Lattice study of dense two-color quark matter at low temperature

N.Yu. Astrakhantsev, V.G. Bornyakov, V.V. Braguta, E.-M. Ilgenfritz, A.Yu. Kotov, A.A. Nikolaev, A.V. Molochkov, A. Rothkopf

IHEP, Protvino, Russia

ITEP, Moscow, Russia

FEFU, Vladivostok, Russia

JINR, Dubna, Russia

University of Stavanger, Norway

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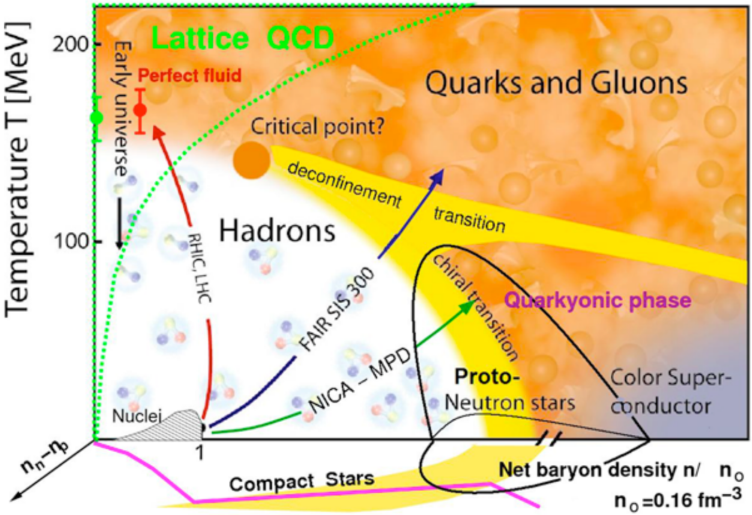
Outline:

1. Introduction
2. Dense quark matter at low and moderate densities
3. Dense quark matter at high density
4. Confinement/deconfinement transition at finite density
5. Polyakov lines correlation functions in dense quark matter
6. Conclusions

Based on papers: PRD**94** (2016) 114510; JHEP 1803 (2018) 161; arXiv:1808.06466

QCD phase diagram



Picture source: The Phases of Dense Matter, INT, July 11 - August 12, 2016

SU(3) QCD

- $Z = \int DUD\bar{\psi}D\psi \;\exp\left(-S_G - \int d^4x\bar{\psi}(\hat{D}+m)\psi\right) =$
  $= \int DU \exp\left(-S_G\right) \times \det\left(\hat{D}+m\right)$
- Eigenvalues go in pairs $\hat{D}: \;\pm i\lambda \Rightarrow$
  $\det\left(\hat{D}+m\right) = \prod_\lambda(\lambda^2+m^2) > 0$
  i.e. one can use lattice simulation
- Introduction of the chemical potential:
  $\det\left(\hat{D}+m\right) \to \det\left(\hat{D}-\mu\gamma_4+m\right) \Rightarrow$
  the determinant becomes complex (**sign problem**)

SU(2) QCD

- $(\gamma_5 C\tau_2)\cdot D^* = D\cdot(\gamma_5 C\tau_2)$
- Eigenvalues go in pairs $\hat{D}-\mu\gamma_4$: λ, λ^*
- For even N_f $\det\left(\hat{D}-\mu\gamma_4+m\right) > 0 \;\Rightarrow$ **free from sign problem**

Differences between SU(3) and SU(2) QCD

- The Lagrangian of the SU(2) QCD has the symmetry: $SU(2N_f)$ as compared to $SU_R(N_f) \times SU_L(N_f)$ for $SU(3)$ QCD
- Goldstone bosons ($N_f = 2$) $\pi^+, \pi^-, \pi^0, d, \bar{d}$

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However, in dense medium:

- **Chiral symmetry is restored**
  symmetry breaking pattern is not important
- **Relevant degrees of freedom are quarks and gluons**
  rather than Goldstone bosons

Global symmetries in SU(2) QCD

$$\mathcal{L} = \overline{\psi}\gamma_\mu D_\mu \psi = i \begin{pmatrix} \psi_L^* \\ \psi_R^* \end{pmatrix}^T \begin{pmatrix} \sigma_\mu D_\mu & 0 \\ 0 & -\sigma_\mu^\dagger D_\mu \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\sigma_\mu = (\sigma_k, -i), \quad \sigma_2 \sigma_\mu \sigma_2 = -\sigma_\mu^T$$

4-spinor may be defined as $\Psi = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ \sigma_2 \tau_2 \psi_R^* \end{pmatrix}$ and

$$\mathcal{L} = i \begin{pmatrix} \psi_L^* \\ \tilde{\psi}_R^* \end{pmatrix}^T \begin{pmatrix} \sigma_\mu D_\mu & 0 \\ 0 & \sigma_\mu D_\mu \end{pmatrix} \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} = i \Psi^\dagger \sigma_\mu D_\mu \Psi$$

- $SU(2N_f)$ symmetry instead of $SU_R(N_f) \times SU_L(N_f)$
- Symmetry breaking scenario is $SU(2N_f) \to Sp(2N_f)$, Goldstone bosons ($N_f = 2$): $\pi^+, \pi^-, \pi^0, d, \bar{d}$

Similarities:

- There are transitions: confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of observables are equal up to few dozens percent:

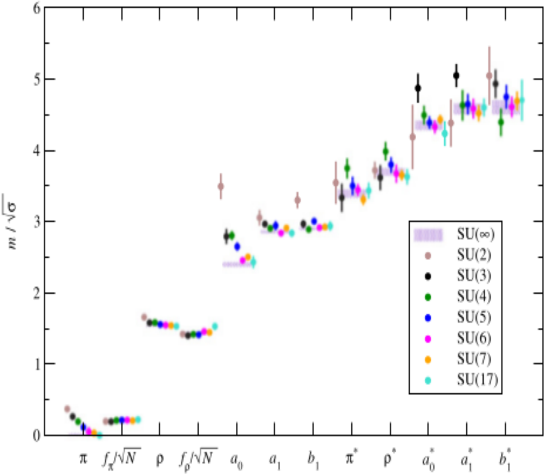
Topological susceptibility [B. Lucini *et. al.*, Nucl. Phys. B715 (2005) 461]:
$\chi^{1/4}/\sqrt{\sigma} = 0.3928(40)\ (SU(2)),\quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35)\ (SU(3))$

Critical temperature [B. Lucini *et. al.*, Phys. Lett. B712 (2012) 279]:
$T_c/\sqrt{\sigma} = 0.7092(36)\ (SU(2)),\quad T_c/\sqrt{\sigma} = 0.6462(30)\ (SU(3))$

Shear viscosity:
$\eta/s = 0.134(57)\ (SU(2))$ [N.Yu. Astrakhantsev *et. al.*, JHEP 1509 (2015) 082]
$\eta/s = 0.102(56)\ (SU(3))$ [H.B. Meyer, PRD 76 (2007) 101701]

Similarities:

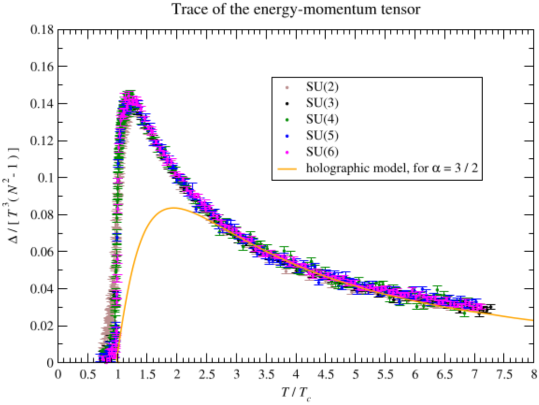
- Spectroscopy (Phys. Rep. 529 (2013) 93; PRD 94, 034506 (2016))



Similarities:

- Thermodynamic properties (JHEP 1205 (2012) 135)
- Some properties of dense medium (PRD 59 (1999) 094019):

$$\Delta \sim \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$



Intermediate summary:

- Dense SU(2) QCD can be used to study dense SU(3) QCD
  - Calculation of different observables
  - Study of various physical phenomena
- Lattice study of SU(2) QCD contains full dynamics of real system (contrary to phenomenological models)

Diquark source

In QC_2D there is a possibility to add diquark source to the action to study spontaneous breakdown of $U(1)_V$:

$$S_F = \sum_{x,y} \left[\overline{\chi}_x M(\mu_q)_{xy} \chi_y + \frac{\lambda}{2} \delta_{xy} \left(\chi^T \tau_2 \chi + \overline{\chi} \tau_2 \overline{\chi}^T \right) \right],$$

which modifies partition function as follows:

$$Z = \int DU \, det \left[M^\dagger(\mu_q) M(\mu_q) + \lambda^2 \right]^{\frac{1}{2}} e^{-S_G[U]}$$

instead of

$$Z = \int DU \, det \, M(\mu_q) e^{-S_G[U]} \, .$$

$\langle qq \rangle$ is colorless, gauge invariant and thus may be measured.

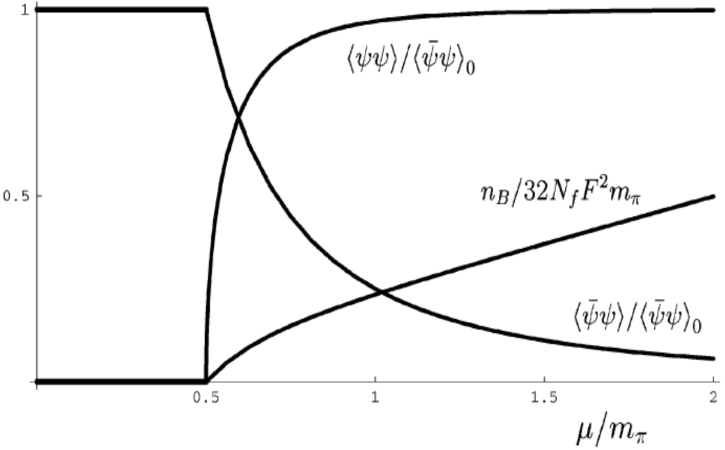
Study of QCD at small and moderate densities

- Staggered fermions discretization
  $S_F = \sum_x (ma)\overline{\chi}_x \chi_x + \frac{1}{2}\sum_{x,\mu} \eta_\mu(x)\left(\overline{\chi}_{x+\mu} U_{x,\mu}\chi_x - \overline{\chi}_x U^\dagger_{x,\mu}\chi_{x+\mu}\right)$
  $\lim_{a\to 0} S_F \to \int d^4x\, \overline{\psi}(\hat{D}+m)\psi$
- Rooting to get $N_f = 2$
- Diquark source in the action $\delta S \sim \lambda \psi^T (C\gamma_5) \times \sigma_2 \times \tau_2 \psi$
- Wilson gauge action
- $a = 0.112$ fm
- Lattice size: $16^3 \times 32$, $(1.75 \text{ fm})^3$
- $m_\pi = 378(4)$ MeV, $m_\pi L_s \approx 3.4$

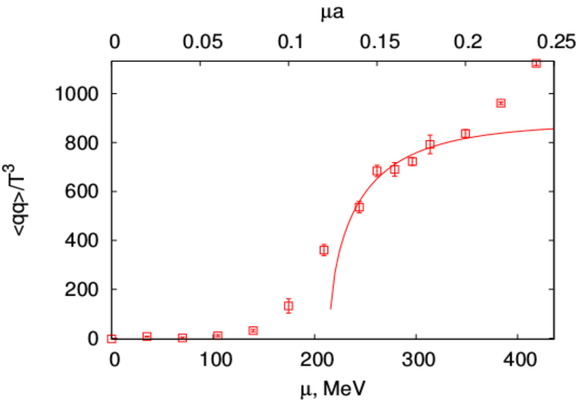
The results are presented in paper PRD**94** (2016) 114510

Small and moderate chemical potentials

Predictions of LO ChPT

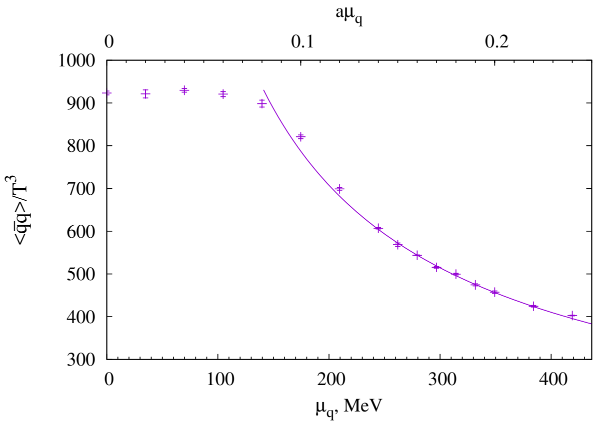


Diquark condensate



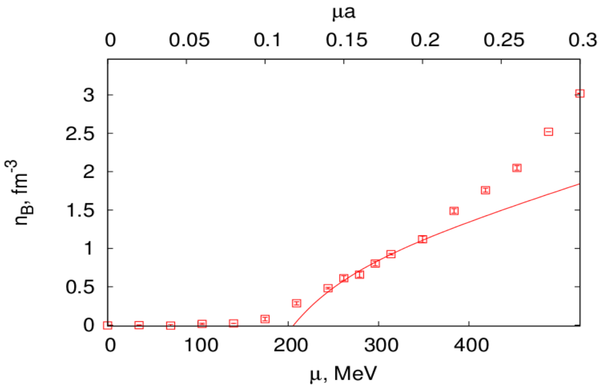
- Good agreement with ChPT $\langle\psi\psi\rangle/\langle\bar{\psi}\psi\rangle_0 = \sqrt{1 - \frac{m_\pi^4}{\mu^4}}$
- Phase transition at $\mu \sim m_\pi/2$
- Bose Einstein condensate (BEC) phase $\mu \in (200, 350)$ MeV

Chiral condensate



- Good fit $\langle \bar{q}q \rangle = A/\mu^\alpha$ with $\alpha = 0.78(2)$, $\chi^2_{dof} = 0.3$
- LO ChPT predicts $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0 = \mu_c^2/\mu^2$
- Similar slower decrease with $\alpha = 1 \ldots 1.3$ was observed in Nucl. Phys. **B 642**, 181 (2002) and PRD **87**, 034507 (2013)

Baryon density



- Good agreement with LO ChPT $n \sim \mu - \frac{m_\pi^4}{\mu^3}$
- At $\mu_0 \sim$ few $\times$ 100 MeV the deviation from CHPT is seen
- The region $\mu < \mu_0$ – **dilute baryon gas**
- The region $\mu > \mu_0$ – **dense quark matter**

Large chemical potentials

Phase diagram for $N_c \to \infty$

L. McLerran, R.D. Pisarski, Nucl.Phys. A796 (2007) 83-100

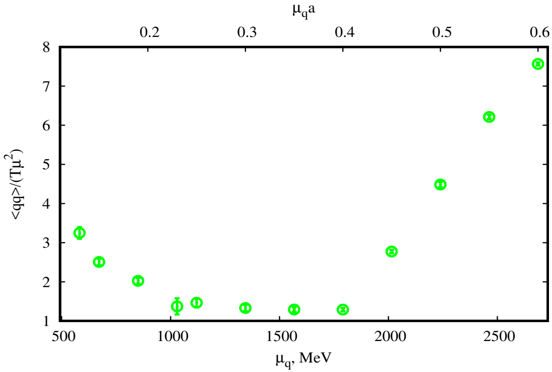
- Hadron phase $\mu < M_N/N_c$ ($p \sim O(1)$)
- Dilute baryon gas $\mu > M_N/N_c$ (width $\delta\mu \sim \frac{\Lambda_{QCD}}{N_c^2}$)
- Quarkyonic phase $\mu > \Lambda_{QCD}$ ($p \sim N_c$)
  - Degrees of freedom:
    - Baryons (on the surface)
    - Quarks (inside the Fermi sphere $|p| < \mu$)
  - Chiral symmetry is restored
  - The system is in confinement phase
- Deconfinement ($p \sim N_c^2$)

Study of QCD at high densities

- Staggered fermions discretization
  $S_F = \sum_x (ma)\overline{\chi}_x \chi_x + \frac{1}{2}\sum_{x,\mu} \eta_\mu(x)\big(\overline{\chi}_{x+\mu} U_{x,\mu}\chi_x - \overline{\chi}_x U^\dagger_{x,\mu}\chi_{x+\mu}\big)$
  $\lim_{a\to 0} S_F \to \int d^4x\, \overline{\psi}(\hat{D}+m)\psi$
- Rooting to get $N_f = 2$
- Diquark source in the action $\delta S \sim \lambda \psi^T (C\gamma_5) \times \sigma_2 \times \tau_2 \psi$
- Symanzik tree-level improved gauge action
- $a = 0.044$ fm
  $\Rightarrow$ **close to continuum limit,**
  one can reach larger density without lattice artifacts
  $\mu > 2000$ **MeV**
- Lattice size: $32^3 \times 32$, $(1.4\ \text{fm})^3$
- $m_\pi = 740(40)$ MeV, $m_\pi L_s \approx 5$

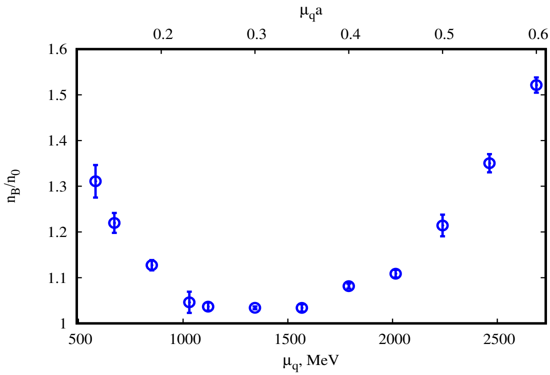
The results are presented in JHEP 1803 (2018) 161; arXiv:1808.06466

Diquark condensate



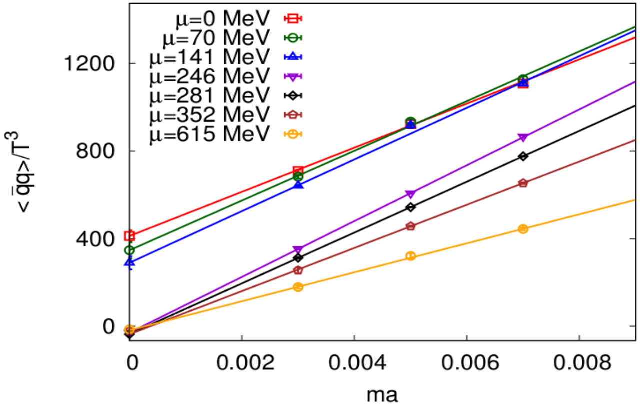
- Bardeen–Cooper–Schrieffer (BCS) phase $\mu > 800$ MeV, $\langle qq \rangle \sim \mu^2$
- **Baryons (on the surface)**

Baryon density



- Free quarks $n_0 = N_f \times N_c \times (2s+1) \times \int \frac{d^3p}{(2\pi)^3}\theta(|p|-\mu) = \frac{4}{3\pi^2}\mu^3$
- **Quarks inside Fermi sphere**
- Quarks inside Fermi sphere dominate over the surface: $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$

Chiral condensate in the chiral limit



Chiral symmetry is restored

Quarkyonic phase:

- Baryons (on the surface)

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- The system is in confinement phase (?)

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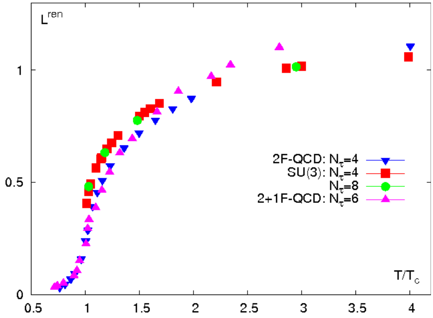
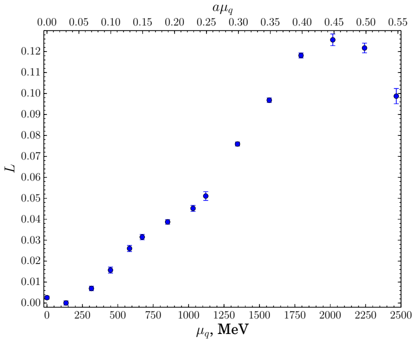
What about confinement in cold dense quark matter?

Polyakov loop at finite temperature and density

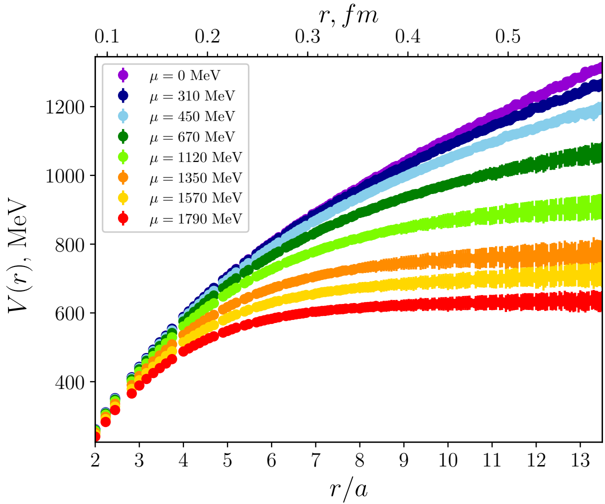




Polyakov loop and confinement/deconfinement transition

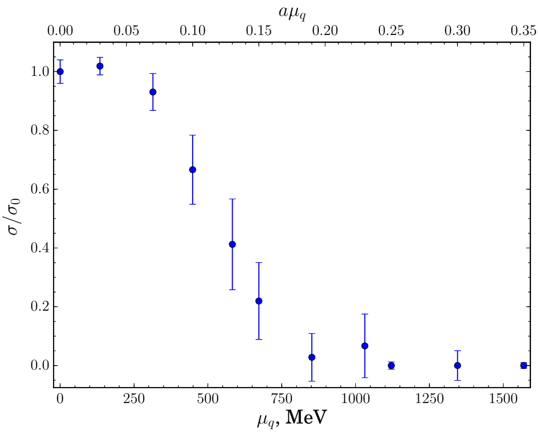
- Rapid transition at finite T
- Smooth transition at finite μ
- Nontrivial physics at $\mu > 2000$ MeV

Interaction potential between static quark-antiquark pair



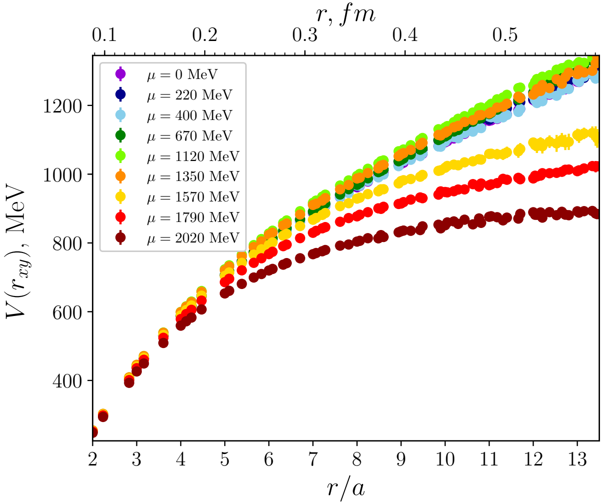
We observe deconfinement in dense medium!

String tension

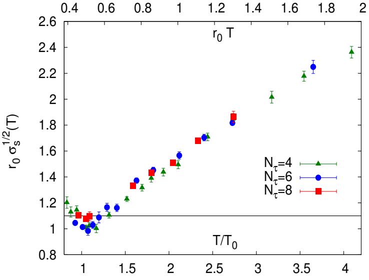


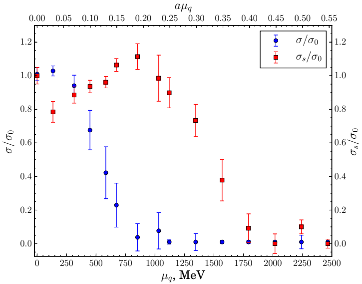
- Good fit by the Cornell potential: $V(r) = A + \frac{B}{r} + \sigma r \quad \mu \leq 1100$ MeV
- Good fit by the Debye potential: $V(r) = A + \frac{B}{r} e^{-m_D r} \quad \mu \geq 850$ MeV
- Confinement/deconfiniment transition in $\mu \in (850, 1100)$ MeV

Spatial quark-antiquark potential in dense medium



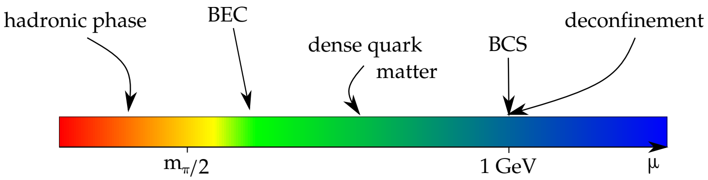
Spatial string tension

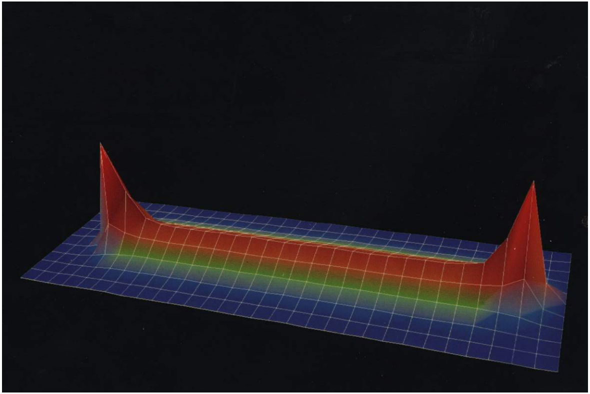




- Deconfinement at $\mu > 900 - 1100$ MeV (?)
- Spatial string tension disappears at $\mu \geq 1800$ MeV ($a\mu > 0.4$)
- Different behaviour of spatial string tension at finite temperature and finite density

Tentative phase diagram





Polyakov lines correlation function

- $\frac{\Omega(r,\mu)}{T} = -(1/N_c^2)\log\left[\langle \mathrm{Tr}L(\vec{r})\,\mathrm{Tr}L^\dagger(0)\rangle\right]$
- Ω is grand potential – fundamental object in QCD
- Describes interaction of quark-antiquark pair
- Sensitive to phase transitions and properties of QCD medium

Grand potentials of a static quark-antiquark pair

In Coulomb gauge:

$$\begin{aligned}
\exp\left[-\Omega_{\bar{q}q}(r,\mu)/T\right] &= (1/4)\left\langle \mathrm{Tr}L(\vec{r})\mathrm{Tr}L^\dagger(0)\right\rangle \\
\exp\left[-\Omega_{\mathbf{1}}(r,\mu)/T\right] &= (1/2)\left\langle \mathrm{Tr}\left[L(\vec{r})L^\dagger(0)\right]\right\rangle \\
\exp\left[-\Omega_{\mathbf{3}}(r,\mu)/T\right] &= (1/3)\left\langle \mathrm{Tr}L(\vec{r})\mathrm{Tr}L^\dagger(0)\right\rangle - (1/6)\left\langle \mathrm{Tr}\left[L(\vec{r})L^\dagger(0)\right]\right\rangle
\end{aligned}$$

Additional relation:

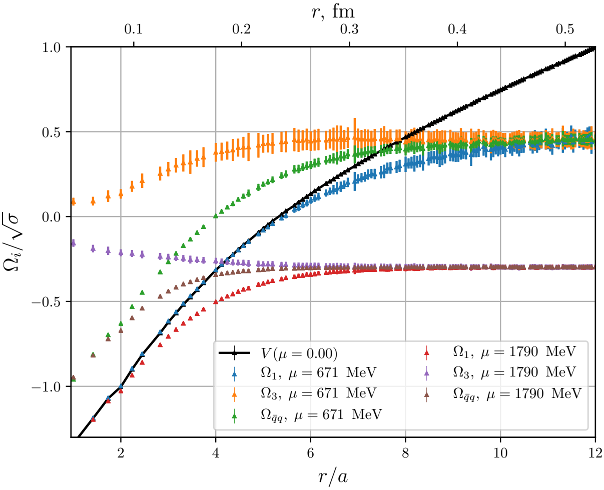
$$exp\left(-\Omega_{\bar{q}q}/T\right) = \frac{1}{4}exp\left(-\Omega_{\mathbf{1}}/T\right) + \frac{3}{4}exp\left(-\Omega_{\mathbf{3}}/T\right)$$

Renormalization:

$$\begin{aligned}
\Omega_{\mathbf{1}}(r\to 0) &= V^{ren.}(r\to 0) \\
\Omega_{\bar{q}q}(r\to\infty) &= \Omega_{\mathbf{1}}(r\to\infty)
\end{aligned}$$

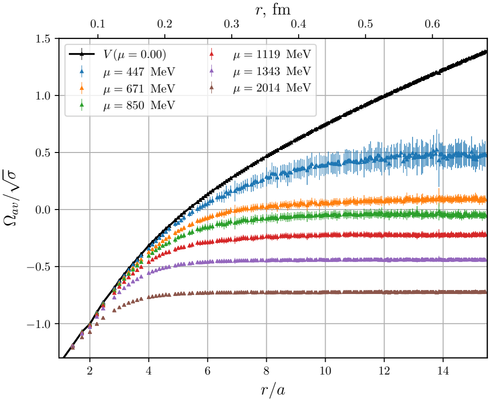
[for details see O. Kaczmarek *et al.*, Phys. Lett. **B543**, 41 (2002)]

Grand potentials of a static quark-antiquark pair



$\Omega_3(r)$ changes the slope at high densities

String breaking in cold dense quark matter

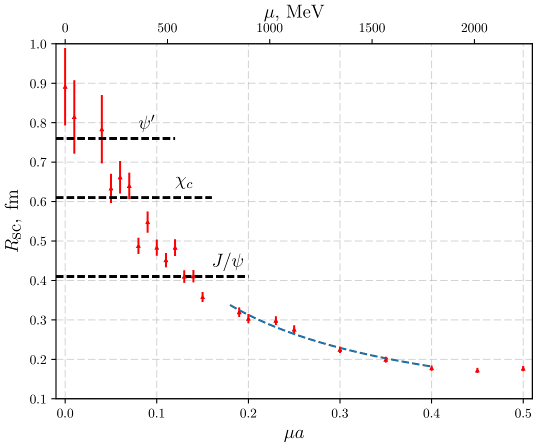


The grand potential and string breaking

- The plateau in the grand potential is the manifestation of the string breaking
- The large the baryon density the smaller the string breaking distance
- Quantitative study of the string breaking phenomenon: the screening length

$$V_{\mu=0}(R_{scr.}) = \Omega_{\bar{q}q}(\infty, \mu)$$

Screening length and quarkonia dissociation

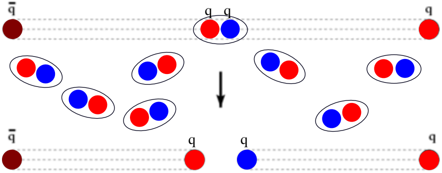


The screening length

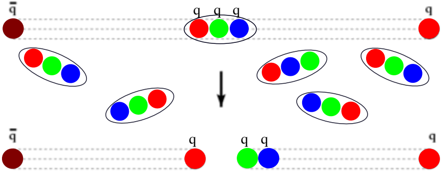
- Onset of quarkonia dissociation (in confinement!)
- In confinement phase the $R_{scr.}$ is described by string breaking
- In deconfinement phase the $R_{scr.}$ is described by Debye screening (blue curve: $R_{SC} = 1/[Am_D(\mu)]$, where $m_D^2(\mu) = (4/\pi)\alpha_s(\mu)\mu^2$)

String breaking in dense medium

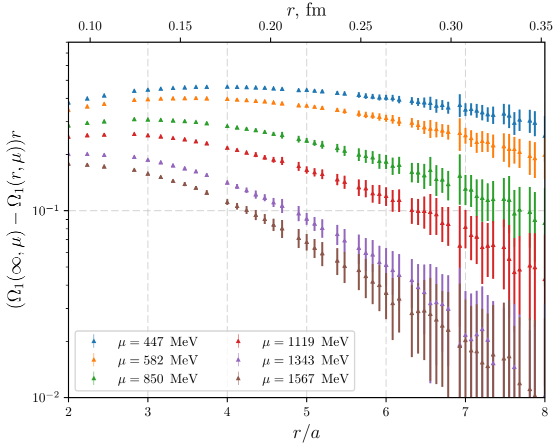
In SU(2) QCD:



Analogous mechanism may be proposed in SU(3) QCD:



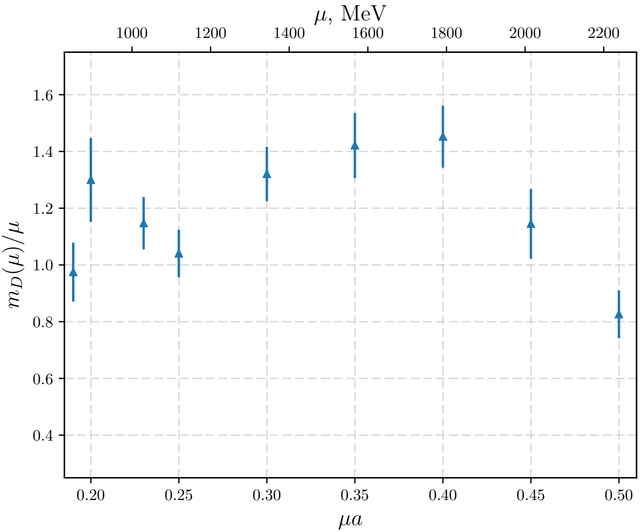
Debye screening in dense medium



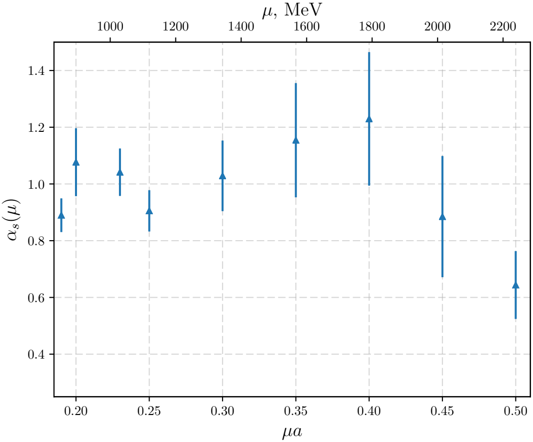
Debye screening in dense cold quark matter

- $\Omega_{\mathbf{1}}(r,\mu) = \Omega_{\mathbf{1}}(\infty,\mu) - \frac{3}{4}\frac{\alpha_s(\mu)}{r} exp(-m_D r)$
- We observe exponential Debye screening
- From the fit we determine the $m_D(\mu)$ and $\alpha_s(\mu)$

Debye mass in cold dense matter

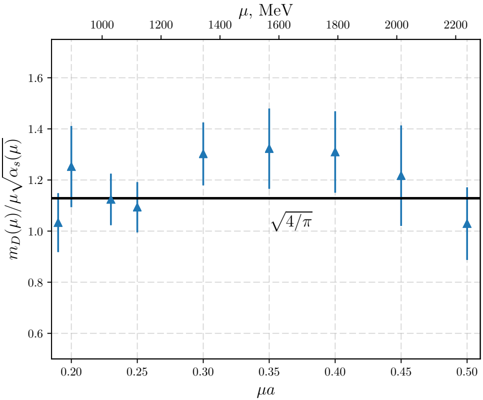


Effective coupling constant in cold dense matter



$\alpha_s \sim 1$ i.e. even at high density QCD is strongly correlated

One-loop formula for the Debye mass



- The one-loop formula:
  $m_D^2(\mu) = \frac{4}{\pi}\alpha_s(\mu)\mu^2 \;\Rightarrow\; \frac{m_D(\mu)}{\mu\sqrt{\alpha_s(\mu)}} = \sqrt{\frac{4}{\pi}}$
- The one-loop formula works well even for the $\alpha_s \sim 1$

Conclusions:

- **First observation of deconfinement in dense medium**
- Difficult to determine critical chemical potential $\mu_c \in (850, 1100)$ MeV
- Spatial string tension disappears $\mu \geq 1800$ MeV
- Deconfinement at finite density is different to deconfinement at finite temperature
- String breaking distance decreases with density
- Heavy quarkonia dissociate at moderate denisties due to string breaking
- We observe Debye screening phenomenon in deconfinement phase