Gravitational axial anomaly and Unruh effect in curved spacetime

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What is this work about?

The work is based on the connection between the quantum UV chiral anomaly and relativistic hydrodynamics.

We have considered the quantum chiral current as a classical hydrodynamical current in almost simplest case of a curved spacetime (Einstein manifolds) and found that the KINEMATICS in such a hydro-current DEFINES THE MAGNITUDE OF THE VIOLATION OF CHIRAL SYMMETRY of the fermionic system in an external gravitational field.

I. Chiral anomalies

Anomalies are violations of some symmetries of the theory, preserved in the classical case, due to quantum effects.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi + A_{\mu}j^{\mu}, \quad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad m = 0.$$

There are some symmetries on classical level, which involves conserving currents

$$\begin{cases} \psi \to \psi e^{ie\alpha} \Longrightarrow \quad \partial_{\mu}j^{\mu} = 0, \\ \psi \to \psi e^{ie\alpha\gamma^{5}} \Longrightarrow \quad \partial_{\mu}j^{\mu}_{A} = 0 \end{cases}$$

But in external fields (EM or gravitational) due to quantum effects the axial current is not conserved.

$$\partial_{\mu}j^{\mu}_{A} = \frac{e^2}{16\pi^2}F_{\mu\nu}\widetilde{F}^{\mu\nu} \neq 0 \to N_R - N_L \neq const$$

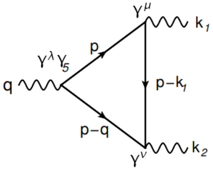
Roughly speaking: At the quantum level, the physics of the world and physics of its mirror reflection do not match. Mirror symmetry is violated

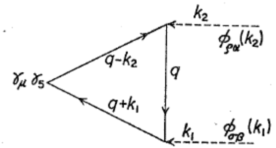
I. Chiral anomalies

Chiral anomalies take place at very high energies - Ultra-Violet phenomena There are at least two types of chiral anomalies whether the mirror symmetry violated via EM or gravitational fields.

Feynman diagrams of electromagnetic and gravitational anomalies

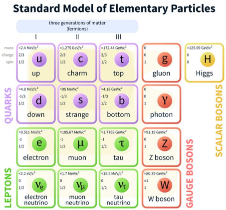
$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu} \widetilde{F}^{\mu\nu} \qquad \nabla_\mu j^\mu_A = \frac{1}{384\pi^2} R_{\mu\nu\lambda\rho} \widetilde{R}^{\mu\nu\lambda\rho}$$

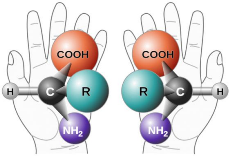


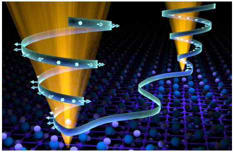


I. Various consequences of chiral anomaly

Our world has an inherent chirality and it manifests itself on different scales







Hydrodynamics operating with a long waves therefore it's an Infra-Red theory

Hydrodynamics is just based on conservation laws

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0$$

Derivative expansion: We can express the hydrodynamical current or the stress energy tensor as a series of derivatives acting on different parameters, such as the 4-velocity of fluid u_μ, temperature T, and space-time metric $g_{\mu\nu}$. All the kinematic variables are derivatives of the 4-velocity

III. Connection between hydrodynamics and EM chiral anomaly

Current of relativistic fluid with the new vorticity term

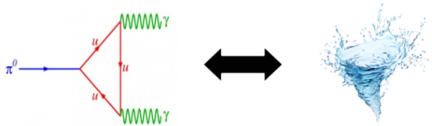
$$j_A^\mu = \underbrace{nu^\mu - \sigma T\left(g^{\mu\nu} + u^\mu u^\nu\right)\partial_\nu\left(\frac{\mu}{T}\right)}_{\textit{Landau and Lifshitz}} + \underbrace{\xi\omega^\mu}_{\textit{New term}}$$

Electromagnetic chiral anomaly

$$\underbrace{\partial_\mu j_A^\mu = -\frac{1}{8}C\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}}_{\textit{Chiral anomaly}}$$

UV anomaly connected with hydrodynamical current: vortical term proportional to the anomaly coefficient [Son, Surowka PRL (2009)]

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + P}\right)$$



In other words, in a hot and dense medium quantum UV anomalies are expressed macroscopically

Hydrodynamics is also related to the gravitational anomaly and its called Kinematical Vortical Effect (KVE).

Current in the 3^{rd} order of gradients

$$j_A^\mu = \left(\lambda_1 \omega^2 + \lambda_2 a^2\right) \omega^\mu$$

Gravitational chiral anomaly

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} R^{\sigma\rho}_{\alpha\beta}$$

There is connection between transport coefficients in flat spacetime and coefficient of anomaly, induced by curvature of spacetime [Prokhorov, Teryaev, Zakharov PRL (2022)]

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Neverthelees, this result was obtained in a case of serious limitation of Ricci-flat spacetime $R_{\mu\nu} = 0$.

III. Generalization of KVE on Einstein manifolds

Consideration of the chiral medium in Einstein manifolds $R_{\mu\nu} = \Lambda g_{\mu\nu}$

The fluid axial current and the axial anomaly

$$j_A^\mu = \Bigg(\lambda_1\omega^2 + \lambda_2 a^2 + \underbrace{\lambda_\Lambda R}_{\textit{Curvature term}}\Bigg)\omega^\mu \qquad\qquad \nabla_\mu j_A^\mu = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}.$$

Lead to relations

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}, \qquad \lambda_\Lambda = -\frac{\lambda_2}{3}.$$

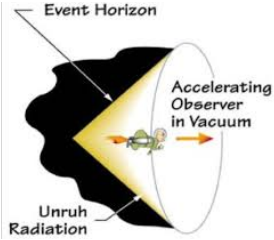
As can be seen, even in this case KVE still exist. Moreover, the curvature of the spacetime connected with the magnitude of acceleration in the current.

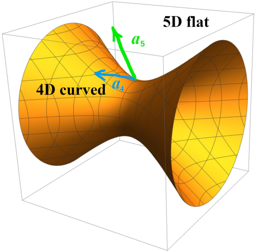
IV. Unexpected Consequence: The Unruh effect in curved spacetime

From the point of view of an accelerating observer, the empty spacetime contains a gas of particles at a temperature proportional to the acceleration.

Hydrodynamic conservation law in curved spacetime

$$\nabla_\mu T^{\mu\nu} = 0$$





$$\langle \hat{T}^{\mu\nu} \rangle (T = T_U) = 0; \quad \rightarrow T_U = \frac{a_5}{2\pi} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$

Discussion. Kinematic nature of the axial anomaly

The KVE is a purely kinematic effect in **flat** spacetime. The relation holds even without an external gravitational field.

$$j^A_\mu = \left(\lambda_1 \omega^2 + \lambda_2 a^2\right) \omega_\mu \quad \rightarrow \quad \frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}.$$

$$\nabla_\mu j^\mu_A = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}.$$

The nature of the chiral (or mirror) symmetry violation may be purely kinematic. External fields only make anomaly 'manifests' through the kinematics

Or perhaps, external gravitational (which violating symmetry) field 'emerges' from the kinematics of the system

Thank you for your attention!