Dependence of chromomagnetic field screening on its color components

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September 4, 2018

Magnetic Mass

Magnetic (electric) mass shows how fast magetic (electric) field decreases with distance in plasma.

$$F_{\mu\nu} = C(r)\,\mathrm{e}^{\,-m^k r^k} \tag{1}$$

$$m = \frac{1}{\lambda}, \qquad \lambda \text{ – screening length} \tag{2}$$

Example: QED

$$m_{\mathsf{D}}^2 = \frac{1}{3}e^2T^2 \qquad \text{– electric (Debye) mass} \tag{3}$$

$$m_{\mathsf{magn}}^2 = 0 \qquad \text{– magnetic mass} \tag{4}$$

Screened

Long range

Magnetic Mass in SU(2) Gauge Theory

$$F_{\mu\nu} = \sum_{a=1}^{3} F_{\mu\nu}^{(a)} \frac{\sigma_a}{2}, \qquad \sigma_a \text{ – the Pauli matrices} \tag{5}$$

In absence of external field:

$$m_{\mathrm{D}}^2 \sim g^2 T^2, \qquad m_{\mathrm{magn}}^2 \sim g^4 T^2 \tag{6}$$

// D. Gross, R. Pisarski, and L. Yaffe, Rev. Mod. Phys. **53**, 43 (1981)

In the presence of external chromomagnetic field H ($gH/T^2 << 1$):

Neutral gluon field:

$$m_{\mathrm{D}}^2 \sim g^2 T^2 \left(1 - C\sqrt{gH}/T\right), \qquad m_{\mathrm{magn}}^2 = 0 \tag{7}$$

// M. Bordag and V. Skalozub, Phys. Rev. D **75**, 125003 (2007) [hep-th/0611256]

// S. Antropov, M. Bordag, V. Demchik and V. Skalozub, Int. J. Mod. Phys. A **26**, 4831 (2011) [arXiv:1011.3147 [hep-ph]]

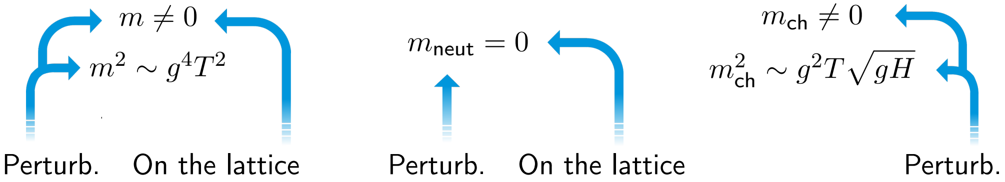
Color-charged gluon fields:

$$m_{\mathrm{D}}^2 \sim g^2 T^2 \left(1 - C\sqrt{gH}/T\right), \qquad m_{\mathrm{magn}}^2 \sim g^2 T \sqrt{gH} \tag{8}$$

// M. Bordag and V. Skalozub, Phys. Rev. D **77**, 105013 (2008) [arXiv:0801.2306 [hep-th]]

// M. Bordag and V. Skalozub, Phys. Rev. D **85**, 065018 (2012) [arXiv:1201.1978 [hep-th]]

Magetic Mass: Analytical Calculations vs Lattice



The aim of this investigation: to show that m_{magn} is produced by the charged components of the gluon field on the lattice

Quantum Gluodynamics on the Lattice

Continuous Minkovsky space-time	$\longrightarrow$	Euclidean 4D discrete lattice
Continuous operators	$\longrightarrow$	Discrete operators on the lattice
Gluon fields	$\longrightarrow$	SU(N) matrices at the links of the lattice

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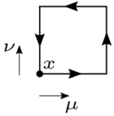
Expectation value of a measured quantity $\mathcal{O}$:

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}U\,\mathcal{O}[U]e^{-S[U]} \quad \longrightarrow \quad \langle\mathcal{O}\rangle \approx \frac{1}{K}\sum_{U_k}\mathcal{O}[U_k] \tag{9}$$

$\mathcal{Z} = \int \mathcal{D}U\, e^{-S[U]}$, $\int \mathcal{D}U = \prod_{x,\mu}\int dU_\mu(x)$, configurations U_k are distributed with probability $\propto e^{-S[U_k]}$.

Lattice Wilson action: $$S_W = \beta\sum_{\mu>\nu}\sum_x\left[1 - \frac{1}{N}\mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu}\right] \tag{10}$$



$$S_W \xrightarrow{a\to 0} \frac{1}{4g^2}\int d^4x\,F^{(c)}_{\mu\nu}(x)F^{(c)}_{\mu\nu}(x) \tag{11}$$

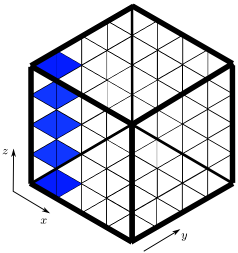
$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U^\dagger_\mu(x+\hat{\nu})U^\dagger_\nu(x) \text{ – plaquette.} \tag{12}$$

Foundations of Our Investigation

1. T. A. DeGrand and D. Toussaint, "The Behavior of Nonabelian Magnetic Fields at High Temperature," Phys. Rev. D **25**, 526 (1982)
   - Screening of the chromomagnetic field of the monopole-antimonopole string was shown
   - Color structure could not be clarified by this method
2. S. Antropov, M. Bordag, V. Demchik and V. Skalozub, "Long range chromomagnetic fields at high temperature," Int. J. Mod. Phys. A **26**, 4831 (2011) [arXiv:1011.3147 [hep-ph]]
   - Zero magnetic mass of the Abelian chromomagnetic field was shown
   - Non-Abelian components of the chromomagnetic field were not investigated

We combine methods of two these papers to separate the contribution of Abelian and non-Abelian components to the m_{magn}

Monopole-Antimonopole String on the Lattice



// T. A. DeGrand and D. Toussaint

$$S = \beta \sum_n \sum_{\mu>\nu} \left[1 - \frac{1}{N} \mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(n) \Xi_{\mu\nu}(n) \right],$$

$$\Xi_{\mu\nu}(n) \in Z(N)$$

Center of the SU(N) group:

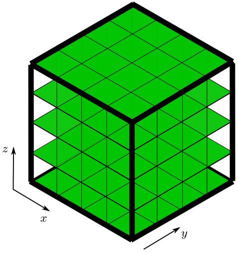
$$Z(N) = \{ \sqrt[N]{1} \cdot I \}$$

SU(2) case: $Z(2) = \{1 \cdot I,\ -1 \cdot I\}$

$\Xi_{\mu\nu}(n) \neq I$ if string $\cap\, U_{\mu\nu}(n)$

$\Xi_{\mu\nu}(n) = -I$ if $x = 0$, $y = 0$, $\forall z, t$

Abelian Field Flux on the Lattice



Plaquette:

// S. Antropov *et al.*

$$\left.\begin{aligned} U_{\mu\nu}(x) &= U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x) \\ U_\mu(n) &= \mathrm{e}^{\,iaA_\mu(n)} \end{aligned}\right\} \Rightarrow U_{\mu\nu}(n) = \mathrm{e}^{\,ia^2F_{\mu\nu}(n)}$$

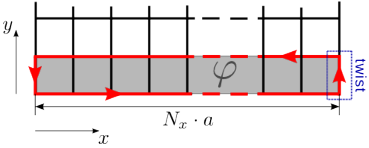
$$U'_{xy} = \mathrm{e}^{\,ia^2(H_z+H_z^{\text{ext}})} = U_{xy}\,\mathrm{e}^{\,ia^2H_z^{\text{ext}}}$$

$$U'_y(0,n_y,n_z,n_t) = U_y(0,n_y,n_z,n_t)\,\mathrm{e}^{\,i\varphi}$$

$$\varphi = a^2N_xH_z^{\text{ext}}$$

Twisted boundary conditions:

$$\begin{cases} U_y(N_x,n_y,n_z,n_t) = U_y(0,n_y,n_z,n_t)\,\mathrm{e}^{\,i\varphi}, \\ U_\mu(N_x,n_y,n_z,n_t) = U_\mu(0,n_y,n_z,n_t), \quad \mu\neq y, \\ \\ U_\mu(n_x,N_y,n_z,n_t) = U_\mu(n_x,0,n_z,n_t), \\ U_\mu(n_x,n_y,N_z,n_t) = U_\mu(n_x,n_y,0,n_t), \\ U_\mu(n_x,n_y,n_z,N_t) = U_\mu(n_x,n_x,n_z,0). \end{cases}$$



$$\mathrm{e}^{\,i\varphi} = \mathrm{e}^{\,i\varphi_3\sigma_3/2} = \begin{pmatrix} \mathrm{e}^{\,i\varphi_3/2} & 0 \\ 0 & \mathrm{e}^{\,-i\varphi_3/2} \end{pmatrix}$$

Outline of the Investigation

Lattices: $N_t \times N^3$, $N_t = \text{const}$

Measured quantity: $\langle U \rangle = \langle \mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu} \rangle$

Investigated quantity: $f(N) = |\langle U \rangle_{\text{field}} - \langle U \rangle_0|$

Possibilities for f:

- $f \sim 1/N^2$ – flux tubes, the flux is conserved;
- $f \sim 1/N^4$ – Coulombic behavior, flux spreads out over the available area;
- $f \sim \mathrm{e}^{-kN^2}$ – screening of the field; $k = m^2_{\text{magn}}$;
- $f \sim 1/N$ – spontaneous field generation, flux increases with distance.

Simulations are performed

- in absence of external Abelian field flux φ;
- in presence of external Abelian field flux φ:

  φ is directed parallel to the monopole-antimonopole string.

Simulations Setup

Lattices used: $4 \times N^3$, $N = 6, 8, \ldots, 72$

External Abelian field flux $\varphi = 0.08$ ($\sim 10^4$ MeV^2)

$\beta = 2.835$ ($T \sim 1.2$ GeV)
$\beta = 3.020$ ($T \sim 1.9$ GeV)
$\beta = 3.091$ ($T \sim 2.3$ GeV)

Simulations are performed with the QCDGPU program
(`https://github.com/vadimdi/QCDGPU`, V. Demchik, N. K., Comp. Sc. and Appl.,**1**, 1 (2014) [arXiv:hep-lat/1310.7087])

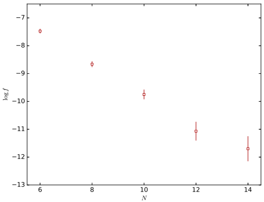
hYBRILIT/JINR

HPC Village

https://openwall.info/wiki/HPC/Village

hgpu.org

χ^2-analysis of the Data



$$f_i = |\langle U\rangle_{\text{field}} - \langle U\rangle_0|_i$$

The data are fitted through minimization of χ^2 function:

$$\chi^2(a) = \sum_{i=1}^{K} \frac{[y_i - \log f(N_i;a)]^2}{\sigma_i^2}, \tag{13}$$

$$y_i = \log f_i, \qquad f(N_i;a) = \frac{A}{N^b}\, \mathrm{e}^{-kN^q}\,.$$

$\chi^2_{min} = \chi^2(\hat{a}) \sim \chi^2_\nu; \qquad \nu = K - L; \qquad L = \text{Length}\, a$

Hypothesis testing:

- H_0: $f(N_i;a)$ describes the data; $\Leftrightarrow$ $\chi^2_{min} \leq \chi^2_{\nu;0.05}$
- H_1: $f(N_i;a)$ does not describe the data. $\Leftrightarrow$ $\chi^2_{min} > \chi^2_{\nu;0.05}$

Functions describing the data

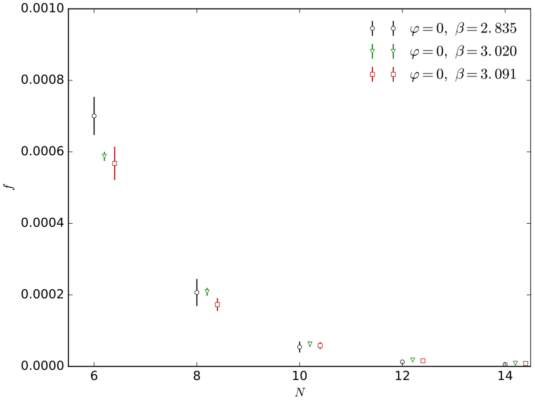
$\Delta\chi^2 = \chi^2(a) - \chi^2_{min} \sim \chi^2_L$

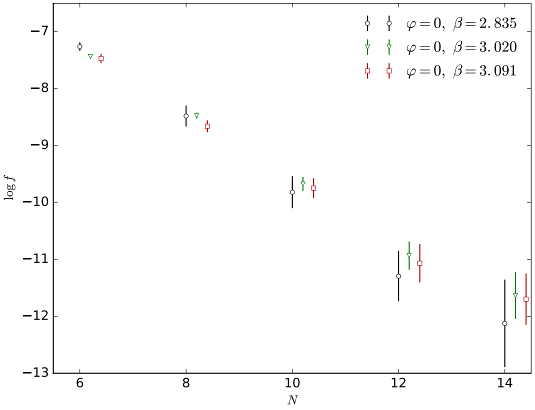
$\chi^2(a) \leq \chi^2_{min} + \chi^2_{L;0.05} \qquad \Rightarrow$ 95% CIs for a

CIs for screening parameters

Results: Data at $\varphi = 0$

$$f(N) = |\langle U\rangle_{\text{field}} - \langle U\rangle_0|$$





Results: Fitting at $\varphi = 0$

Function	$\beta = 2.835$: χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	$\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$	$\beta = 3.020$: χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	$\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$	$\beta = 3.091$: χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	$\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$
A/N	137	9.49	✗	–	509	9.49	✗	–	190	9.49	✗	–
A/N^2	80.4	9.49	✗	–	247	9.49	✗	–	102	9.49	✗	–
A/N^4	14.0	9.49	✗	–	19.5	9.49	✗	–	9.53	9.49	✗	–
$A\,e^{-kN}$	0.40	7.81	✓	63.9 ± 10.9	2.19	7.81	✓	54.4 ± 4.5	0.98	7.81	✓	56.8 ± 8.0
$A\,e^{-kN^2}$	3.18	7.81	✓	3.68 ± 0.63	11.8	7.81	✗	3.33 ± 0.28	10.3	7.81	✗	3.18 ± 0.45
$(A/N)\,e^{-kN}$	0.60	7.81	✓	51.7 ± 10.9	4.14	7.81	✓	41.5 ± 4.5	0.64	7.81	✓	44.8 ± 8.0
$(A/N)\,e^{-kN^2}$	1.49	7.81	✓	2.99 ± 0.63	4.09	7.81	✓	2.55 ± 0.28	5.10	7.81	✓	2.52 ± 0.45
$(A/N^2)\,e^{-kN}$	0.99	7.81	✓	39.5 ± 10.9	7.29	7.81	✓	28.6 ± 4.5	0.77	7.81	✓	32.7 ± 8.0
$(A/N^2)\,e^{-kN^2}$	0.63	7.81	✓	2.30 ± 0.63	2.16	7.81	✓	1.78 ± 0.28	1.89	7.81	✓	1.85 ± 0.45
$(A/N^4)\,e^{-kN}$	2.32	7.81	✓	15.1 ± 10.9	17.2	7.81	✗	2.80 ± 4.52	2.45	7.81	✓	8.65 ± 7.96
$(A/N^4)\,e^{-kN^2}$	1.36	7.81	✓	0.91 ± 0.63	15.5	7.81	✗	0.23 ± 0.28	1.58	7.81	✓	0.52 ± 0.45

Results: Fitting at $\varphi = 0$

	$\beta = 2.835$				$\beta = 3.020$				$\beta = 3.091$			
Function	χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	$\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$	χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	$\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$	χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	$\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$
A/N	137	9.49	✗	–	509	9.49	✗					
A/N^2	80.4	9.49	✗	–	247	9.49	✗					
A/N^4	14.0	9.49	✗	–	19.5	9.49	✗	–	9.53			
$A\,e^{-kN}$	0.40	7.81	✓	63.9 ± 10.9	2.19	7.81	✓	54.4 ± 4.5	0.98			
$A\,e^{-kN^2}$	3.18	7.81	✓	3.68 ± 0.63	11.8	7.81	✗	3.33 ± 0.28	10.3			
$(A/N)\,e^{-kN}$	0.60	7.81	✓	51.7 ± 10.9	4.14	7.81	✓	41.5 ± 4.5	0.64			
$(A/N)\,e^{-kN^2}$	1.49	7.81	✓	2.99 ± 0.63	4.09	7.81	✓	2.55 ± 0.28	5.10			
$(A/N^2)\,e^{-kN}$	0.99	7.81	✓	39.5 ± 10.9	7.29	7.81	✓	28.6 ± 4.5	0.77			
$(A/N^2)\,e^{-kN^2}$	0.63	7.81	✓	2.30 ± 0.63	2.16	7.81	✓	1.78				
$(A/N^4)\,e^{-kN}$	2.32	7.81	✓	15.1 ± 10.9	17.2	7.81	✗	2.80				
$(A/N^4)\,e^{-kN^2}$	1.36	7.81	✓	0.91 ± 0.63	15.5	7.81	✗	0.23				

PHYSICAL REVIEW D VOLUME 25, NUMBER 2 15 JANUARY 1982

Behavior of non-Abelian magnetic fields at high temperature

T. A. DeGrand* and D. Toussaint

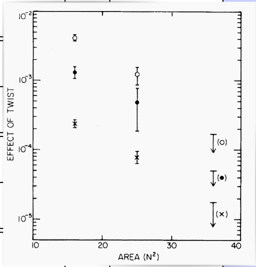
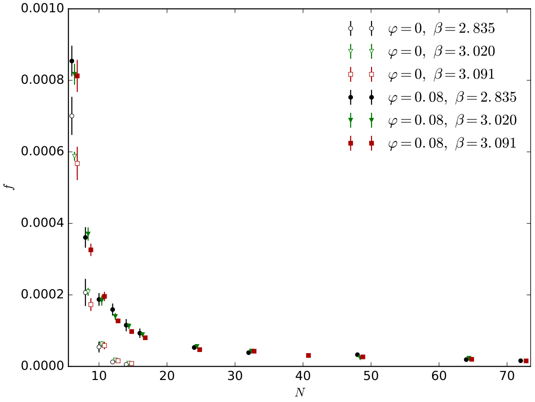


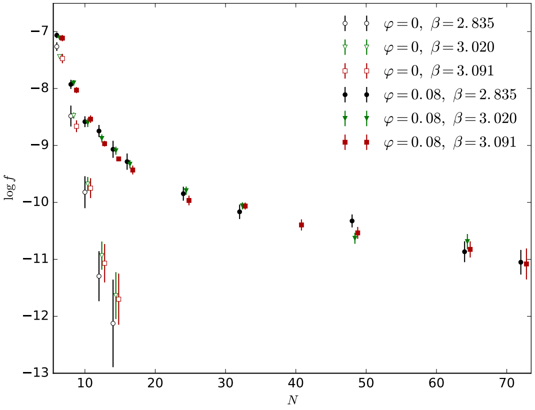
TABLE II. Fits to quantities to a/N^2, b/N^4, or Ce^{-kN^2}, with χ^2.

Quantity	a	χ^2	b	χ^2	c	k	χ^2
$\langle U_{xy} - \frac{1}{2}(U_{xz}+U_{yz})\rangle_{tw}$	0.0216 ± 0.0027	26.5	0.53 ± 0.06	7.5	0.021	0.136 ± 0.021	1.1
$\langle U\rangle_{tw} - \langle U\rangle_{no}$	0.0112 ± 0.0028	12.6	0.28 ± 0.06	5.1	0.0187	0.165 ± 0.054	1.3
$\langle U_{xy}\rangle_{tw} - \langle U_{xy}\rangle_{no}$	0.0367 ± 0.045	38.6	0.89 ± 0.09	12.3	0.0507	0.157 ± 0.031	1.5

Results: Data at $\varphi = 0.08$

$$f(N) = |\langle U \rangle_{\text{field}} - \langle U \rangle_0|$$





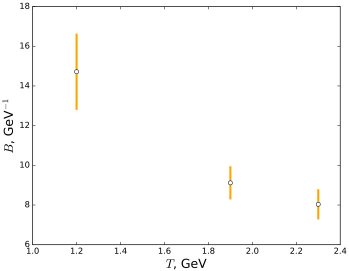
Results: Fitting at $\varphi = 0.08$

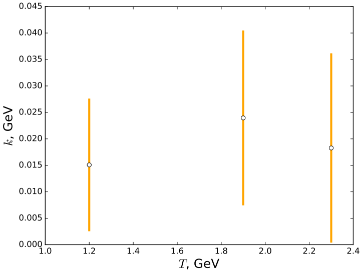
Function	$\beta = 2.835$: χ^2_{min}	$\beta = 2.835$: $\chi^2_{\nu;0.05}$	$\beta = 2.835$: a/r	$\beta = 3.020$: χ^2_{min}	$\beta = 3.020$: $\chi^2_{\nu;0.05}$	$\beta = 3.020$: a/r	$\beta = 3.091$: χ^2_{min}	$\beta = 3.091$: $\chi^2_{\nu;0.05}$	$\beta = 3.091$: a/r
A/N^b	91.2	16.9	✗	170	15.5	✗	223	18.3	✗
$(A/N^b)\,e^{-kN}$	30.0	15.5	✗	44.9	14.1	✗	69.0	16.9	✗
$(A/N^b)\,e^{-kN^2}$	47.2	15.5	✗	73.7	14.1	✗	118	16.9	✗
$A\,e^{B/N}\,e^{-kN}$	5.33	15.5	✓	7.14	14.1	✓	7.00	16.9	✓

$\beta = 2.835$: $B = 20.3 \pm 2.64$, $k = (1.09 \pm 0.91) \times 10^{-2}$

$\beta = 3.020$: $B = 20.1 \pm 1.84$, $k = (1.08 \pm 0.75) \times 10^{-2}$

$\beta = 3.091$: $B = 21.3 \pm 2.01$, $k = (6.90 \pm 6.76) \times 10^{-3}$

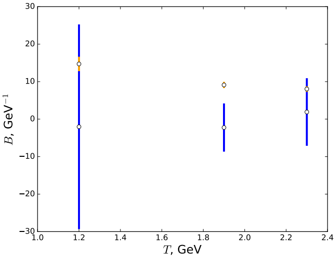


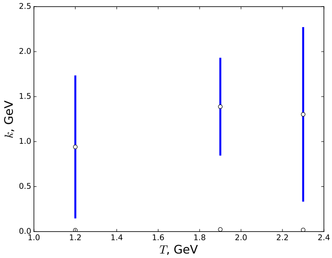


Comparison of the results

$f(N) = A\,\mathrm{e}^{B/N}\,\mathrm{e}^{-kN}$

	$\beta = 2.835$			$\beta = 3.020$			$\beta = 3.091$		
	χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r	χ^2_{min}	$\chi^2_{\nu;0.05}$	a/r
$\varphi = 0$	0.36	5.99	✓	1.34	5.99	✓	0.64	5.99	✓
	$B = -2.81 \pm 37.6$ $k = (6.83 \pm 5.76) \times 10^{-1}$			$B = -4.95 \pm 14.2$ $k = (6.29 \pm 2.46) \times 10^{-1}$			$B = 5.04 \pm 23.9$ $k = (4.92 \pm 3.66) \times 10^{-1}$		
$\varphi = 0.08$	5.33	15.5	✓	7.14	14.1	✓	7.00	16.9	✓
	$B = 20.3 \pm 2.64$ $k = (1.09 \pm 0.91) \times 10^{-2}$			$B = 20.1 \pm 1.84$ $k = (1.08 \pm 0.75) \times 10^{-2}$			$B = 21.3 \pm 2.01$ $k = (6.90 \pm 6.76) \times 10^{-3}$		





$$m_0 = 1.26 \pm 0.41 \text{ GeV}$$

$$m_{0.08} = (1.83 \pm 0.87) \times 10^{-2} \text{ GeV}$$

at 95% CL

Conclusions

- Both monopole-antimonopole string and external Abelian field flux are introduced on the lattice.
- Results of the previous investigations are reproduced.
- It is shown that adding of the Abelian field flux weakens the screening of the string field. This confirms that
  - for the Abelian field $m_{\mathsf{magn}} = 0$;
  - m_{magn} of the monopole-antimonopole string field is produced by its non-Abelian components.

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Thank you for your attention!