$\mathcal{N} = 2$ supersymmetry and higher-spin theories

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Gauge fields

- **Spin** 0 (ϕ) $\rightarrow$ $$S_{scalar} = \int d^4x \left\{ \partial^\mu \phi \partial_\mu \phi - m^2\phi^2 - \underbrace{V(\phi)}_{interactions} \right\}$$

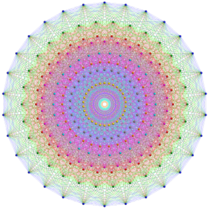
- **Spin** 1 (A_μ) $\rightarrow$ Gauge principle: compact Lie algebra

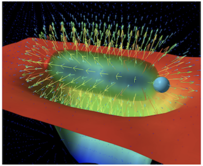
$$S_{Maxwell} = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

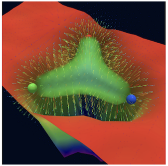
$$S_{Yang-Mills} = -\frac{1}{4}\int d^4x\, Tr\, G^{\mu\nu}G_{\mu\nu}, \quad G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig[G_\mu, G_\nu].$$

Symmetry incorporates *Yang-Mills theory + matter*. Non-dynamical spacetime!

G	$SU(N)$	$SO(N)$	$Sp(N)$	E_6	E_7	E_8	F_4	G_2
$\dim G$	N^2-1	$\frac{1}{2}N(N-1)$	$N(2N+1)$	78	133	248	52	14
$\dim F$	N	N	$2N$	27	56	248	6	7



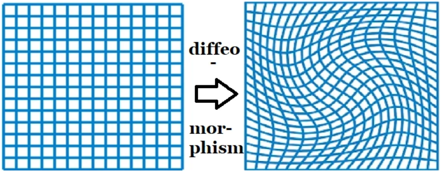


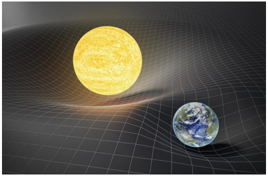


- **Spin** 2 ($g_{(\mu\nu)}$) $\rightarrow$ Gauge principle: non-compact Lie algebra

$$S_{Einstein} = -\frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda), \qquad \underbrace{R}_{\text{Ricci scalar}} \sim \partial^2 g + \dots$$

Incorporates Gravity + Yang-Mills + matter. Dynamical spacetime!





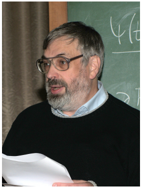
- **Spin** 3 ($h_{(\mu\nu\rho)}$) $\rightarrow$???

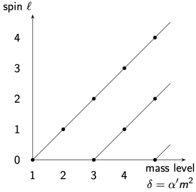
[Berends, Burgers, van Dam 1985], [Bekaert, Boulanger, Leclercq 2010]

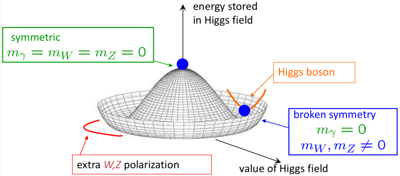
Such theories suffers incurable obstruction in quartic order!

- Gauge Principle: Higher-spin gauge theories $\leftrightsquigarrow$ infinite-dimensional nonabelian gauge algebras containing the spacetime isometry algebra as maximal finite-dimensional subalgebra. Gravity is a subsector.
- Unification of fundamental interactions. Fields with arbitrarily high spin most probably necessary for consistency, as in string theory.
- Vasiliev's unfolding: Geometric approach to field theory. Manifest differomorphism covariance. Gauge invariance is a consequence of Cartan's integrability.
- String theory first appeared as a model reproducing the Regge trajectories of massive hadronic resonances with increasing spin. *Veneziano amplitudes*: extremely soft UV behaviour thanks to the exchange of infinitely many massive HS states!

String (field) theory $\leftrightsquigarrow$ broken phase of higher-spin gauge theory?

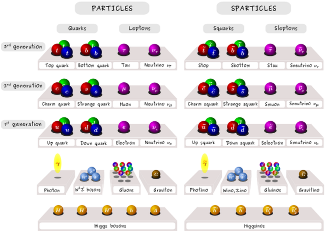






- Supersymmetry (SUSY) is a theoretical framework in physics that proposes a symmetry between fermions (matter particles with half-integer spin) and bosons (force carrier particles with integer spin).

$$S = -\frac{1}{2\pi} \int d^2\sigma \left(\partial_\alpha X_\mu \partial^\alpha X^\mu + \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right)$$



String theories

Type	Spacetime dimensions	SUSY generators	chiral	open strings	heterotic compactification	gauge group	tachyon
Bosonic (closed)	26	N = 0	no	no	no	none	yes
Bosonic (open)	26	N = 0	no	yes	no	U(1)	yes
I	10	N = (1,0)	yes	yes	no	SO(32)	no
IIA	10	N = (1,1)	no	no	no	U(1)	no
IIB	10	N = (2,0)	yes	no	no	none	no
HO	10	N = (1,0)	yes	no	yes	SO(32)	no
HE	10	N = (1,0)	yes	no	yes	$E_8 \times E_8$	no
M-theory	11	N = 1	no	no	no	none	no

Supersymmetry and harmonic superspace

- Supersymmetric theorise naturally can be naturally formulated in the superspace – which is the extension of Minkowski space-time:

$$\mathbb{R}^4 = \{x^m\} \quad \to \quad \mathbb{R}^{4|4\mathcal{N}} = \{x^m, \theta^{\alpha i}, \bar{\theta}^{\dot{\alpha}}_i\}.$$

- Superfields collect and unify ordinary fields:

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \frac{i}{\sqrt{2}}\theta^2\sigma^\mu\bar{\theta}\partial_\mu\psi(x) - \frac{1}{4}\theta^4\Box\psi(x).$$

- Supergravity in superspace:

$$S_{sugra} = \frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\mathfrak{D}^{++}q^+_a - \frac{1}{4\kappa^2}\int d^4x d^8\theta du\, E\, H^{+5}H^{--5}.$$

- Supergravity in components:

$$\begin{aligned}
e^{-1}\mathcal{L} = {} & \kappa^{-2}\left(\tfrac{1}{2}R - \bar{\psi}_{i\mu}\gamma^{\mu\nu\rho}D_\nu\psi^i_\rho\right) - N_{IJ}D_\mu X^I D^\mu\bar{X}^J - \tfrac{1}{2}g_{XY}D_\mu q^X D^\mu q^Y \\
& + \left\{-\tfrac{1}{4}\mathrm{i}\bar{F}_{IJ}F^{+I}_{\mu\nu}F^{+\mu\nu J} + \tfrac{1}{16}N_{IJ}X^I X^J T^+_{ab}T^{+ab} + \mathrm{h.c.}\right\} \\
& - N_{IJ}\vec{Y}^I\cdot\vec{Y}^J - N^{-1|IJ}P^0_I P^0_J - 2\bar{X}^I X^J k_I{}^X k_{JX} + \tfrac{2}{3}C_{I,JK}e^{-1}\varepsilon^{\mu\nu\rho\sigma}W_\mu{}^I W_\nu{}^J\left(\partial_\rho W_\sigma{}^K + \tfrac{3}{8}f_{LM}{}^K W_\rho{}^L W_\sigma{}^M\right) \\
& + \tfrac{1}{2}\bar{\psi}_{ai}\gamma^{abc}\psi^j_b\left(\delta^i_j N_{IJ}\bar{X}^I D_c X^J + D_c q^X\vec{k}_X\cdot\vec{\tau}_j{}^i\right) + \left\{-\tfrac{1}{4}N_{IJ}\bar{\Omega}^{iI}\hat{\not{D}}\Omega^J_i - \bar{\zeta}_{\bar{\mathcal{A}}}\hat{\not{D}}\zeta^{\mathcal{B}}d^{\bar{\mathcal{A}}}{}_{\mathcal{B}} + \tfrac{1}{2}N_{IJ}\bar{\psi}_{ia}\not{D}X^I\gamma^a\Omega^{iJ}\right. \\
& + \mathrm{i}\bar{\psi}_{ia}\not{D}q^X\gamma^a\zeta_{\bar{\mathcal{A}}}f^{i\mathcal{B}}{}_X d^{\bar{\mathcal{A}}}{}_{\mathcal{B}} - \tfrac{1}{2}N_{IJ}\varepsilon_{ij}\left(\bar{\Omega}^{iI}\gamma_\mu - \bar{X}^I\bar{\psi}^i_\mu\right)\psi^j_\nu F^{-\mu\nu J} - \tfrac{1}{16}\mathrm{i}F_{IJK}\bar{\Omega}^I_i\gamma^{\mu\nu}\Omega^J_j\varepsilon^{ij}F^{-K}_{\mu\nu} \\
& + \tfrac{1}{2}\bar{\psi}_{ai}\gamma^a\left[\Omega^I_j P_I{}^{ij} + \Omega^I_j N_{IJ}f_{KL}{}^J X^L\bar{X}^K\varepsilon^{ij} - 4\mathrm{i}X^I k_I{}^X f^{i\mathcal{B}}{}_X C_{\mathcal{B}\mathcal{A}}\zeta^{\mathcal{A}}\right] + \tfrac{1}{2}N_{IJ}\bar{\Omega}^I_i f_{KL}{}^J\Omega^L_j\bar{X}^K\varepsilon^{ij} + \tfrac{1}{2}\bar{X}^I\bar{\psi}^i_a\gamma^{ab}\psi^j_b P_{Iij} \\
& \left. + 2X^I\bar{\zeta}^{\mathcal{A}}\zeta^{\mathcal{B}}t_{I\mathcal{A}\mathcal{B}} + 2\mathrm{i}k^X_I f^{i\mathcal{B}}{}_X\varepsilon_{ij}d^{\bar{\mathcal{A}}}{}_{\mathcal{B}}\bar{\zeta}_{\bar{\mathcal{A}}}\Omega^{jI} + \mathrm{h.c.}\right\} + \text{4-fermion terms.}
\end{aligned}$$

- For theories with extended supersymmetry ($\mathcal{N} \geq 2$) ordinary is not adequate! (no-go "theorems" for extended supersymmetry)
- Harmonic superspace [Galperin, Ivanov, Kalitsyn, Ogievetsky, Sokatchev 1984] is the universal method to deal with off-shell $\mathcal{N} = 2$ supersymmetry theories:

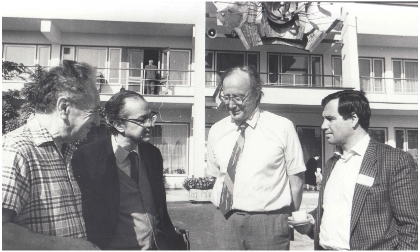
$$\mathbb{HR}^{4+2|8} = \mathbb{R}^{4|8} \times S^2 = \{x^{\alpha\dot{\alpha}}, \theta^{\alpha i}, \bar{\theta}^{\dot{\alpha} i}, u_i^{\pm}\}.$$

- Introduction of harmonic coordinates lead to the presence of a new supersymmetric invariant superspace – analytic superspace, with coordinates $\zeta = \{x_A^{\alpha\dot{\alpha}}, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u_i^{\pm}\}$, where analytic coordinates defined as

$$x_A^{\alpha\dot{\alpha}} := x^{\alpha\dot{\alpha}} - 4i\theta^{\alpha(i}\bar{\theta}^{\dot{\alpha}j)}u_i^+ u_j^-, \quad \theta^{+\alpha} := \theta^{\alpha i}u_i^+, \quad \bar{\theta}^{+\dot{\alpha}} := \bar{\theta}^{\dot{\alpha} i}u_i^+.$$

- One can also define *harmonic derivatives* $\partial^{\pm\pm} = u^{\pm i}\frac{\partial}{\partial u^{\mp i}}$, which in the analytical basis take the form:

$$\mathcal{D}^{\pm\pm} := \partial^{\pm\pm} - 4i\theta^{\alpha\pm}\bar{\theta}^{\dot{\alpha}\pm}\partial_{\alpha\dot{\alpha}} + \theta^{\pm\hat{\alpha}}\partial^{\pm}_{\hat{\alpha}}.$$



$\mathcal{N}=2$ higher-spin theories

- $\mathcal{N}=2$ higher-spin theories (see talk by E.Ivanov) have natural geometric formulation in terms of unconstrained analytical prepotentials [Buchbinder, Ivanov, N.Z. 2021]:

$$\left\{h^{++\alpha(s-1)\dot\alpha(s-1)}, h^{++\alpha(s-1)\dot\alpha(s-2)+}, h^{++\alpha(s-2)\dot\alpha(s-1)+}, h^{++\alpha(s-2)\dot\alpha(s-2)}\right\}.$$

- These prepotentials are the higher-spin generalization of linearized (over flat $\mathcal{N}=2$ harmonic superspace) $\mathcal{N}=2$ "minimal" Einstein supergravity ($s=2$ case).
- Analytic prepotentials are defined up to gauge transformations:

$$\delta_\lambda h^{++\alpha(s-1)\dot\alpha(s-1)} = \mathcal{D}^{++}\lambda^{\alpha(s-1)\dot\alpha(s-1)} + 4i\left[\lambda^{+\alpha(s-1)(\dot\alpha(s-2)}\bar\theta^{+\dot\alpha)} + \theta^{+(\alpha}\bar\lambda^{\alpha(s-2))\dot\alpha(s-1)}\right],$$

$$\delta_\lambda h^{++\alpha(s-1)\dot\alpha(s-2)+} = \mathcal{D}^{++}\lambda^{+\alpha(s-1)\dot\alpha(s-2)}, \qquad \dots$$

- These higher-spin prepotentials covariantize harmonic derivative with respect to the higher-spin $\mathcal{N}=2$ supersymmetry:

$$\mathcal{D}^{++} \quad\to\quad \mathcal{D}^{++} + \kappa_s h^{++\alpha(s-2)\dot\alpha(s-2)M}\partial_M\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}(J)^{P(s)},$$

and so naturally interact with the hypermultiplet.

- In the Wess-Zumino type gauge

$$h^{++\alpha(s-1)\dot\alpha(s-1)} = -4i\theta^+_\beta\bar\theta^+_{\dot\beta}\Phi^{(\alpha(s-1)\beta)(\dot\alpha(s-1)\dot\beta)} - 4i\left(\frac{s-1}{s}\right)^2\theta^{+(\alpha}\bar\theta^{+(\dot\alpha}\Phi^{\alpha(s-2))\dot\alpha(s-2))} + \dots$$

$$h^{++\alpha(s-2)\dot\alpha(s-2)} = -4i\theta^+_\beta\bar\theta^+_{\dot\beta}C^{(\alpha(s-2)\beta)(\dot\alpha(s-2)\dot\beta)} - 4i\left(\frac{s-2}{s-1}\right)^2\theta^{+(\alpha}\bar\theta^{+(\dot\alpha}C^{\alpha(s-3))\dot\alpha(s-3))} + \dots$$

we obtain content of $\mathcal{N}=2$ higher-spin off-shell supermultiplet:

$$\boxed{s \;\leftrightarrow\; 2\times(s-\frac{1}{2}) \;\leftrightarrow\; s-1} \qquad + \qquad \text{auxiliary fields}$$

- These fields corresponds to the Fronsdal higher-spin fields:

$$\delta_\lambda\Phi^{\alpha(s)\dot\alpha(s)} = \partial^{(\alpha(\dot\alpha}a^{\alpha(s-1))\dot\alpha(s-1))}, \qquad \delta_\lambda\Phi^{\alpha(s-2)\dot\alpha(s-2)} = \partial_{\beta\dot\beta}a^{(\alpha(s-2)\beta)(\dot\alpha(s-2)\dot\beta)}.$$

- Totally there are $8(s^2+(s-1)^2)_B + 8(s^2+(s-1)^2)_F$ off-shell degrees of freedom.

$\mathcal{N}=2$ higher-spin theories

- Rigid $\mathcal{N}=2$ supersymmetry transformations have non-standard form:

$$\delta_\epsilon h^{++\alpha(s-1)\dot\alpha(s-1)} = -4i\left[h^{++\alpha(s-1)(\dot\alpha(s-2)+}\bar\epsilon^{-\dot\alpha)} - \epsilon^{-(\alpha}h^{++\alpha(s-2))\dot\alpha(s-1)}\right], \quad \dots$$

- To construct the invariant action one need to introduce supersymmetry-invariant superfields:

$$G^{++\alpha(s-1)\dot\alpha(s-1)} = h^{++\alpha(s-1)\dot\alpha(s-1)} + 4i\left[h^{++\alpha(s-1)(\dot\alpha(s-2)+}\bar\theta^{-\dot\alpha)} - h^{++(\alpha(s-2)\dot\alpha(s-1)+}\theta^{-\alpha)}\right],$$

$$G^{++\alpha(s-2)\dot\alpha(s-2)} = h^{++\alpha(s-2)\dot\alpha(s-2)} - 2i\left[h^{++(\alpha(s-2)\beta)\dot\alpha(s-2)+}\theta^{-}_{\beta} - h^{++(\alpha(s-2)(\dot\alpha(s-2)\dot\beta)+}\bar\theta^{-}_{\dot\beta}\right].$$

- Invariant action have universal form for any spin!:

$$S_{(\mathrm{s})} = (-1)^{s+1}\int d^4x d^8\theta du\, \Big[G^{++\alpha(s-1)\dot\alpha(s-1)}G^{--}_{\alpha(s-1)\dot\alpha(s-1)} + 4G^{++\alpha(s-2)\dot\alpha(s-2)}G^{--}_{\alpha(s-2)\dot\alpha(s-2)}\Big].$$

- Where G^{--}-superfields are defined as the solutions of zero-curvature equations:

$$\mathcal{D}^{++}G^{--\dots} = \mathcal{D}^{--}G^{++\dots}.$$

- In component fields, these action leads to Fronsdal action for spins s, $s-1$ and Fang-Fronsdal action for doublet of $s-1/2$ spins.

Outlook

[I] I. Buchbinder, E. Ivanov and N. Zaigraev, *Unconstrained off-shell superfield formulation of 4D, $\mathcal{N}=2$ supersymmetric higher spins*, JHEP **12** (2021), 016, [arXiv:2109.07639 [hep-th]].

[II] I. Buchbinder, E. Ivanov and N. Zaigraev, *Off-shell cubic hypermultiplet couplings to $\mathcal{N}=2$ higher spin gauge superfields*, JHEP **05** (2022), 104, [arXiv:2202.08196 [hep-th]].

[III] I. Buchbinder, E. Ivanov and N. Zaigraev, *$\mathcal{N}=2$ higher spins: superfield equations of motion, the hypermultiplet supercurrents, and the component structure*, JHEP **03** (2023), 036, [arXiv:2212.14114 [hep-th]].

[IV] I. Buchbinder, E. Ivanov and N. Zaigraev, *$\mathcal{N}=2$ superconformal higher-spin multiplets and their hypermultiplet couplings*, JHEP **08** (2024), 120 [arXiv:2404.19016 [hep-th]].

[V] E. Ivanov and N. Zaigraev, *Off-shell invariants of linearized 4D, $\mathcal{N}=2$ supergravity in the harmonic approach*, Phys. Rev. D **110** (2024) no.6, 066020 [arXiv:2407.08524 [hep-th]].

[VI] N. Zaigraev, *$\mathcal{N}=2$ higher-spin supercurrents*, Phys. Lett. B **858** (2024), 139056 [arXiv:2408.00668 [hep-th]].

[VII] E. Ivanov and N. Zaigraev, *$\mathcal{N}=2$ superconformal gravitino in harmonic superspace*, Phys. Lett. B **862** (2025), 139333 [arXiv:2412.14822 [hep-th]].

[VIII] I. Buchbinder, E. Ivanov and N. Zaigraev, *Towards $\mathcal{N}=2$ higher-spin supergravity*, Invited Contribution to the A.A. Starobinsky Memorial Volume, Springer 2025 [arXiv:2503.02438 [hep-th]].

Thank you for your attention!