Spin observables in pd->pd and dd->npd processes

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CONTENT

- Motivation:
  NN forces is a basis of nuclear and hadronic physics. **Spin–dependent pp- and pn-elastic scattering amplitudes are not derived from theory**, **but necessary for theoretical interpretation of many nuclear data on spin observables .**

- Phenomenological models for spin-dependent pN elastic scattering.

- Glauber spin-dependent theory of pd-elastic scattering and pN-amplitudes.

- Relations between spin observables in dd→n+p+d and in pd->pd.

- Preliminary summary

- **Amaresh Datta**: about MC simulations for dd →npd with separation of pd→pd.

pN ELASTIC SCATTERING

NN forces is a basis of nuclear and hadronic physics.
NN-> NN is still not well understood, at T>1-3 GeV data on spin dependence of pn-, pp- amplitudes is very noncomplete .
Important TASK: Measurement @ test of spin amplitudes of NN elastic scattering in soft and hard NN- collisions.

$$\phi_1(s,t) = \langle + + |M| + + \rangle,$$
$$\phi_2(s,t) = \langle + + |M| - - \rangle,$$
$$\phi_3(s,t) = \langle + - |M| + - \rangle,$$
$$\phi_4(s,t) = \langle + - |M| - + \rangle,$$
$$\phi_5(s,t) = \langle + + |M| + - \rangle.$$

$$\frac{d\sigma}{dt} = \frac{2\pi}{s^2}\{|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2\}.$$

$$A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2}\,\mathrm{Im}\{\phi_5^*(\phi_1 + \phi_2 + \phi_3 - \phi_4)\},$$
$$A_{NN} \frac{d\sigma}{dt} = \frac{4\pi}{s^2}\{2|\phi_5|^2 + \mathrm{Re}(\phi_1^*\phi_2 - \phi_3^*\phi_4)\},$$

Amplitudes: 5 – pp, 6 – pn, TV gives additional 2

>10 Observables is required

Complete polarization experiment
for pp-elastic requires 9 independent observables.
PWA GWU is performed for pp-elastic up to 3.8 GeV/c (SAID webpage:http://gwdac.phys.gwu.edu.
R.A. Arndt, I.I. Strakovsky, B.L. Workman PRC 56, 3005 (1997);
PWA for pn- elastic – up 1.2 GeV/c

Concerning SPD NICA, above 3 GeV/c $d\sigma/dt$ and mainly A_N (up to 50 GeV/c) and A_{NN}, C_{LL} (up to 6 GeV/c, 12 GeV/c) are measured. Data on double-spin observables D_{NN}, K_{NN} are rather poore in the region of forward angles.

Parametrizations (fit) of the pp- data:
Regge: W.P. Ford, J.W. Van Orden, Phy.Rev. **C87** (2013) 014004;
A. Sibirtsev et al. Eur.Phys.J. **A 45** (2010) 357;
Eikonal: S. Wakaizumi, M. Sawamoto,Prog. Theor. Phys. v.64 (1980) 1699
Regge-eikonal : O. V. Selyugin , Symmetry., 13 N2 (2021) 164;
Phys.Rev.D 110 (2024) 11, 114028

pd elastic scattering within the spin-dependent Glauber model as a test of pN amplitudes

pd-pd: The simplest process with both **pp-** and pn-amplitudes involved.

dd-dd elastic is much more complicated, spin-dependent Glauber formalism is not yet developed.

Elastic $pd \to pd$ transitions

$$\hat{M}(\mathbf{q},\mathbf{s}) =$$

$$\exp(\frac{1}{2}i\mathbf{q}\cdot\mathbf{s})M_{pp}(\mathbf{q}) + \exp(-\frac{1}{2}i\mathbf{q}\cdot\mathbf{s})M_{pn}(\mathbf{q}) +$$

Single pN-scattering

$$+\frac{i}{2\pi^{3/2}}\int \exp(i\mathbf{q}'\cdot\mathbf{s})\Big[M_{pp}(\mathbf{q}_1)M_{pn}(\mathbf{q}_2) + p \leftrightarrow n\Big]d^2\mathbf{q}'.$$

Double scattering

On-shell elastic pN scattering amplitude (**T-even, P-even**)

$$\begin{aligned} M_{pN} &= A_N + (C_N\boldsymbol{\sigma}_1 + C'_N\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}} + B_N(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{k}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{k}}) + \\ &+ (G_N - H_N)(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{n}}) + (G_N + H_N)(\boldsymbol{\sigma}_1\cdot\hat{\mathrm{q}})(\boldsymbol{\sigma}_2\cdot\hat{\mathrm{q}}) \end{aligned}$$

M. Platonova, V. Kukulin, PRC **81** (2010) 014004:

GENERAL SPIN STRUCTURE OF THE pd-pd AMPLITUDES AND SPIN OBSERVABLES

A.A. Temerbayev, Yu. N. U. Yad. Fiz. 78 (2015) 38; Bull. Rus, Ac. Sci. v.80 №3 (2016) 242

Yu. N. Uzikov, A. Bazarova, A.A. Temerbaev,

Physics of Particles and Nuclei, 2022, Vol. 53, No. 2, pp. 419—425.

$$\langle p'\mu', d'\lambda'|T|p\mu, d\lambda\rangle = \varphi_{\mu'}^{+} e_{\beta}^{(\lambda')*} T_{\beta\alpha}(\mathbf{p}, \mathbf{p}', \boldsymbol{\sigma}) e_{\alpha}^{(\lambda)} \varphi_{\mu}, \quad (1)$$

$$T_{xx} = M_1 + M_2\sigma_y \quad T_{xy} = M_7\sigma_z + M_8\sigma_x \quad T_{xz} = M_9 + M_{10}\sigma_y$$
$$T_{yx} = M_{13}\sigma_z + M_{14}\sigma_x \quad T_{yy} = M_3 + M_4\sigma_y \quad T_{yz} = M_{11}\sigma_x + M_{12}\sigma_z$$
$$T_{zx} = M_{15} + M_{16}\sigma_y \quad T_{zy} = M_{17}\sigma_x + M_{18}\sigma_z \quad T_{zz} = M_5 + M_6\sigma_y,$$

12 independent spin amplitudes (i=1,…,12) M_i for P-and T-invariance included

All spin observables can be calculated

$$\frac{d\sigma}{dt} = \frac{1}{6}\mathrm{Tr}MM^+, \quad \mathrm{Tr}MM^+ = 2\sum_{i=1}^{18}|M_i|^2, \qquad (3)$$

$$A_y^d = \mathrm{Tr}MS_yM^+/\mathrm{Tr}MM^+ = -\frac{2}{\sum_{i=1}^{18}|M_i|^2}\mathrm{Im}(M_1M_9^* + M_2M_{10}^* + M_{13}M_{12}^* + M_{14}M_{11}^* + M_{15}M_5^* + M_{16}M_6^*),$$

$$A_y^p = \mathrm{Tr}M\sigma_yM^+/\mathrm{Tr}MM^+ = \frac{2}{\sum_{i=1}^{18}|M_i|^2}[\mathrm{Re}(M_1M_2^* + M_9M_{10}^* + M_3M_4^* + M_{15}M_{16}^* + M_5M_6^*) - \mathrm{Im}(M_8M_7^* + M_{14}M_{13}^* + M_{11}M_{12}^* + M_{17}M_{18}^*)],$$

$$A_{yy} = \mathrm{Tr}MP_{yy}M^+/\mathrm{Tr}MM^+ = 1 - \frac{3}{\sum_{i=1}^{18}|M_i|^2} \times (|M_3|^2 + |M_4|^2 + |M_7|^2 + |M_8|^2 + |M_{17}|^2 + |M_{18}|^2),$$

$$A_{xx} = \mathrm{Tr}MP_{xx}M^+/\mathrm{Tr}MM^+ = 1 - \frac{3}{\sum_{i=1}^{18}|M_i|^2} \times (|M_1|^2 + |M_2|^2 + |M_{13}|^2 + |M_{14}|^2 + |M_{15}|^2 + |M_{16}|^2),$$

$$C_{y,y} = \mathrm{Tr}MS_y\sigma_yM^+/\mathrm{Tr}MM^+ = -\frac{2}{\sum_{i=1}^{18}|M_i|^2} \times [\mathrm{Im}(M_2M_9^* + M_1M_{10}^* + M_{16}M_5^* + M_{15}M_6^*) + \mathrm{Re}(M_{14}M_{12}^* - M_{13}M_{11}^*)],$$

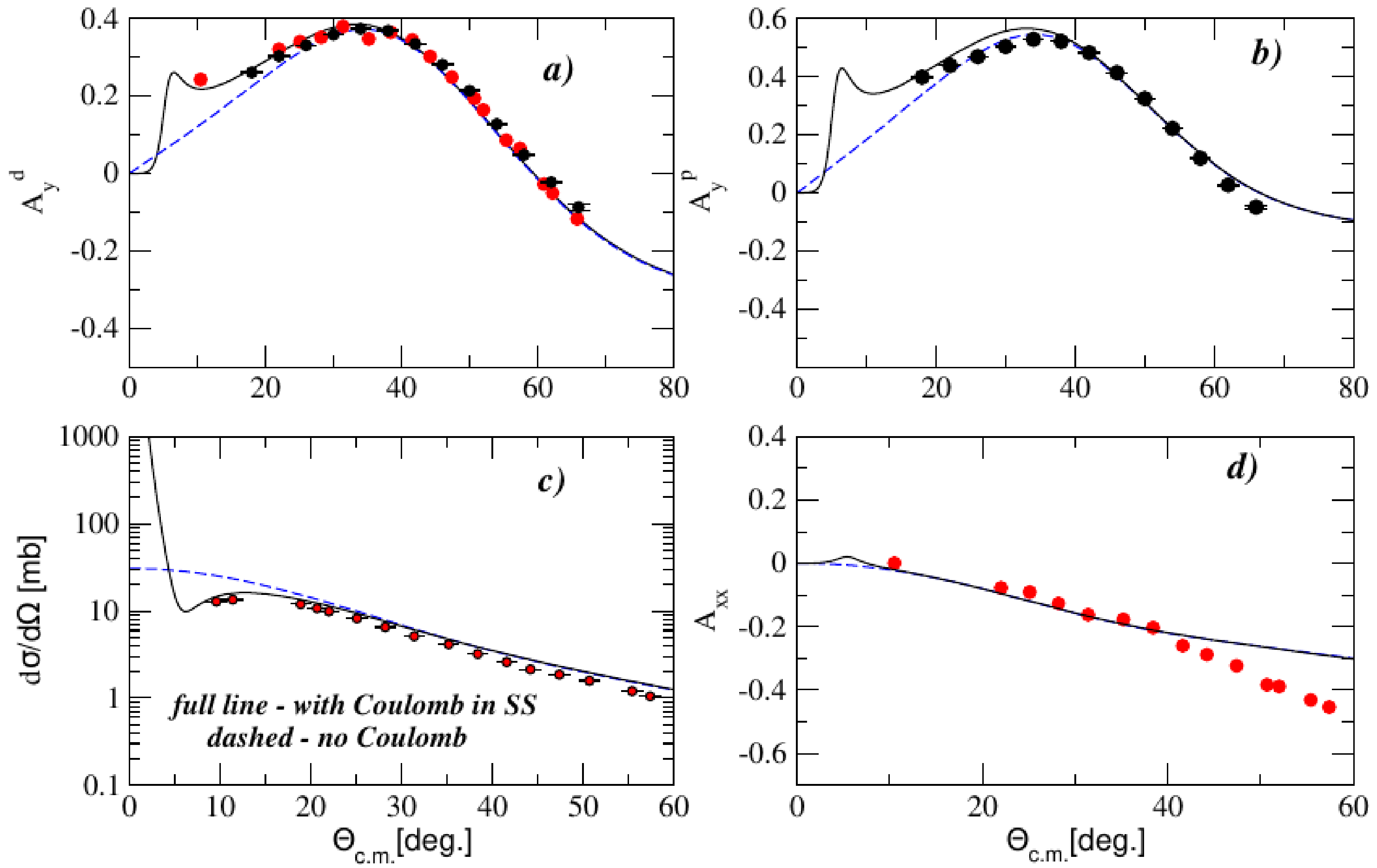
$$C_{x,x} = \mathrm{Tr}MS_x\sigma_xM^+/\mathrm{Tr}MM^+ = -\frac{2}{\sum_{i=1}^{18}|M_i|^2}[\mathrm{Im}(M_8M_9^* + M_3M_{11}^* + M_{17}M_5^*) + \mathrm{Re}(M_7M_{10}^* - M_4M_{12}^* + M_{18}M_6^*)],$$

$$C_{xx,y} = \mathrm{Tr}M_{xx}\sigma_yM^+/\mathrm{Tr}MM^+ = A_y^p - \frac{6}{\sum_{i=1}^{18}|M_i|^2} \times [\mathrm{Re}(M_2M_1^* + M_{16}M_{15}^*) - \mathrm{Im}(M_{14}M_{13}^*),$$

Test calculations: pd elastic scattering at 135 MeV

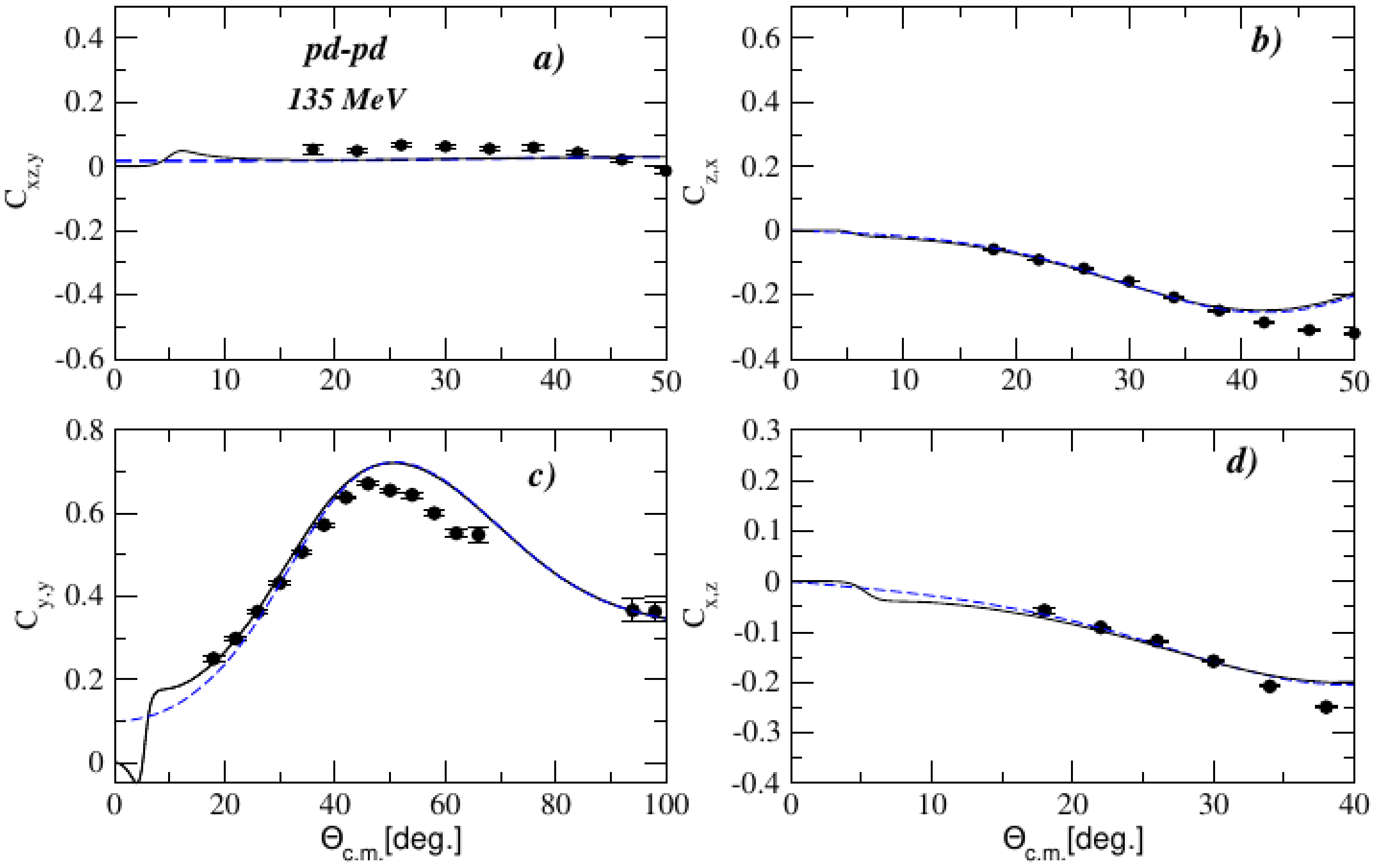
A.A. Temerbayev, Yu.N.Uzikov, Yad. Fiz. **78** (2015) 38

Comparable with results of Faddeev calculations



Data: K. Sekiguchi et al. PRC (2002); B. von Przewoski et al. PRC (2006)
See also Faddeev calculations: A.Deltuva, A.C. Fonseca, P.U. Sauer, PRC 71 (2005) 054005.

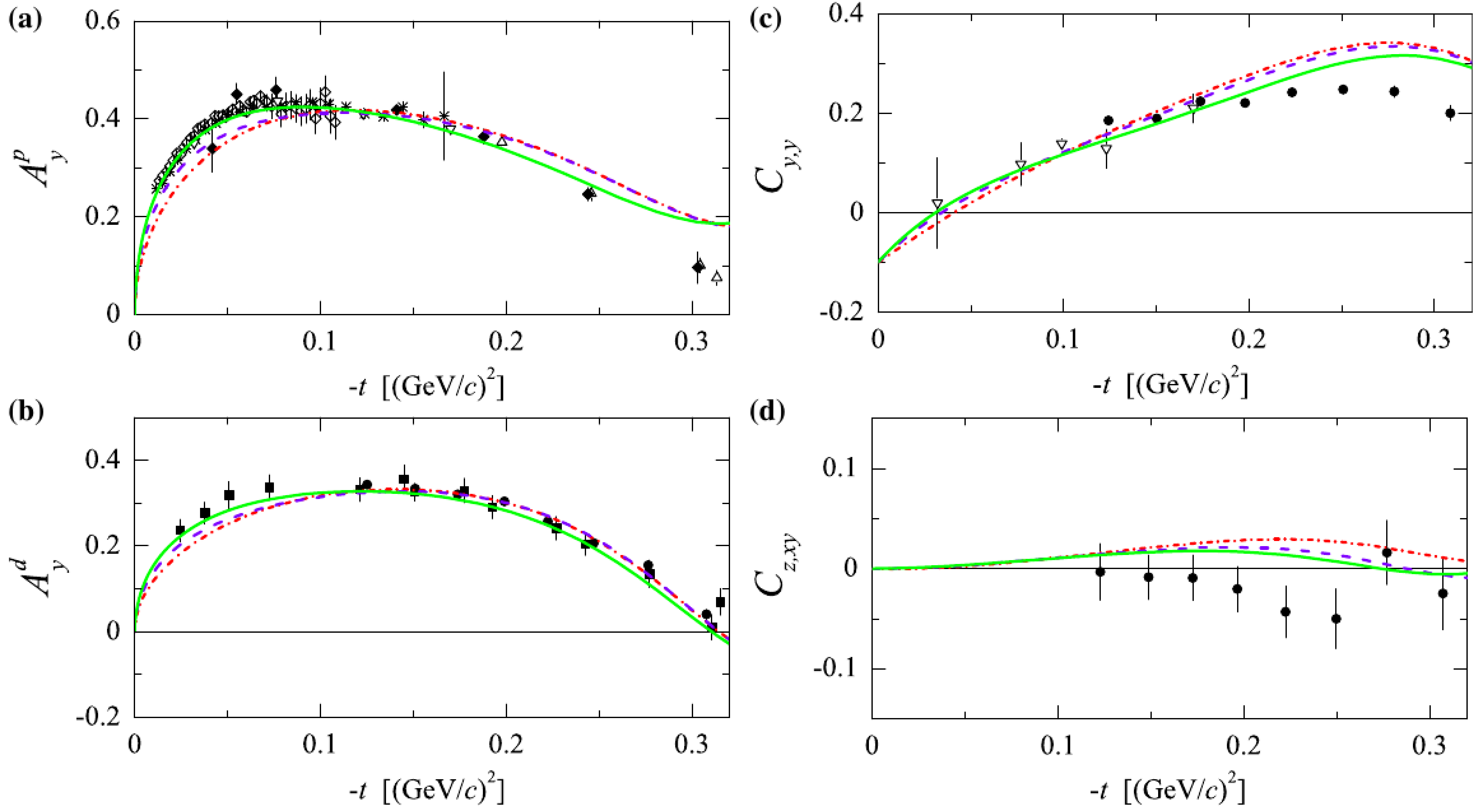
A.A. Temerbayev, Yu.N. Uzikov, Yad. Fiz, 78 (2015) 38



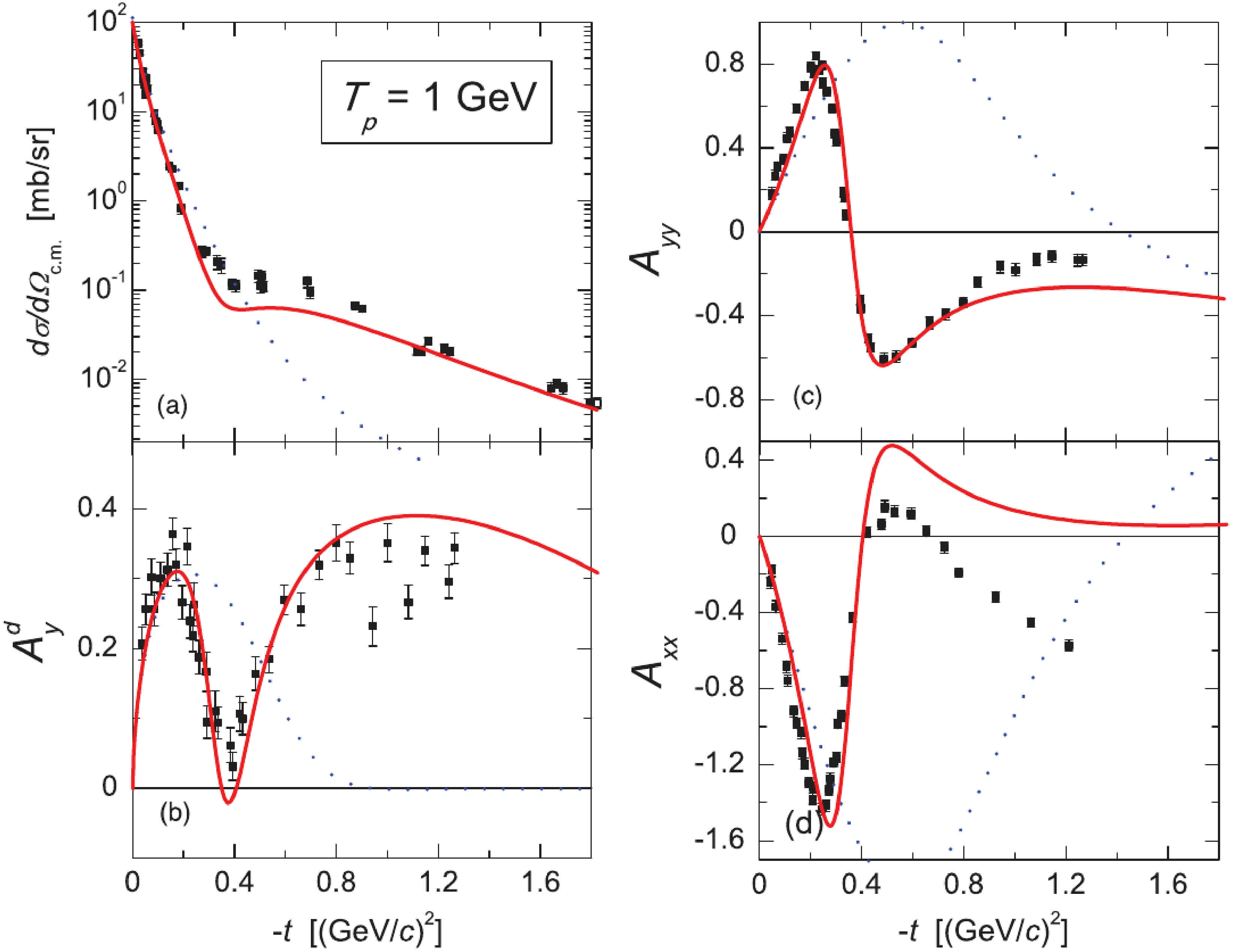
$C_{y,y}$, $C_{x,z}$, $C_{z,x}$, $C_{xz,y}$

Figure 1: Spin correlation coefficients $C_{xz,y}$ (a), $C_{z,x}$ (b), $C_{y,y}$ (c), $C_{x,z}$ (d) at 135 MeV versus the c.m.s. scattering angle calculated within the modified Glauber model [15] without (dashed lines) and with (full) Coulomb included in comparison with the data from [22].

M.N. Platonova, V.I. Kukulin, *Eur.Phys.J.A* 56 (2020) 5, 132

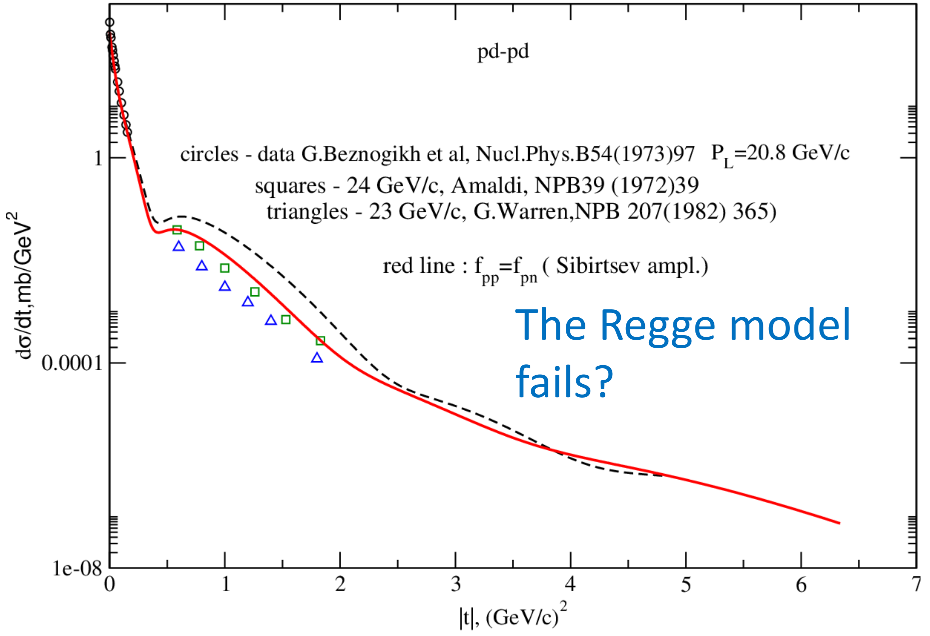


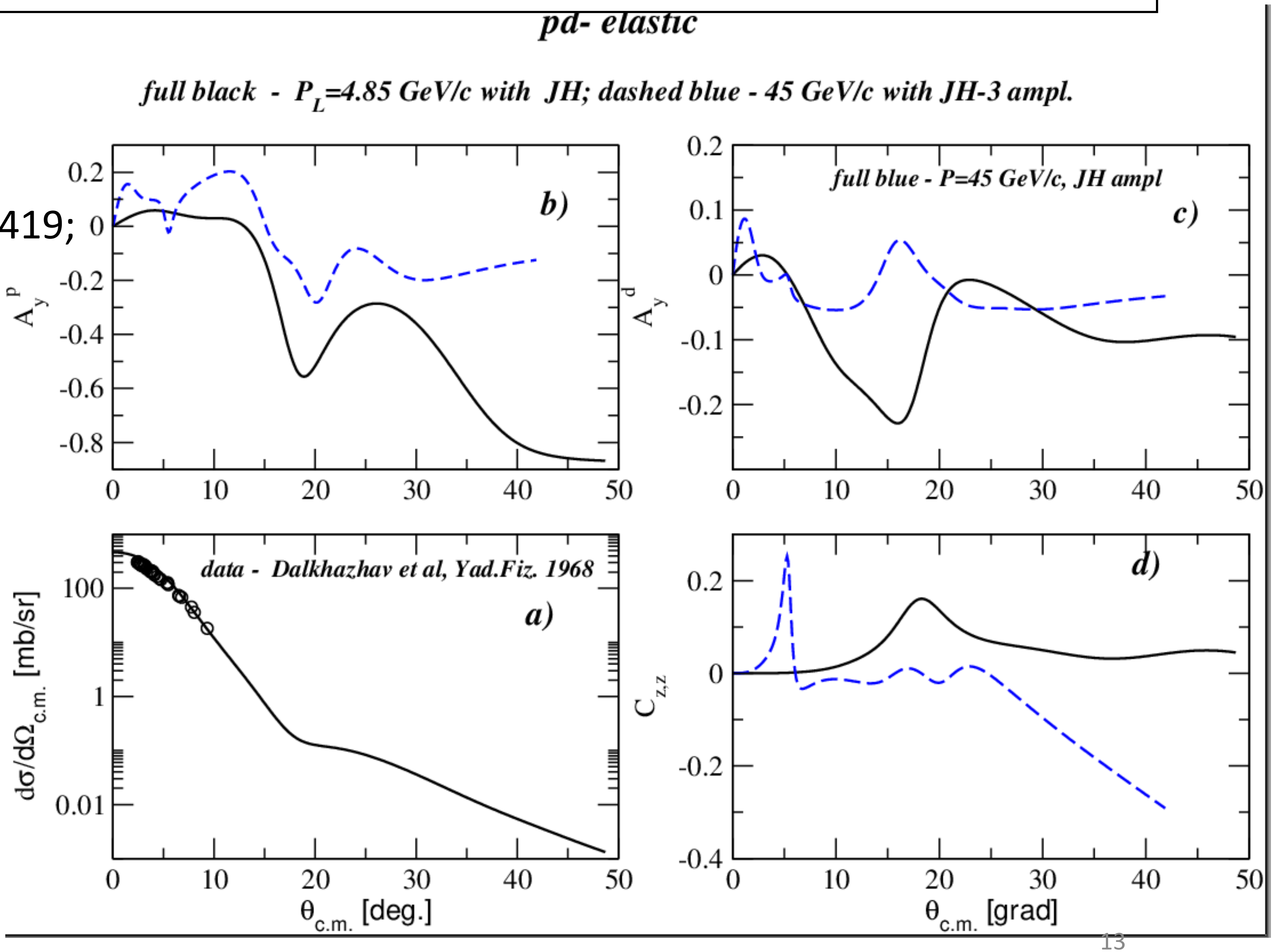
M.N. Platonova, V.I. Kukulin, PRC 81(2010) 014004



Towards test of pN amplitudes at higher energies in *pd* elastic scattering within the Glauber model

Yu.N. U., J. Haidenbauer, A. Temerbayev, A. Bazarova, Phys.Part. Nucl. 53 (2022) 419;
NN-Regge: A.Sibirtsev et al. EPJ A(2010)





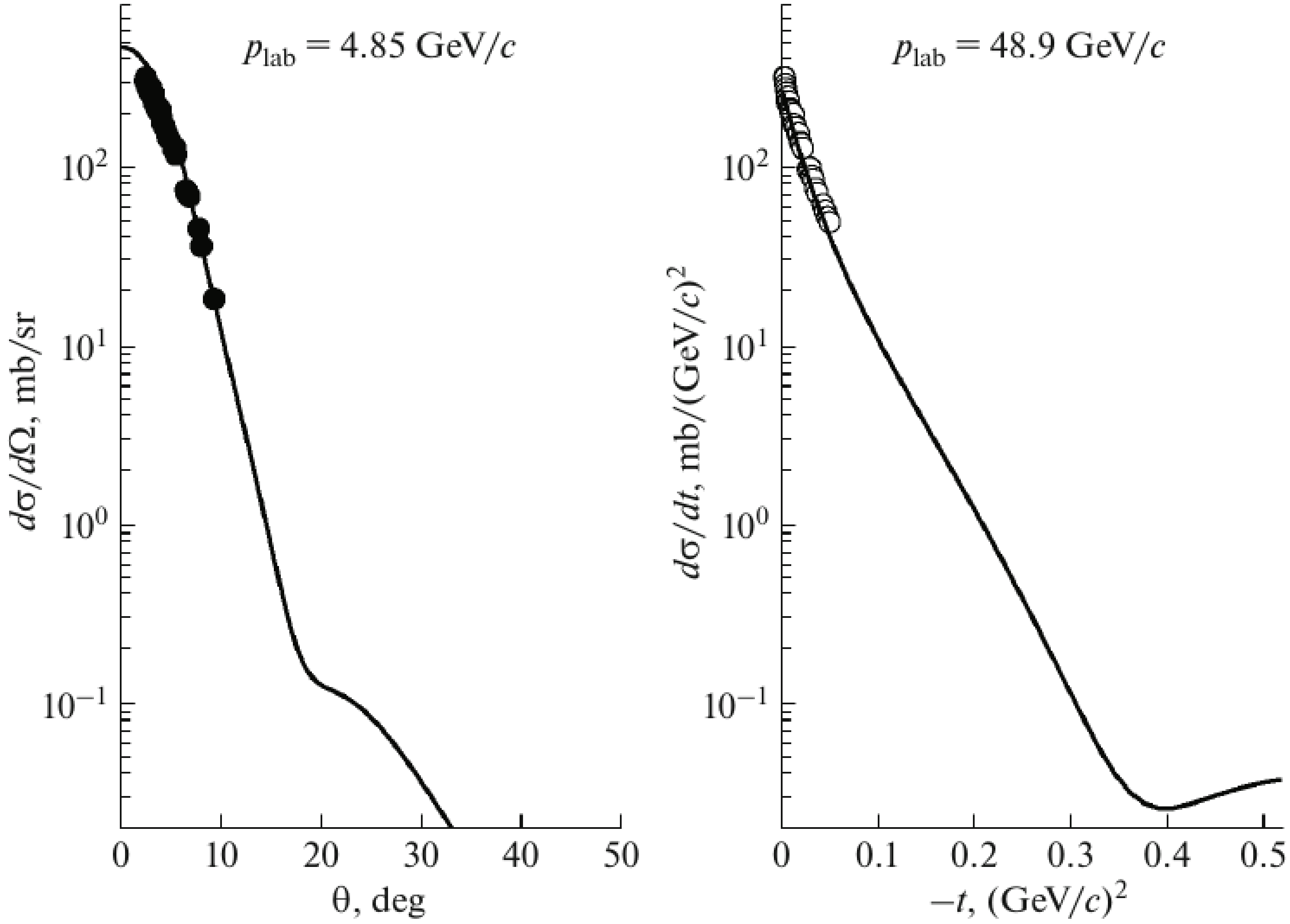


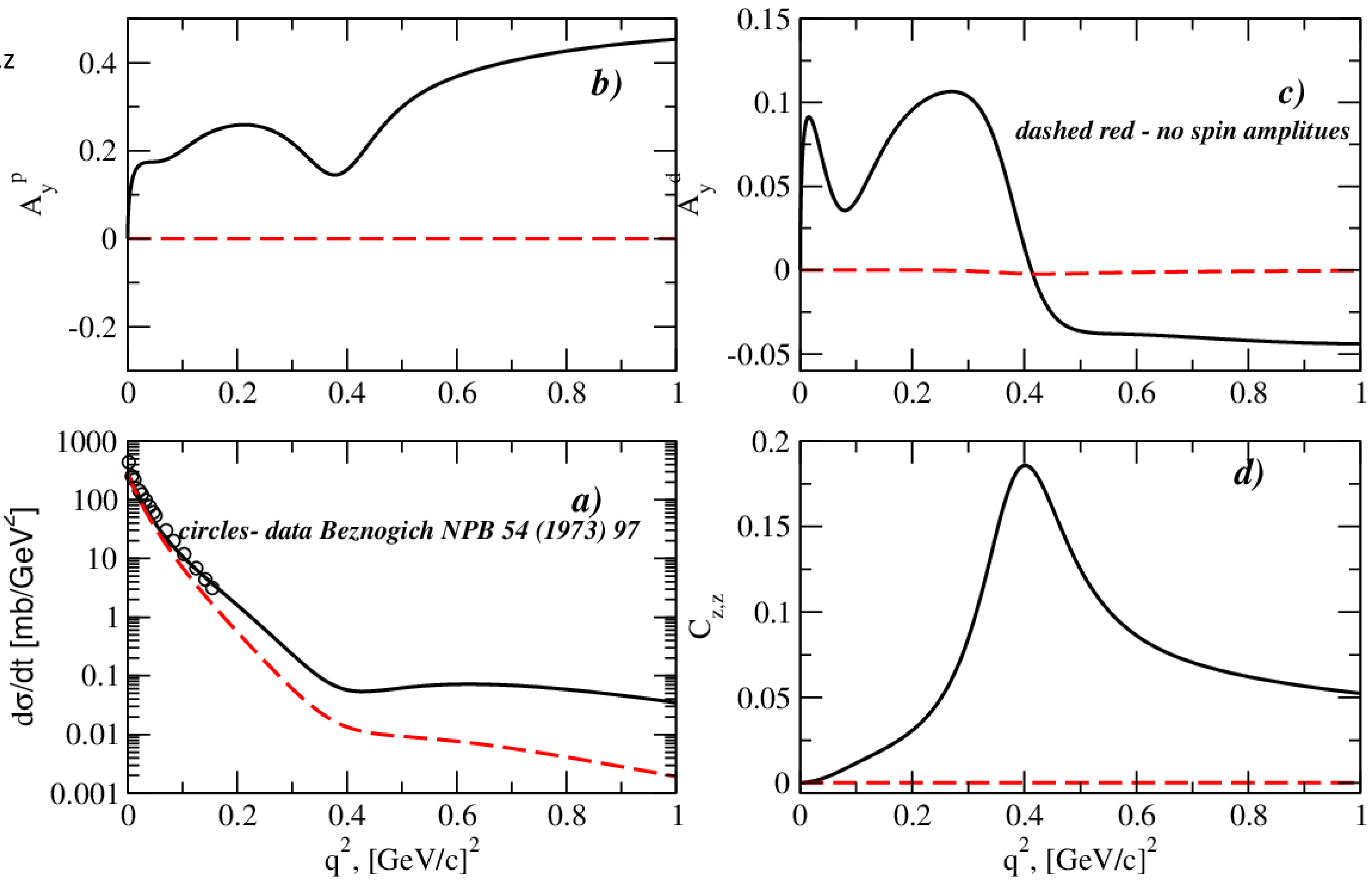
Fig. 2. Differential cross section for *pd* elastic scattering. Predictions are shown for p_{lab} = 4.8 (left) and 45 GeV/*c* (right). Data are taken from [18] (4.8 GeV/*c*) and [19] (48.9 GeV/*c*).

pd- elastic

High sensitivity to spins:

A_y^p A_y^d $C_{z,z}$

full black - P_L=20.4 GeV/c with Sibirtsev amplitudes

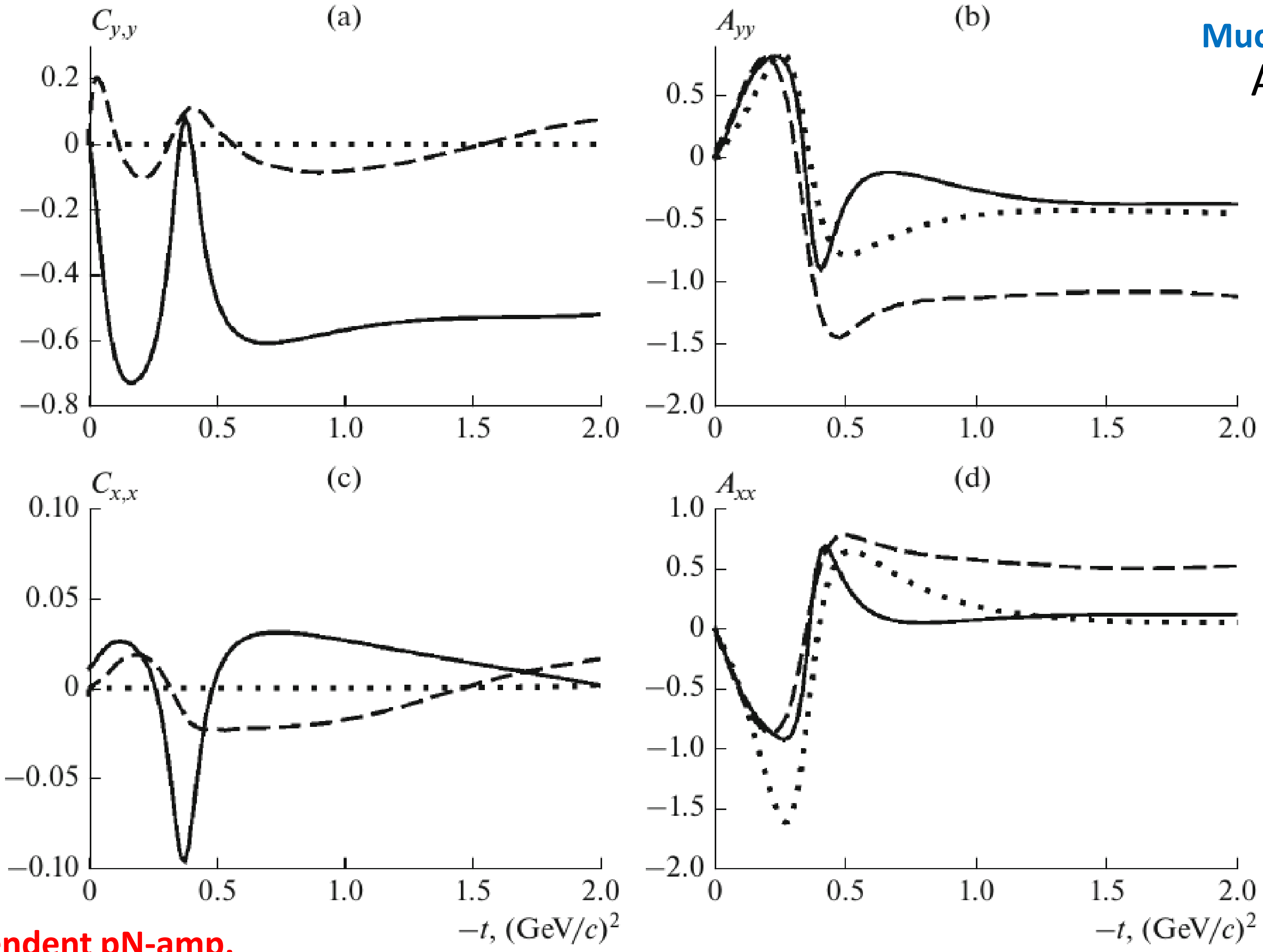


High sensitive:

$C_{y,y}$, $C_{x,x}$

Much less sensitive:

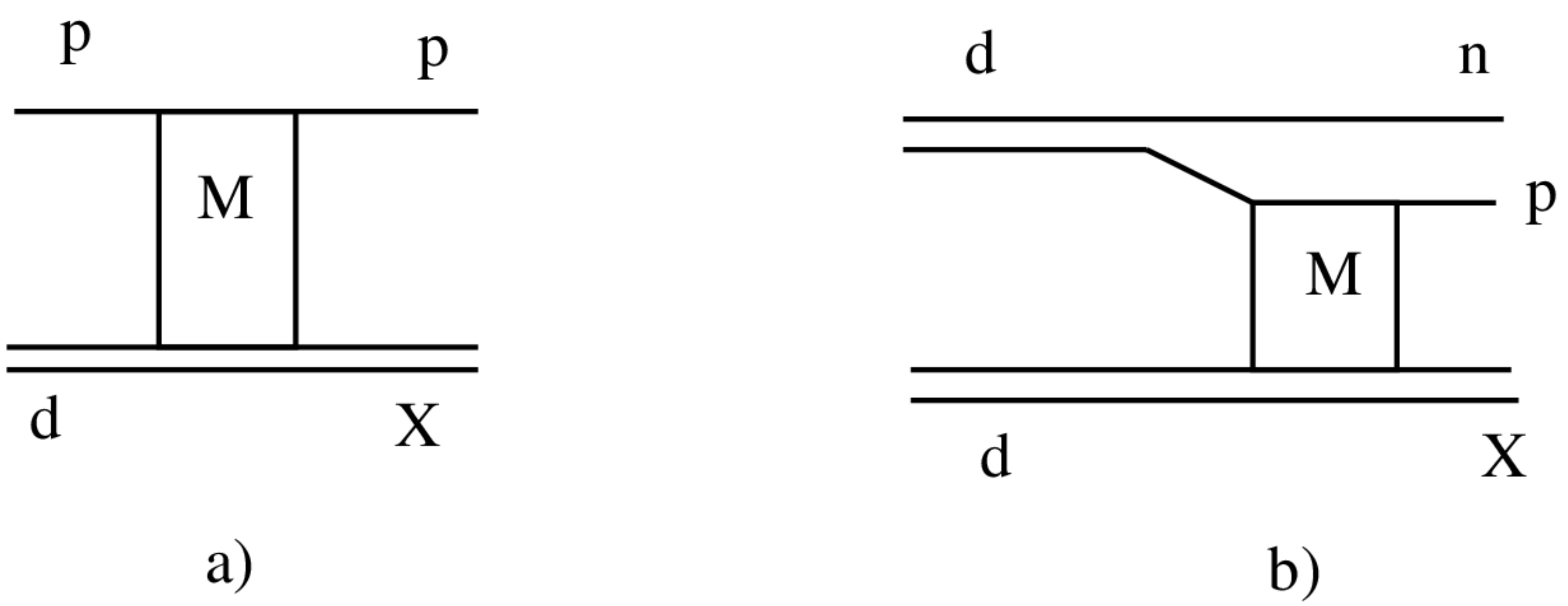
A_{yy} A_{xx}



Dotted : spin-independent pN-amp.

Fig. 4. Results for spin-dependent *pd* observables. Same description of curves as in Fig. 3. The dotted lines are results where the spin-dependent *pN* amplitudes have been omitted in the calculation.

From dd-npd to pd-pd



$$T(dd \to n + pX) = \Sigma_{\sigma'} < \sigma_n, \sigma_{p'} \,|\psi_d^{\lambda}(\vec{q}) > T_{\lambda\sigma'}^{M_X \sigma_p}(pd \to pX)$$

When the final neutron takes one half of the deuteron momentum (S-wave dominance, suppressed p_T momenta), then the pd->pd amplitude can be extracted with minimum distortions,

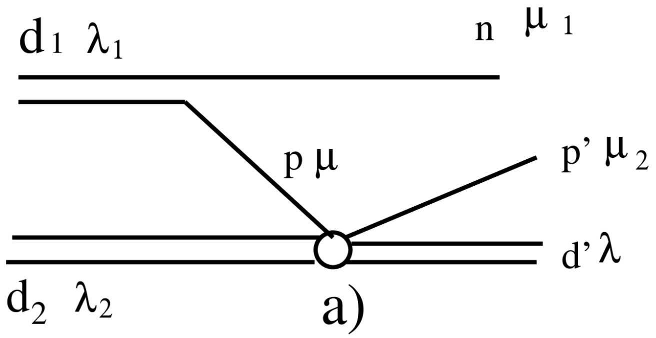
$$\vec{p}_n = \vec{p}_d / 2$$

Transition matrix element for the $dd \to npd$ in IA, S-wave :

$$M^{\mu_1\mu_2\lambda'}_{\lambda_1\lambda_2} = K\sum_{\mu}(\frac{1}{2}\mu_1\frac{1}{2}\mu|1\lambda_1)u(q)M^{\lambda'\mu_2}_{\lambda_2\mu}(pd \to pd). \quad (1)$$

where $K = \sqrt{2m_d/4\pi}$.

IA =



$$\overline{|M^{\mu_1\mu_2\lambda'}_{\lambda_1\lambda_2}|^2} = K^2\overline{|M^{\lambda'\mu_2}_{\lambda_2\mu}(pd \to pd)|^2}. \quad (2)$$

$$d\sigma_{\lambda_2} = \frac{1}{3}\sum_{\lambda_1}\sum_{\mu_1\mu_2\lambda'}|M^{\mu_1\mu_2\lambda'}_{\lambda_1\lambda_2}(dd \to npd)|^2 = K^2\frac{1}{2}\sum_{\mu\lambda'\mu_2}|M^{\lambda'\mu_2}_{\lambda_2\mu}(pd \to pd)|^2.$$

The differential cross section for collision of two spin-1 particles:

H. Ohlsen, Rep. Prog. Phys. 35 (1972) 717

$$I = I_0\left(1 + \frac{3}{2}P_y A_y + \frac{3}{2}P_y^T A_y^T + \frac{9}{4}P_y P_y^T C_{y,y}\right), \qquad (3)$$

Vector analyzing power $A_y^{d_2}$:

$$A_y^{d_2}(d_1\vec{d}_2 = npd) = \frac{d\sigma_{\lambda_2=+1} - d\sigma_{\lambda_2=-1}}{d\sigma_{\lambda_2=+1} + d\sigma_{\lambda_2=0} + d\sigma_{\lambda_2=-1}} = A_y^d(p\vec{d} \to pd)$$

Vector analyzing power $A_y^{d_1}$:

$$A_y^{d_1}(\vec{d}_1 d_2 \to npd) = \frac{d\sigma_{\lambda_1=+1} - d\sigma_{\lambda_1=-1}}{d\sigma_{\lambda_1=+1} + d\sigma_{\lambda_1=0} + d\sigma_{\lambda_1=-1}} = \frac{2}{3}A_y^p(\vec{p}d \to pd)$$

Tenzor analyzing power

$OZ \uparrow\uparrow \vec{p}_d$

$OY \uparrow\uparrow [\vec{p}_d \times \vec{p}_{d'}]$

$$A_y^d(d_1\vec{d}_2 \to npd) = \frac{d\sigma_{\lambda_2=+1} + d\sigma_{\lambda_2=-1} - 2d\sigma_{\lambda_2=0}}{d\sigma_{\lambda_2=+1} + d\sigma_{\lambda_2=0} + d\sigma_{\lambda_2=-1}} = A_{yy}^d(p\vec{d} \to pd)$$

$C_{y,y}$ needs four options for dd-collision: (i) $P_y = P_y^T = \frac{2}{3}$ (ii) $P_y = \frac{2}{3}$, $P_y^T = -\frac{2}{3}$ and the same for $P_y = -\frac{2}{3}$. The cross section $I_{\uparrow\uparrow}$ for the option (i), a $I_{\uparrow\downarrow}$ for (ii) From Eq.(3) one can find:

$$C_{y,y} = \frac{(I_{\uparrow\uparrow} - I_{\uparrow\downarrow}) + (I_{\downarrow\downarrow} - I_{\downarrow\uparrow})}{(I_{\uparrow\uparrow} + I_{\uparrow\downarrow}) + (I_{\downarrow\downarrow} + I_{\downarrow\uparrow})}, \qquad (4)$$

Yu.N. Uzikov, A.A. Temerbayev, Phys. Part. Nucl. 55 (2024) 895;
e-Print: 2311.12605 [nucl-th]; dd-> npnp and hard pN-elastic scattering

PHYSICAL REVIEW C **82**, 014003 (2010)

Vector and tensor analyzing powers in deuteron-proton breakup at 130 MeV

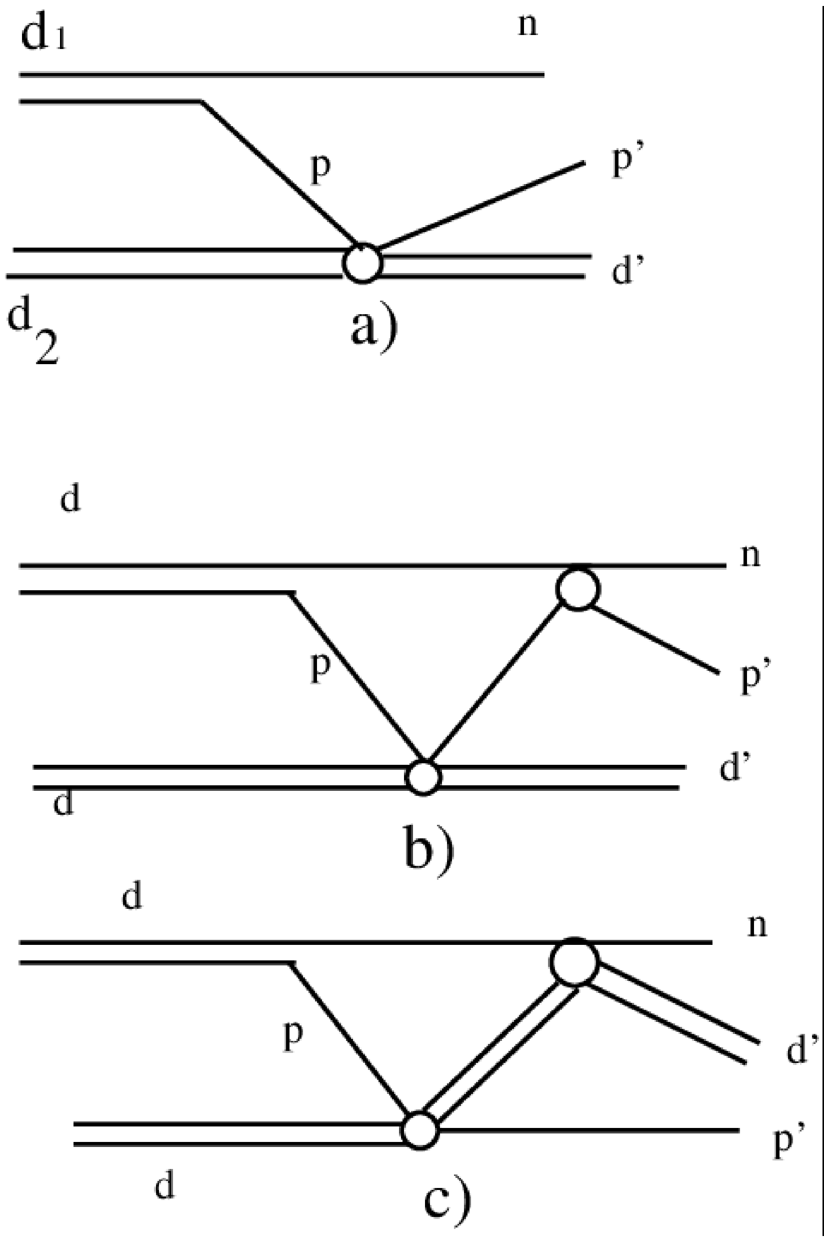
E. Stephan,[1,*] St. Kistryn,[2] R. Sworst,[2] A. Biegun,[1] K. Bodek,[2] I. Ciepał,[2] A. Deltuva,[3] E. Epelbaum,[4] A. C. Fonseca,[5] J. Golak,[2] N. Kalantar-Nayestanaki,[6] H. Kamada,[7] M. Kiš,[6] B. Kłos,[1] A. Kozela,[8] M. Mahjour-Shafiei,[6,†] A. Micherdzińska,[1,‡] A. Nogga,[9] R. Skibiński,[2] H. Witała,[2] A. Wrońska,[2] J. Zejma,[2] and W. Zipper[1]

TABLE I. Set of the polarization states used in the $^1\mathrm{H}(\vec{d},pp)n$ breakup experiment. The maximum polarizations P_Z, P_{ZZ} (for 100% efficiency of transitions in the ion source) and corresponding combinations of the magnetic fields are shown. The x indicates that the magnetic field is switched on, whereas the—indicates that the magnetic field is switched off. I_f denotes the full beam intensity. In the case of transitions with medium field on, the beam intensity is reduced to 2/3 of I_f in the case of 100% efficient transitions.

Polarization states		Magnetic fields				Beam intensity
P_Z	P_{ZZ}	SF1	SF2	MF	WF	
0	0	–	–	–	–	I_f
$+\frac{1}{3}$	$+1$	x	–	–	–	I_f
$+\frac{1}{3}$	-1	–	x	–	–	I_f
0	$+1$	x	–	x	–	$\frac{2}{3}I_f$
0	-2	–	x	x	–	$\frac{2}{3}I_f$
$+\frac{2}{3}$	0	x	x	–	–	I_f
$-\frac{2}{3}$	0	–	–	–	x	I_f

P_{yy}=-1 or P_{yy}=+1 for P_y=1/3

P_{yy}=0 for P_y=+2/3, -2/3



Relations between dd->n+p+d and pd->pd

$$|M(dd \to npd)|^2 = K[u^2(q) + w^2(q)]\,|M(pd \to pd)|^2$$

$d_2^{\uparrow}$: Polarized

$$A_Y^d(dd_2^{\uparrow} \to npd) = A_Y^d(pd^{\uparrow} \to pd),$$

$$A_{YY} = (dd_2^{\uparrow} \to npd) = A_{YY}(pd^{\uparrow} \to pd)$$

$d_1^{\uparrow}$: Polarized

$$A_Y^d(d_1^{\uparrow}d \to npd) = \frac{2}{3}A_Y^p(p^{\uparrow}d \to pd)$$

Double polarized:

$$C_{Y,Y}(d^{\uparrow}d^{\uparrow} \to npd) = \frac{2}{3}C_{y,y}(p^{\uparrow}d^{\uparrow} \to pd)$$

Rescatterings b), c) will be taken into account

NN-elastic data and phenomenological models

NN helicity amplitudes:

SAID: Arndt R.A. et al. PRC 76 (2007) 025209; $\sqrt{s_{NN}} = 1.9 - 2.4 GeV$

A. Sibirtsev et al., Eur. Phys. J. A 45 (2010) 357; arXiv:0911.4637 [hep-ph] (pp, Regge-type parametrization); $\sqrt{s_{NN}} = 2.5 - 15 GeV$

W.P. Ford, J. van Orden, Phys. Rev. C 87 (2013) $\sqrt{s_{pN}} = 2.5 - 3.5 GeV$
(pp-, pn; Regge);

O.V. Selyugin, Symmetry., 13 N2 (2021) 164; (Regge –eikonal); $\sqrt{s_{NN}} = 5 - 25 GeV$
Phys.Rev.D 110 (2024) 11, 114028 ; e-Print: 2407.01311 [hep-ph
arxiv:2407.01311[hep-ph]

Preliminary SUMMARY

OBSERVABES: in IA spin observables A_y, A_{yy} ,$C_{y,y}$ **of the reaction dd→npd are directly related to those for pd->pd.**

- Actual problem important for hadron spin physics.
- Can be studied at the first stage of SPD.
- Large cross section does, most likely, not require a large beam time.
- This study can be extended to the dd- elastic scattering at SPD.

What has to be done:

(i) Estimation for FSI effects.

(ii) Calculations of A_y, A_{yy} ,$C_{y,y}$ for pd-pd with different pN models.

(...) Theory of dd-elastic with full spin-dependence.

References

[1] M.N. Platonova, V.I. Kukulin,
Refined Glauber model versus Faddeev calculations and experimental data for pd spin observables
Phys.Rev.C 81 (2010) 014004, *Phys.Rev.C* 94 (2016) 6, 069902; (erratum) e-Print: 1612.08694 [nucl-th]

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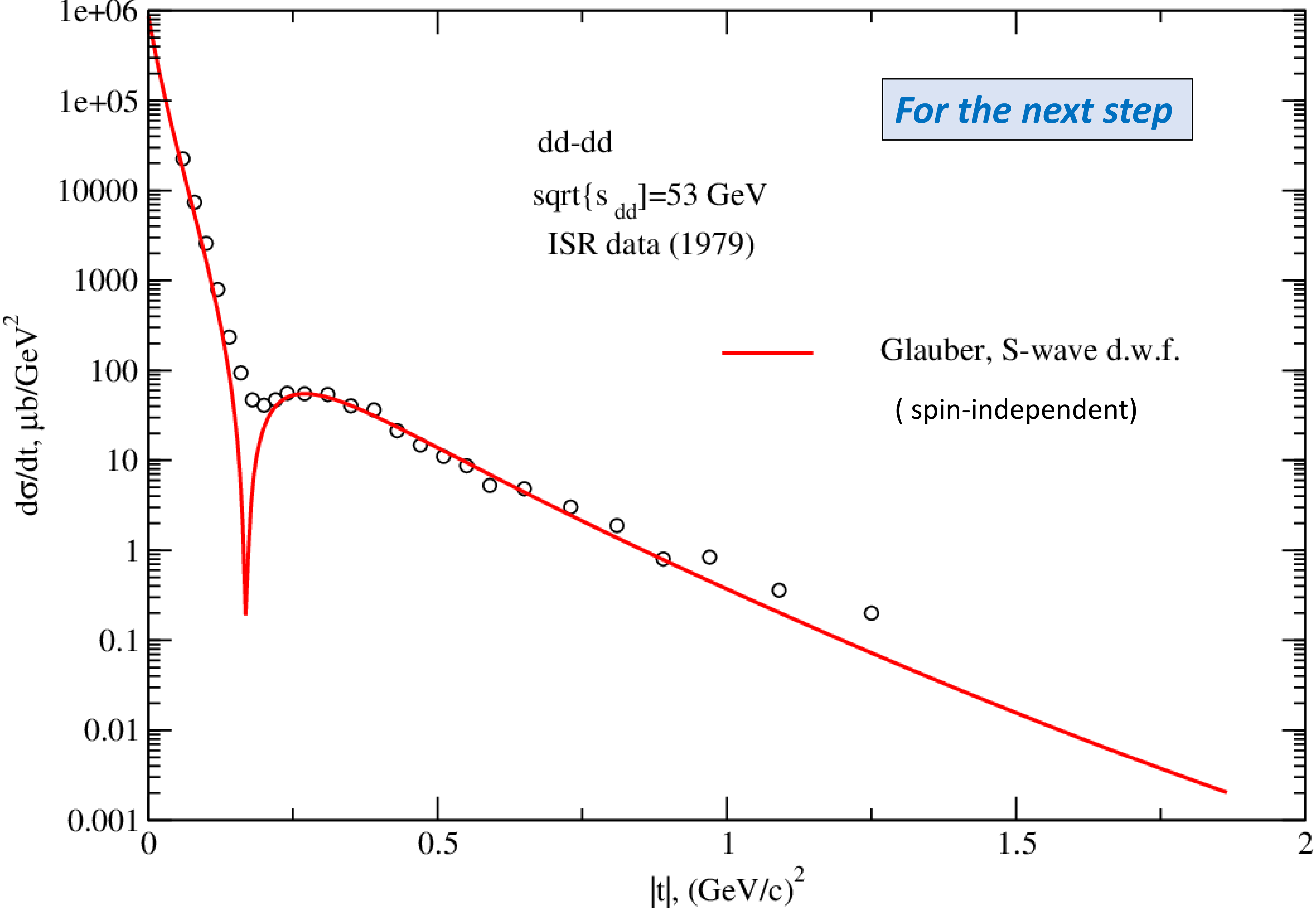
[4] W.P. Ford, J. van Orden, Regge model for NN spin-dependent amplitudes, Phys. Rev. C87(2013) (pp, pn).

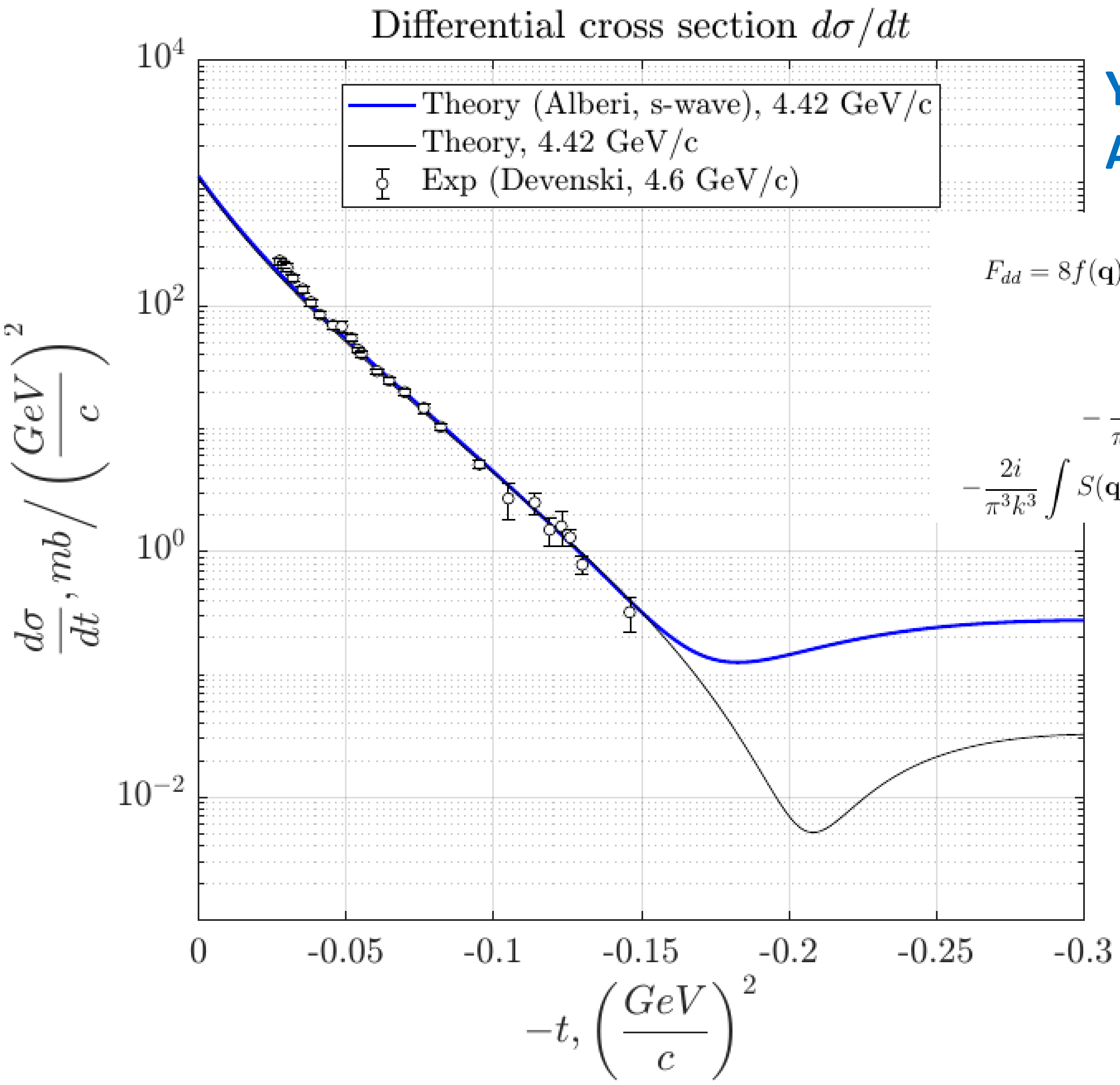
[5] O.V. Selyugin, Elastic scattering at sqrt{s}=6 GeV up to sqrt{s}=13TeV: Proton-proton; proton-antiproton, and proton-neutron, *Phys.Rev.D* 110 (2024) 11, 114028 ; e-Print: 2407.01311 [hep-ph].

[6] Yu. Uzikov, J. Haidenbauer, A. Bazarova, A. Temerbaev,
Spin Observables of Proton–Deuteron Elastic Scattering at SPD NICA Energies within the Glauber Model and pN Amplitudes - *Phys.Part.Nucl.* 53 (2022) 2, 419-425, e-Print: 2011.04304 [nucl-th]

THANK YOU FOR ATTENTION!


For the next step
dd-dd
sqrt{s dd]=53 GeV
ISR data (1979)
Glauber, S-wave d.w.f.
(spin-independent)
dσ/dt, μb/GeV2
|t|, (GeV/c)2
1e+06
1e+05
10000
1000
100
10
1
0.1
0.01
0.001
0
0.5
1
1.5
2

dd→dd, Glauber model



Yu.N.U, M. Platonova, A. Kornev, A. Klimochkina, IJMP E (2024)

$$F_{dd} = 8f(\mathbf{q})S^2\left(\tfrac{1}{2}\mathbf{q}\right) + \frac{2i}{\pi k}\Big[4S\left(\tfrac{1}{2}\mathbf{q}\right)\int S(\mathbf{q}_1) f\left(\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) f\left(-\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) \mathrm{d}^2\mathbf{q}_1 +$$
$$+ 2\int S^2(\mathbf{q}_1) f\left(\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) f\left(-\mathbf{q}_1 + \tfrac{1}{2}\mathbf{q}\right) \mathrm{d}^2\mathbf{q}_1\Big] -$$
$$- \frac{8}{\pi^2 k^2}\int S(\mathbf{q}_1)S(\mathbf{q}_2) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_1\right) f\left(\mathbf{q}_1 + \mathbf{q}_2\right) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_2\right) \mathrm{d}^2\mathbf{q}_1\, \mathrm{d}^2\mathbf{q}_2 -$$
$$- \frac{2i}{\pi^3 k^3}\int S(\mathbf{q}_1)S(\mathbf{q}_2) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_1 - \mathbf{q}_3\right) f(\mathbf{q}_3) f(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3) f\left(\tfrac{1}{2}\mathbf{q} - \mathbf{q}_2 - \mathbf{q}_3\right) \mathrm{d}^2\mathbf{q}_1\, \mathrm{d}^2\mathbf{q}_2\, \mathrm{d}^2\mathbf{q}_3.$$

G. Alberi et al. NPB 17 (1970) , 621

Without spins in pN

NN-amplitudes

T-even P-even

$$M_{N(ij)}(\mathbf{q}) = A_N + C_N(\boldsymbol{\sigma}_i \cdot \hat{n}) + C'_N(\boldsymbol{\sigma}_j \cdot \hat{n}) + B_N(\boldsymbol{\sigma}_i \cdot \hat{k})(\boldsymbol{\sigma}_j \cdot \hat{k}) + (G_N + H_N)(\boldsymbol{\sigma}_i \cdot \hat{q})(\boldsymbol{\sigma}_j \cdot \hat{q}) + (G_N - H_N)(\boldsymbol{\sigma}_i \cdot \hat{n})(\boldsymbol{\sigma}_j \cdot \hat{n}),$$

$$C_N' \approx C_N + i\frac{q}{2m}A_N$$

$$\hat{k} = \frac{\mathbf{p}+\mathbf{p}'}{|\mathbf{p}+\mathbf{p}'|}, \quad \hat{q} = \frac{\mathbf{p}-\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|}, \quad \hat{n} = (\hat{k} \times \hat{q}),$$

C. Sorensen , PRD 19 (1979)

Deuteron w.f.

$$\Psi^d_{(ij)} = \frac{1}{\sqrt{4\pi}r}\left(u(r) + \frac{1}{2\sqrt{2}}w(r)\hat{S}_{12}(\hat{r};\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j)\right)$$

$$\hat{S}_{12}(\hat{r};\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j) = 3(\boldsymbol{\sigma}_i \cdot \hat{r})(\boldsymbol{\sigma}_j \cdot \hat{r}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

AT HIGHER ENERGIES $\sqrt{s_{pN}} = 3 - 10 GeV^2$

A.Sibirtsev et al., Eur.Phys. J. A 45 (2010) 357

$$\phi_{ai}(s,t) = \pi\beta_{ai}(t)\frac{\xi_i(s,t)}{\Gamma(\alpha(t))}; i = \rho, \omega, a_2, f_2, P; a = 1-5;$$

$$\xi_i(t,s) = \frac{1 + S_i \exp[-i\pi\alpha(t)]}{\sin[\pi\alpha_i(t)]}\left[\frac{s}{s_0}\right]^{\alpha_i(t)},$$

$$\alpha_i(t) = \alpha_i^0 + \alpha_i^{'} t,$$

$$\beta_{1i}(t) = c_{1i}\exp(b_{1i}t),$$

$$\beta_{2i}(t) = c_{2i}\exp(b_{2i}t)\frac{-t}{4m_N^2},$$

$$\beta_{3i}(t) = c_{3i}\exp(b_{3i}t),$$

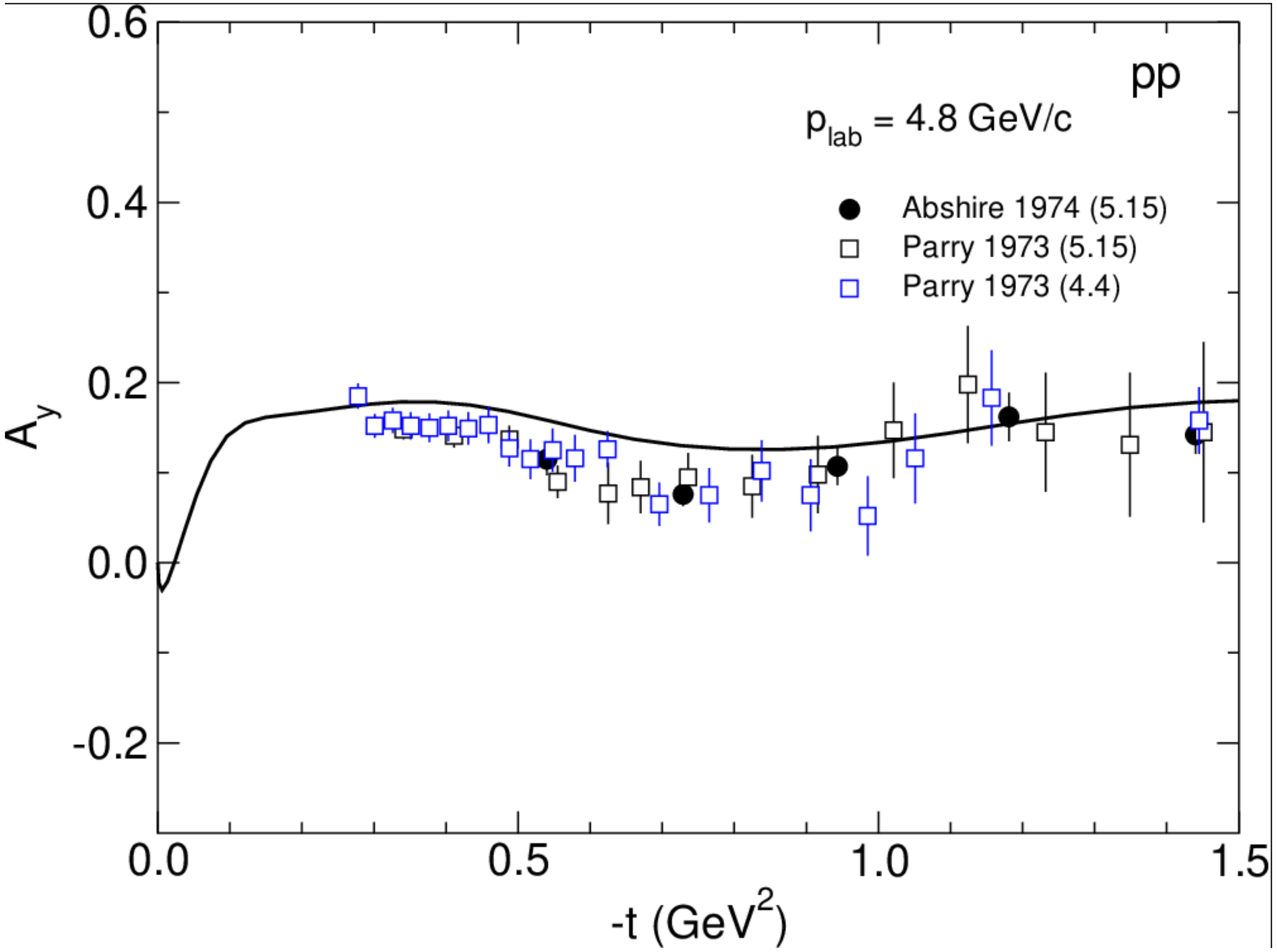
$$\beta_{4i}(t) = c_{4i}\exp(b_{4i}t)\frac{-t}{4m_N^2},$$

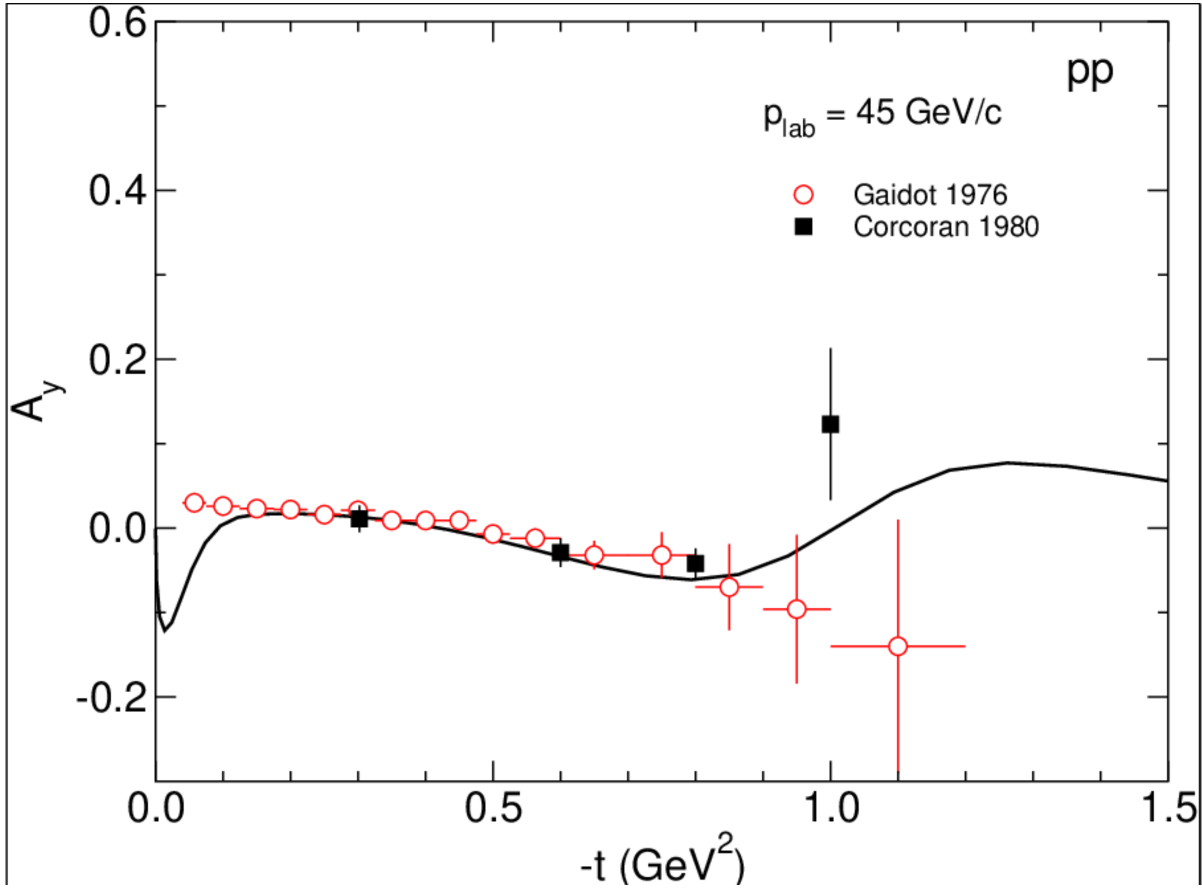
$$\beta_{5i}(t) = c_{5i}\exp(b_{5i}t)\left[\frac{-t}{4m_N^2}\right]^{1/2}.$$

The Regge formalism for pp-helicity amplitudes at proton beams momenta p_L= 3-50 GeV/c includes single- Pomeron exchange and trajectories ρ, ω, f_2, a_2

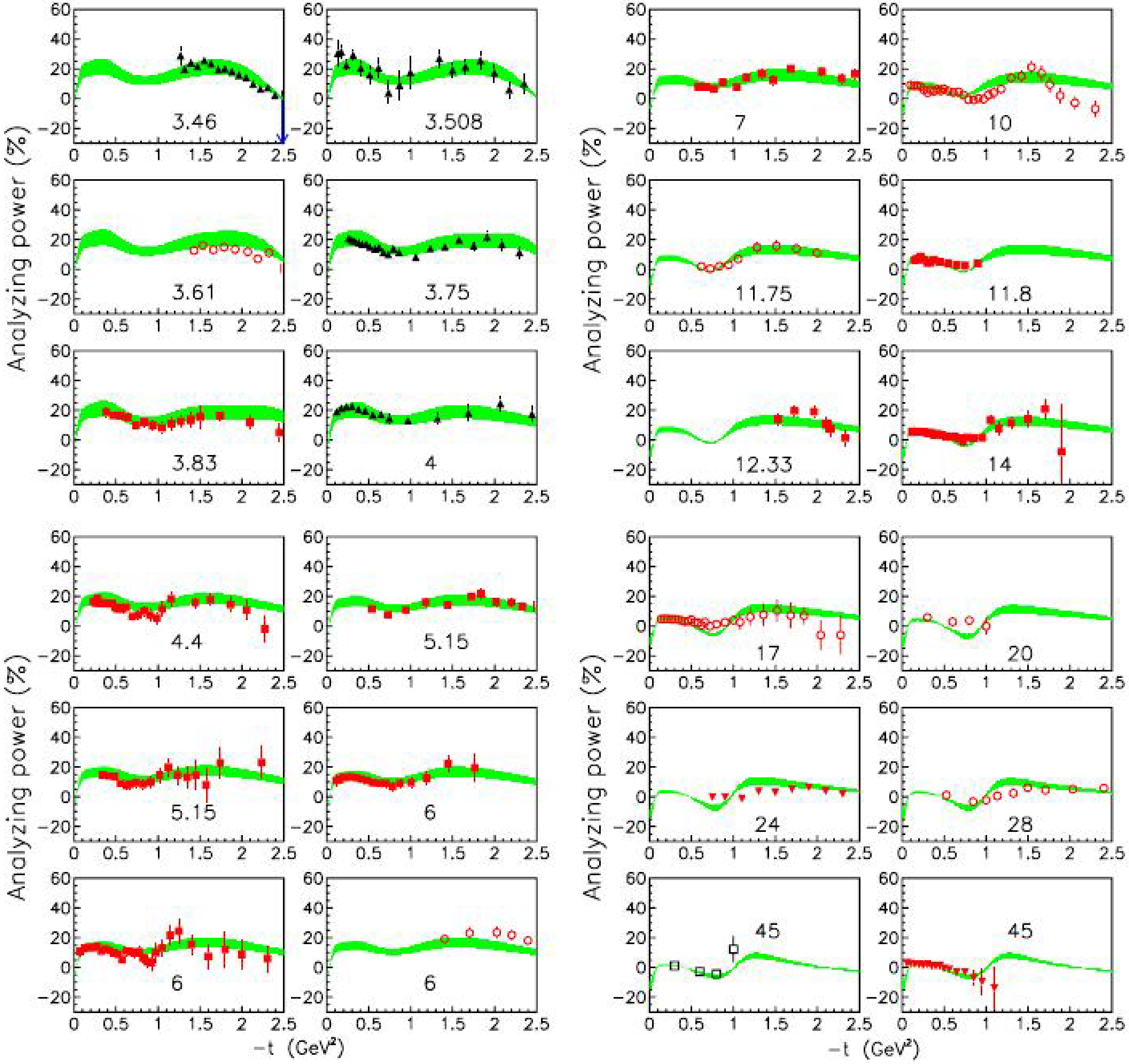
Data on $d\sigma / dt$, A_N, A_{NN}

A. Sibirtsev et al, EPJA (2010)





A. Sibirtsev et al, EPJA (2010)



Two sets of deuterons beams:

$$P_1 = +\frac{2}{3}, P_2 = +\frac{2}{3}$$

$$P_1 = +\frac{2}{3}, P_2 = -\frac{2}{3}$$

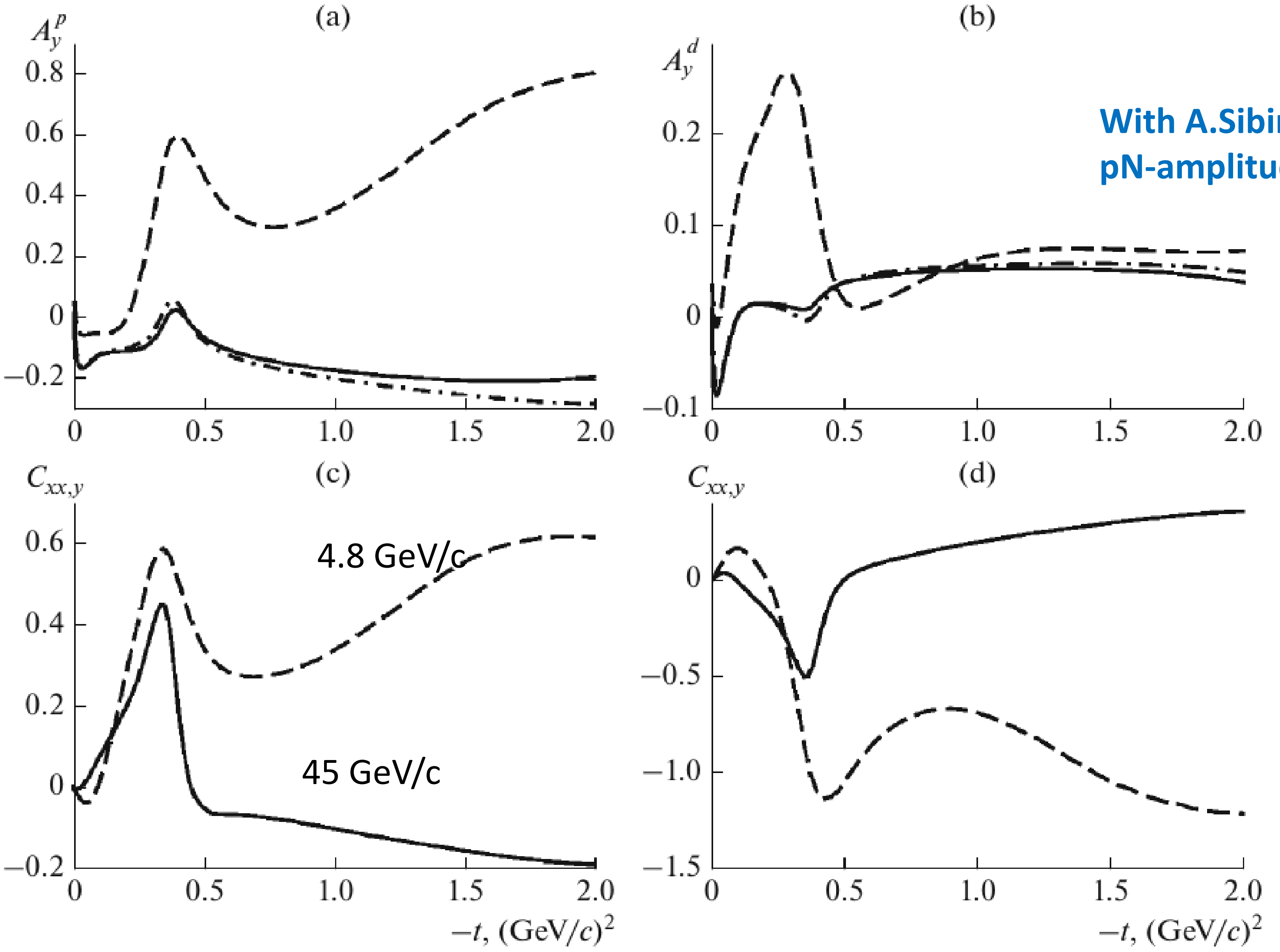
N_1 **N_2**

$$A^{dd}_{YY} = \frac{\mathcal{N}_1 - \mathcal{N}_2}{\mathcal{N}_1 + \mathcal{N}_2}$$

In terms of $d\sigma_{\lambda_1\lambda_2}$

$$A^{dd}_{YY} = \frac{2 \cdot 2d\sigma_{++} + 2d\sigma_{+0} + 2d\sigma_{0+} + d\sigma_{00} - (2 \cdot 2d\sigma_{+-} + 2d\sigma_{+0} + 2d\sigma_{0-} + d\sigma_{00})}{2 \cdot 2d\sigma_{++} + 2d\sigma_{+0} + 2d\sigma_{0+} + d\sigma_{00} + (2 \cdot 2d\sigma_{+-} + 2d\sigma_{+0} + 2d\sigma_{0-} + d\sigma_{00})}$$

Yu.N. Uzikov, A.A. Temerbayev, Phys. Part. Nucl. 55 (2024) 895;
e-Print: 2311.12605 [nucl-th]



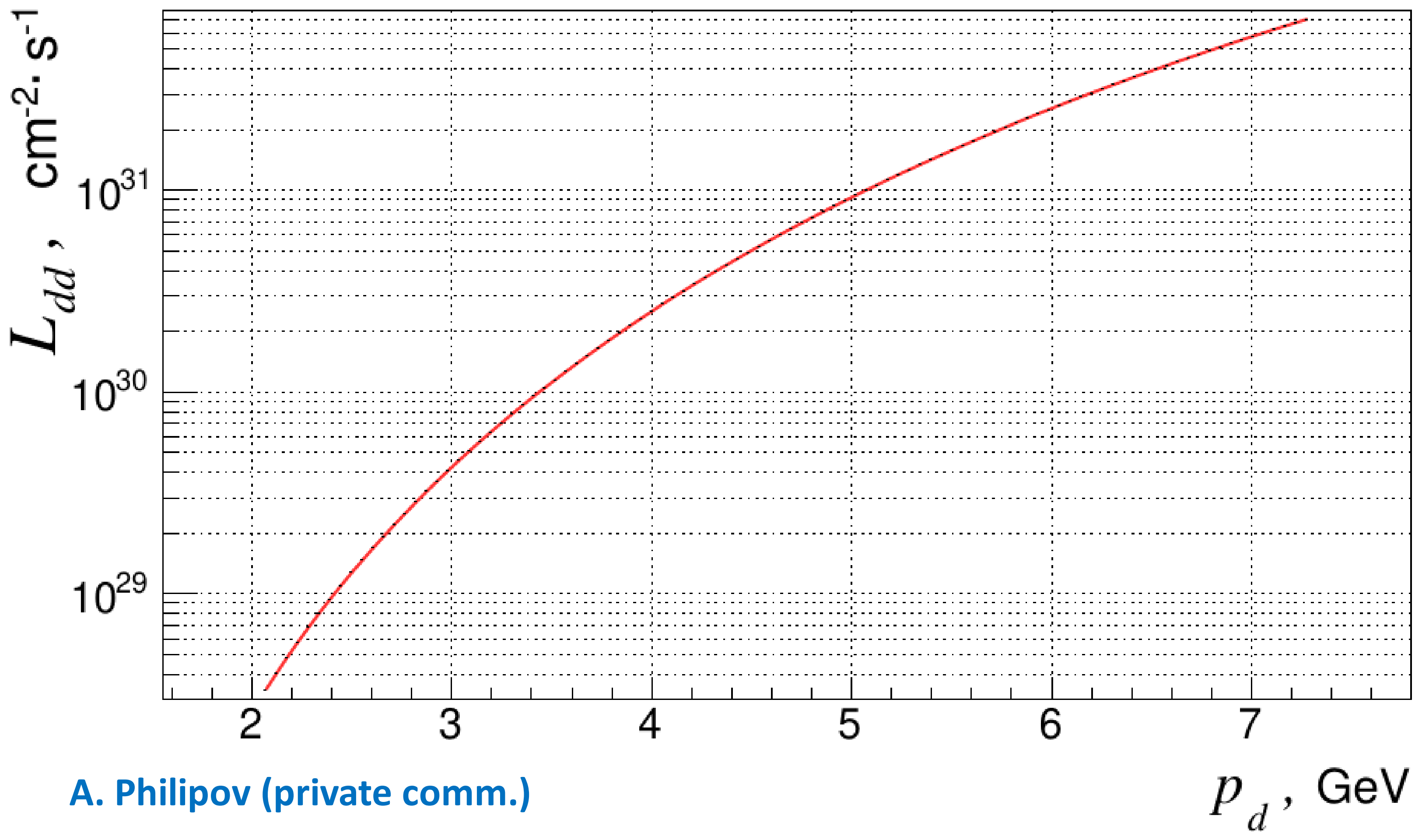
With A.Sibirtsev et al. (2010) pN-amplitudes

Fig. 3. Results for spin-dependent *pd* observables. Predictions for $p_{\mathrm{lab}} = 4.8$ GeV/*c* are shown by dashed lines while those at 45 GeV/*c* correspond to the solid lines. For the latter, the effect of the Coulomb interaction is indicated by the dash-dotted lines.

Luminosity in dd- collision,

p_d is the c.m.s. momentum of the deuteron

$$\sqrt{s_{dd}} = 14.5 GeV$$



A. Philipov (private comm.)