Phase transitions in Nature: from water to quark-gluon plasma

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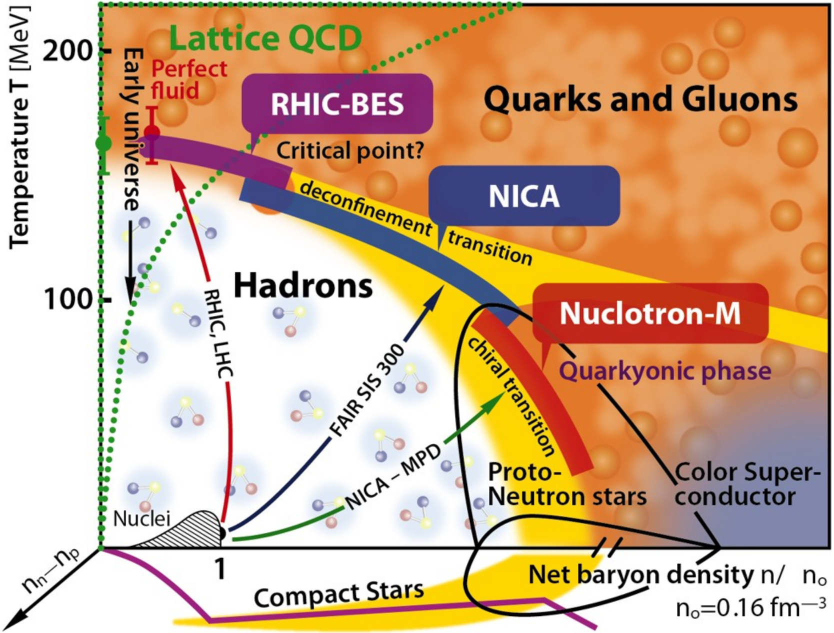
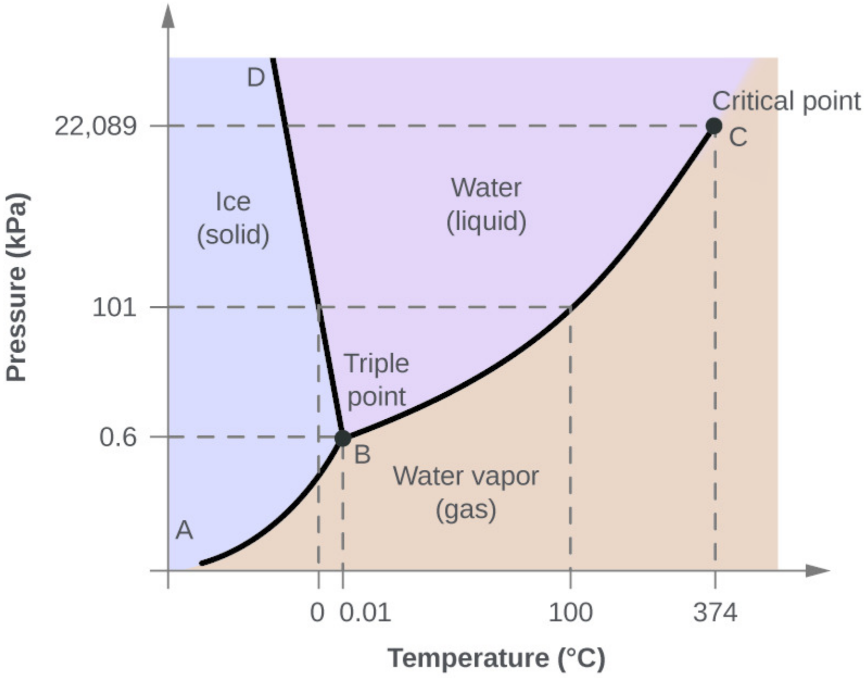
Basic Experimental Facility of BLTP


SPD
(Detector)
BM@N (Detector)
Extracted beam
Collider
MPD
(Detector)
E-cooling
Heavy Ion
Linac
Nuclotron
LU-20
Booster
NICA


Lattice QCD
Perfect fluid
RHIC-BES
Critical point?
Quarks and Gluons
Temperature T [MeV]
200
100
Early universe
deconfinement
transition
NICA
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
Nuclotron-M
Quarkyonic phase
chiral transition
Proto-
Neutron stars
Color Super-
conductor
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_o
n_o=0.16 fm^{-3}


Pressure (kPa)
22,089
101
0.6
D
Ice
(solid)
Water
(liquid)
Critical point
C
Triple
point
B
Water vapor
(gas)
A
0
0.01
100
374
Temperature (°C)

Outline

- Types of phase transitions
- Phase diagram of water and beyond
- How well can we predict the phase diagram
- Expected phases of strongly interacting matter
- Where to find and how to look at hot dense nuclear matter
- Theoretical approaches to studying the QCD phase diagram
- Why the critical point is discussed: hopes and challenges

Ehrenfest Classification (1933)

Classifies phase transitions by the lowest derivative of Gibbs free energy that shows discontinuity:

1st Order Transition (Melting, boiling)

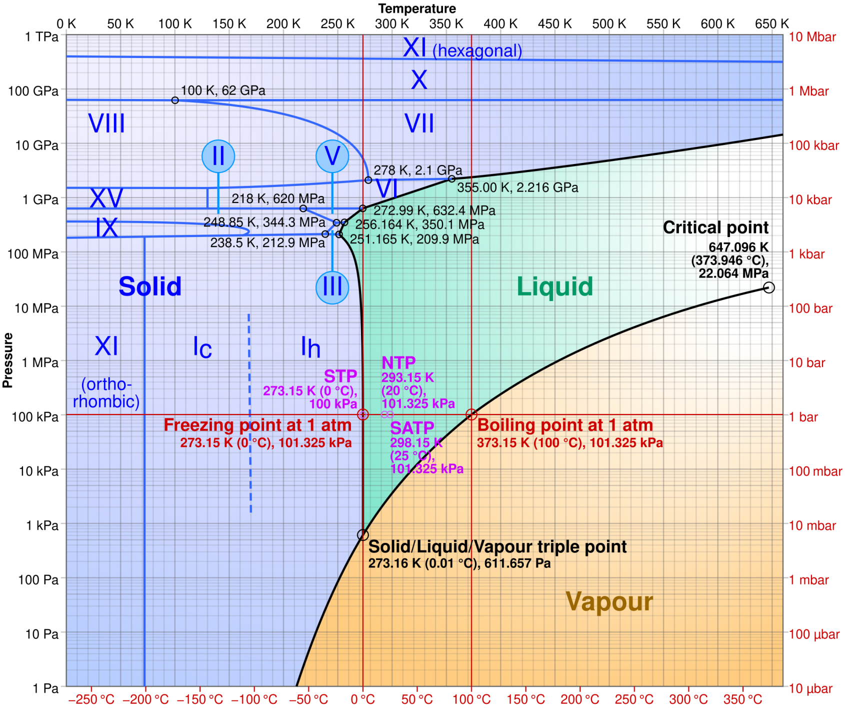
- Discontinuity in 1st derivatives:
  - Entropy $S = -(\partial G/\partial T)_P$ (latent heat)
  - Volume $V = (\partial G/\partial P)_T$

2nd Order Transition (Ferromagnetic transition, superconductivity)

- Continuous 1st derivatives, discontinuity in 2nd derivatives
  - Heat capacity $C_P = -T(\partial^2 G/\partial T^2)_P$
  - Compressibility $\kappa_T = -\frac{1}{V}\left(\partial^2 G/\partial P^2\right)_T$

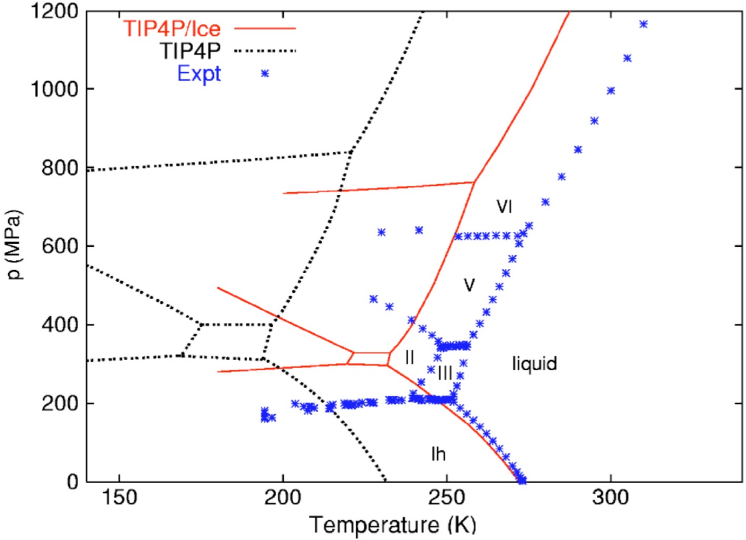
Missing Cases:

- Critical points (liquid-gas)
- Berezinskii-Kosterlitz-Thouless (BKT) transitions
- Quantum phase transitions ($T = 0$)

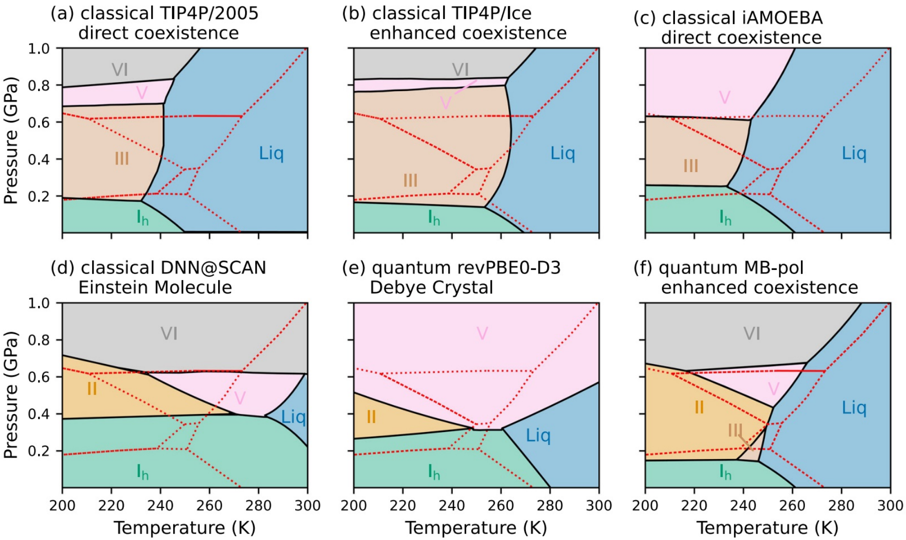

Temperature
0 K
50 K
100 K
150 K
200 K
250 K
300 K
350 K
400 K
450 K
500 K
550 K
600 K
650 K
1 TPa
100 GPa
10 GPa
1 GPa
100 MPa
10 MPa
1 MPa
100 kPa
10 kPa
1 kPa
100 Pa
10 Pa
1 Pa
Pressure
10 Mbar
1 Mbar
100 kbar
10 kbar
1 kbar
100 bar
10 bar
1 bar
100 mbar
10 mbar
1 mbar
100 µbar
10 µbar
−250 °C
−200 °C
−150 °C
−100 °C
−50 °C
0 °C
50 °C
100 °C
150 °C
200 °C
250 °C
300 °C
350 °C
XI (hexagonal)
X
100 K, 62 GPa
VIII
VII
II
V
278 K, 2.1 GPa
VI
355.00 K, 2.216 GPa
XV
218 K, 620 MPa
272.99 K, 632.4 MPa
IX
248.85 K, 344.3 MPa
256.164 K, 350.1 MPa
238.5 K, 212.9 MPa
251.165 K, 209.9 MPa
Critical point
647.096 K
(373.946 °C),
22.064 MPa
Solid
III
Liquid
XI
Ic
Ih
(ortho-
rhombic)
STP
273.15 K (0 °C),
100 kPa
NTP
293.15 K
(20 °C),
101.325 kPa
Freezing point at 1 atm
273.15 K (0 °C), 101.325 kPa
SATP
298.15 K
(25 °C),
101.325 kPa
Boiling point at 1 atm
373.15 K (100 °C), 101.325 kPa
Solid/Liquid/Vapour triple point
273.16 K (0.01 °C), 611.657 Pa
Vapour

How Do We Know the Structure of the Phase Diagram of Water?

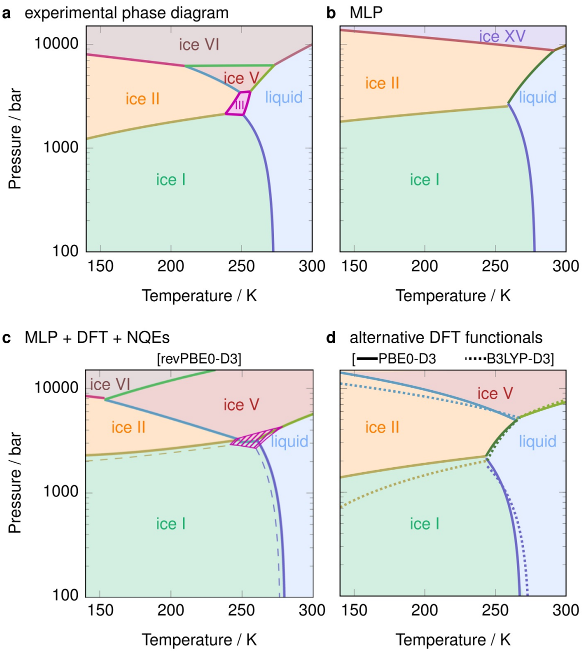
- Experiment is still most reliable sours of quantitative data
- Theoretical approach
  - *Ab initio* calculations
    - Density Functional Theory (Approximate)
    - Quantum Monte-Carlo (Computationally expensive)
    - Density Functional Theory + Molecular Dynamics
  - Effective potentials
    - Lennard-Jones potential: $U(r) \sim Ar^{-12} - Br^{-6}$
    - Corrections accounting for symmetries and molecule spatial structure
  - Modern techniques
    - Construction of effective potential via Deep Learning



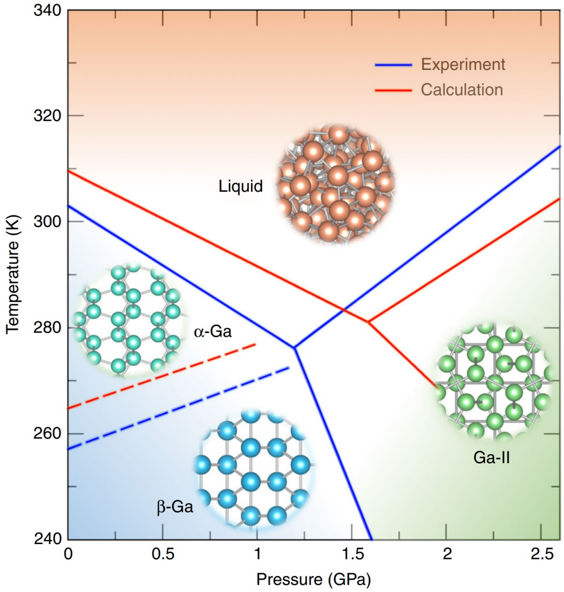
Picture from *Abascal et al., J. Chem. Phys. (2005)*



Picture from *Bore, Paesani, Nature Communication (2023)*



Picture from *Reinhardt, Cheng, Nature Communication (2021)*



Picture from *Niu el al., Nature Communication (2020)*

Main Phases of Strongly Interacting Matter

Quark-Gluon Plasma

- Presence of non-confined quarks
  - Polyakov loop $L(x) = \frac{1}{N_c}\mathrm{tr}\,\mathcal{P}\,\exp\left(ig\int_0^{1/T} A_0(x,\tau)d\tau\right)$ evolution of static quark
  - $\langle L\rangle = Z_q/Z = \exp[-F_q/T] \neq 0$ indicates possibility of free quarks

Chiral condensate

- $SU(N_f)_L \times SU(N_f)_R = SU(N_f)_A \times SU(N_f)_V \to SU(N_f)_V$
  - Chiral condensate $\langle \bar{q}q\rangle \neq 0$ contributes to hadron masses
    $m_u \sim m_d = 3-5\mathrm{MeV}$ while $m_\pi \sim 140\mathrm{MeV}$ and $m_p \sim m_n \sim 1\mathrm{GeV}$
  - $\dim SU(N_f)_A = N_f^2 - 1 = 3 \Rightarrow$ three massless Goldstone bosons (π^+, π^-, π^0)

Color superconductivity

- Analogy of the usual superconductivity via formation of Cooper pair
  - Color condensate $\langle qq\rangle = \langle q_f^a q_{f'}^b\rangle \neq 0$
  - Gluons acquire mass and hence longitudinal component, massless phonon, pseudo-scalar states

Neutron Stars: Nature's Ultimate Density Labs

Extreme Matter in a Teaspoon

To create **1 cm³** of quark-gluon plasma, you would need to:

- Compress a **small mountain** ($\sim$3 Billion tons) into a sugar cube
- Heat it to **1.75 trillion K** (100,000$\times$ Sun's core temperature)

QGP Parameters at Critical Point

Temperature:	150 MeV ($\approx 1.75 \times 10^{12}$ K)
Chem. potential:	300 MeV
Energy density:	$\mathbf{3.2 \times 10^{29}}$ **J/cm**^{3}

These are approximately the conditions inside **neutron star cores!**

Signatures of Phase Transitions in Neutron Stars

1. Equation of State Anomalies

- **Mass-Radius relation changes**:
  - Stars with similar masses but different radii
  - Modified maximum mass limit ($\geq 2.3 M_{\odot}$?)
  - Abnormally compact or inflated stars

2. Neutrino Cooling Anomalies

- **Modified particle spectrum**:
  - Enhanced cooling via quark direct URCA processes
  - Enhanced cooling via increased meson component
  - Gapped modes near Fermi surface (reduced cooling)

3. Gravitational Wave Features

- Tidal deformability
- Post-merger oscillation

4. Magnetic & Rotational Effects

- Differential rotation (crust vs quark core)
- Magnetic field anomalies
- Glitched behavior

Current observational data lacks conclusive evidence for any of these signatures

Probing QCD Matter: Colliders as Our Only Hope

Collider Experiments: A Microscopic Window

- **Advantage**: Can try to create conditions with various T and μ_B
- **Challenges**:
  - Fleeting lifetime ($< 10^{-22}$ s) - shorter than hadronization timescale
  - Tiny volume involved ($\sim$fm^3)
  - Final state always hadrons (QGP signals indirect)

The Ice Collider Analogy

Colliding tiny ice cubes $\Rightarrow$ Studying resulting tiny ice fragments

Initial state is a solid phase $\Rightarrow$ Final state also a solid phase

One sees no liquid remains, but asks whether melting had occurred during an impact?

What Sings of Possible Transitions We Expect

- J/ψ **suppression** since $c\bar{c}$ pairs tend to dissolve in QGP due to screening of the color field
- **Jet quenching** (suppression of particle production with high p_T)
- **The angular distribution** of particles indicates that the medium in the collision region behaves like an ideal fluid.
- **Enhanced production of strange particles**, because drcease of $\langle q\bar{q}\rangle$ condensate makes effective m_s smaller than typical temperature while in QGP reactions $gg \to s\bar{s}$ and $q\bar{q} \to s\bar{s}$ are also enhanced
- In-medium mass shifts and corresponding **change in meson decay widths**
- Peaks of dilepton $\bar{e}e$ and diphoton at relatively **small energies** $\sim$ **100 Mev** may indicate formation of light modes in color superconducting phase

Phase Transition vs. Crossover

- **Phase coexistence and latent heat release**
  - Sharp multiplicity change in narrow energy range
- **Hysteresis effects: System properties depend on temperature change rate**
  - Particle spectra/correlations at gradual collision energy changes
- **Drastic change in heavy quark production**
  - Strange-charmed particle correlations

Current collider data support the crossover scenario for both the chiral transition and the confinement-deconfinement transition, while providing almost no evidence for a transition to the color superconductivity state

Lattice Quantum Chromodynamics (LQCD)

Definition

Lattice QCD is a non-perturbative approach to QCD where:

- Continuous spacetime is discretized on a 4D Euclidean lattice
- Gluon fields are represented by SU(3) link variables $U_\mu(x)$
- Quark fields reside on lattice sites

Path Integral on the Lattice

The partition function in continuum QCD:

$$Z = \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,e^{-S[A,\psi,\bar{\psi}]} \quad \rightarrow \quad Z_{\text{lat}} = \int \prod_{x,\mu} dU_\mu(x)\,\det(M[U])e^{-S_G[U]}$$

$$U_\mu(x) \approx e^{igA_\mu(x)}$$

Problems and Predictions

Chemical Potential μ and the Sign Problem

- For $\mu \neq 0$, the fermion determinant becomes complex:

$$\det(M[U, \mu]) \in \mathbb{C}$$

- The weight $e^{-S} \det(M)$ cannot be interpreted as probability
- Standard Monte Carlo methods fail (require real positive weights)
- Calculations with imaginary μ_B and analytic continuation for $\mu_B/T \lesssim 2$

Phase Diagram Predictions (Small μ)

LQCD reliably predicts:

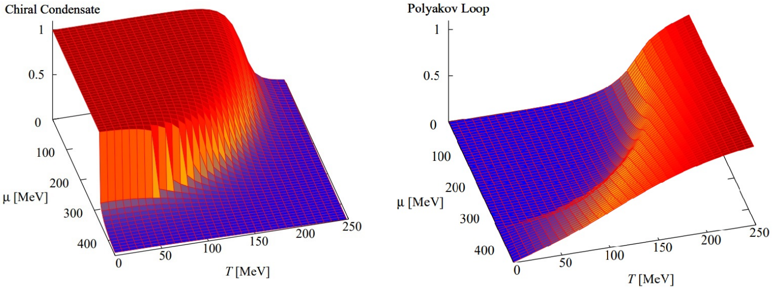
- Crossover for both the chiral transition and the confinement-deconfinement transition at $T_c \approx 156$ MeV ($\mu = 0$)
- No first-order transition at small μ
- Critical point location remains uncertain
- Transition to the phase of color superconductivity remains uncertain

Polyakov–Nambu–Jona–Lasinio (PNJL) Model

$$\mathcal{L}_{\mathrm{PNJL}} = \bar{q}(i\gamma^{\mu} D_{\mu} - m_0)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}(\Phi, \bar{\Phi}, T)$$

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln\left[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2\right]$$

$$\Phi = \frac{1}{N_c}\langle \mathrm{tr}_c L\rangle, \quad L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^{\beta} A_4(\vec{x},\tau)d\tau\right], \quad D_{\mu} = \partial_{\mu} - i\delta_{\mu 0}A_4$$



Picture from *Fukushima, Phys. Rev. D (2008)*

Core Ideas of FRG

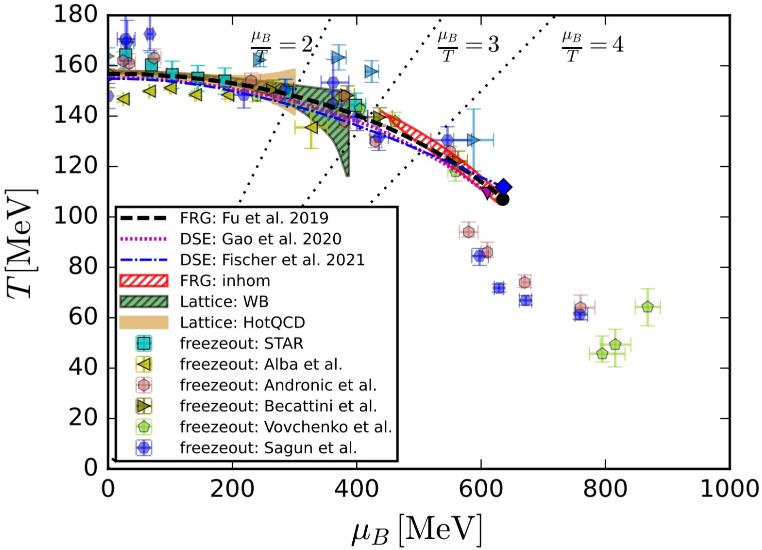
Key Concepts

- **Mode separation** via regulator function $R_k(p)$: suppresses fluctuations with $p^2 < k^2$ (slow modes) while keeps $p^2 > k^2$ (fast modes) integrated out
- **Effective average action** Γ_k Interpolates between bare action $(k = \Lambda)$ and full free energy $(k = 0)$
- **Ansatz** for Γ_k must capture relevant physics while remaining tractable

Wetterich Equation

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k \right]$$

- Controls the flow of Γ_k
- Numerically solved from UV $(k = \Lambda)$ to IR $(k \to 0)$
- Works for any T and μ but gives rather uncontrolled approximation



Picture from *Fu, Phys. Rev. D (2020)*

Universality of Critical Behavior

- Near the critical point, second derivatives of the free energy exhibit **non-analytic behavior**.
- A **universality class** is defined by systems sharing the same **dimensionality, symmetry, order parameter nature**
  These systems show identical singular behavior near criticality.
- The **order parameter** is a collective mode whose correlation length diverges at the critical point.

Critical Exponent	**Liquid-Gas Transition**	**Curie Point**
$\alpha \approx 0.11$	$C = -T\frac{\partial^2 F}{\partial T^2} \sim (T - T_c)^{-\alpha}$	$C = -T\frac{\partial^2 F}{\partial T^2} \sim (T - T_c)^{-\alpha}$
$\beta \approx 0.326$	$\rho_L - \rho_G = -\left.\frac{\partial F}{\partial \mu}\right\|_T \sim (T_c - T)^{\beta}$	$M = -\left.\frac{\partial F}{\partial H}\right\|_T \sim (T_c - T)^{\beta}$
$\gamma \approx 1.237$	$\chi_T = -\left.\frac{\partial^2 F}{\partial \mu^2}\right\|_T \sim \|T - T_c\|^{-\gamma}$	$\chi = -\left.\frac{\partial^2 F}{\partial H^2}\right\|_T \sim \|T - T_c\|^{-\gamma}$
$\delta \approx 4.790$	$P - P_c = \left.\frac{\partial F}{\partial V}\right\|_T \sim \|\rho - \rho_c\|^{\delta}$	$H = \left.\frac{\partial F}{\partial M}\right\|_T \sim \|M\|^{\delta}$
$\eta \approx 0.036$	$G(r) = \langle \delta\rho(r)\delta\rho(0)\rangle \sim r^{-(d-2+\eta)}$	$G(r) = \langle S_i S_j \rangle \sim r^{-(d-2+\eta)}$
$\nu \approx 0.630$	$\xi \sim \|T - T_c\|^{-\nu}$	$\xi \sim \|T - T_c\|^{-\nu}$

Landau Theory Approach to 3D Ising Magnet

Partition Function Representation

$$Z = \sum_{\{S_i\}} e^{-H/kT} = \sum_{M} \sum_{\{S_i\}|_M} e^{-H/kT} = \sum_{M} e^{-F_{mf}(M)/kT}$$

where $F_{mf}(M)$ is the mean-field free energy and $M = \frac{1}{N}\sum_i S_i$.

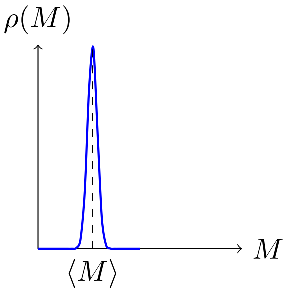
- The magnetization distribution:

$$\rho(M) \propto e^{-F_{mf}(M)/kT}$$

- Key assumption: $\rho(M)$ is sharply peaked at $\langle M \rangle$

$$\frac{\partial F_{mf}(M)}{\partial M} = 0$$

For small M near the critical point, it is sufficient to consider the leading terms in the expansion of F_{mf}



Limitations of Landau Theory

- **Non-Gaussian fluctuations**: Near T_c, the order parameter distribution becomes **wide (large variance)**, **asymmetric (non-Gaussian tails)**, dominated by **long-wavelength fluctuations**
- **Modern approach**: Keep Landau-like free energy *functional* but integrate over all fluctuations:

$$Z = \int \mathcal{D}M(\mathbf{x})\, e^{-S[M]}, \quad S[M] = \int d^d r \left[\frac{1}{2}(\nabla M)^2 + \frac{a}{2}M^2 + \frac{b}{4}M^4 \right]$$

Renormalization Group Solution

- Perturbation theory fails due to strong coupling near T_c
- By eliminating UV divergences, the renormalization group (RG) provides a systematic way to handle fluctuations
- Yields critical exponents as fixed point properties

Critical Fluctuations in QCD Matter

Cumulants as Fluctuation Measures

- $\kappa_2 = \langle X^2 \rangle - \langle X \rangle^2 = \sigma^2$
- $\kappa_4 = \langle X^4 \rangle - 3\langle X^2 \rangle^2$
- $\kappa_6 = \langle X^6 \rangle - 15\langle X^4 \rangle \langle X^2 \rangle + 30\langle X^2 \rangle^3$

Where X can relate to **particle multiplicity, electric and baryon charges**

For baryon charge fluctuations

$$\kappa_{2n} \sim |T - T_c|^{2-n-\alpha}$$

Universality Classes

- **Deconfinement transition** – 3D Ising universality class (same as liquid-gas and ferromagnetic transitions)
- **Chiral transition** – O(4) universality class

Critical Slowing Down and Finite Size Efects

The collision of nuclei in a collider cannot be considered an equilibrium process

Langevin equation:

$$\frac{\partial \phi}{\partial t} = -\Gamma \frac{\delta F[\phi]}{\delta \phi} + \eta(\mathbf{r}, t), \qquad \langle \eta \eta' \rangle = 2\Gamma \delta(t - t')\delta(\mathbf{r} - \mathbf{r}')$$

Key Concept

Near T_c, relaxation times diverge due to growing correlation length ξ:

$$\tau \sim \xi^z \sim |T - T_c|^{-z\nu}$$

For different models $z \sim 2 - 3$

If the system size L_{sys} is smaller than the correlation length ξ, and its lifetime τ_{sys} is shorter than the relaxation time τ, then critical fluctuations will be suppressed. The system will exhibit behavior similar to that far from the critical point, which may mask true critical behavior as a smooth crossover.

Thank you for attention!