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# Generalized Parton Model (GPM) and it's application to calculation of TSSA

Generalized Parton Model (GPM) and it's application to calculation of TSSA

Factorization formula for the GPM

### Factorization formula for the GPM

Within the GPM we can write the following expression for the differential crosssection of  $2 \rightarrow 1$  hard subprocess  $g(q_1) + g(q_2) \rightarrow C(k)$ :

$$d\sigma(pp \to \mathcal{C}X) = \int dx_1 \int d^2 \mathbf{q_{1T}} \int dx_2 \int d^2 \mathbf{q_{2T}} \times F_g(x_1, q_{1T}, \mu_F) F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to \mathcal{C}), \quad (1)$$

where  $C = J/\psi, \psi(2S)$  or  $\chi_c(1P)$ , and

$$d\hat{\sigma}(gg \to \mathcal{C}) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - k) \frac{\overline{|M(gg \to \mathcal{C})|^2}}{2x_1 x_2 s} \frac{d^4k}{(2\pi)^3} \delta_+(k^2 - m_{\mathcal{C}}^2).$$
(2)

In a case of  $2 \to 2$  subprocess  $g(q_1) + g(q_2) \to C(k) + g(q_3)$ ,  $C = J/\psi, \psi(2S)$  in formula (1)  $d\hat{\sigma}(gg \to C)$  must be replaced by:

$$d\hat{\sigma}(gg \to \mathcal{C}g) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - k - q_3) \frac{\overline{|M(gg \to \mathcal{C}g)|^2}}{2x_1 x_2 s} \frac{d^3k}{(2\pi)^3 2k_0} \frac{d^4q_3}{(2\pi)^3} \delta_+(q_3^2).$$
(3)

$$q_1^{\mu} = \left( x_1 \frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{1T}^2}{2\sqrt{s}x_1}, \mathbf{q}_{1T}, x_1 \frac{\sqrt{s}}{2} - \frac{\mathbf{q}_{1T}^2}{2\sqrt{s}x_1} \right)^{\mu}, \tag{4}$$

$$q_2^{\mu} = \left( x_2 \frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{2T}^2}{2\sqrt{s}x_2}, \mathbf{q}_{2T}, -x_2 \frac{\sqrt{s}}{2} + \frac{\mathbf{q}_{2T}^2}{2\sqrt{s}x_2} \right)^{\mu}.$$
(5)

Generalized Parton Model (GPM) and it's application to calculation of TSSA

Polarized production. TSSA

### Transverse Single Spin Asymmetry (TSSA)

In inclusive process  $p^{\uparrow}p \to \mathcal{C}X \ \mathcal{C} = J/\psi, \chi_c, \psi(2S))$  TSSA is defined as:

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\Delta\sigma}{2d\sigma}.$$
 (6)

The numerator and denominator of  $A_N$  have the form:

$$d\sigma \propto \int dx_1 \int d^2 q_{1T} \int dx_2 \int d^2 q_{2T} F_g(x_1, q_{1T}, \mu_F) F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to \mathcal{C}X), \quad (7)$$
  
$$d\Delta \sigma \propto \int dx_1 \int d^2 q_{1T} \int dx_2 \int d^2 q_{2T} [\hat{F}_g^{\uparrow}(x_1, \mathbf{q}_{1T}, \mu_F) - \hat{F}_g^{\downarrow}(x_1, \mathbf{q}_{1T}, \mu_F)] \times F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to \mathcal{C}X), \quad (8)$$

where  $\hat{F}_{g}^{\uparrow,\downarrow}(x,q_{T},\mu_{F})$  is the distribution of unpolarized gluon (or quark) in polarized proton,  $\hat{F}_{g}^{(\uparrow)}(x_{1},\mathbf{q}_{1T},\mu_{F}) - \hat{F}_{g}^{(\downarrow)}(x_{1},\mathbf{q}_{1T},\mu_{F}) \equiv \Delta \hat{F}_{g}^{\uparrow}(x_{1},\mathbf{q}_{1T},\mu_{F})$  – the gluon Sivers function (GSF).

Generalized Parton Model (GPM) and it's application to calculation of TSSA

LIncluding ISI and FSI – Color Gauge Invariant formulation of GPM

### TSSA within the CGI-GPM framework



Figure 1: LO diagrams for the process  $p^{\uparrow}p \rightarrow J/\psi X$ , assuming a color-singlet production mechanism, within the GPM (a) and the CGI-GPM (b), (c). It turns out that only initial state interactions depicted in (b) contribute to the TSSA. Figure is from [D'Alesio *et.al.*, Phys. Rev. D **96**, 036011 (2017)].

In GPM (Fig. 1 (a)) we can write the numerator of the asymmetry as follows:

$$d\Delta\sigma \propto \Delta \hat{F}_g^{\uparrow}(x_1, \mathbf{q}_{1T}, \mu_F) \otimes F_g(x_2, q_{2T}, \mu_F) \otimes H^U_{gg \to cd}, \tag{9}$$

where  $H^U_{gg \to cd} = \overline{|M(gg \to cd)|^2}$ .

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Figure 2: LO diagrams for the process  $p^{\uparrow}p \rightarrow J/\psi X$ , assuming a color-singlet production mechanism, within the GPM (a) and the CGI-GPM (b), (c). It turns out that only initial state interactions depicted in (b) contribute to the TSSA. Figure is from [D'Alesio *et.al.*, Phys. Rev. D **96**, 036011 (2017)].

Formally, the numerator of the asymmetry in the CGI-GPM approach ([L. Gamberg and Z. B. Kang, Phys. Lett. B **696**, 109 (2011)]) can be obtained from eq. (9) by with the substitution:

$$F_{1T}^{\perp g} H_{gg \to J/\psi g}^{U} \to \frac{C_{I}^{(f)} + C_{F_{c}}^{(f)}}{C_{U}} F_{1T}^{\perp g(f)} H_{gg \to J/\psi g}^{U} + \frac{C_{I}^{(d)} + C_{F_{c}}^{(d)}}{C_{U}} F_{1T}^{\perp g(d)} H_{gg \to J/\psi g}^{U} \equiv \\ \equiv F_{1T}^{\perp g(f)} H_{gg \to J/\psi g}^{Inc(f)} + F_{1T}^{\perp g(d)} H_{gg \to J/\psi g}^{Inc(d)}.$$
(10)

TSSA in charmonium production at NICA

# TSSA in charmonium production at NICA

TSSA in charmonium production at NICA

└─Numerical results. Predictions for NICA

### TSSA of $\chi_{c_0}$ at NICA (D'Alesio), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 3: Predictions for TSSA  $A_N^{\chi_{C0}}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The D'Alesio *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

### TSSA of $\chi_{c_0}$ at NICA (SIDIS1), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 4: Predictions for TSSA  $A_N^{\chi_{c0}}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SIDIS1 *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

### TSSA of $\chi_{c_0}$ at NICA (SIDIS2), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 5: Predictions for TSSA  $A_N^{\chi_{C0}}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SIDIS2 *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

└─Numerical results. Predictions for NICA

### TSSA of $\chi_{c_2}$ at NICA (D'Alesio), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 6: Predictions for TSSA  $A_N^{\chi_{C_2}}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The D'Alesio *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

### TSSA of $\chi_{c_2}$ at NICA (SIDIS1), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 7: Predictions for TSSA  $A_N^{\chi_{C_2}}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SIDIS1 *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

### TSSA of $\chi_{c_2}$ at NICA (SIDIS2), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 8: Predictions for TSSA  $A_N^{\chi_{C_2}}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SIDIS2 *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

### TSSA of $\psi(2S)$ at NICA (D'Alesio), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 9: Predictions for TSSA  $A_N^{\psi(2S)}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The D'Alesio *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

### TSSA of $\psi(2S)$ at NICA (SIDIS1), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 10: Predictions for TSSA  $A_N^{\psi(2S)}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SIDIS1 *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

### TSSA of $\psi(2S)$ at NICA (SIDIS2), $|y| \leq 3$ , $\sqrt{s} = 24$ GeV.



Figure 11: Predictions for TSSA  $A_N^{\psi(2S)}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SIDIS2 *et al.* parametrisation of GSFs is used.

TSSA in charmonium production at NICA

LNumerical results. Predictions for NICA

TSSA of  $\chi_{c_1}$  at NICA,  $|y| \leq 3$ ,  $\sqrt{s} = 24$  GeV.



Figure 12: Predictions for TSSA  $A_N^{\chi_{c_1}}$  as function of  $p_T$  and  $x_F$  at  $\sqrt{s} = 24$  GeV within ICEM approach.

TSSA in charmonium production at SpinQuest

## TSSA in charmonium production at SpinQuest (Fermilab NM4)

TSSA in charmonium production at SpinQuest



### About SpinQuest Main goal of SpinQuest

spin axis of the nucleon?

#### Physics of SpinQuest

Polarized target

Spectrometer

Collaboration

Presentations and publications C

Data Management Plan

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#### Related Links

- Website
- University of Michigan Wiki (login required)



Every proton and neutron is made up of tinier particles called quarks and gluons, and we continue to explore how the quarks and gluons

A fixed-target experiment using the 120 GeV Main Injector beam and a polarized frozen-ammonia target.

$$\sqrt{s} = 15 \text{ GeV}, \qquad |x_F| \ge 0.4$$

TSSA in charmonium production at SpinQuest

└─Numerical results. Predictions for SpinQuest

### TSSA of $J/\psi$ at SpinQuest: $\sqrt{s} = 15$ GeV, $|x_F| \ge 0.4$ .



Figure 13: Predictions for TSSA  $A_N^{J/\psi}$  (prompt) as function of  $p_T$  (left) and  $x_F$  (right) at  $\sqrt{s} = 15$  GeV within GPM+CSM approaches.

TSSA in charmonium production at SpinQuest

└─Numerical results. Predictions for SpinQuest

### Comparison to NICA: $\sqrt{s} = 27$ GeV, $|y| \le 3$ .



Figure 14: Predictions for TSSA  $A_N^{J/\psi}$  (prompt) as function of  $p_T$  (left) and  $x_F$  (right) at  $\sqrt{s} = 27$  GeV within GPM+CSM approaches.

TSSA in charmonium production at SpinQuest

-Numerical results. Predictions for SpinQuest

### TSSA of $J/\psi$ at SpinQuest: $\sqrt{S} = 15$ GeV, $|x_F| \ge 0.4$ .



Figure 15: Predictions for TSSA  $A_N^{J/\psi}$  (prompt) as function of  $p_T$  (left) and  $x_F$  (right) at  $\sqrt{s} = 15$  GeV within CGI-GPM+CSM approaches.

# A small announcement: predictions for $J/\psi$ within Soft Gluon Resummation model.

Calculations of Kirill Shilyaev (PhD student, Samara University):



Figure 16: Predictions for cross sections for prompt  $J/\psi$ :  $\frac{d\sigma}{dp_T}$  at NICA  $\sqrt{s} = 27$  GeV (left) and  $E \frac{d^3\sigma}{dp^3}$  at PHENIX  $\sqrt{s} = 200$  GeV (right) within the SGR model.

▶ For all the *P*-wave charmonia states we see significant discrepancies between predictions within CSM and ICEM. It is especially noticeable for  $x_F$ -distribution.

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- At SpinQuest kinematics we see quite similar results of  $A_N$  for  $J/\psi$ .

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- ▶ SIDIS1 parametrization of the GSF gives the biggest value of the TSSA.
- At SpinQuest kinematics we see quite similar results of  $A_N$  for  $J/\psi$ .
- ▶ CGI-GPM can reproduce **negative** values of the TSSA.

### Thank you for your attention!

Backup: Color factors and Feynman rules in the CGI-GPM framework



Figure 17: CGI-GPM color rules for the eikonal three-gluon (a), quark-gluon (b) and antiquark-gluon (c) vertices. The color projectors for the gluon (d) and the QSF (e) are shown as well. The eikonal gluon has color index c. Figure is from [D'Alesio *et.al.*, Phys. Rev. D **96**, 036011 (2017)].

The color factors are:

$$\mathcal{T}_{aa'}^c = \mathcal{N}_{\mathcal{T}} T_{aa'}^c, \mathcal{D}_{aa'}^c = \mathcal{N}_{\mathcal{D}} D_{aa'}^c, \mathcal{Q}_{ij}^c = \mathcal{N}_{\mathcal{Q}} t_{ij}^c, \tag{11}$$

where  $T_{cb}^a \equiv -if_{acb}$ ,  $D_{bc}^a \equiv d_{abc}$ ,  $\mathcal{N}_{\mathcal{T}} = \frac{1}{Tr[T^cT^c]} = 1/(N_c(N_c^2 - 1))$ ,  $\mathcal{N}_{\mathcal{D}} = \frac{1}{Tr[D^cD^c]} = 1/((N_c^2 - 4)(N_c^2 - 1))$ ,  $\mathcal{N}_{\mathcal{Q}} = \frac{1}{Tr[t^ct^c]} = 2/(N_c^2 - 1)$ . So, correspondingly, for the *f*- and *d*-type GSF, the relative color factor is therefore calculated from Fig. 1(b) as follows:

$$C_I^{(f)} = -\frac{1}{2}C_U, C_I^{(d)} = 0.$$
(12)

And in CSM of the heavy quark-antiquark pair to the FSI, depicted in Fig. 1(c):

$$C_{F_c}^{(f)} = C_{F_c}^{(d)} = 0. (13)$$