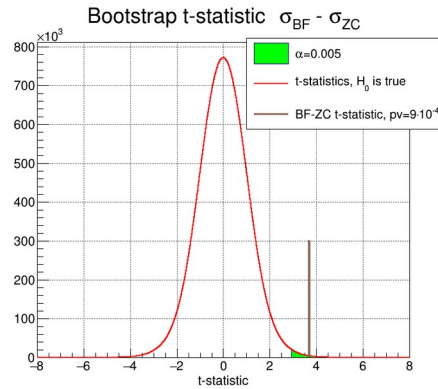
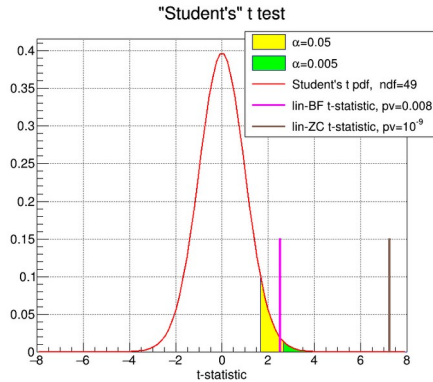


# Software development to improve PHOS time resolution (in collaboration with NRC “Kurchatov Institute” - IHEP)

Time values were extracted from the raw data of the currently operating PHOS using our proposed method (“ZC”) and some other known algorithms. The correspondent time distributions were compared with each other. The preliminary estimations was made using small amount of data available. A single channel time resolution of about 900 ps has been achieved for photons. It corresponds to a time resolution of about 400-500 ps after taking into account all the channels belonging to the electromagnetic shower. It was done without a preliminary calibration. After obtaining more raw data and the calibration, the time resolution is expected to improve even more.

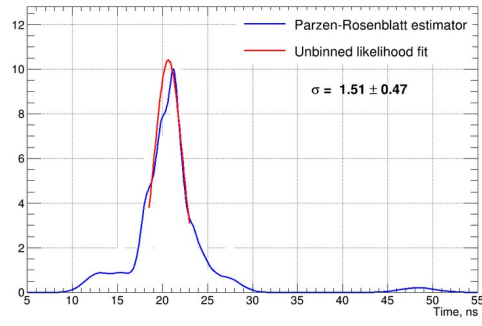


“Student’s” t-test and Bootstrap were used to compare the algorithms. Differences of variances of the times extracted from channels of 50 channels groups were analyzed simultaneously using t-statistic.

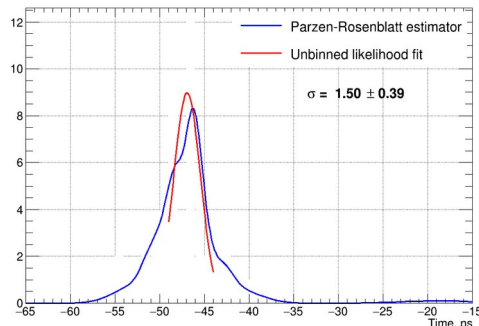
“Linear” method is based on linear fitting of the front samples of signals;  
“BF” method is based on fitting of the signal samples by semi-Gaussian function;  
“ZC” method is based on linear fitting of transformed signal samples.

## Time distributions. Channel 46x31 from the third module

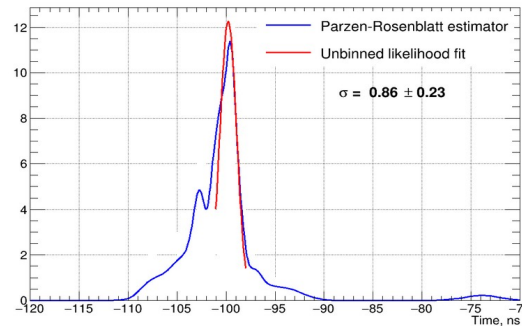
Linear algorithm



BF algorithm

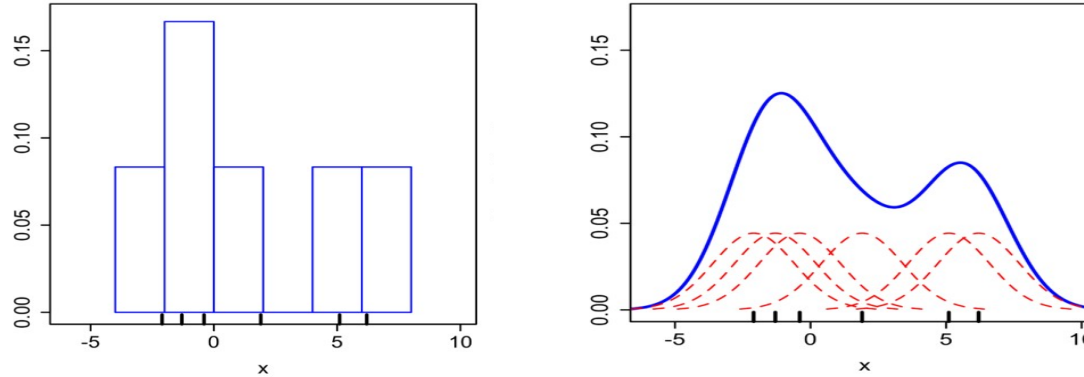


ZC algorithm



During the study, an analysis of the time resolution was performed for the first and third modules of the PHOS detector.

## Parzen – Rosenblatt estimator vs histogram



Parzen - Rozenblatt estimator (solid blue curve) converge faster (compared to the histogram) to the true underlying density for continuous random variables.

Let  $(x_1, x_2, \dots, x_n)$  be independent and identically distributed samples drawn from some univariate distribution with an unknown density  $f$  at any given point  $x$ . Its Parzen - Rosenblatt estimator is

$$\hat{f}(t) = \frac{1}{nh} \sum_{i=1}^n \left( K\left(\frac{t - x_i}{h}\right) \right)$$

where  $K$  is the kernel, a non-negative function, and  $h > 0$  is a parameter called the bandwidth (or window). In the RooFit algorithm  $K$  is Gaussian.

The details of the RooFit algorithm are described here:

Cranmer KS, Kernel Estimation in High-Energy Physics. Computer Physics Communications 136:198-207,2001 - e-Print Archive: hep-ex/0011057

Original works:

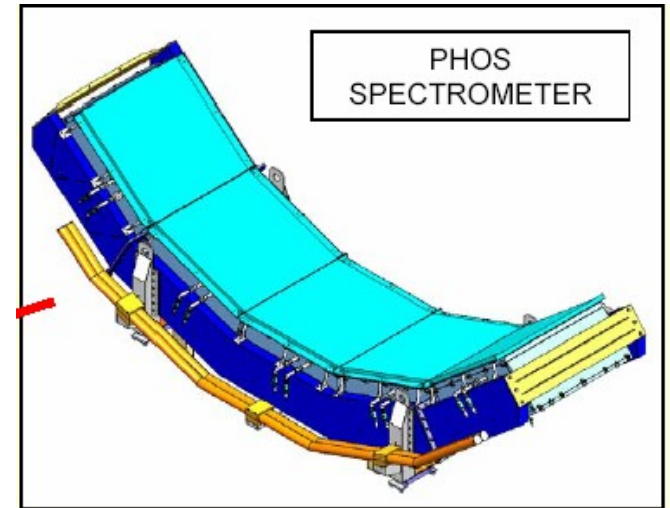
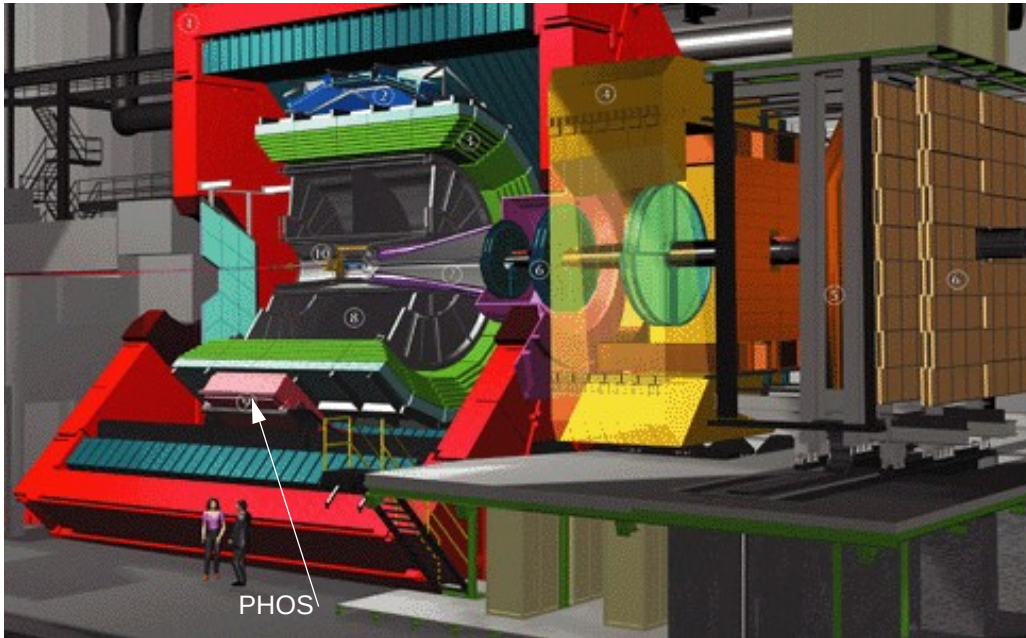
Rosenblatt M. Remarks on Some Nonparametric Estimates of a Density Function. The Annals of Mathematical Statistics. 1956

Parzen E. On Estimation of a Probability Density Function and Mode. The Annals of Mathematical Statistics. 1962



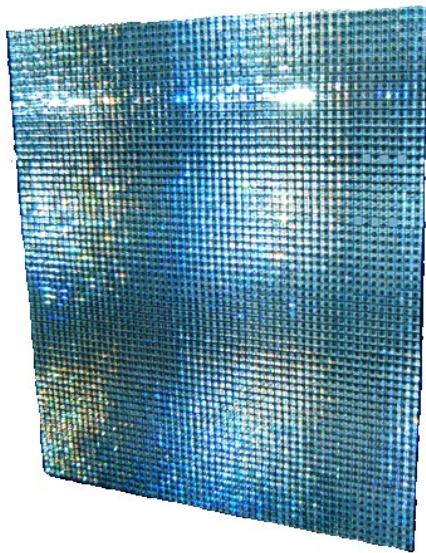
## Bootstrap resampling

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
⑨	⑨	②	③	⑦	①	④	②	⑥	③
⑧	④	②	⑧	③	②	④	①	⑤	⑦



**64 crystals in  $\eta$**

$$\Delta\eta = 0.24 \text{ (1.263 m)}$$



**56 crystals in  $\varphi$**

$$\Delta\varphi = 20^\circ \text{ (0.722 m)}$$

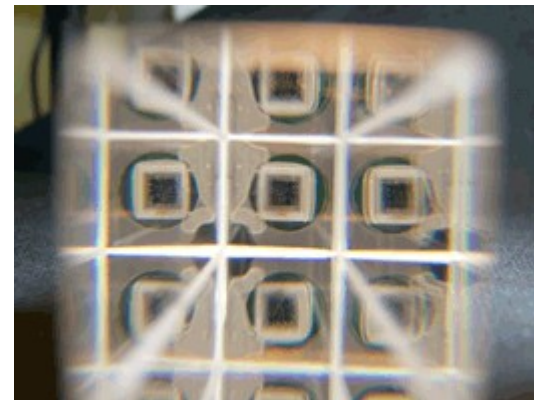


Photo of 1 PHOS Module with  $56 \times 64 = 3584$  PWO crystals

Hans Muller, Zhongbao Yin for PHOS Collaboration [https://alice-phos.web.cern.ch/sites/default/files/documents/Manuals/PHOS-User-Manual\\_2007.pdf](https://alice-phos.web.cern.ch/sites/default/files/documents/Manuals/PHOS-User-Manual_2007.pdf)

J. Grahl et al. / Nuclear Instruments and Methods in Physics Research A 504 (2003) 44–47

“The bootstrap is a computer-based technique for estimating standard errors, biases, confidence intervals and other measures of statistical accuracy. It automatically produces accuracy estimates in almost any situation, including very complicated ones, without requiring much thought from the statistician. This is a considerable virtue, but a virtue that can be abused. The danger lies in the possibility that the bootstrap estimates of accuracy, so easily produced, might be accepted uncritically.”

J. Grahl et al. / Nuclear Instruments and Methods in Physics Research A 504 (2003) 44–47

Peter Hall “The Bootstrap and Edgeworth Expansion” (1992)

Bradley Efron, Robert J. Tibshirani “An Introduction to the Bootstrap” (1993)

Tim C. Hesterberg “Bootstrap Tilting Diagnostics” (2001)

Student “The probable Error of a Mean” Biometrika, Volume 6, Issue 1 (Mar., 1908), 1-25.

Bradley Efron “Jackknife-after-Bootstrap Standard Errors and Influence Functions” Journal of the Royal Statistical Society (1992), 54, No. 1, pp. 83-127

B. Efron. "Bootstrap Methods: Another Look at the Jackknife." The Annals of Statistics, 7 (1) 1 - 26, January, 1979.



## Glivenko-Cantelli Theorem

$X_1, \dots, X_n$  are i.i.d. with distribution  $F$

$F$  is a cumulative distribution function

The empirical cumulative distribution function is

$$F_n(x) = \frac{1}{n} \sum_{i=1}^n I(X_i \leq x)$$

## Bootstrap resampling

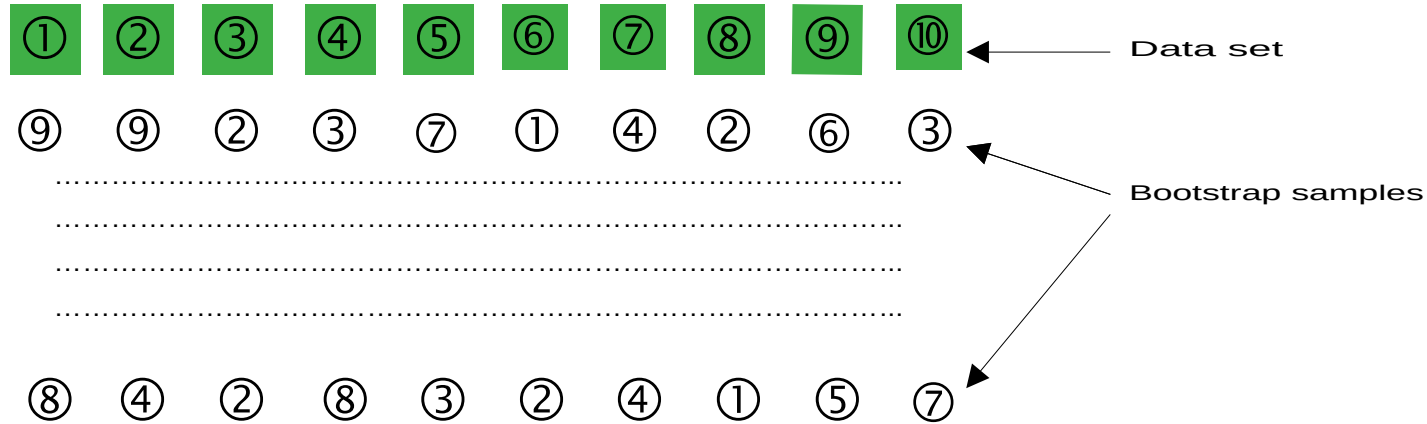
①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
⑨	⑨	②	③	⑦	①	④	②	⑥	③
⑧	④	②	⑧	③	②	④	①	⑤	⑦

.....

## Bootstrap resampling

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
⑨	⑨	②	③	⑦	①	④	②	⑥	③
⑧	④	②	⑧	③	②	④	①	⑤	⑦

## Bootstrap resampling



A random number generator independently selects integers each of which equals any value between 1 and  $N$  with probability  $1/N$ . These integers determine which observations are selected to be in the bootstrap sample. Some observations can appear more than once in the sample. Easy to see that a bootstrap sample contains approximately  $2/3$  unique observations of the original data set on average. For big data number of copies of each observation in a bootstrap sample is distributed by Poisson with Mean=1.

## Bootstrap resampling

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
⑨	⑨	②	③	⑦	①	④	②	⑥	③
⑧	④	②	⑧	③	②	④	①	⑤	⑦

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Then

$$\lim_{n \rightarrow \infty} \sup_x |F_n(x) - F(x)| \stackrel{as}{=} 0$$

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