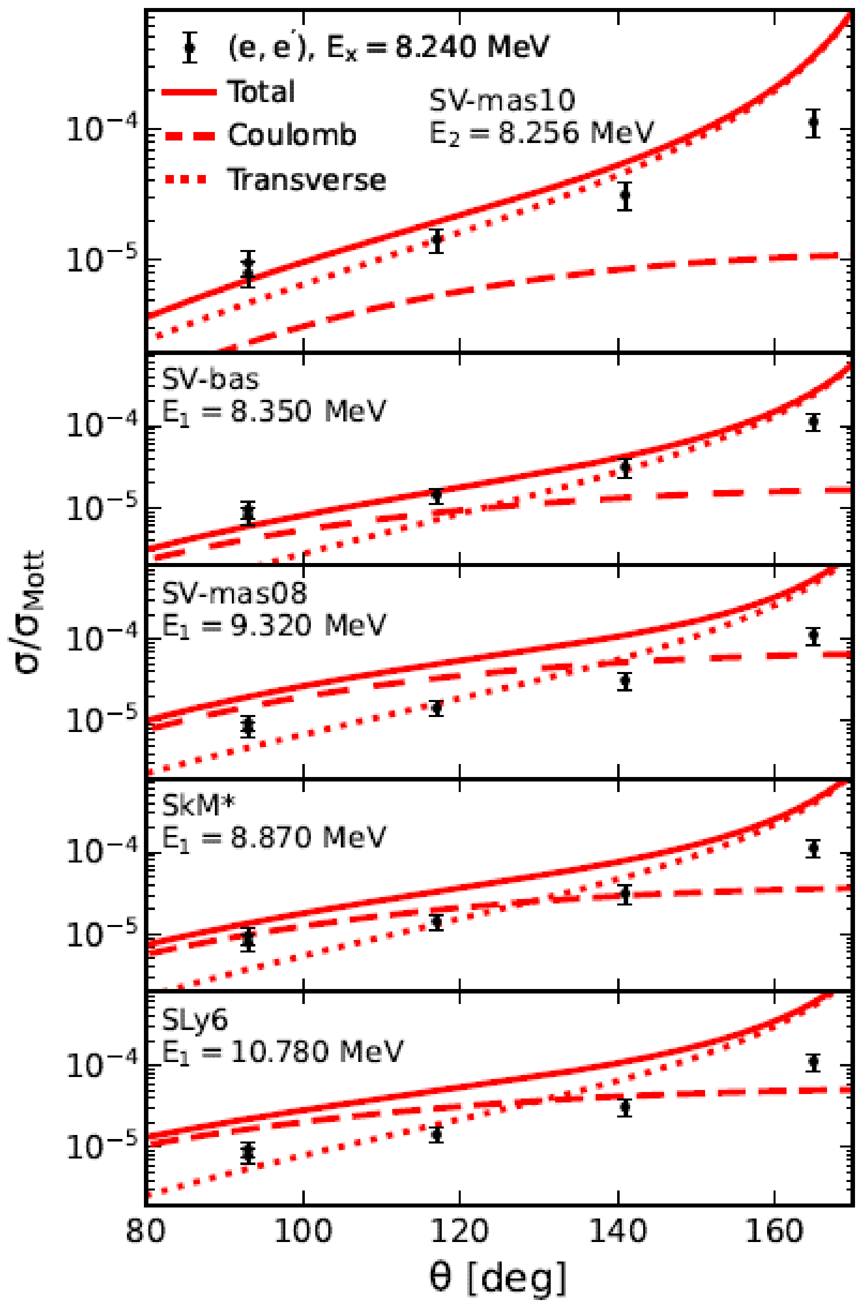

Electron scattering cross sections of the toroidal candidate at 8.240 MeV compared to QRPA predictions using the SV-mas10 interaction.

The strong slope of the cross section for toroidal state in comparison with the GDR.

# Conclusions

- $1^-$ states from 6 to 11 MeV in $^{58}Ni$ with strong slope of transversal form factors were analyzed as possible candidate for toroidal dipole excitations
- We have shown the toroidal nature of these states.
- The vorticity is produced by the dominant 2qp components
- This study is the **first prediction of the individual toroidal states in spherical nuclei** supported by detailed calculations and experimental data from different reaction

**Thanks for your attention!**



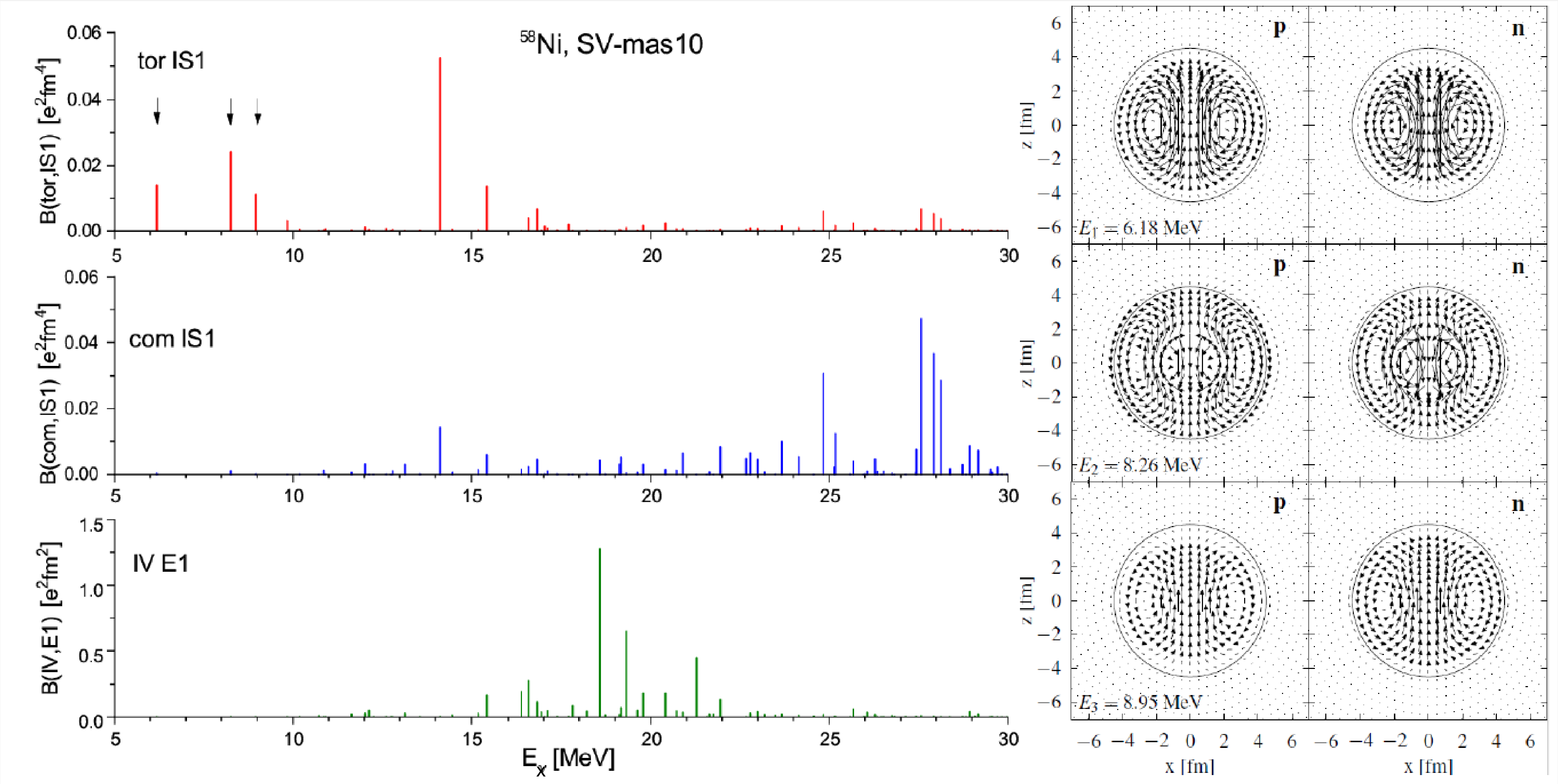

# The dependence of the cross sections depends at θ

A superior agreement with experiment is obtained for SV-mas10 and SV-bas.

The description gets worse with further decreasing effective mass.

The angular dependence of the transverse part is similar for all Skyrme parametrizations.

# SV-mas10: B(E1) and lowest states



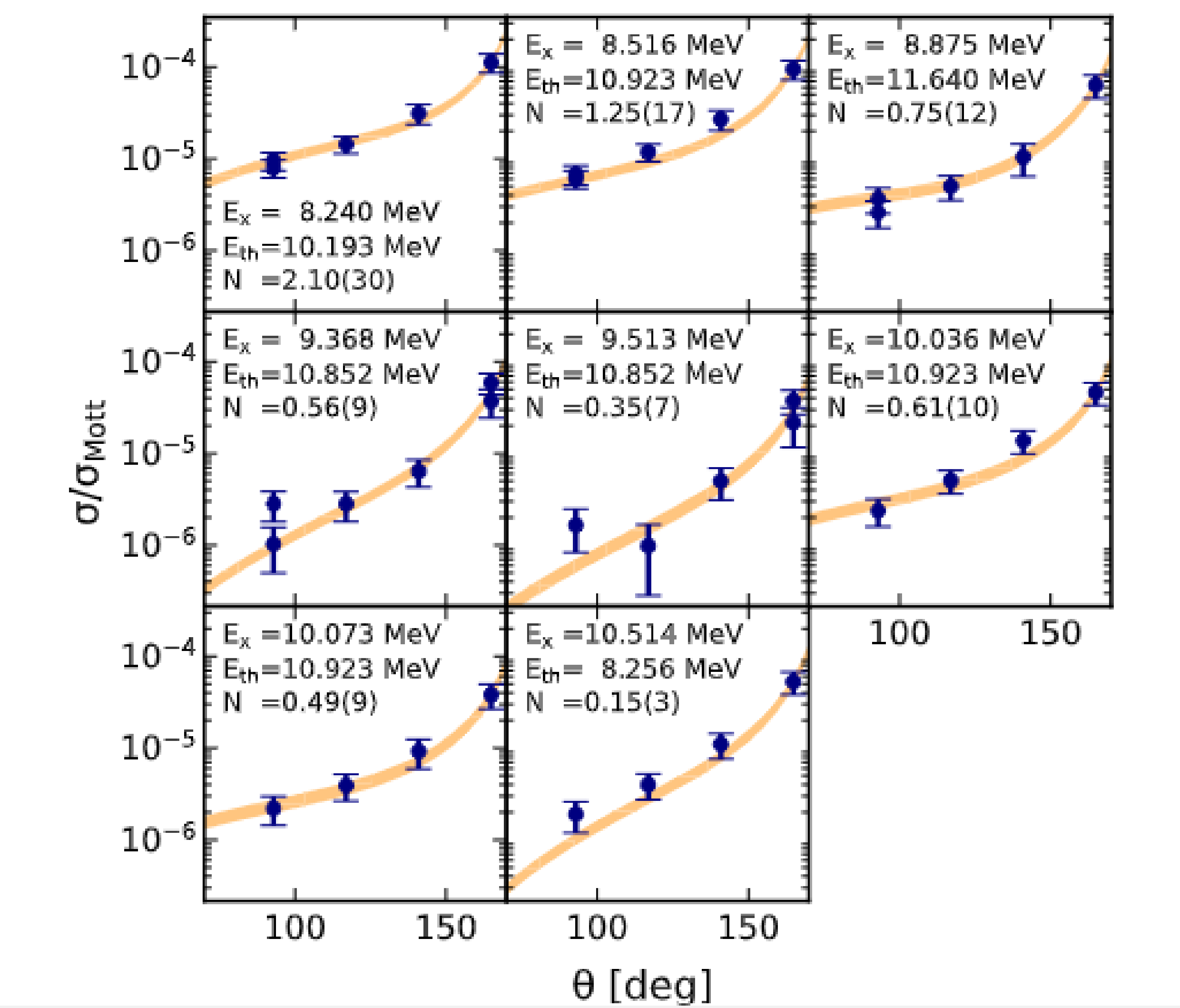

All three lowest states have a toroidal nature.

# Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes:

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).
M.N. Harakeh et al, PRL 38, 676 (1977).
S.F. Semenko, SJNP 34 356 (1981).
J. Heisenberg, Adv. Nucl. Phys. 12, 61 (1981).
S. Stringari, PLB 108, 232 (1982).
E. Wust et al, NPA 406, 285 (1983).
E.E. Serr, T.S. Dumitrescu, T.Suzuki, NPA 404 359 (1983).
D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
E.B. Balbutsev and I.N. Mikhailov, JPG 14, 545 (1988).
E.B.Balbutsev, I.V.Molodtsova, and A.V.Unzhakova, Europhys. Lett. 26, 499 (1994).
S.I. Bastrukov, S. Misicu, A. Sushkov, NPA 562, 191 (1993).
G.N. Afanasiev and Yu.P. Stepanovsky, J. Phys. A 28, 4565 (1995).
I.N. Mikhailov, Ch. Brianson, P. Quentin, SJPN, 27, 303 (1996)
I. Hamamoto, H.Sagawa, X.Z. Zang, PRC 53 765 (1996).
E.C.Caparelli, E.J.V.de Passos, JPG 25, 537 (1999).
N.Ryezayeva et al, PRL 89, 272502 (2002).
G.Colo, N.Van Giai, P.Bortignon, M.R.Quaglia, PLB 485, 362 (2000).
D. Vretenar, N. Paar, P. Ring,T. Nikshich, PRC 65, 021301(R) (2002).
V.Yu. Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).
J. Kvasil, N. Lo Iudice, Ch. Stoyanov, P. Alexa, JPG 29, 753 (2003).
A. Richter, NPA 731, 59 (2004).
S. Misicu, PRC 73, 024301 (2006).
X. Roca-Maza et al, PRC 85, 024601 (2012).
M. Urban, PRC, 85, 034322 (2012).
…….

# Cross sections of all torodal candidates




The strong slope of the transversal cross section is observed for all states.

# (e,e') data

## Exp. data for 8.24 MeV

| E [MeV] | θ [deg] | q_eff [$fm^{-1}$] | $\sigma/\sigma_M$ |
|---|---|---|---|
| 49.4 | 92.9 | 0.4358 | 7.943E-06 |
| 50.4 | 92.9 | 0.4359 | 9.525E-06 |
| 49.9 | 116.9 | 0.5111 | 1.434E-05 |
| 50.4 | 140.9 | 0.5647 | 3.128E-05 |
| 50.4 | 164.9 | 0.5937 | 1.124E-04 |

W. Mettner, A. Richter et al, NPA 473, 160 (1987).

| E [MeV] | θ [deg] | q_eff [$fm^{-1}$] | $\|F_T\|^2$ |
|---|---|---|---|
| 56.6 | 178.5 | 0.65 | 5.13E-06 |
| 65.4 | 178.5 | 0.74 | 6.1E-06 |

B. Reitz, (1986) private communication

## Exp. data for 10.04 MeV

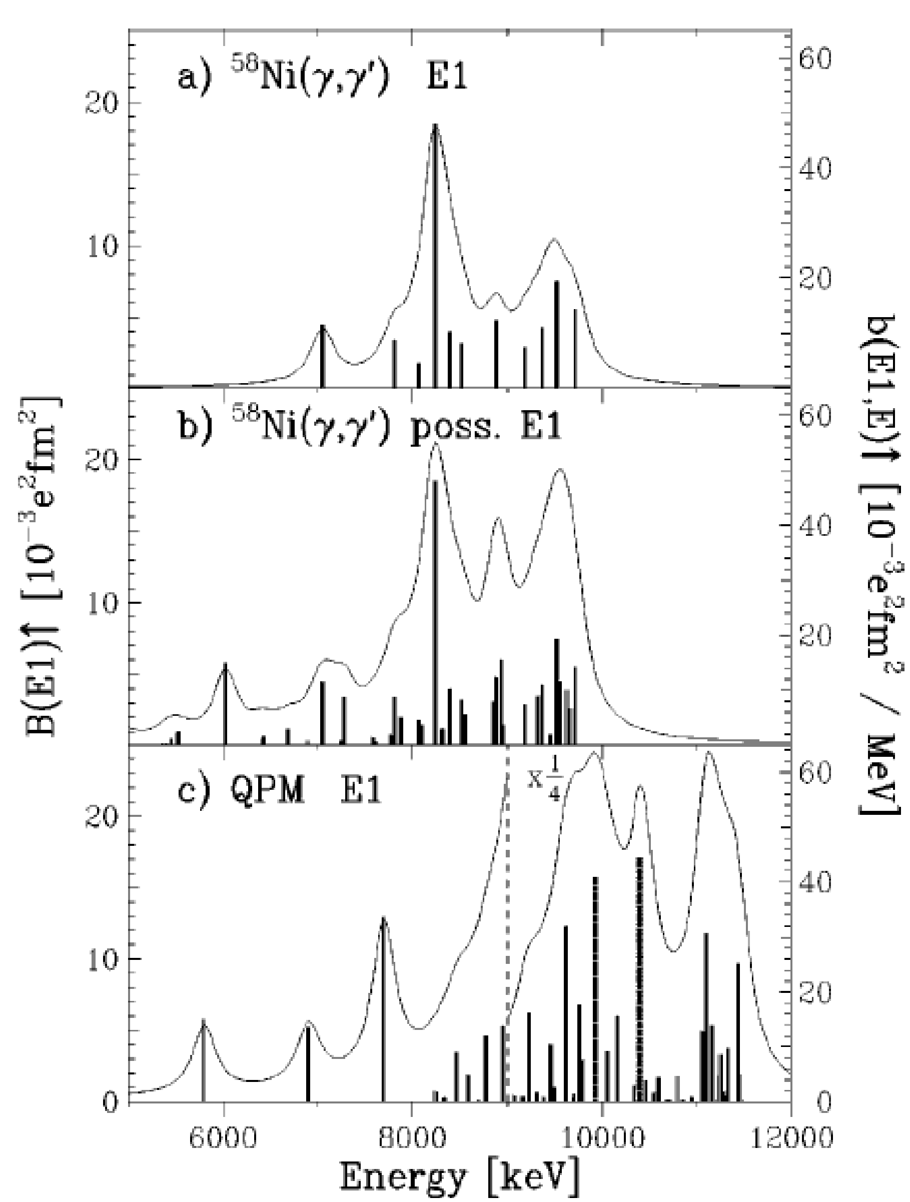
| E [MeV] | θ [degrees] | q_eff [$fm^{-1}$] | $\sigma/\sigma_M$ |
|---|---|---|---|
| 50.4 | 92.9 | 0.423 | 2.373E-06 |
| 49.9 | 116.9 | 0.493 | 5.054E-06 |
| 49.4 | 140.9 | 0.540 | 1.374E-05 |
| 50.4 | 164.9 | 0.579 | 4.613E-05 |
| 52.4 | 140.9 | 0.570 | 1.493E-05 |
| 57.4 | 116.9 | 0.560 | 6.561E-06 |

W. Mettner, A. Richter et al, NPA 473, 160 (1987).

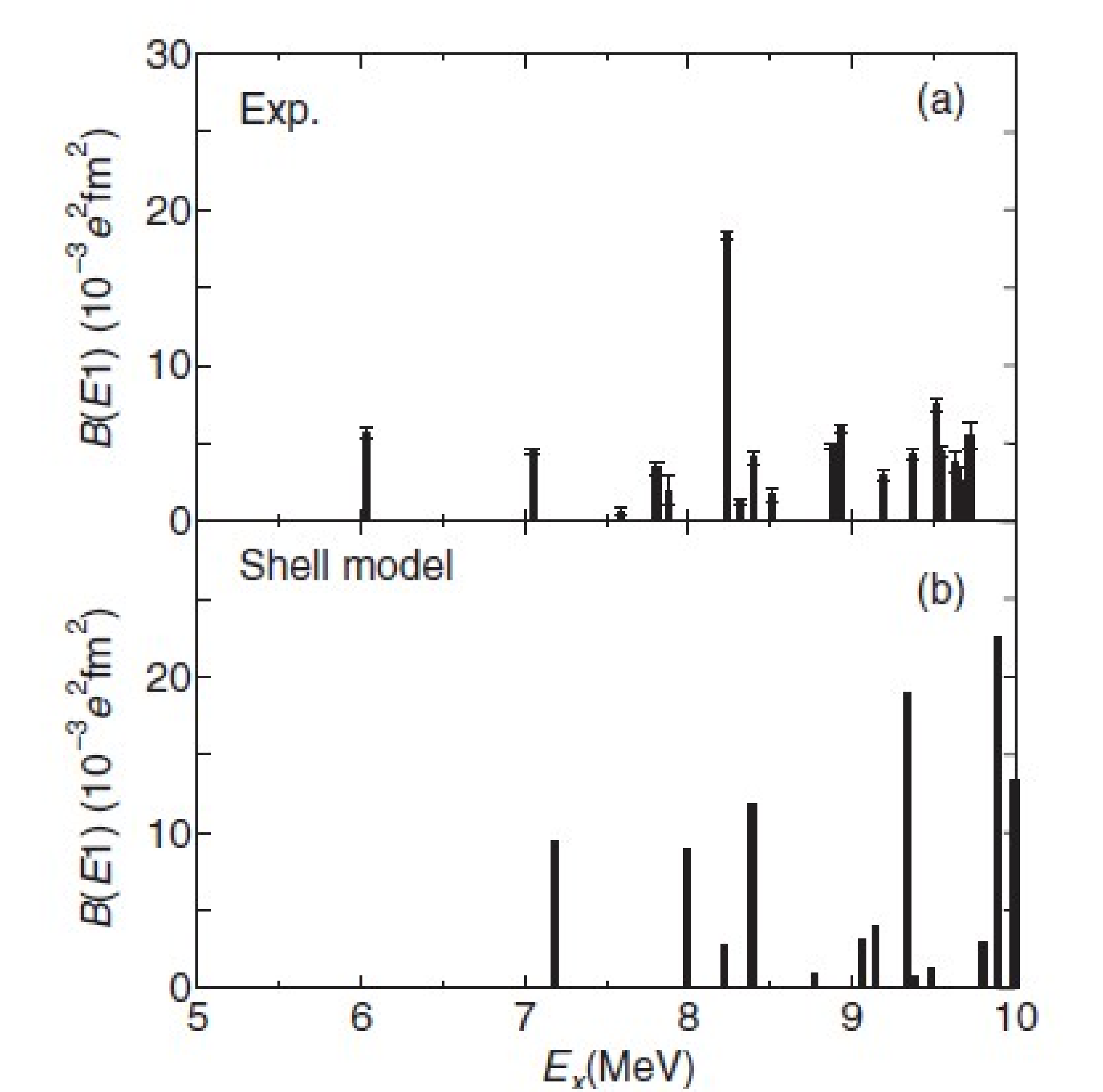
| E [MeV] | θ [deg] | q_eff [$fm^{-1}$] | $\|F_T\|^2$ |
|---|---|---|---|
| 56.6 | 178.5 | 0.65 | 1.33E-06 |
| 65.4 | 178.5 | 0.74 | 1.17E-06 |

B. Reitz, (1986) private communication

# B(E1) in other models




F. Bauwens, et al., PRC 62, (2000)




Shizuma, et al., PRC 109, (2024)

## Dubna-Prague-Bratislava-Erlangen:


J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, and P. Vesely,
"General treatment of vortical, toroidal, and compression modes",
**Phys. Rev.** C84, n.3, 034303 (2011)


General treatment of TDR


A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",
**Phys. Rev.** C87, 024305 (2013).


TDR vs pygmy resonance


P.-G. Reinhard, V.O. Nesterenko, A. Repko, and J. Kvasil,
"Nuclear vorticity in isoscalar E1 modes: Skyrme-RPA analysis",
**Phys. Rev.** C89, 024321 (2014).


TDR as a measure of vorticity


J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
"Deformation effects in toroidal and compression dipole excitations of 170Yb: Skyrme-RPA analysis",
**Phys. Scri.**, v.89, n.5, 054023 (2014).


Deformation effects in TDR


V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal resonances",
EPJ Web of Conferences, v.93, 01020(1-4) (2015)

V.O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig, and P.-G. Reinhard,
"Toroidal resonance: relation to pygmy mode, vortical properties and anomalous deformation splitting",
Phys. Atom. Nucl., v.79, n.6, 842-850 (2016)

A. Repko, J. Kvasil, V.O. Nesterenko, and P.-G. Reinhard,
"Pairing and deformation effects in nuclear excitation spectra",
Eur. Phys. J. v.53, 221 (2017).


154Sm


**V.O. Nesterenko, A. Repko, J. Kvasil, and P.-G. Reinhard,**
**"Individual low-energy toroidal dipole state in 24Mg",**
**PRL 120, 182501 (2018)**


24Mg