Quantum-Inspired Algorithm for Solving the Production Planning Problem

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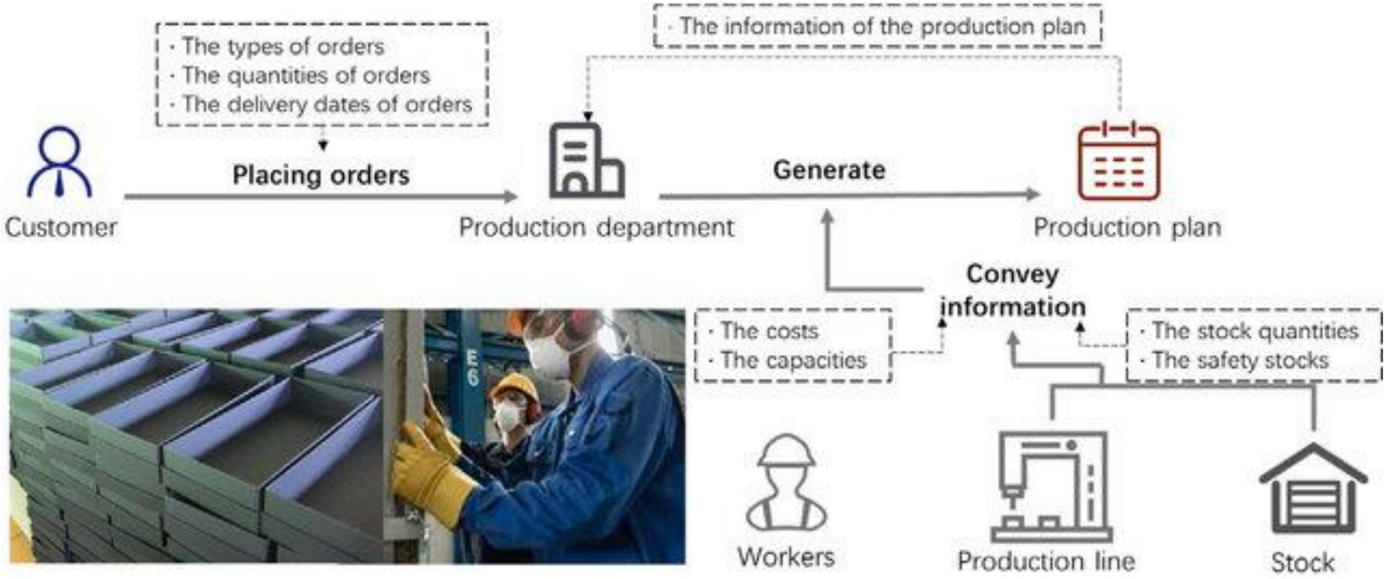
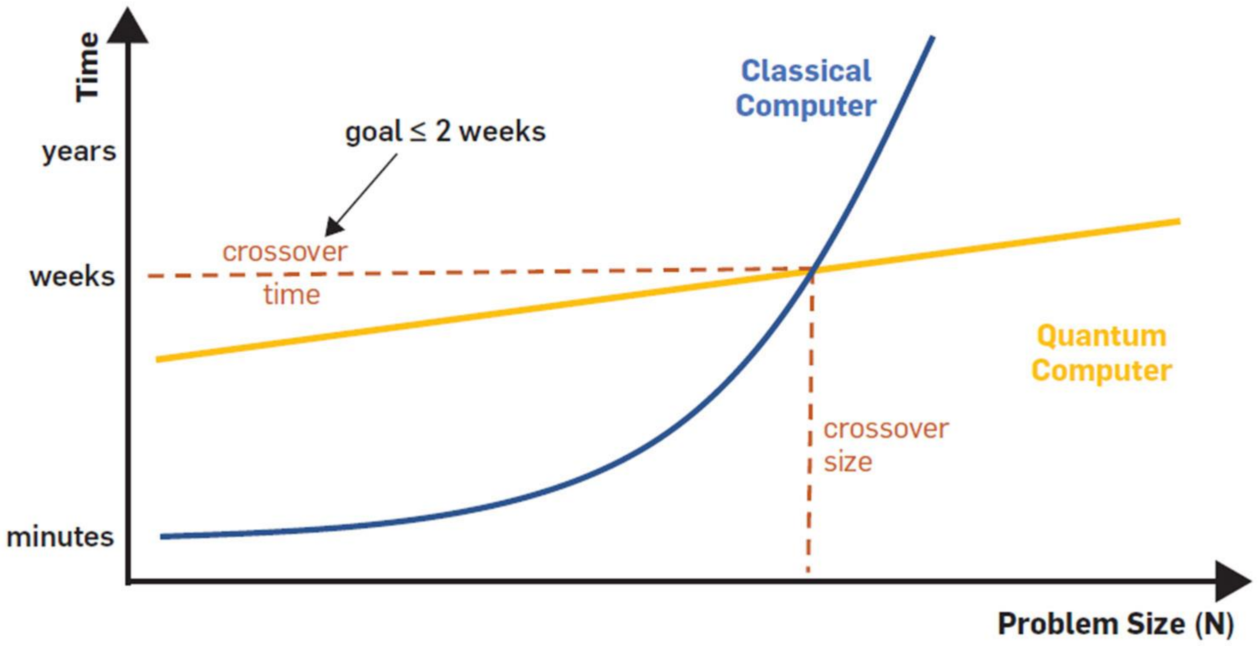
Production planning

- **Input:** customer orders — type, quantity, due-date
- **Decision:** production over the planning horizon
- **Key limits:** machine capacity, worker shifts & skills, warehouse space / safety stock
- **Goal:** minimize total cost of lateness + set-ups + inventory
- **Complexity:** reduces to Job/Flow-Shop or RCPSP ⇒ **NP-hard**; search space explodes for large plants

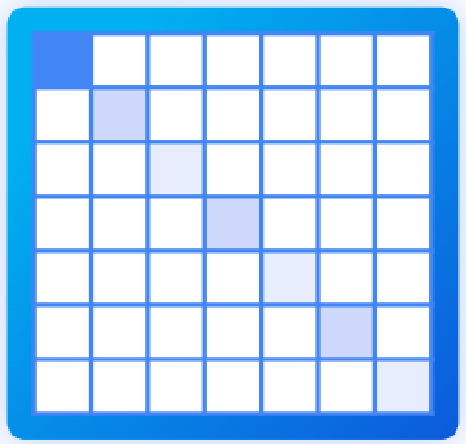


Quantum Computing

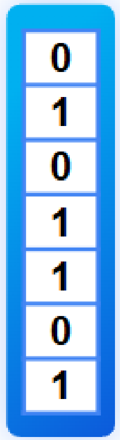


- **Explores many solutions at once** via superposition
- **Escapes local minima** better than classical heuristics (quantum tunneling)
- **Good fit for combinatorial problems** like scheduling, routing, packing
- **Scales better** for large instances where classical solvers slow down rapidly
- **Hybrid solvers** already show speed gains on mid-size problems (thousands of variables)

QUBO



Data + constraints + objectives



Decision vars

- **QUBO** = Quadratic Unconstrained Binary Optimization

$\mathbf{x^T Q x + c^T x}$ to **Min**, where $\mathbf{x \in \{0,1\}^n}$

- **All constraints are encoded as penalties** in the objective (e.g., capacity limits, precedence, timing)
- **Universal format** for quantum annealing and hybrid solvers
- Used in scheduling, routing, packing, allocation, etc.

MILP		**QUBO**
Min $c^T x$	→	Min $x^T Q x + c^T x$
s.t. $Ax \le b$	→	$+ \lambda \|Ax - b\|^2$ as penalty

Quantum Algorithms in Production Scheduling

- **Aggoune & Deleplanque (2023)**

  Solved job-shop scheduling with 4 jobs × 4 machines using QUBO on D-Wave.

- **Pérez Armas et al. (2024)**

  Solved RCPSP instances with up to 30 activities and 2 resources.

- **Pakhomchik et al. (2023)**

  Applied hybrid QUBO method to real automotive workflows with ~100 operations.

- **Toma et al. (2023)**

  Encoded flexible job-shop problems with up to 40 operations and multiple machines.

- **Zhang et al. (2022)**

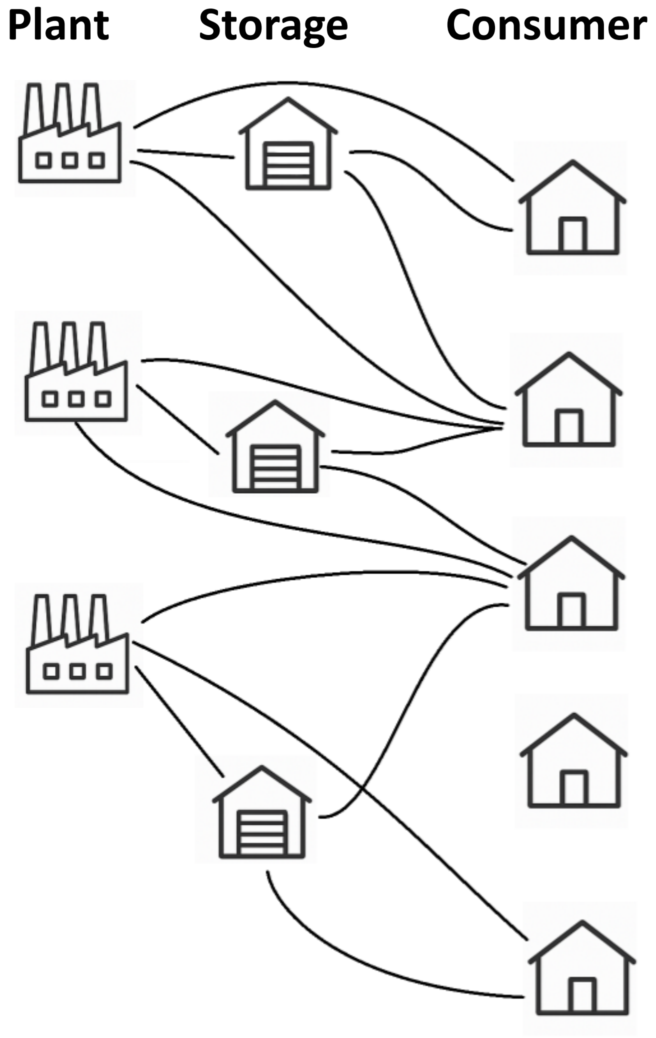
  Solved nurse scheduling instances with ~60 shifts using QUBO + local search on Fujitsu DA.

Production problem

- **Objective**: Develop an optimal production and distribution plan to minimize total costs.

- **Plants**: Multiple with varying production capacities and costs, some existing, others new or upgradable.
- **Consumers**: Several consumers with specific annual demand for products.
- **Storage Constraints**: Products have limited storage times, impacting inventory management.
- **Planning Horizon**: Multi-year period (10+ years).

- **Constraints**:
  - Meet consumer demand annually.
  - Respect plant production capacities and compatibility with consumers.
  - Adhere to storage duration limits and delivery schedules.
  - Account for construction/upgrade costs for new or existing plants.



Plant Capacity Constraint

Ensure production at plant z in year t does not exceed capacity c_z.

MILP Format

- $y_{zt} \ \leq \ c_z, for\ all\ z, t$
- Vector form: $y_j \ \leq \ c_{zj}, where\ j \ = \ Z(t-1) \ + \ z$

QUBO Format

- Binarize: $y_{zt} \ = \ \Sigma\, 2^k\, y_{jk}, y_{jk} \ \in \ \{0,1\}, k \ = \ 0\ to\ \lfloor \log_2 c_{zj} \rfloor$
- Penalty: $\rho\, \Sigma\, (\Sigma\, 2^k\, y_{jk} \ - \ c_{zj} \ + \ \varepsilon_j)\text{^}2, \varepsilon_j \ = \ slack$

QUBO: $\min y^T\, Q\, y \ + \ c^T\, y, Q$ encodes penalty

Supply Plan Constraint

Supply from plant z to consumer r in year t must not exceed demand d_{rt}.

MILP Format

- $x_{rzt} \leq d_{rt}, for\ all\ r, z, t$
- Vector form: $x_i \leq d_{r_i t_i}, where\ i = RZ(t-1) + R(z-1) + r$

QUBO Format

- Binarize: $x_{rzt} = \Sigma\, 2^k\, x_{ik}, x_{ik} \in \{0,1\}, k = 0\ to\ \lfloor \log_2 d_{r_i t_i} \rfloor$
- Penalty: $\rho\, \Sigma \left(\Sigma\, 2^k\, x_{ik} - d_{r_i t_i} + \varepsilon_i\right)^2, \varepsilon_i = slack$

QUBO: $\min x^T\, Q\, x + c^T\, x, Q$ encodes penalty

Compatibility Constraint

No supply from plant z to consumer r if incompatible ($p_{rz} = 0$).

MILP Format

- $x_{rzt} = 0 \; if \; p_{rz} = 0, for \; all \; t$
- Vector form: $x_i = 0 \; if \; p_{r_i z_i} = 0, i = RZ(t_i - 1) + R(z_i - 1) + r_i$

QUBO Format

- Binarize: $x_{rzt} = \Sigma \, 2^k \, x_{ik}, x_{ik} \in \{0,1\}$
- Penalty: $\rho \, \Sigma_{i:\, p_{r_i z_i}=0} \left(\Sigma \, 2^k x_{ik}\right)^2$

QUBO: $\min x^T \, Q \, x, Q$ penalizes non-zero x_{ik} when $p_{r_i z_i} = 0$

Delivery Timing Constraint

No supply from plant z to consumer r during restricted years after start ($t \ in \ [g_r, g_r + h_{rz} - 1]$).

MILP Format

- $x_{rzt} = 0 \ if \ t \in [g_r, g_r + h_{rz} - 1], for \ all \ r, z$
- Vector form: $x_i = 0 \ if \ t_i \in \left[g_{r_i}, g_{r_i} + h_{r_i z_i} - 1\right]$

QUBO Format

- Binarize: $x_{rzt} = \Sigma \, 2^k \, x_{ik}, \ \ x_{ik} \in \{0,1\}$
- Penalty: $\rho \, \Sigma_{i:\, t_i \in \left[g_{r_i}, g_{r_i} + h_{r_i z_i} - 1\right]} \left(\Sigma \, 2^k x_{ik}\right)^2$

QUBO: $\min x^T Q \, x, Q$ penalizes non-zero x_{ik} in restricted periods

Production Balance Constraint

Total supply from plant z across all consumers and years equals its total production.

MILP Format

- $\Sigma_{r,t}\, x_{rzt} \ = \ \Sigma_t\, y_{zt}, for\ all\ z$
- Vector form: $A_1\, w \ = \ b_1, A_1 \ = \ (I_{1xT} \otimes \ E_Z \otimes \ I_{1xR}, -I_{1xT} \otimes \ E_Z), b_1 = \ 0$

QUBO Format

- Binarize: $x_{rzt} \ = \ \Sigma\, 2^k\, x_{ik}, y_{zt} \ = \ \Sigma\, 2^k\, y_{jk}$
- Penalty: $\rho_1 \Sigma_z \left(\Sigma_i A_{1i} \Sigma\, 2^k x_{ik} - \ \Sigma_j A_{1j} \Sigma\, 2^k y_{jk}\right)^2$

QUBO: $\min w^T\, Q\, w, Q \ = \ \rho_1\, A_1^T A_1, w = (x, y)$

Demand Satisfaction Constraint

Total supply to consumer r in year t meets demand d_{rt}.

MILP Format

- $\Sigma_z x_{rzt} \; = \; d_{rt}, for \; all \; r, t$
- Vector form: $A_2 w \; = \; b_2, \;\; A_2 = \; (E_T \otimes \; I_{1xZ} \otimes \; E_R, 0_{RT} x \; ZT), b_2 \; = \; d$

QUBO Format

- Binarize: $x_{rzt} \; = \; \Sigma \, 2^k x_{ik}$
- Penalty: $\rho_1 \Sigma_{r,t} \left(\Sigma_i A_{2i} \Sigma \, 2^k x_{ik} - \; d_{rt} \right)^2$

QUBO: $\min x^T \; Q \; x, Q \; = \; \rho_1 A_2^T A_2$

Storage Duration Constraint

Production at plant z up to year t must not exceed supply within storage limit s_z.

MILP Format

- $\Sigma_{s=1}^{t} y_{zs} \leq \Sigma_r \Sigma_{s=1}^{t+s_z} x_{rzs},\ \ t \leq T - s_z - 1, for\ all\ z$
- Vector form: $C_2^z\, w \leq 0, C_2^z = (-M^z \otimes e^z \otimes I_{1xR}, M(1{:}T - s_z - 1, :) \otimes e^z)$

QUBO Format

- Binarize: $x_{rzs} = \Sigma\, 2^k\, x_{ik}, y_{zs} = \Sigma\, 2^k y_{jk}$
- Penalty: $\rho_2 \Sigma_{z,t} \left(\Sigma_j C_{2j}^z \Sigma\, 2^k y_{jk} - \Sigma_i C_{2i}^z \Sigma\, 2^k x_{ik} + \varepsilon_{zt}\right)^2$

QUBO: $\min w^T Q\, w, Q = \rho_2 (C_2^z)^T C_2^z$

Real problem

- 5 consumers
- 3 plants
- Storage times 1, 3, 5 years

In MILP:

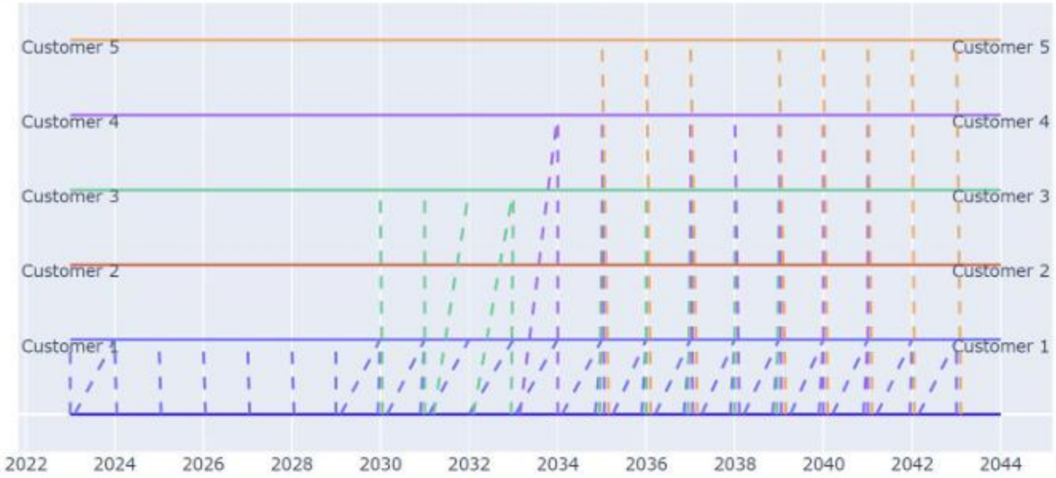
- 378 integer variables
- 108 equality constraints
- 111 inequality constraints

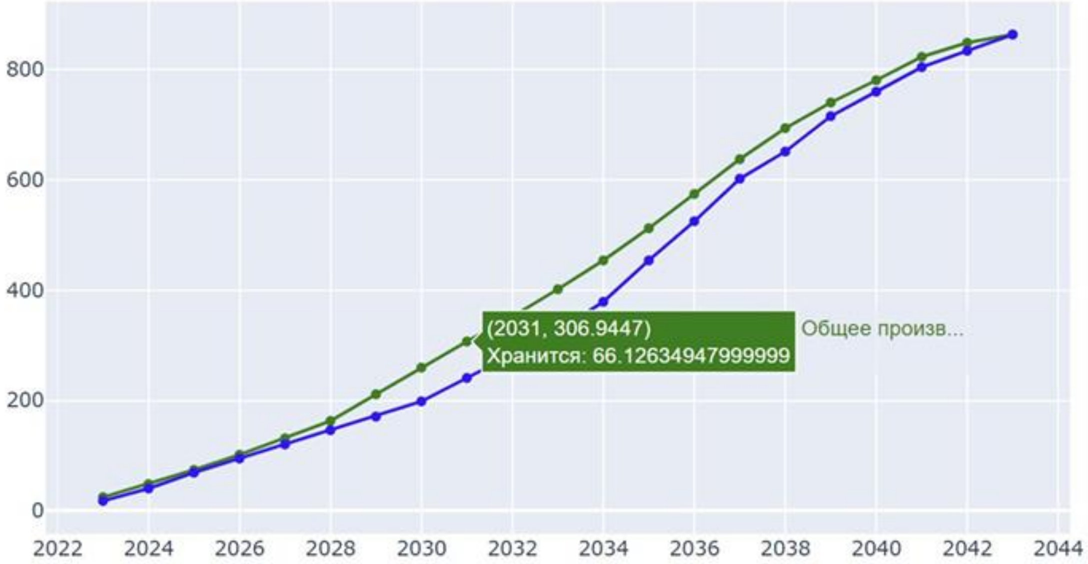
In QUBO:

- 1233 binary decision variables

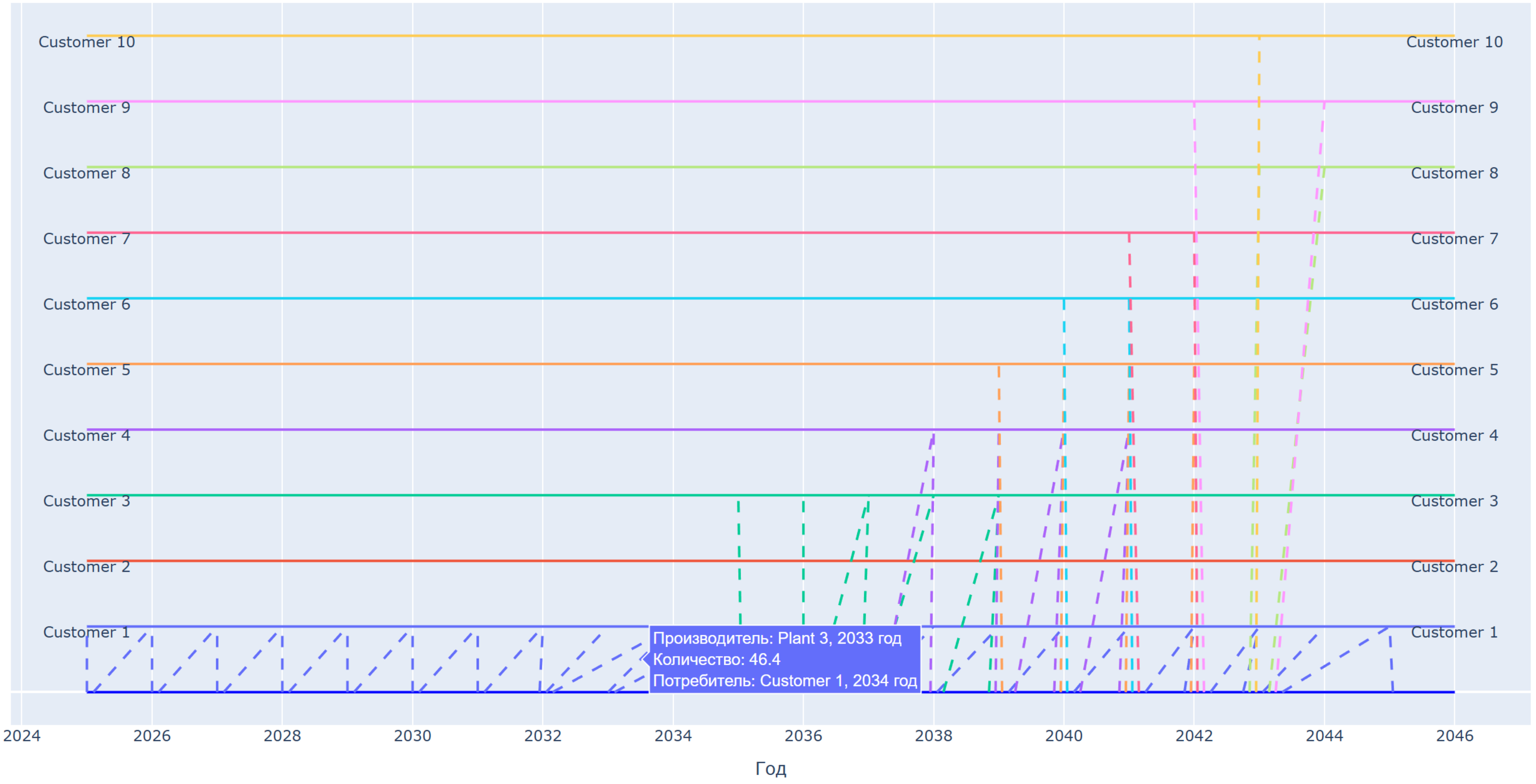
8 hours of manual planning to

5 min quantum algorithm run



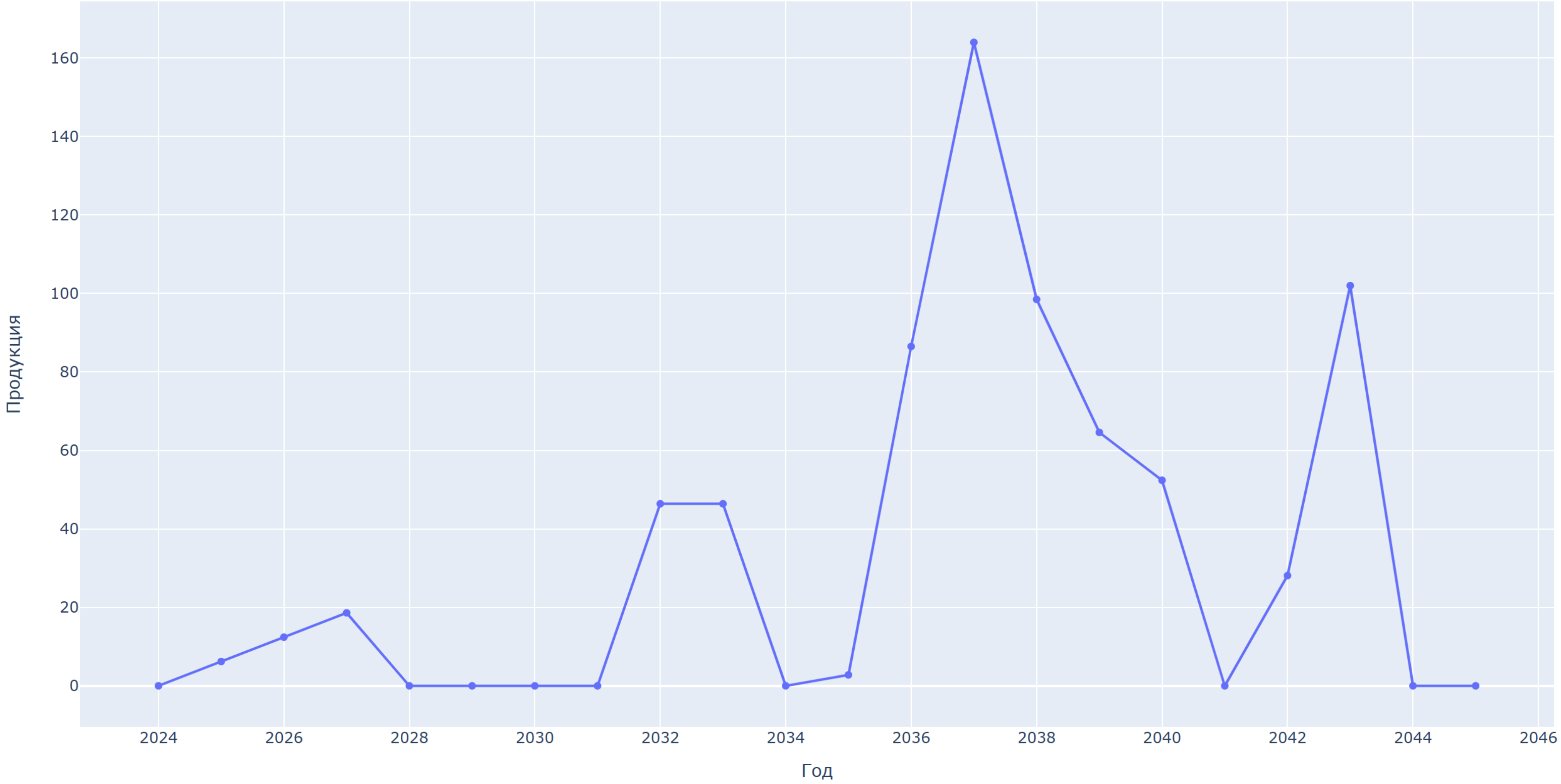


Production plan

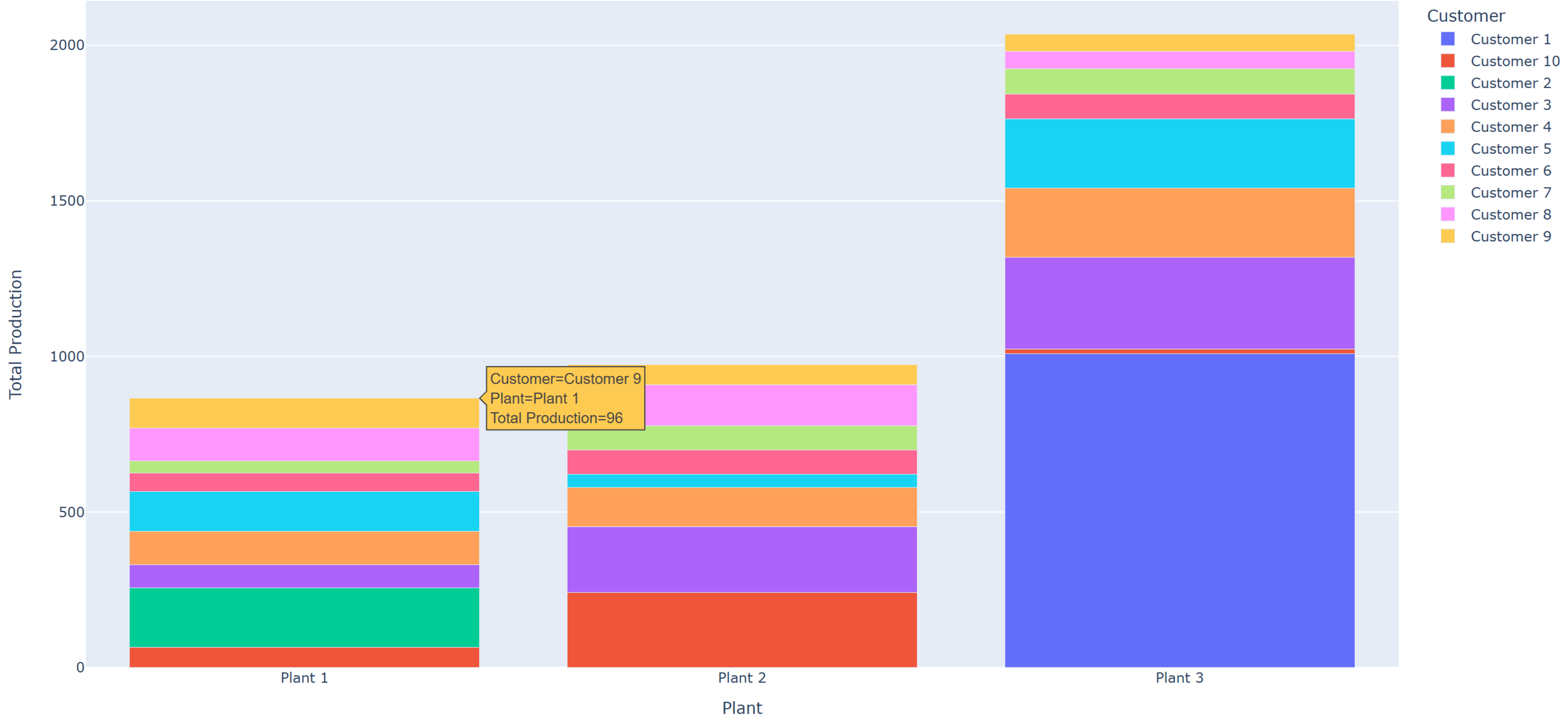


Customer consumption

Plant storage



Production distribution



Results

- Provided verification of quantum inspired algorithms for real world problems without decomposition
  - In the literature often small test problems are solved.
- Solved the problem on real data
  - Quantum algorithms provided several solutions which could be used for stability analysis
- Rather small problem size for testing on real quantum computer
  - In the future could be run on real machine
- Comparison with HIGHS solver was provided
  - Results are identical

Our plan:

- New model with additional constraints was developed
- New real data with bigger scale obtained

Thanks for your attention!

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