Rotating Black Holes in Extended Theories of Gravity: Constraints Using EHT Data

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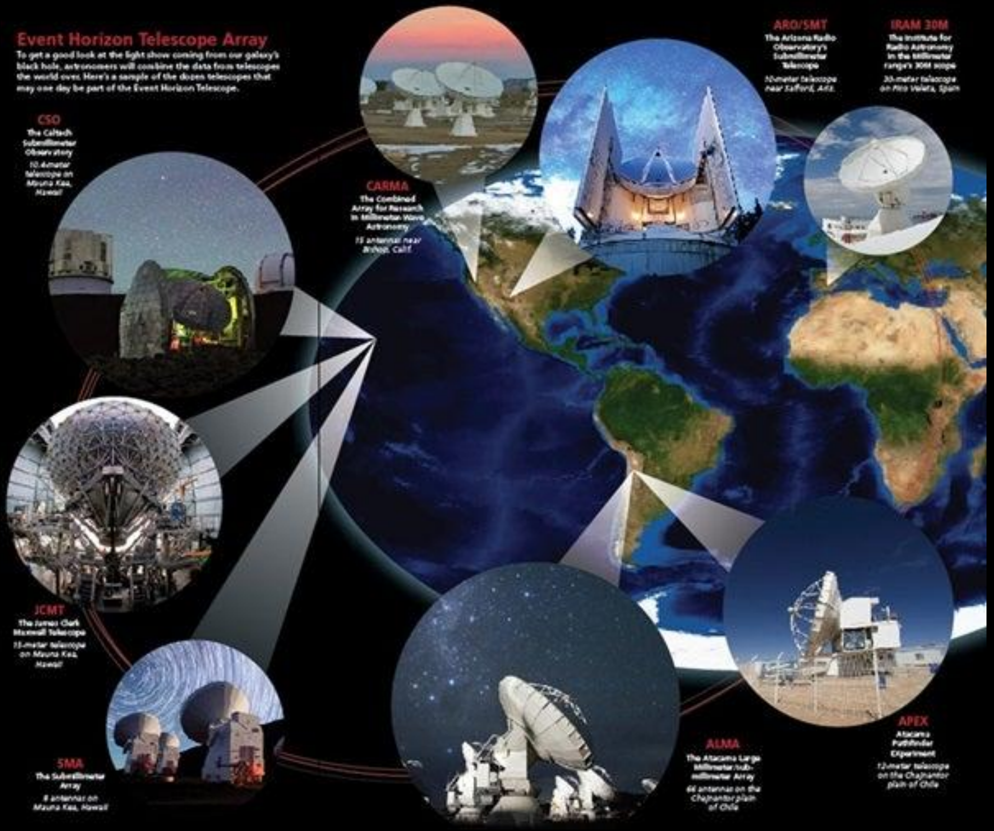
Alushta-2025

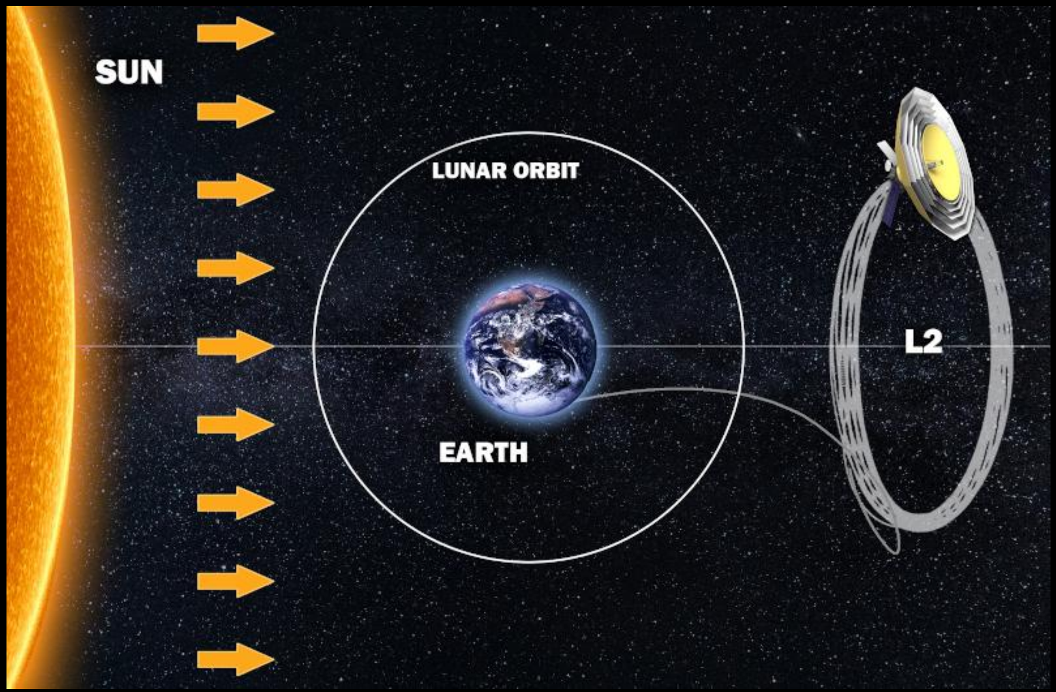
Introduction

Event Horizon Telescope (EHT)



K. Akiyama, et al., Astrophys. J. 875 (1) L5 (2019).

Millimetron space observatory



https://millimetron.ru

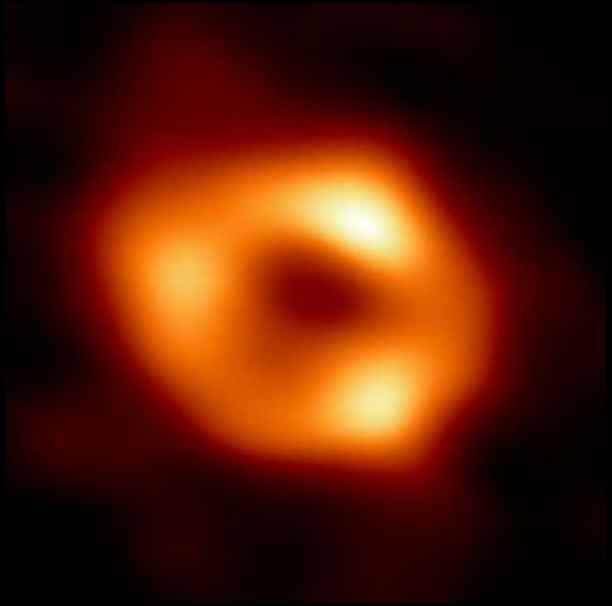
Introduction

M 87*

BH shadow size from 4.3M to 6.1M
Rotational parameter a=0.9375

K. Akiyama, et al., Astrophys. J. 875 (1) L5 (2019).
Cui, Y.; others. Nature 621, 711–715, (2023).

Sgr A*



BH shadow size from 4.3M to 5.3M
Rotational parameter a=0.5 or a=0.94

The Event Horizon Telescope Collaboration, The Astrophysical Journal Letters 930 L17 (2022).

The basic theory of black hole shadows was presented in the works (A. F. Zakharov, Sov. Phys. JETP, 64, 1 (1986)) and (Zakharov, A.F. Int. J. Mod. Phys. A 20,2321–2325, (2005)). In the work (Zakharov, A.F. Phys. Rev. D 90, 062007, (2014)) it was proposed to supplement the metric with a tidal charge.

Newman-Janis algorithm improved version*

Metric function :

$$ds^2 = -G(r)dt^2 + \frac{1}{F(r)}dr^2 + H(r)d\Omega^2. \qquad (1)$$

The components of the axially symmetric metric :

$$\begin{aligned} g_{tt} &= -\frac{FH + a^2\cos^2\theta}{(K + a^2\cos^2\theta)^2}\Psi, \\ g_{t\phi} &= -a\sin^2\theta\frac{K - FH}{(K + a^2\cos^2\theta)^2}\Psi, \\ g_{\theta\theta} &= \Psi, \\ g_{rr} &= \frac{\Psi}{FH + a^2}, \qquad (2) \\ g_{\phi\phi} &= \Psi\sin^2\theta(1 + a^2\sin^2\theta\frac{2K - FH + a^2\cos^2\theta}{(K + a^2\cos^2\theta)^2}). \end{aligned}$$

$$K = H(r)\sqrt{F(r)/G(r)}.$$

$$y \equiv \cos\theta.$$

$\Psi(r, y^2, a)$ conditions :

$$\lim_{a\to 0}\Psi(r, y^2, a) = H(r),$$

$$\begin{aligned} &(K + a^2y^2)^2(3\Psi_r\Psi_{y^2} - 2\Psi\Psi_{r,y^2}) = 3a^2K_r\Psi^2, \\ &\Psi\left[K_r^2 + K(2 - K_{rr}) - a^2y^2(2 + K_{rr})\right] + \\ &+ (K + a^2y^2)[(4y^2\Psi_{y^2} - K_r\Psi_r] = 0. \qquad (3) \end{aligned}$$

$$ds_c^2 = \Psi_c/\Psi_n ds_n^2. \qquad (4)$$

Solution of eq. (3):

$$\begin{aligned} \Psi_c &= H(r)\exp\left[a^2 f(r, a^2y^2, a)\right] \approx \\ &\approx H(r) + a^2X(y^2, r) + o(a^2), \qquad (5) \end{aligned}$$

$$A_r = \partial A/\partial r$$

$$KH_rK_r + HK_r^2 + HK(K_{rr} - 2) = 0,$$

$$X(y^2, r) = \frac{H^2(8K - K_r^2)y^2}{K^2(8H - H_rK_r)},$$

$$\begin{aligned} &K_r(8K - K_r^2)K_{rrr} + K_r^2(K_{rr} - 2)^2 - \\ &- 4KK_{rr}(K_{rr} + 4) + 48K = 0. \qquad (6) \end{aligned}$$

*Azreg-Aïnou, M. Eur. Phys. J. C 74, 2865,(2014).

Black Hole Shadows Modelling: Rotation Accounting

The coordinates of the shadow on plane normal to the direction on a distant observer:

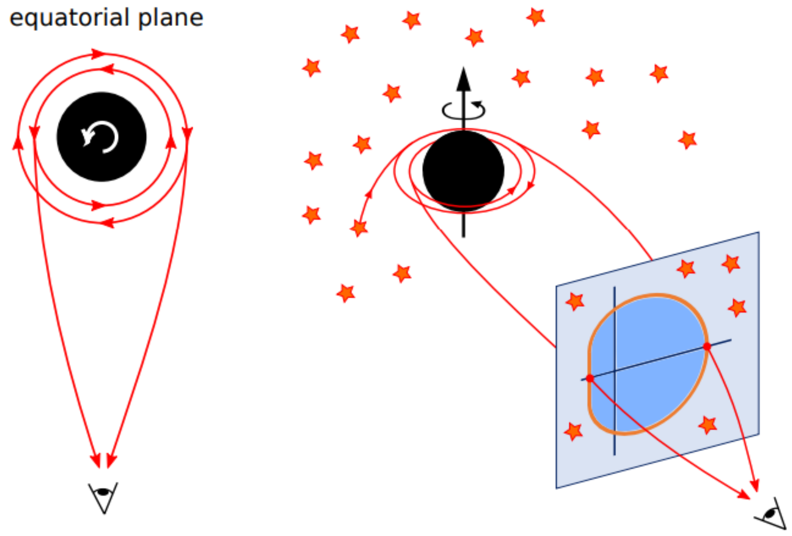
$$\lambda = \frac{K + a^2}{a} - \frac{2K'}{a}\frac{(FH + a^2)}{(HF)'},$$

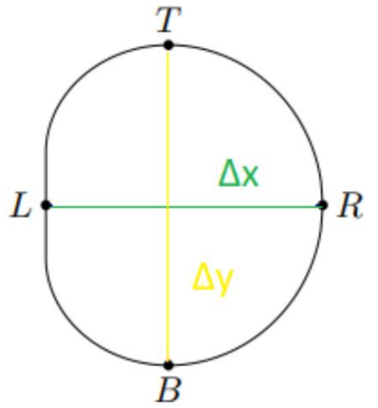
$$\eta = \frac{4(a^2 + FH)}{((HF)')^2}(K')^2 - \frac{1}{a^2}\left[K - \frac{2(FH + a^2)}{(HF)'}K'\right].$$

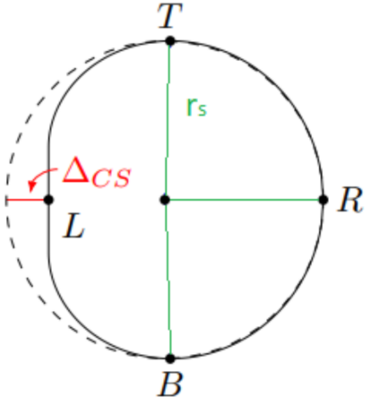
$$x' = -\frac{\lambda}{\sin\theta_0},$$

$$y' = \pm\sqrt{\eta + a^2\cos^2\theta_0 - \frac{\lambda^2}{\tan^2\theta_0}},$$

Image of the shadow of rotating BH and photonic geodesics







$$D = \frac{x_{min} + x_{max}}{2},$$

$$\delta_{cs} = \Delta_{cs}/r_s,$$

D – displacement parameter

δ_{cs} – distortion parameter

Volker Perlick, Oleg Yu. Tsupko, Physics Reports, Volume 947, 2022, Pages 1-39

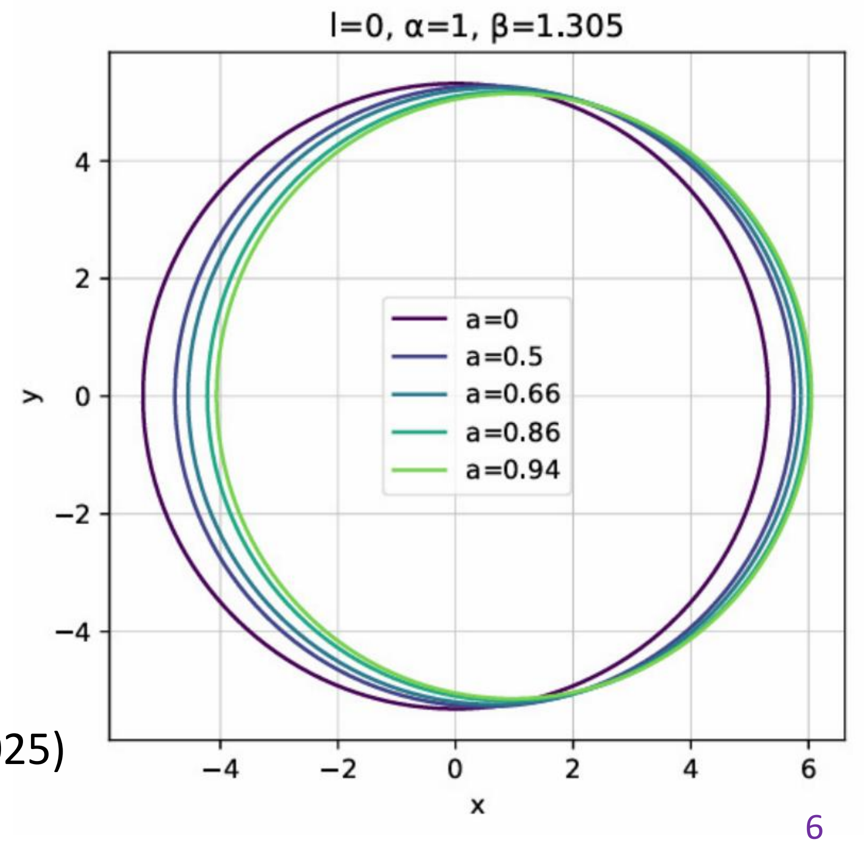
K. Hioki and Kei-ichi Maeda, Phys. Rev. D **80**, 024042 (2009).

Black hole shadows

Extended theories of gravity used in the presentation*:

1. Horndeski theory
2. Bumblebee model
3. Gauss-Bonnet scalar gravity
4. Loop Quantum Gravity (modified Hayward metric and BH metric with transition surface**)
5. Conformal Gravity
6. f(Q) gravity

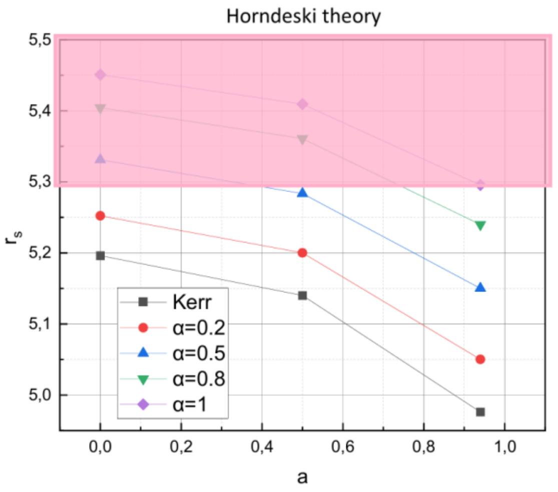
An example of constructing a BH shadow profile for the Loop Quantum Gravity case (modified Hayward metric). The most probable values of the rotation parameters a for Sgr A* and the static case (0, 0.5 and 0.94) were used for the construction. However, some models contain critical values of the rotation parameters for certain parameter configurations:

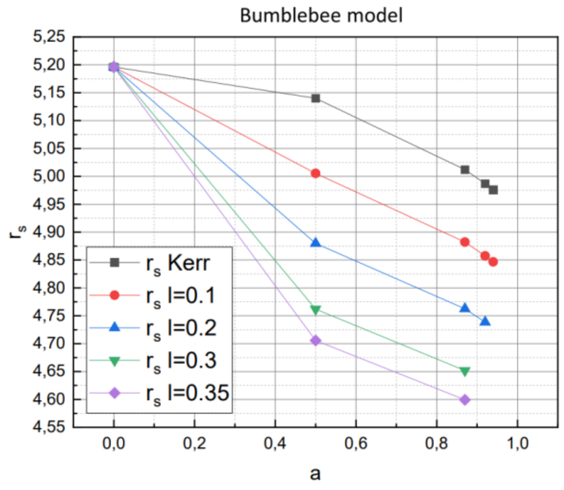


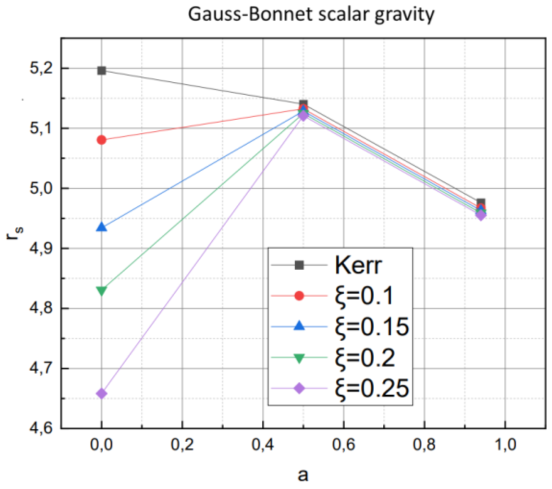
*S. Alexeyev, O. Zenin, A. Baiderin, JETP, 167, N. 2 (2025) (russian version)

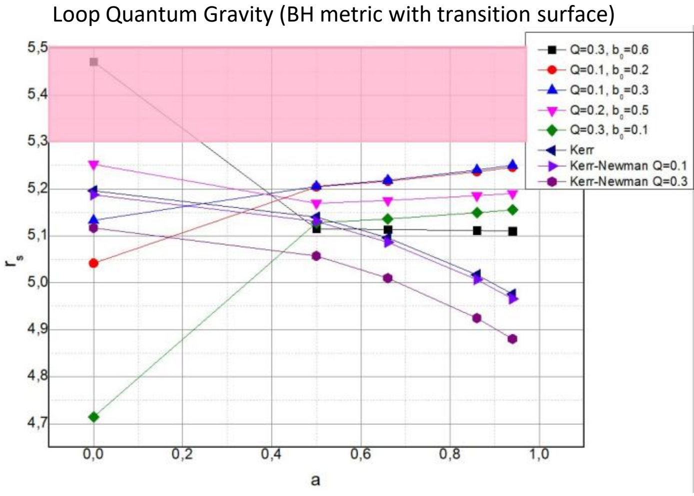
** S. Alexeyev, E. Antonov, O. Zenin, Moscow University Physics Bulletin (2025) (russian version) (accepted for publication)

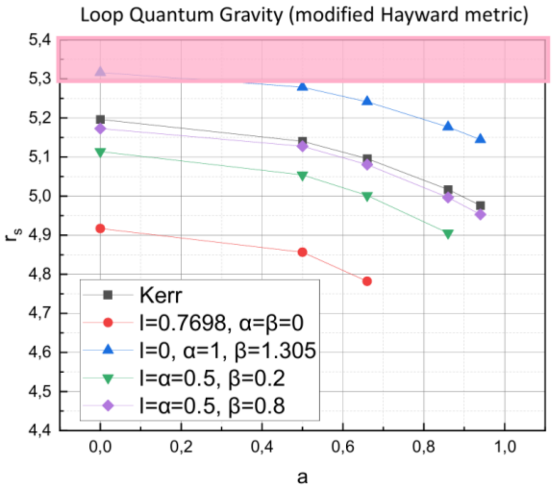
Shadow size

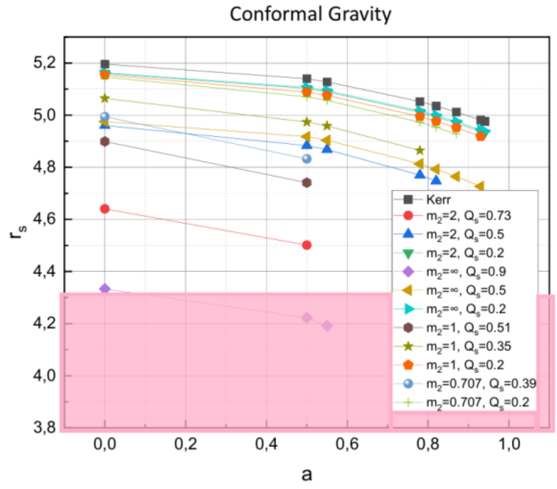


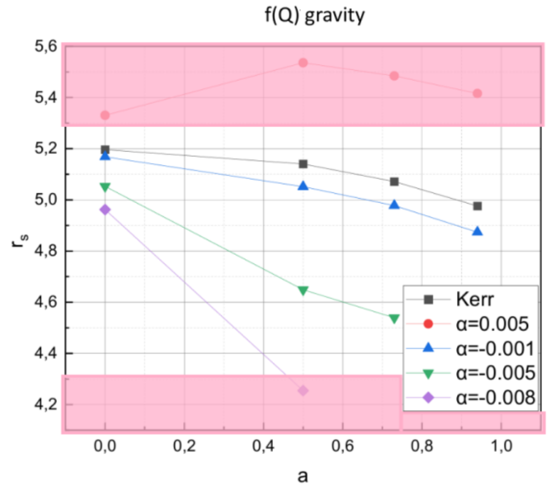






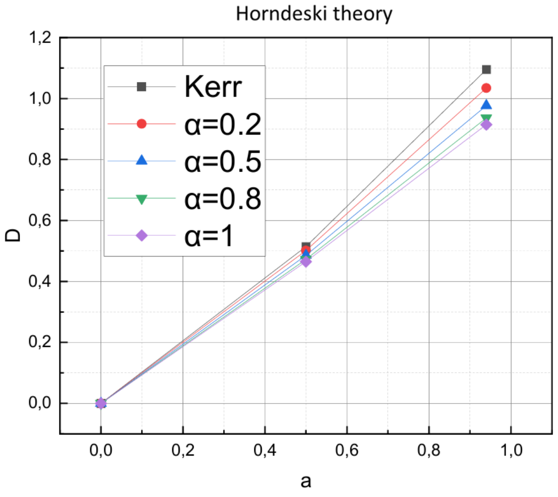


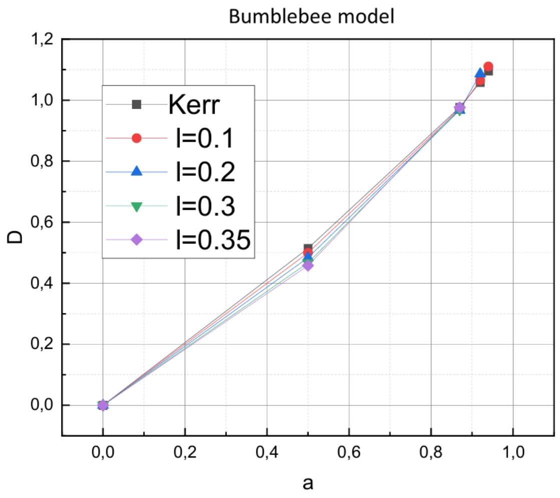


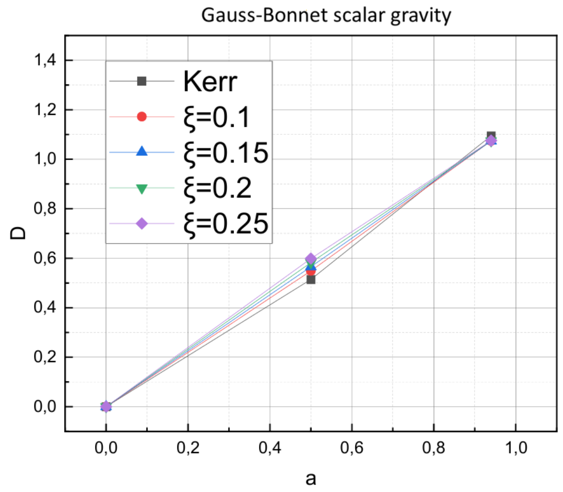


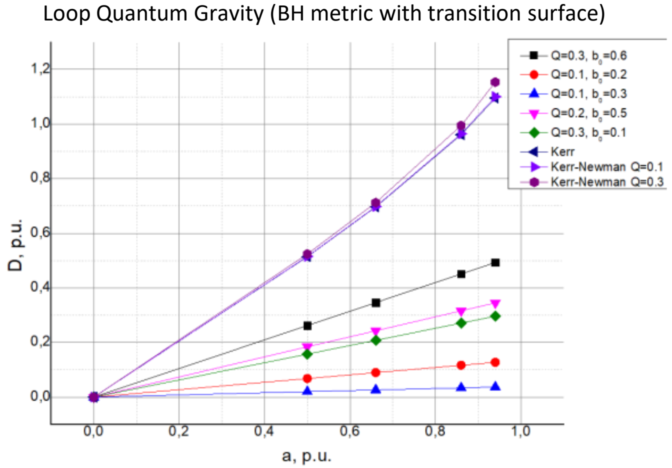
Red zone – EHT Constraints

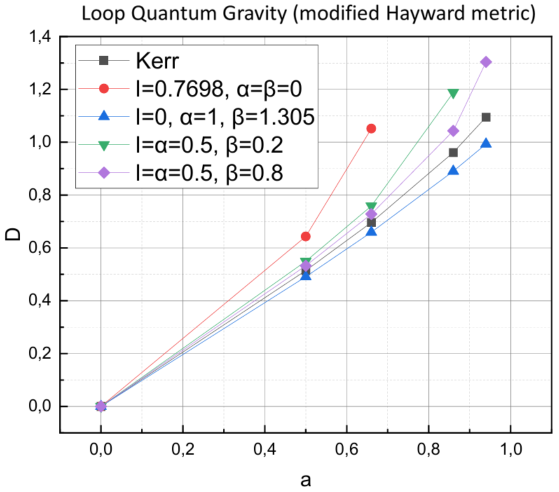
Displacement parameter

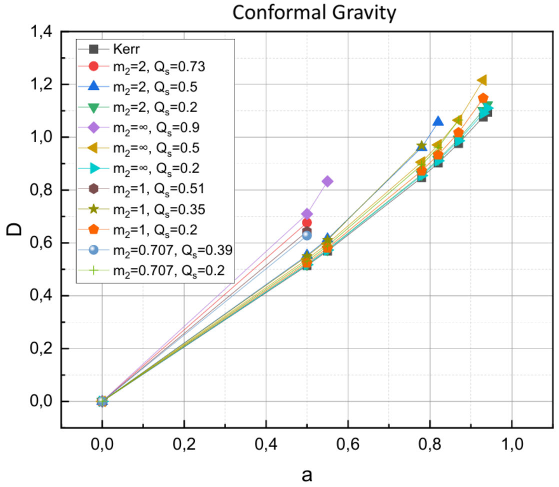


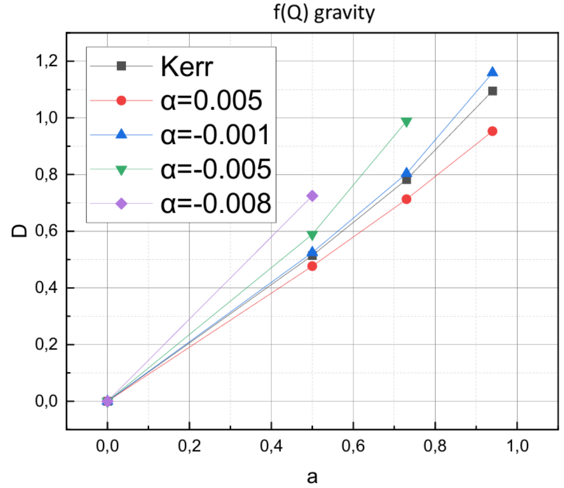






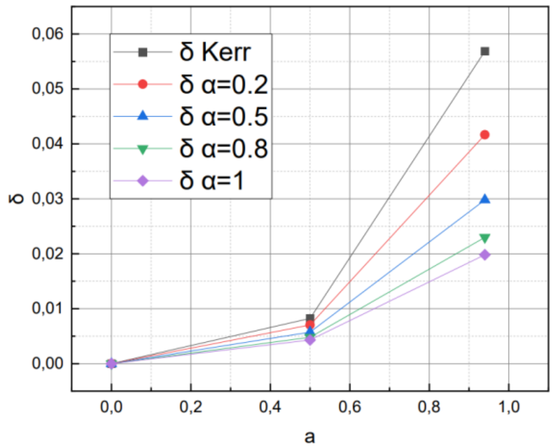




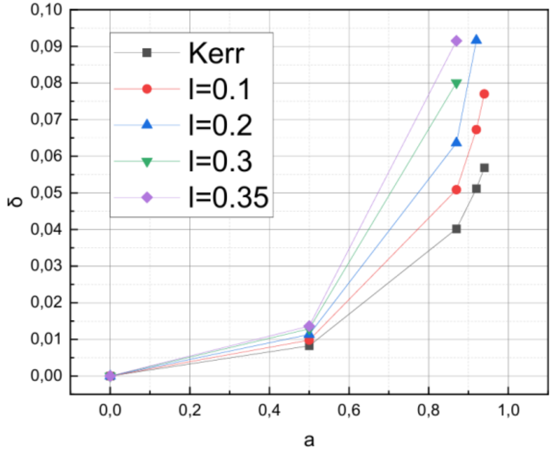


Distortion parameter

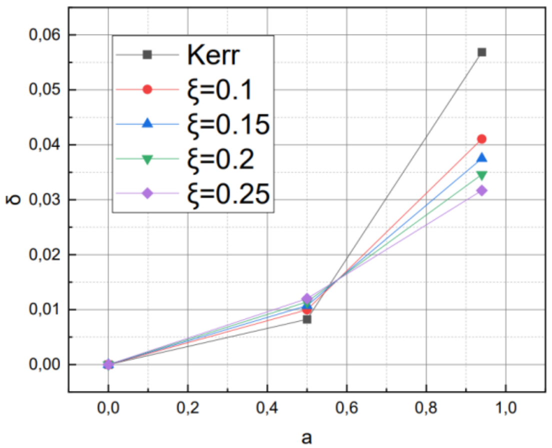
Horndeski theory



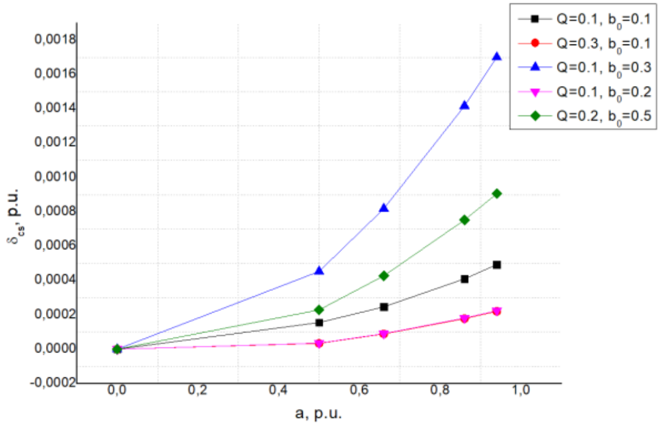
Bumblebee model



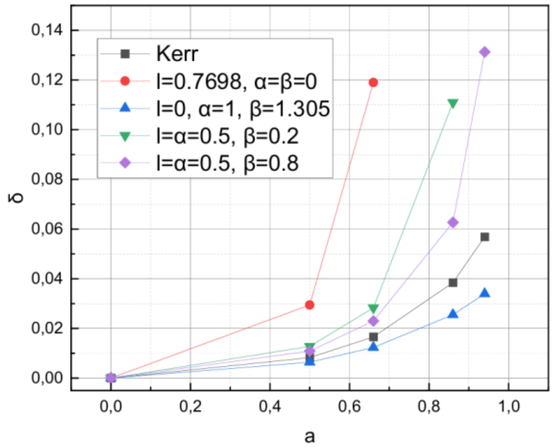
Gauss-Bonnet scalar gravity



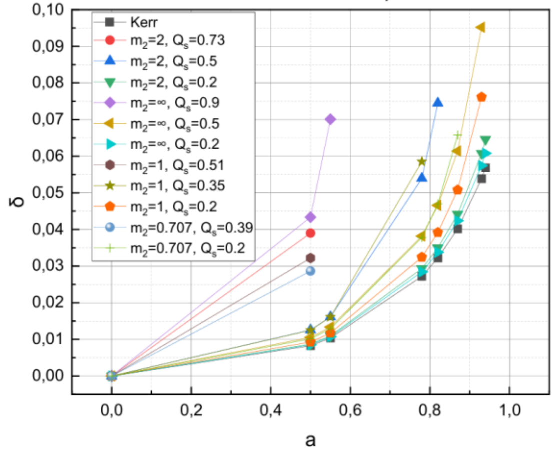
Loop Quantum Gravity (BH metric with transition surface)



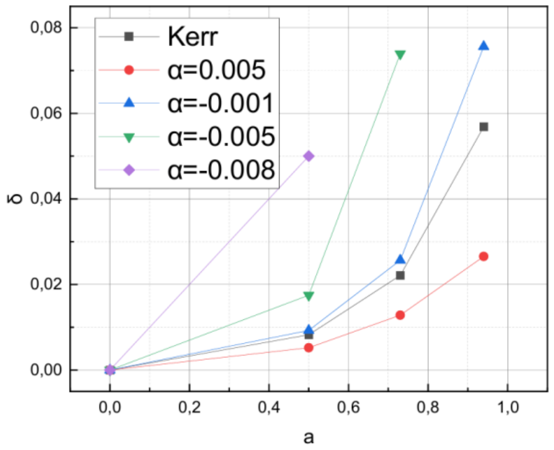
Loop Quantum Gravity (modified Hayward metric)

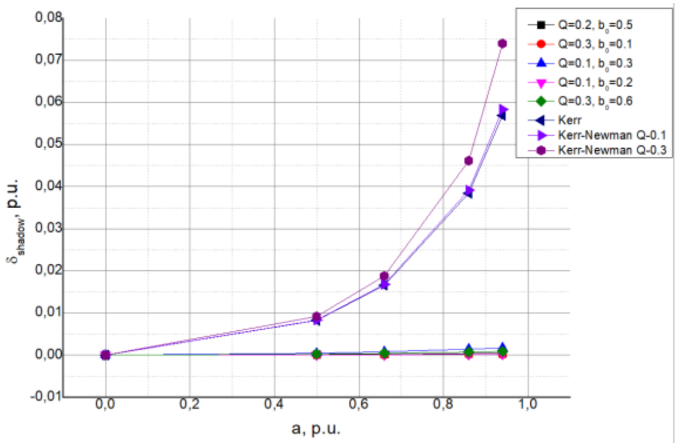


Conformal Gravity



f(Q) gravity





Discussion and conclusions 1

- Spherically symmetric solutions for extended gravity theories contain a number of additional parameters that are not present in the simplest solution of general relativity. Furthermore, these solutions, in addition to having one or more additional parameters, have a more complex structure compared to the Reissner-Nordström metric.

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- The presence of additional parameters of the theory due to a more complex structure of the solution gives rise to the presence of critical values of the angular momentum $\boldsymbol{a}_{crit}$. Such values exist in all theories considered, except for the Horndeski model and, partially, the Gauss-Bonnet scalar-tensor gravity (for $\xi<0.3$).

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- The previously made conclusion is confirmed that for some of the models considered, taking into account the parameters of the theory either slows down the rotation and the effects associated with it (this is most clearly manifested in the Horndeski theory and the Gauss-Bonnet scalar gravity), or enhances them (this is most clearly manifested in the Bumblebee model). For the other models considered, this effect is also present, but it works less linearly.

Discussion and conclusions 2

- Considering the dependence of the shift parameter and its closeness to the Kerr value, we can conclude that the Horndeski, Bumblebee, and Gauss-Bonnet scalar gravity models work best and with a minimum number of additional parameters and restrictions as a basis for modeling black hole shadow profiles. Apparently, the best results should be expected from the Horndeski model. The Bumblebee model provides the best match with the Kerr metric.

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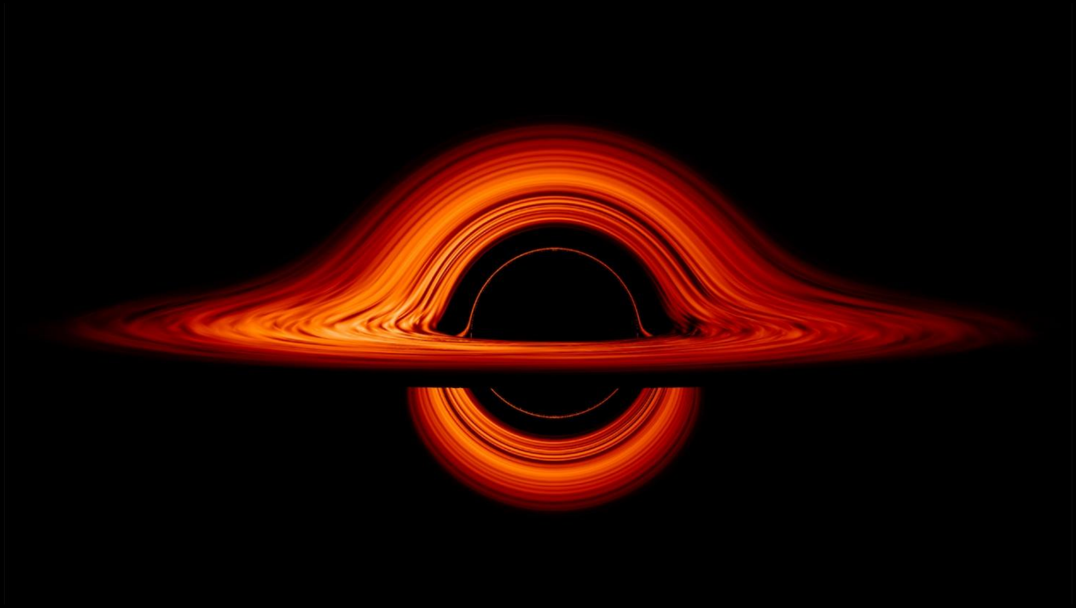
- In the BH metric with transition surface there is also no central singularity, but in this metric the shadow shape is close to spherically symmetric even with strong rotation. Therefore, this metric cannot describe rapidly rotating BH.

Thank you for your attention!!!



The work of Oleg Zenin was supported in part by the Foundation for the Advancement of Theoretical Physics and Mathematics “BASIS” grant number 22-2-2-11-1