Bayesian approach for centrality determination in nucleus-nucleus collisions at the BM@N experiment

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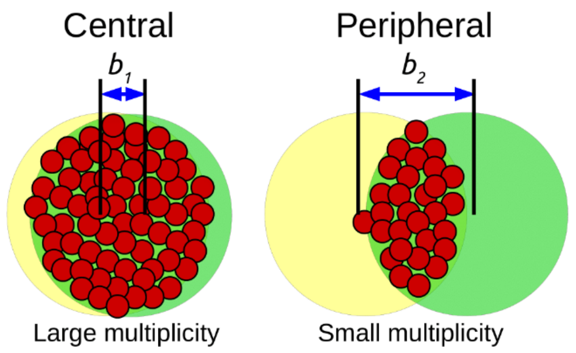
Centrality

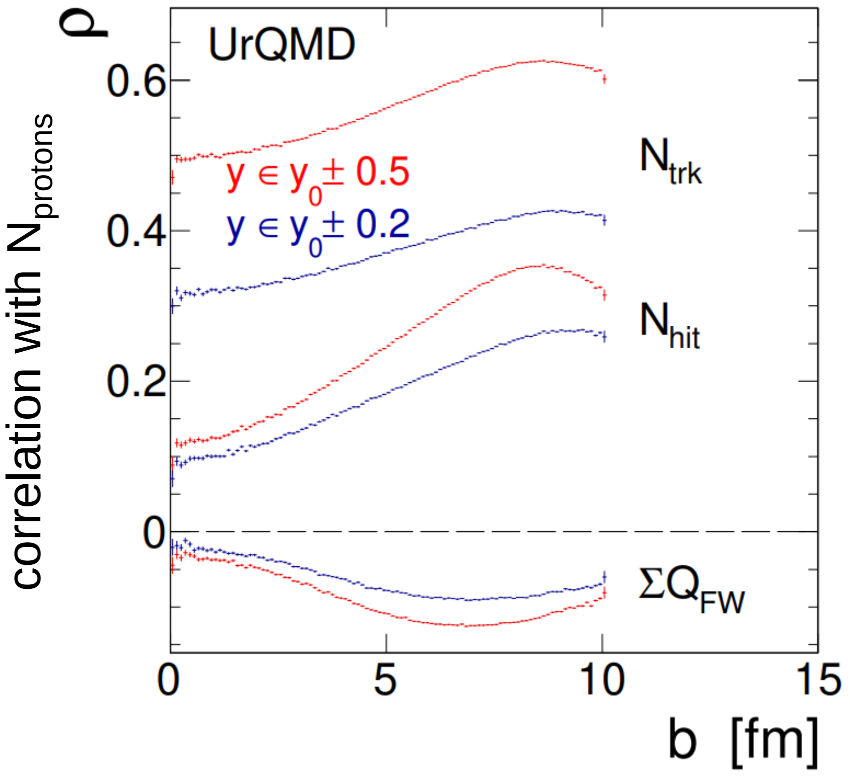
- Evolution of matter produced in heavy-ion collisions depend on its initial geometry

- Centrality procedure maps initial geometry parameters with measurable quantities (multiplicity or energy of the spectators)

- **This allows comparison of the future BMAN results with the data from other experiments (STAR BES, NA49/NA61 scans) and theoretical models**

$$c(b) = \frac{\int_0^b \frac{d\sigma}{db'} db'}{\int_0^\infty \frac{d\sigma}{db'} db'} = \frac{1}{\sigma_{A-A}} \int_0^b \frac{d\sigma}{db'} db'$$



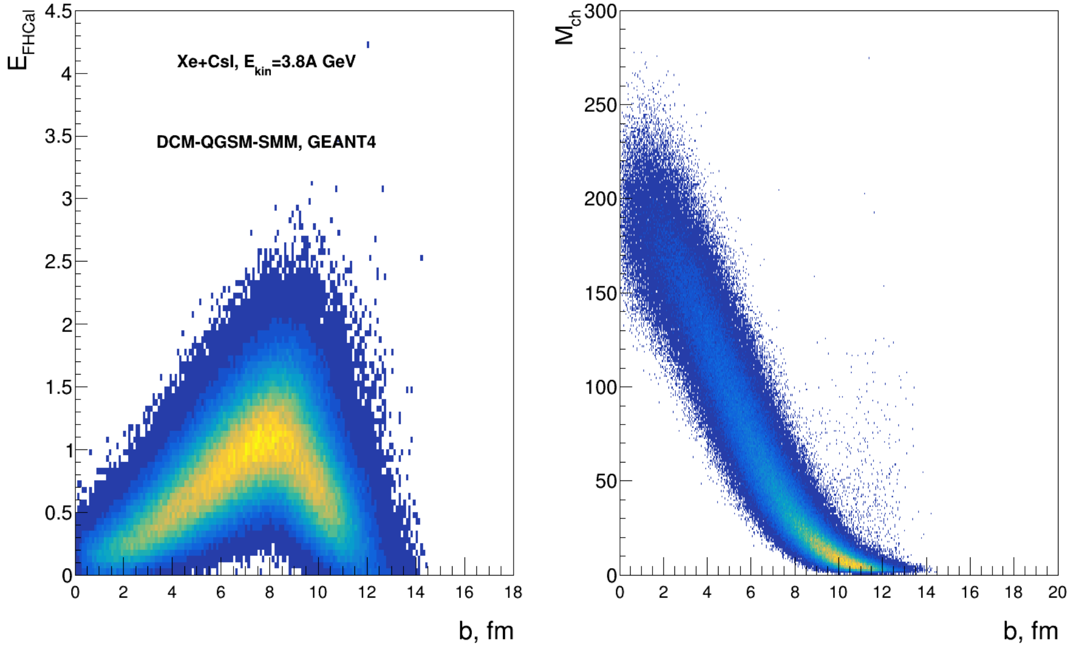


HADES; Phys.Rev.C 102 (2020) 2, 024914

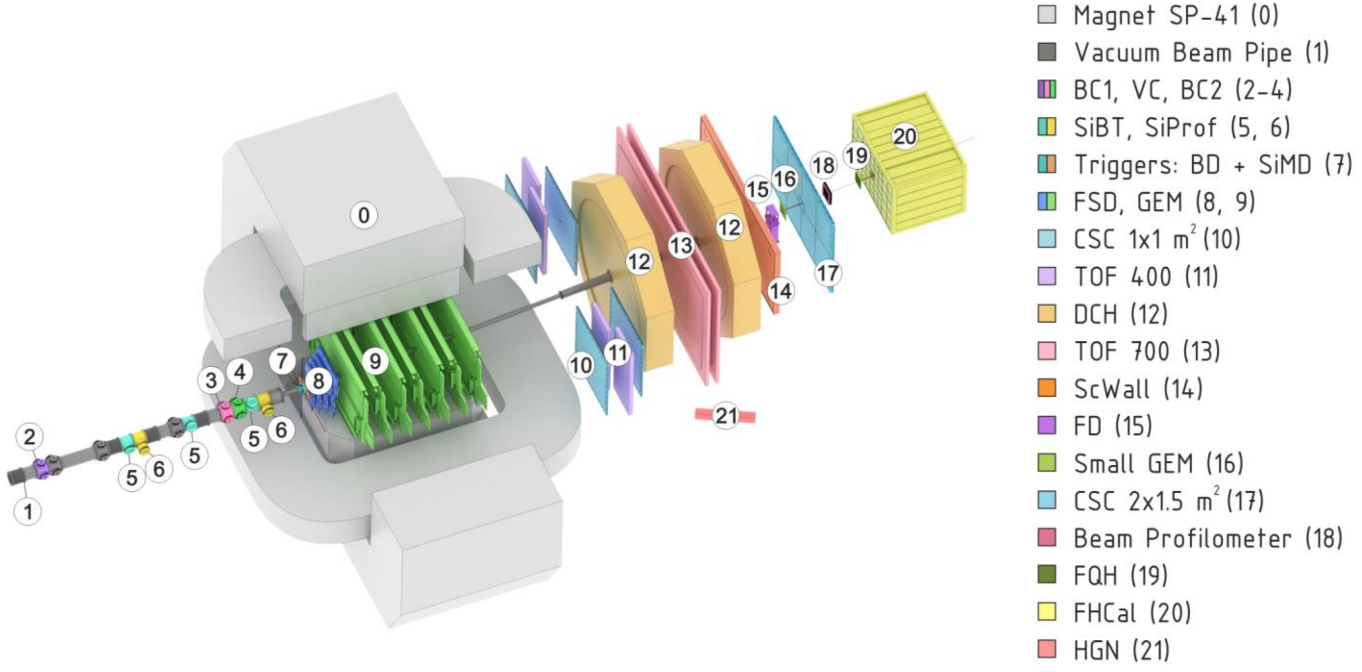
- A number of produced protons is stronger correlated with the number of produced particles (track & RPC+TOF hits) than with the total charge of spectator fragments (FW)
- to suppress self-correlation biases, it is necessary to use spectators fragments for centrality estimation

Centrality determination in BM@N

DCM-QGSM-SMM → GEANT4 → Reconstruction → Centrality determination



Dependence of energy in FHCal and track multiplicity on the impact parameter



BM@N setup overview

The Bayesian inversion method (Γ-fit): DCM-QSM-SMM based

- The fluctuation kernel Gamma distr.:

$$P(M \mid c_b) = \frac{1}{\Gamma(k(c_b))\theta^2} M^{k(c_b)-1} e^{-M/\theta}$$

$$c_b = \int_0^b P(b')db'$$ – centrality based on impact parameter

$$\theta = \frac{D(M)}{\langle M \rangle}, \quad k = \frac{\langle M \rangle}{\theta}$$

$\langle M \rangle, D(M)$ – average and variance of Multiplicty

$$P(M) = \int_0^1 P(M \mid c_b) dc_b$$

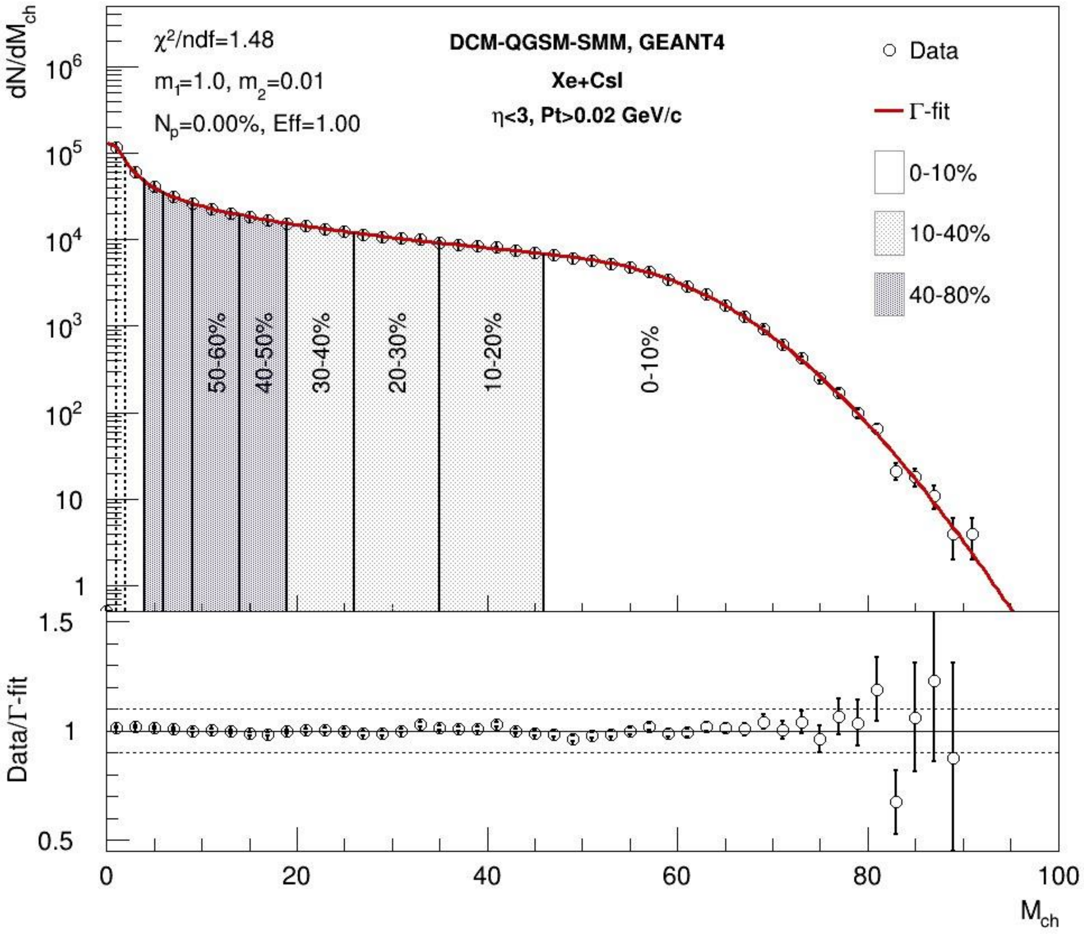
$$\langle M \rangle = m_1 \cdot \langle M' \rangle$$

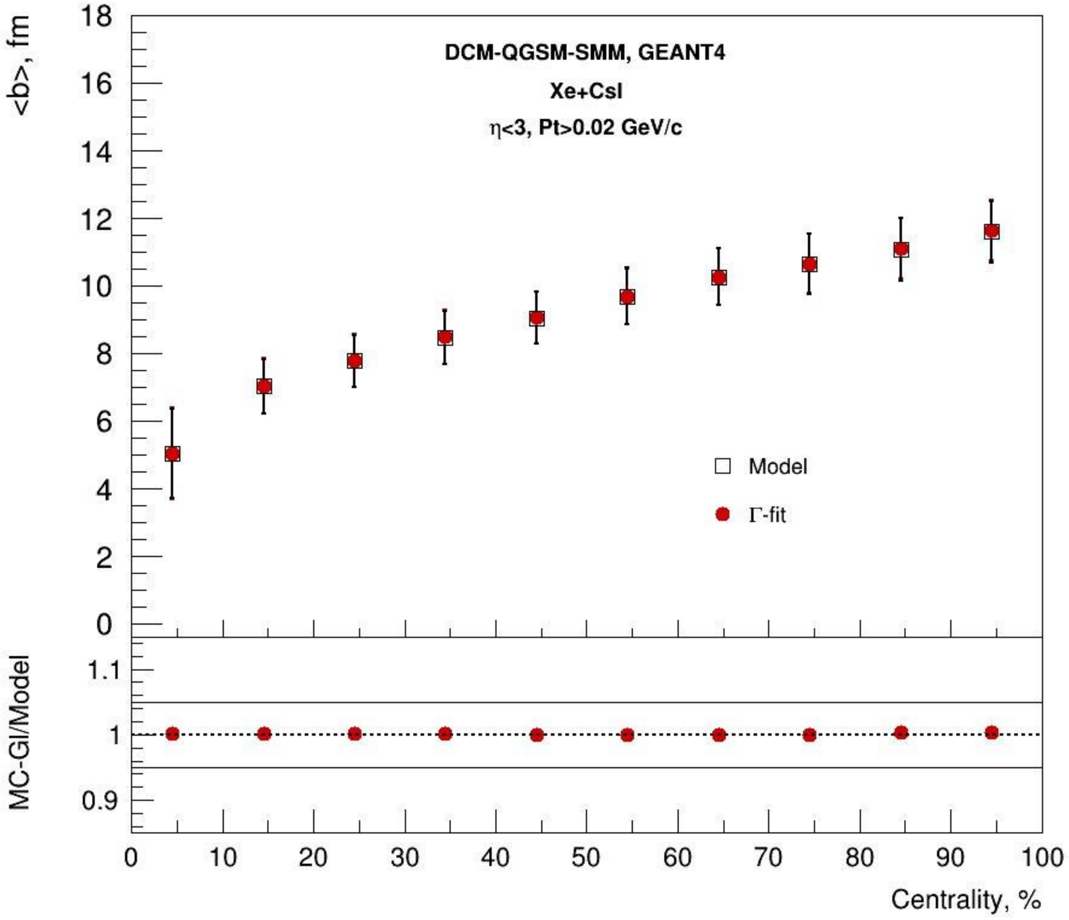
$$D(M) = m_1^2 \cdot D(M') + m_1 \cdot m_2 \langle M' \rangle$$

$\langle M'(c_b) \rangle$, $D(M'(c_b))$ – average value and var. of energy/mult. from the rec. model data

- can be approximated by polynomials and exponential polynomial

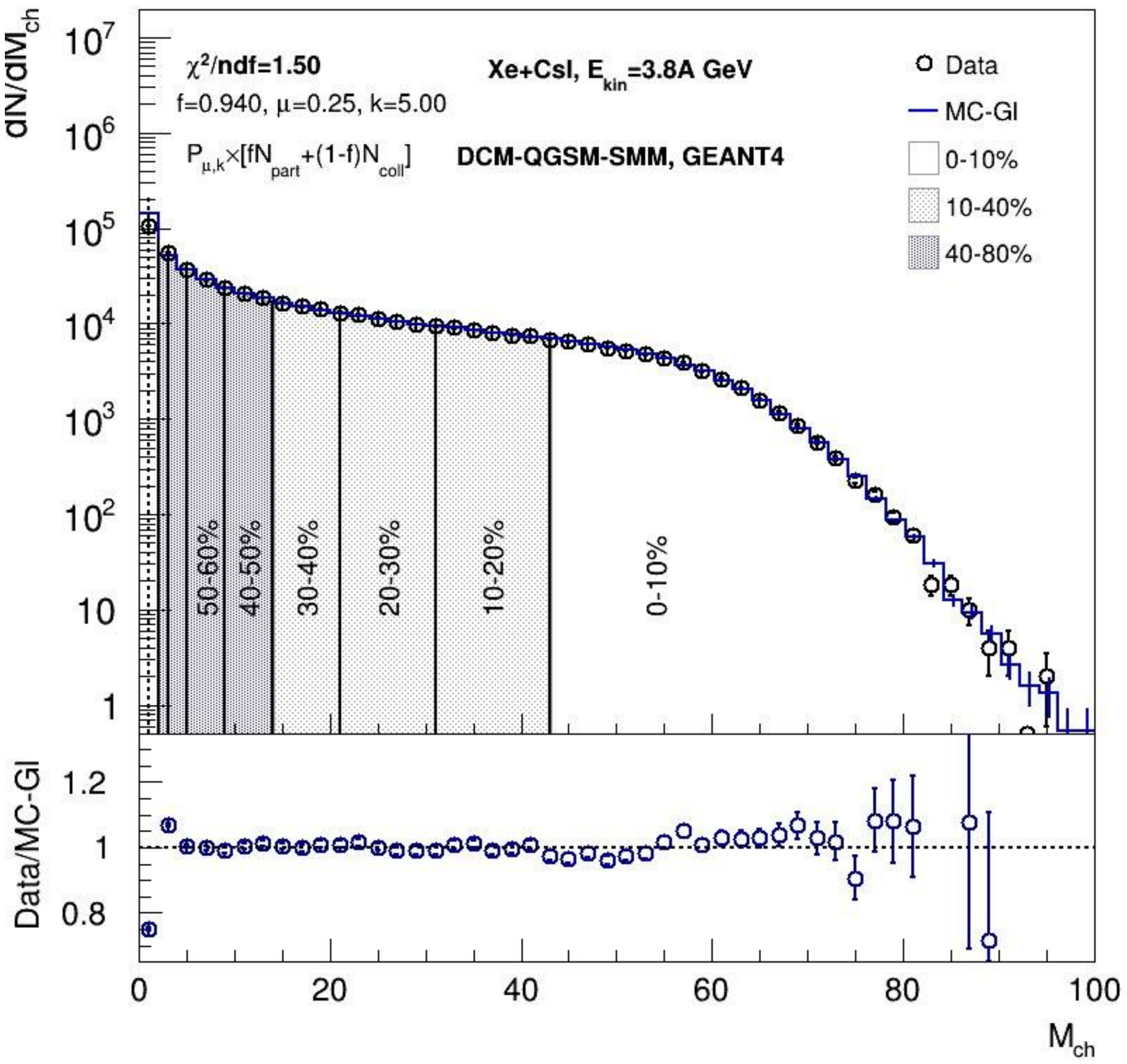
Fit results: reconstructed model data

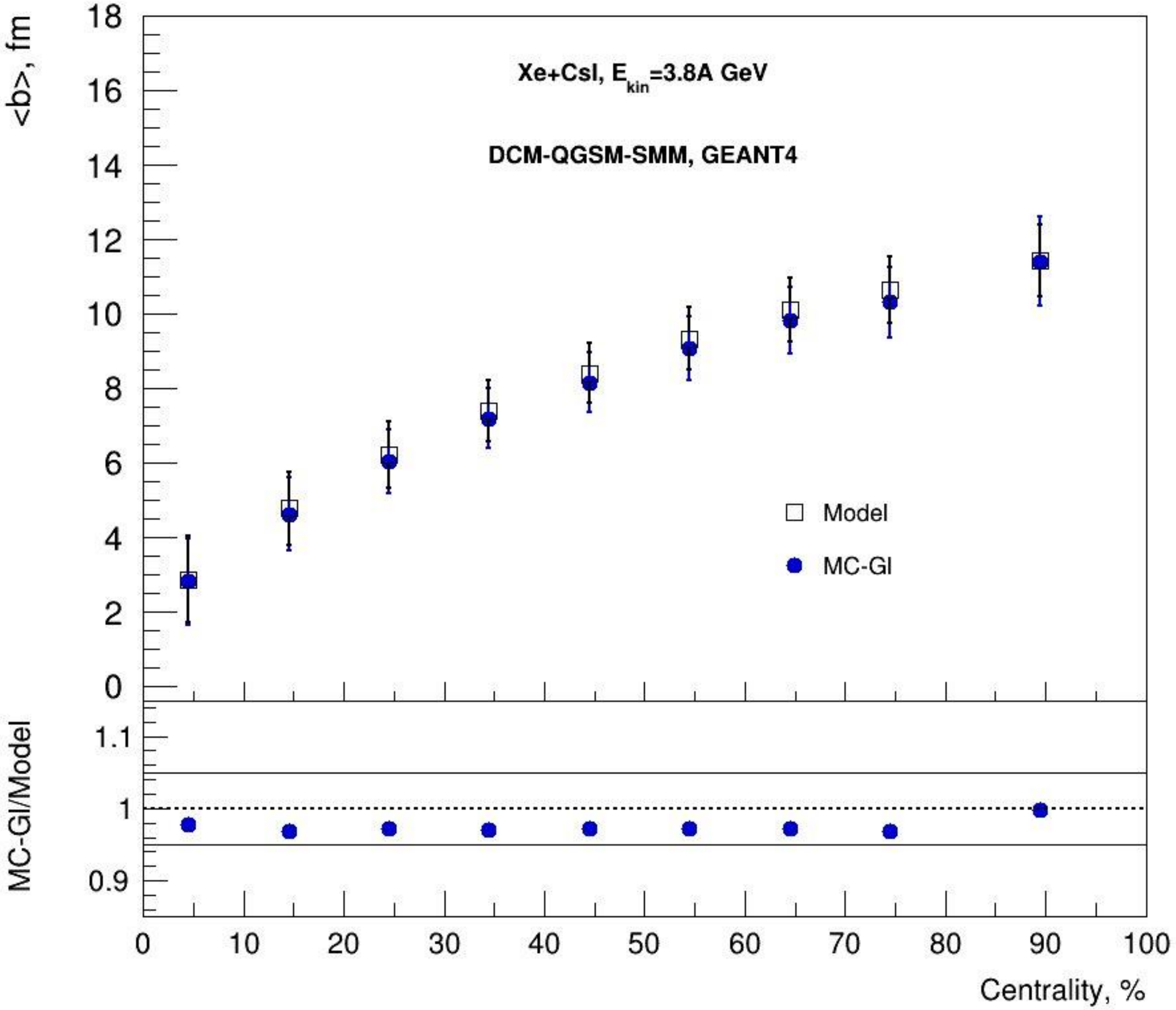




Good agreement with data

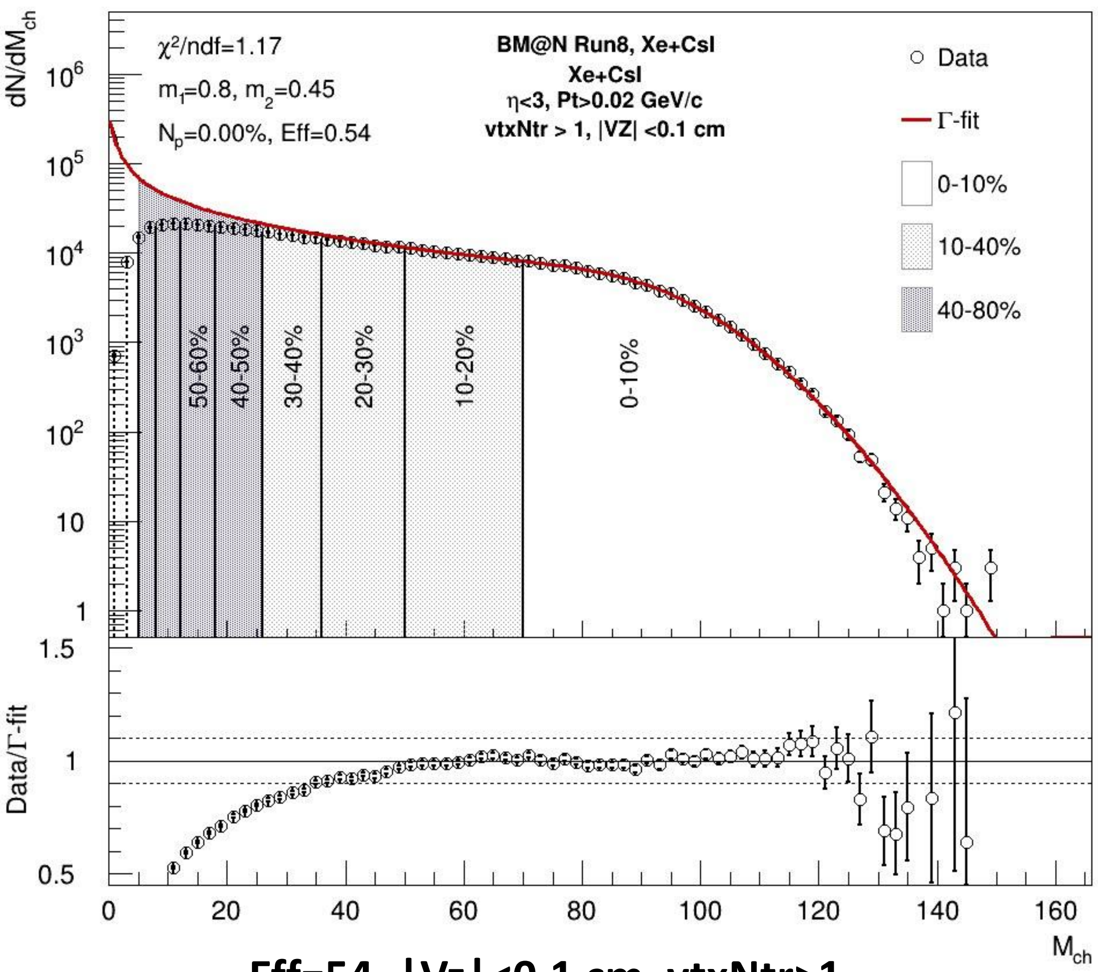
Fit results: reconstructed model data



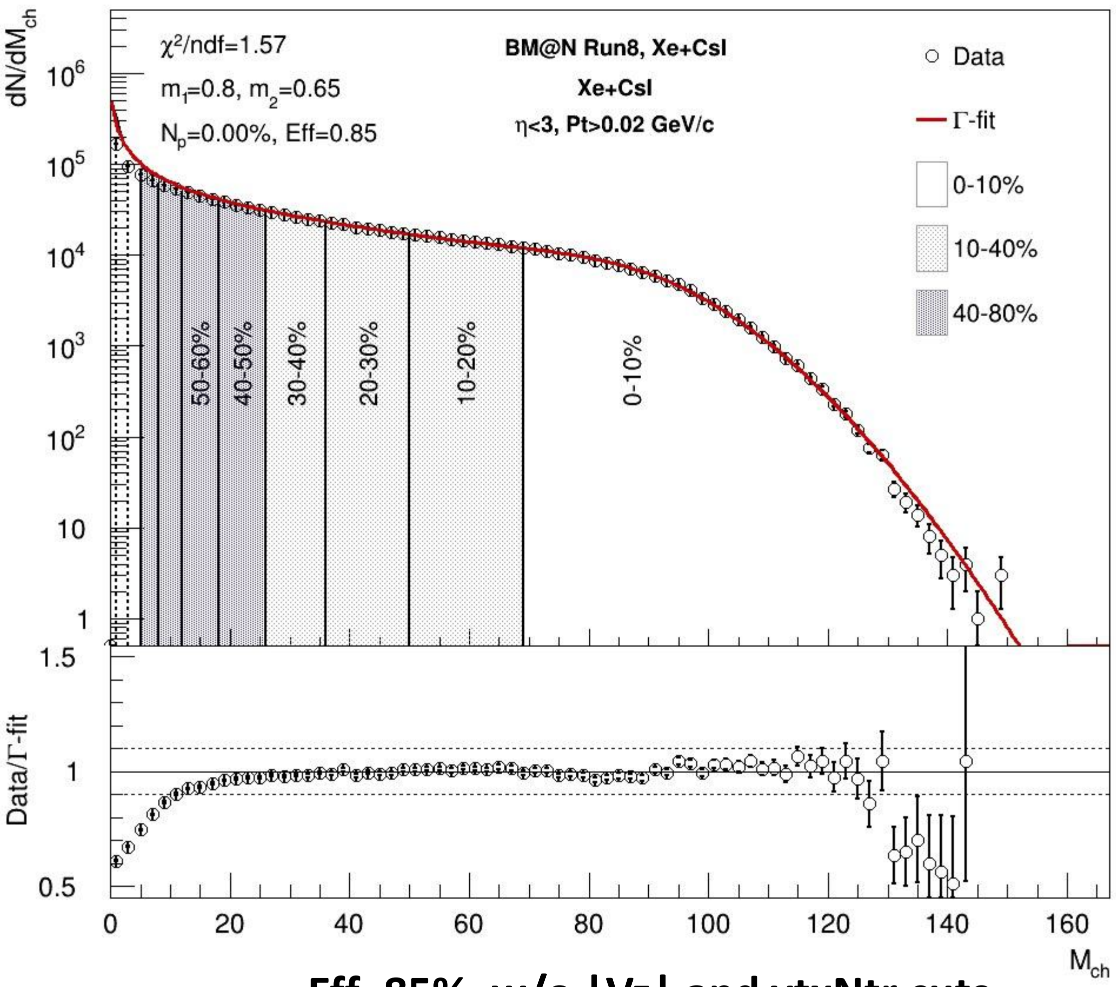


Good agreement with data

Fit results: experimental data



Eff=54, |Vz|<0.1 cm, vtxNtr>1



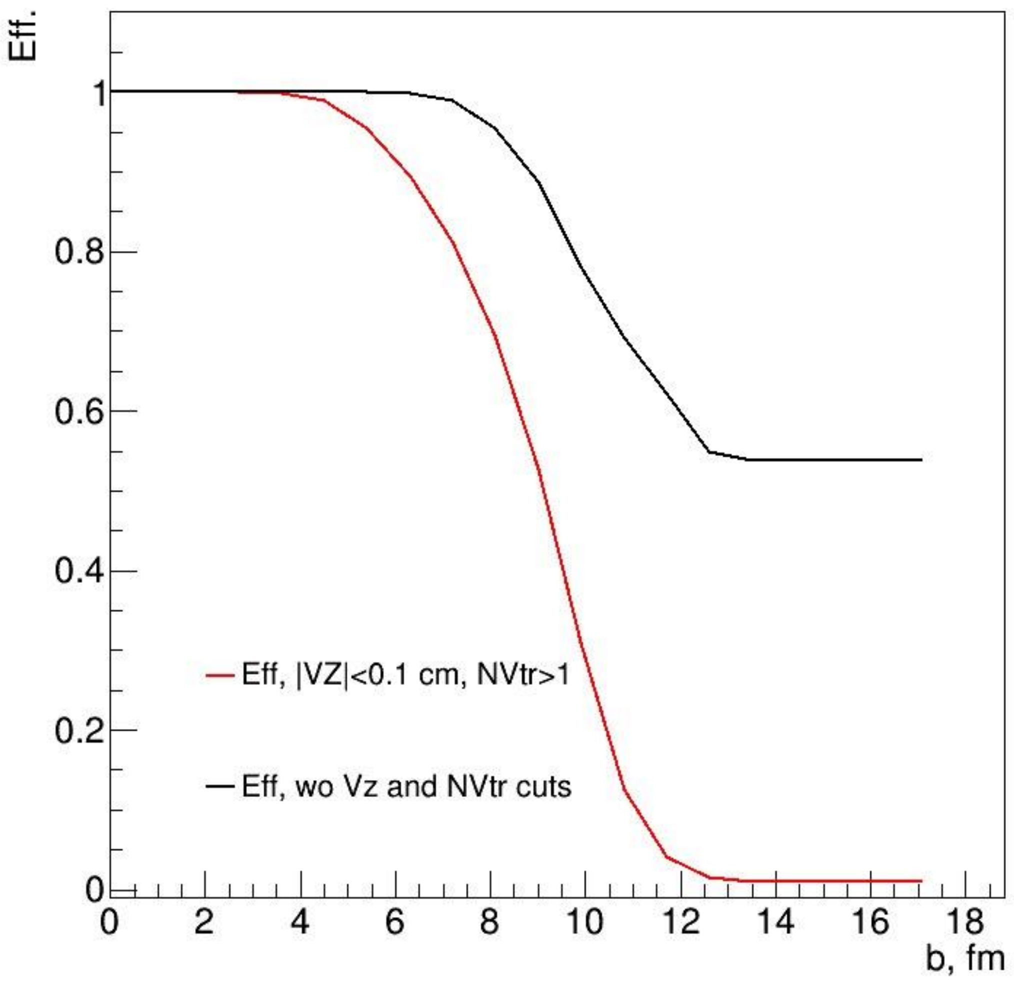
Eff=85%, w/o |Vz| and vtxNtr cuts

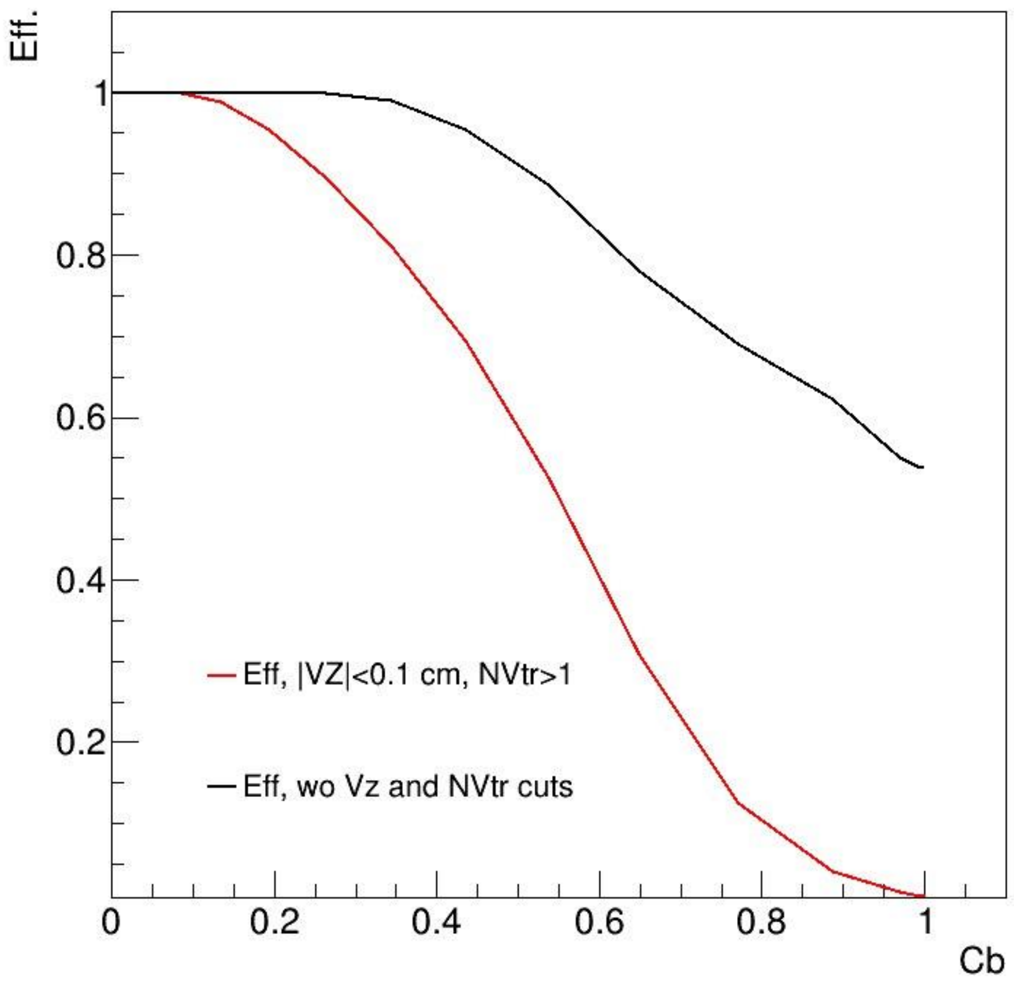
Vertex Cuts: CCT2, 14e3<BC1<40e3, FD<4250

Track selection: DCA<2 cm, η<3, Pt>0.02 GeV/c

Convoluted trigger efficiency

$$P_{eff}(b) = \int_0^{M_{\max}} P_{eff}(\mathrm{M})P(\mathrm{M}\,|\,\mathrm{b})dM$$





The Bayesian inversion method (Γ-fit): 2D fit

- The fluctuation kernel for energy and multiplicity at fixed impact parameter can be describe by 2D Gamma distr.:

$$P(E,M \mid c_b) = G_{2D}(E,M,\langle E\rangle,\langle M\rangle,D(E),D(M),R)$$

$$c_b = \int_0^b P(b')db'$$ – centrality based on impact parameter

$\langle E\rangle, D(E)$ – average value and variance of energy

$\langle M\rangle, D(M)$ – average value and variance of mult.

$R(E,M)$ – Pirson correlation coefficient

$$R(E,M) = \frac{\varepsilon_1^2 m_1^2}{\varepsilon_2 m_2} R(E',M')$$

$\varepsilon_1, \varepsilon_2, m_1, m_2$ - fit parameters

$\langle E'(c_b)\rangle$, $D(E'(c_b))$ – average value and var. of energy/mult. from the rec. model data

$$\langle E\rangle = \varepsilon_1 \langle E'(c_b)\rangle, \quad D(E) = \varepsilon_2 D(E'(c_b))$$
$$\langle M\rangle = m_1 \langle M'(c_b)\rangle, \quad D(M) = m_2 D(M'(c_b))$$

$\langle E'(c_b)\rangle, \quad D(E'(c_b))$ - can be approximated by polynomials

$$\langle E'(c_b)\rangle = \sum_{j=1}^{8} a_j c_b^j, \quad D(E'(c_b)) = \sum_{j=1}^{6} b_j c_b^j$$

$$\langle M'(c_b)\rangle = \sum_{j=1}^{8} a_j c_b^j, \quad D(M'(c_b)) = \sum_{j=1}^{6} b_j c_b^j$$

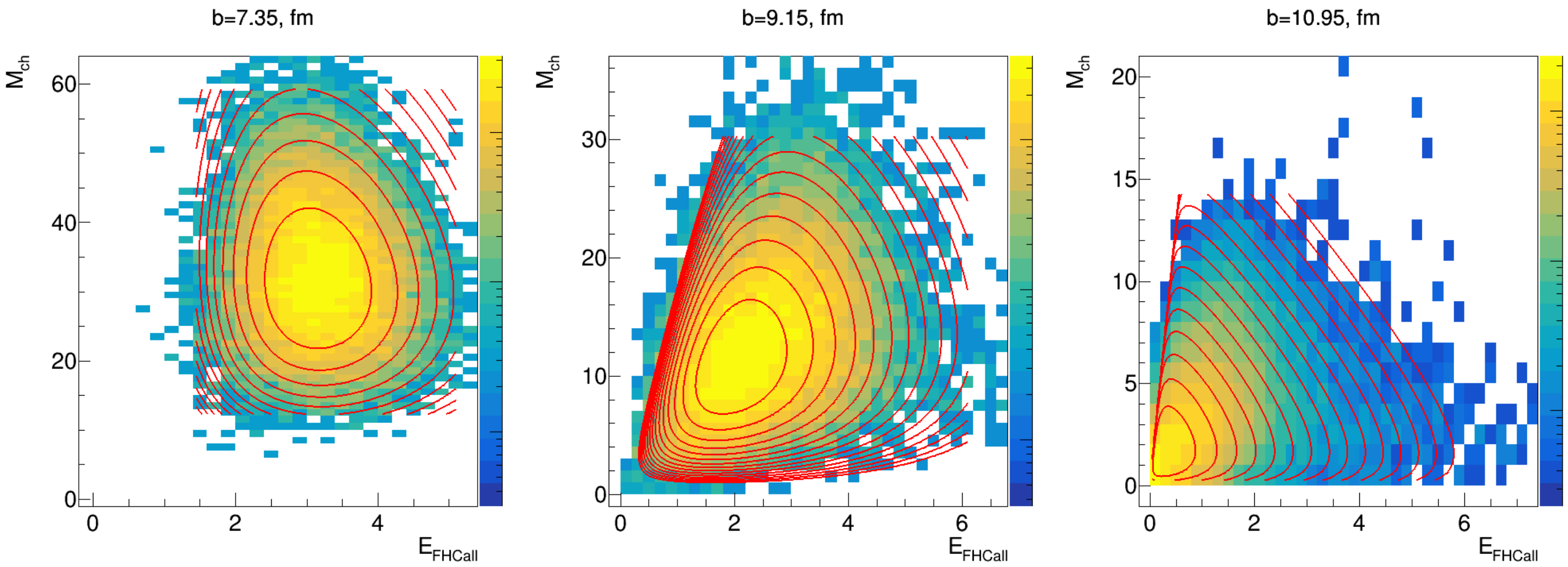
The fluctuation of energy and multiplicity at fixed impact parameter

It is possible to find such a rotation angle of the system that $\mathrm{cov}(x,y)=0$

$$\langle x\rangle = \cos(\alpha)\langle E\rangle + \sin(\alpha)\langle M\rangle$$
$$\langle y\rangle = -\sin(\alpha)\langle E\rangle + \cos(\alpha)\langle M\rangle$$

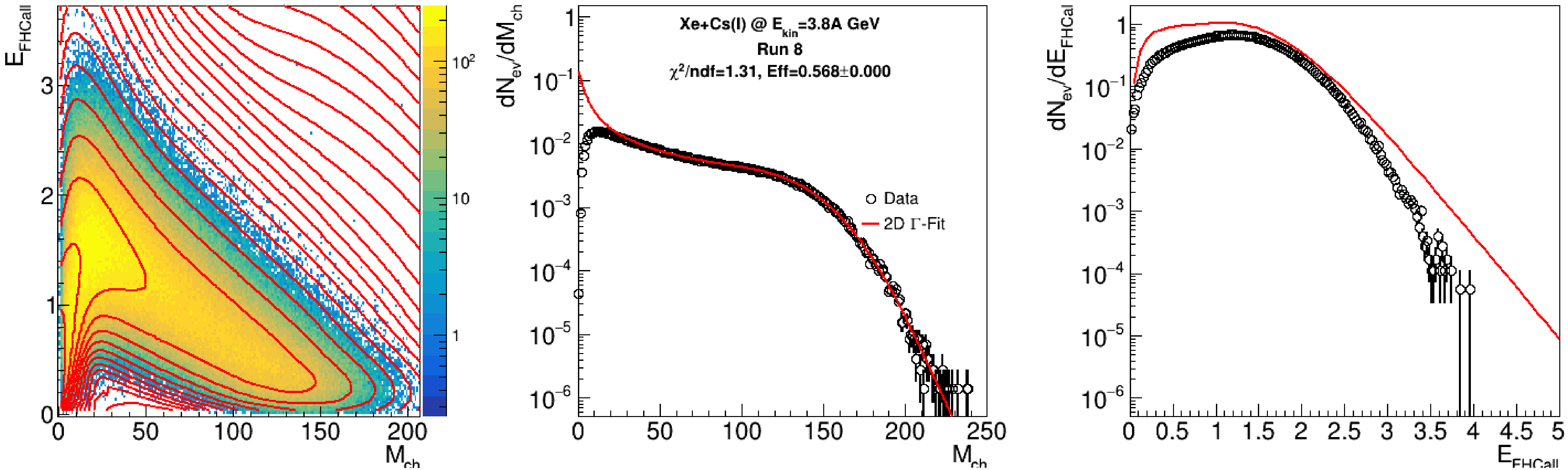
$$\alpha = \arctan\left(\frac{2\sqrt{D(E)D(M)}R(E,M)}{D(E)-D(M)}\right)$$

$$G_{2D}(E_{FH}, M_{ch}, \langle E\rangle, \langle M\rangle, D(E), D(M), R) = G(x,\theta_x,k_x)\cdot G(y,\theta_y,k_y)$$



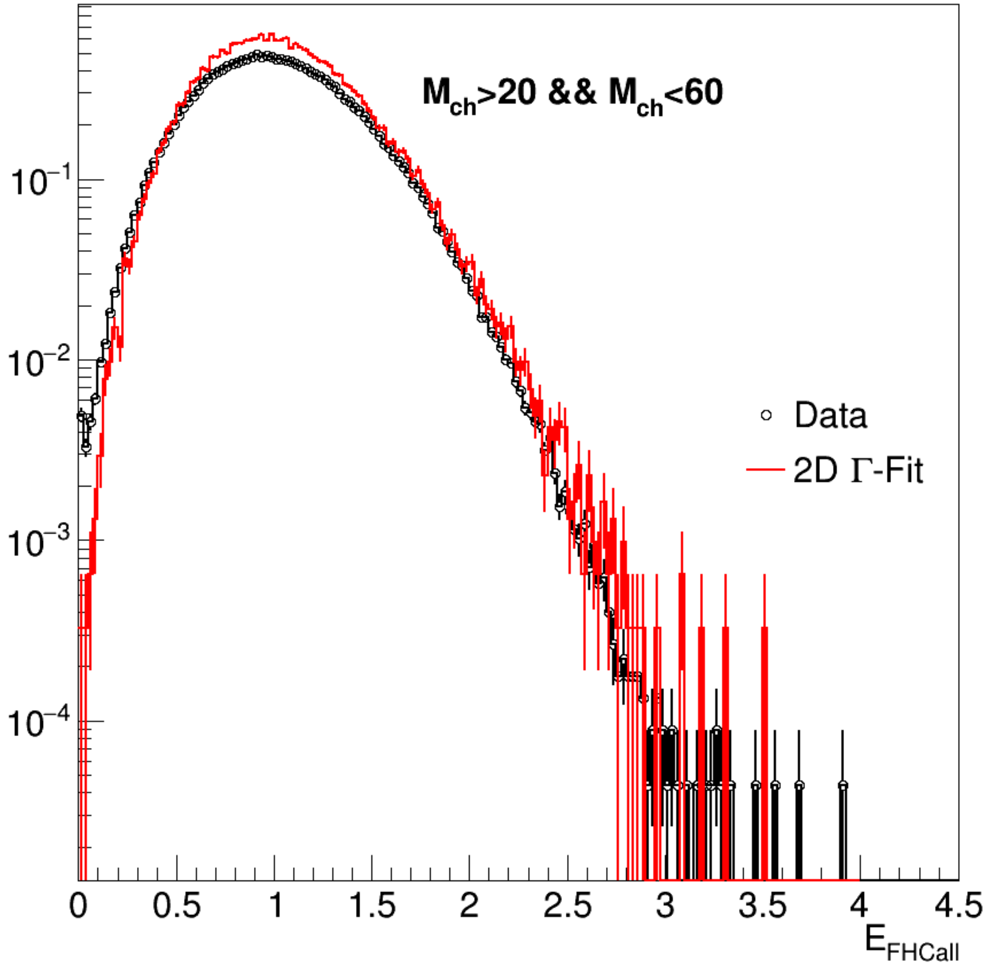
The distribution of energy and multiplicity at a fixed impact parameter is well described by the gamma distribution

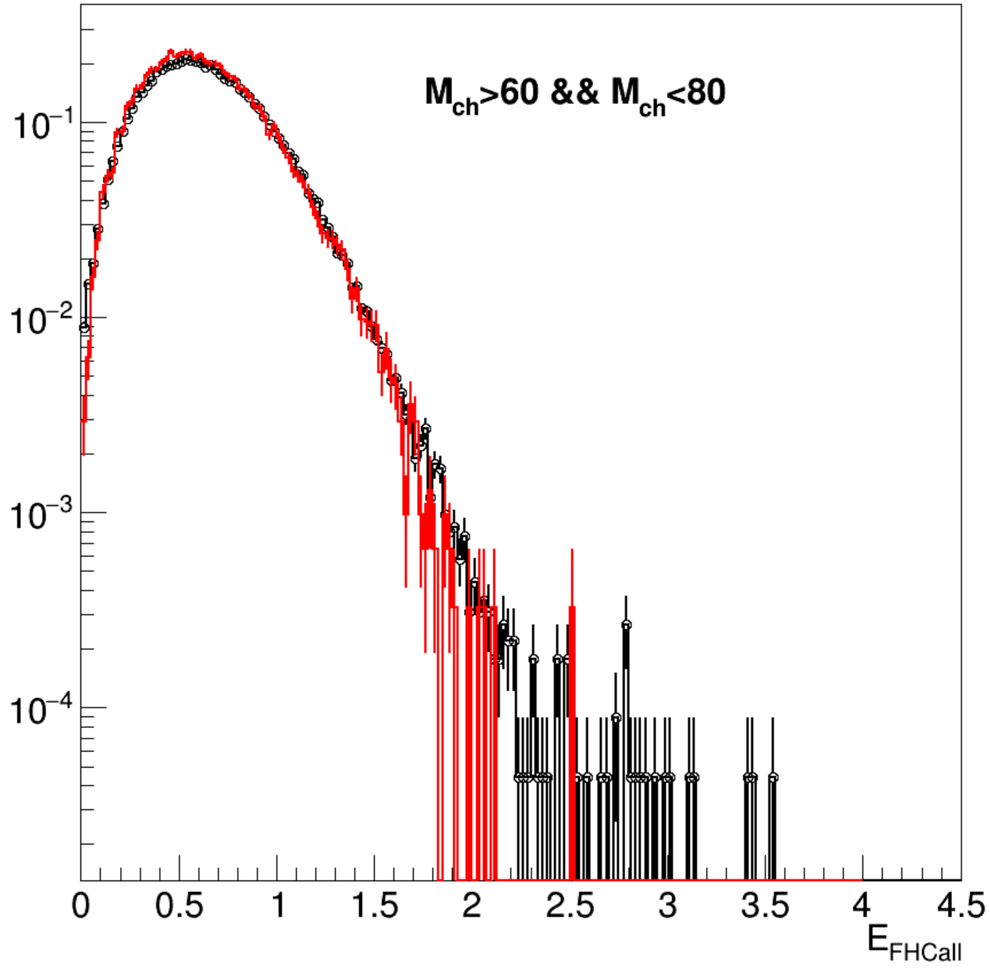
2D fit results



The fit function qualitatively reproduces the multiplicity-energy correlation from FHCal

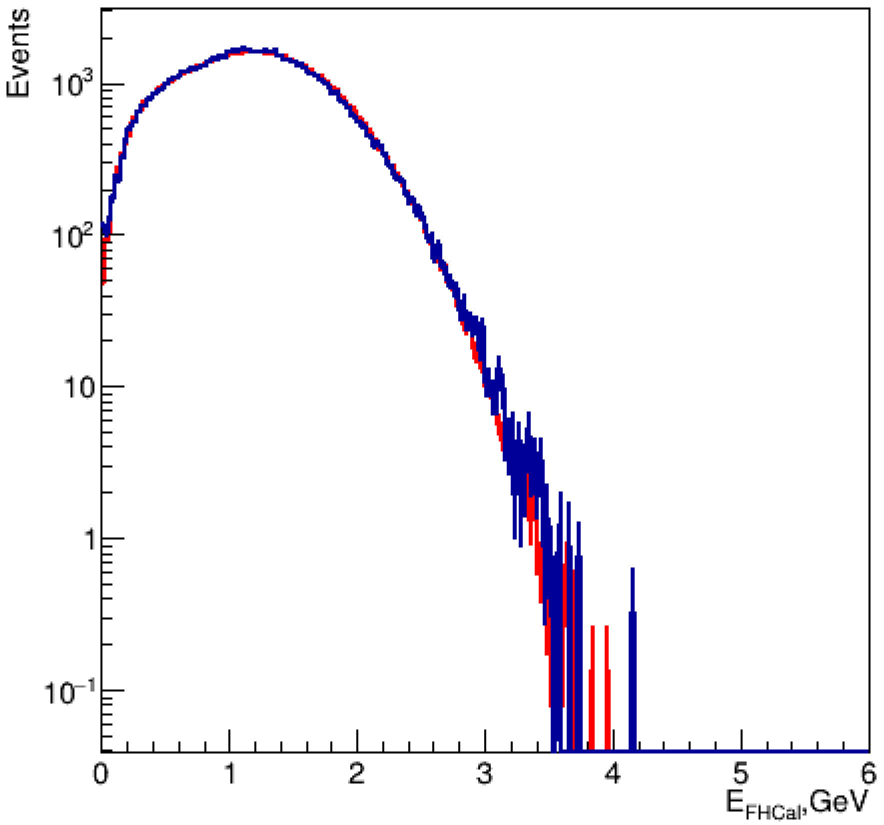
Energy distributions from FHCal for different multiplicity cuts

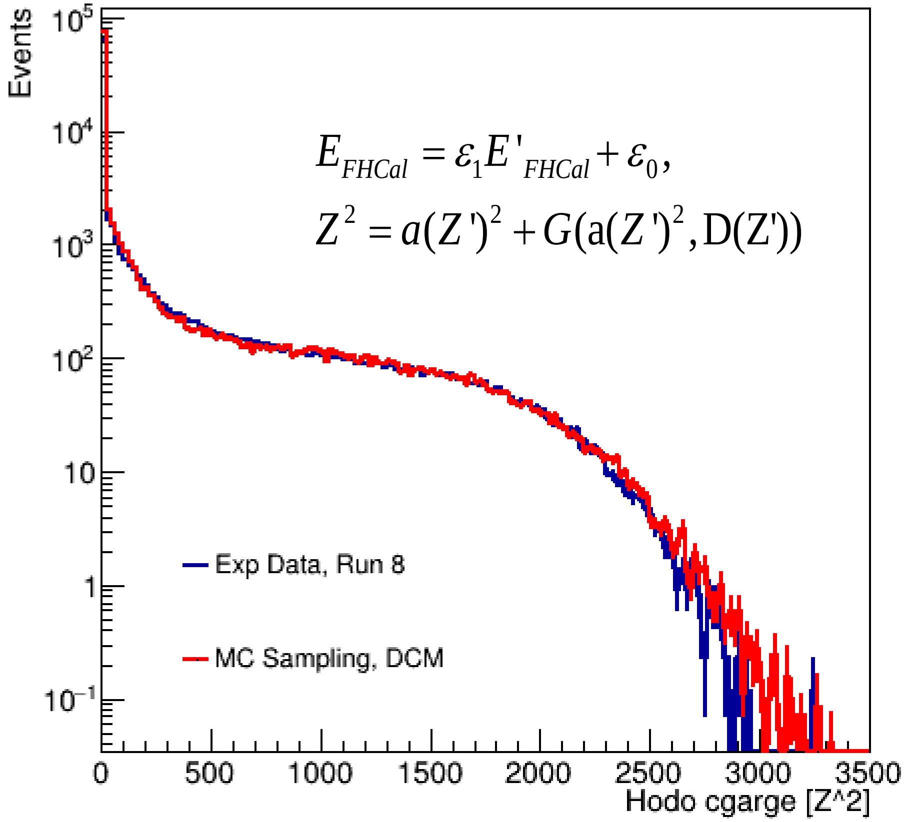




Good agreement between fit and data for the area below the anchorpoint

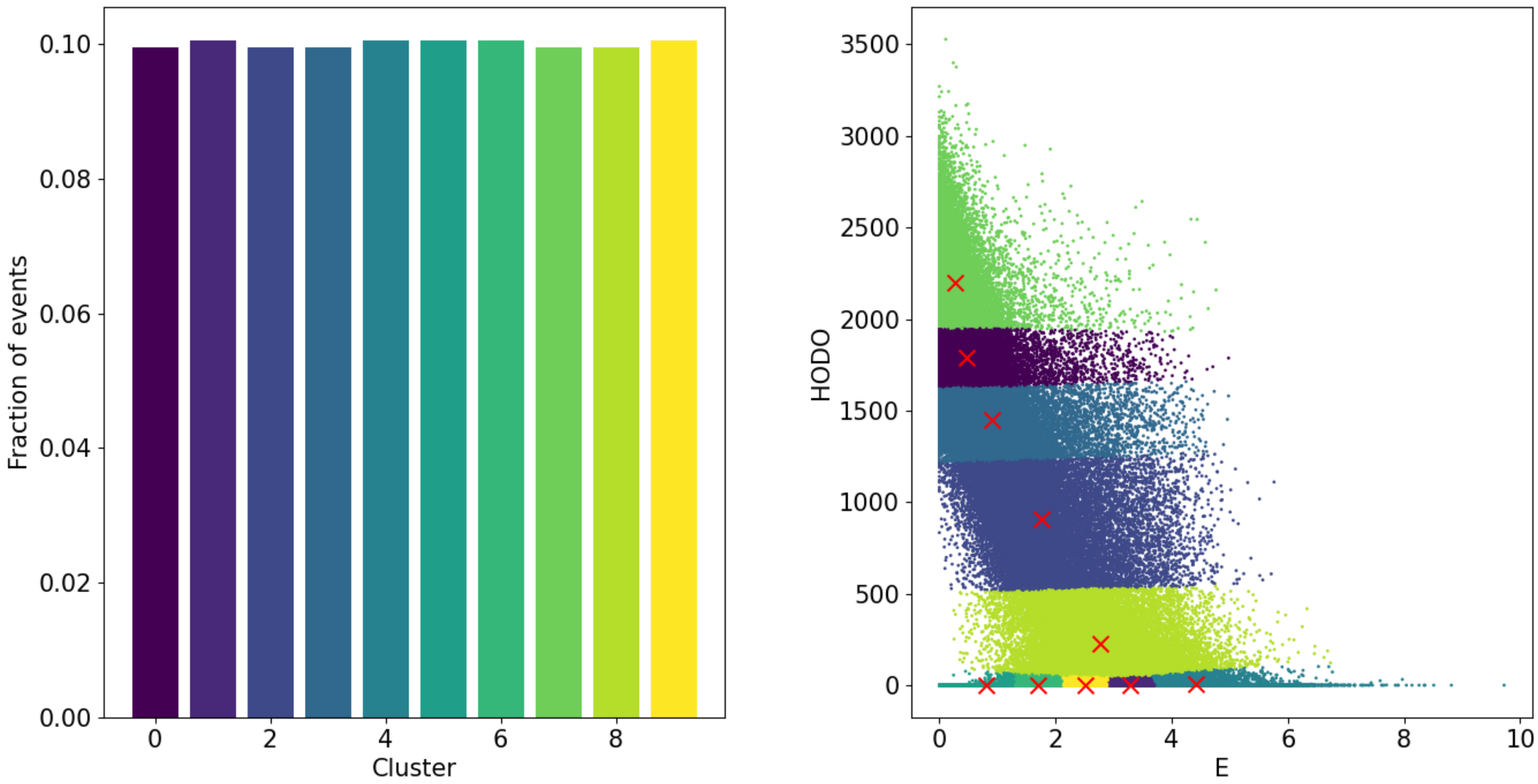
The results of the fit signals from the calorimeter and hodoscope





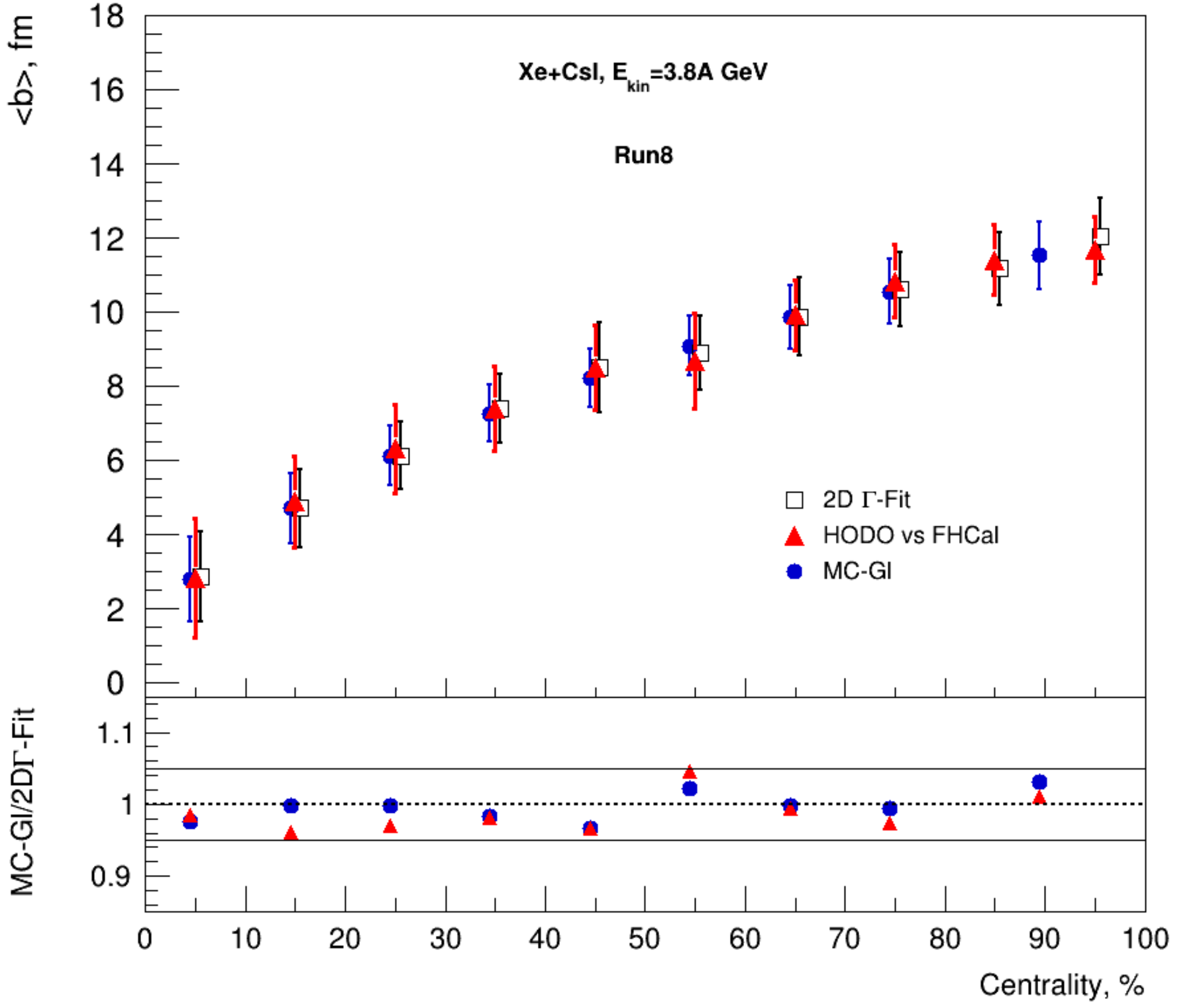
Good agreement of fit results for the calorimeter
The fit procedure for the hodoscope is in the process of developing

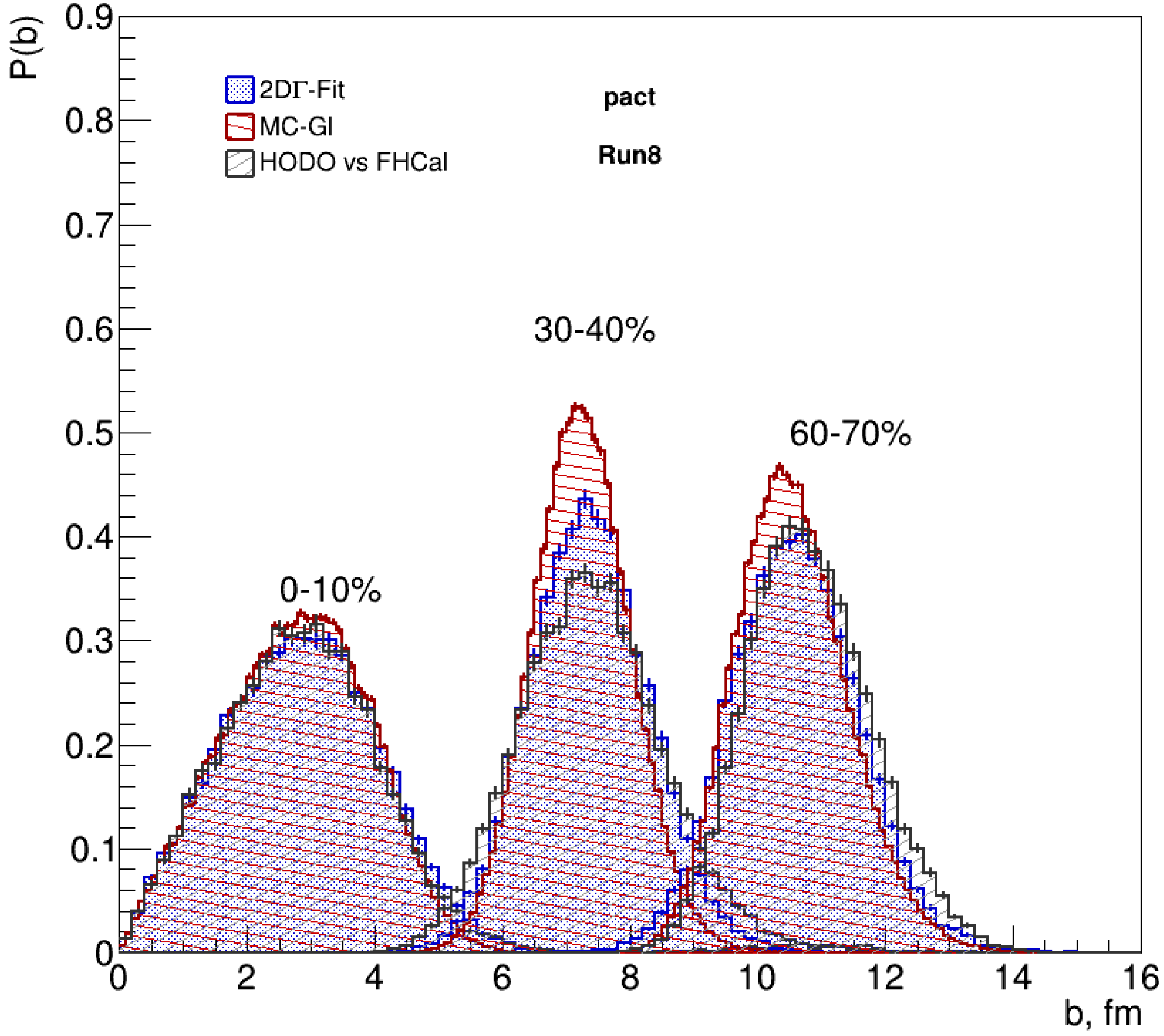
Centrality determination using an forward calorimeter and hodoscope



The K-means method allows to divide a two-dimensional distribution into centrality classes.
In order to correctly apply the class boundaries, it is necessary to match the simulation results with the experiment

Comparison with MC-Glauber fit





There is agreement within 5%.

Summary and Outlook

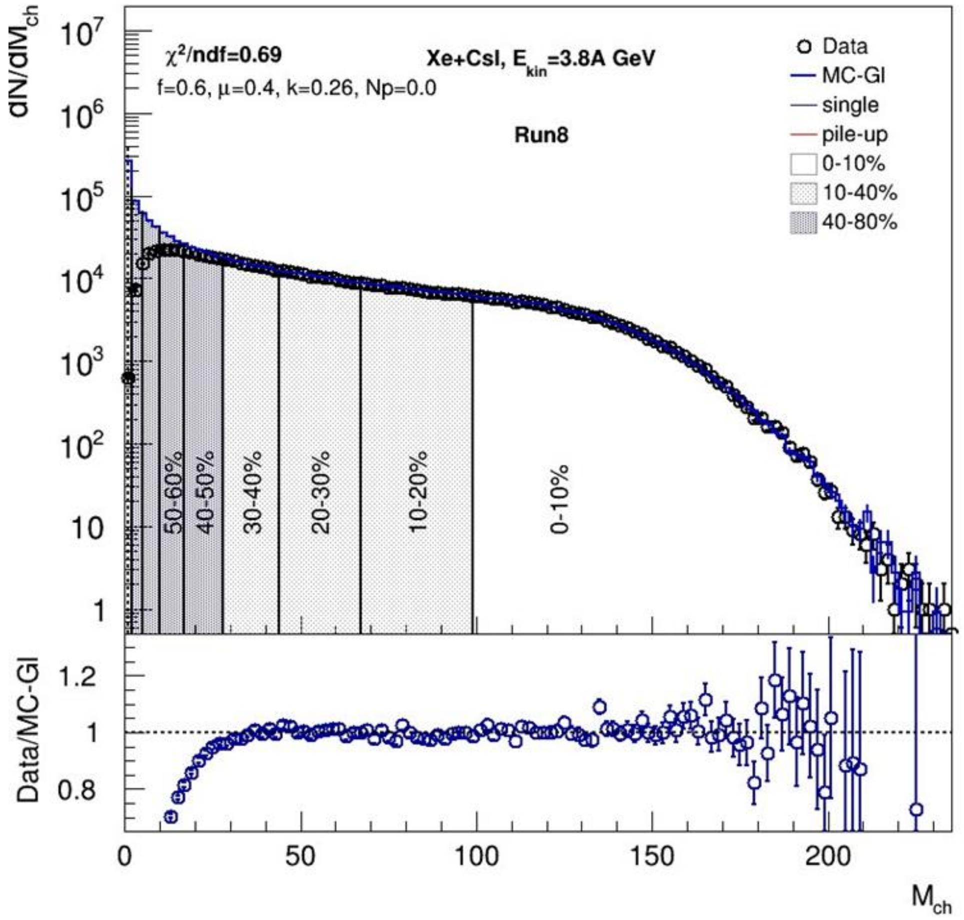
- Both methods the Bayesian inversion and MC-Glauber are provide consistent results
- Convoluted trigger efficiency (CTE) depends on event selection criteria
    - CTE with $|Vz|<0.1$ cm and vtxNtr>1 is 55%, w/o 85%
- A new approach for centrality determination with energy of spectators(FHCal) and the signal from the hodoscope was developed
    - The proposed methods are in good agreement with the classical approach based on the Glauber model
- Robust study using different models (DCM, UrQMD, etc.) and observables (TOF hit multiplicity, etc.)
- Check up the upcoming production

Thank you for your attention!

MC-Glauber based centrality framework

MC Glauber data → Evaluate N_a: $N_a = fN_{part}+(1-f)N_{coll}$ → Call $NBD(\mu,k) \times N_a$ → Build multiplicity fitting function

Build multiplicity fitting function + Input multiplicity distribution → Evaluate χ^2 → Minimize χ^2 to find α, μ, k



NBD – negative binomial distribution
Parameters of the fit:

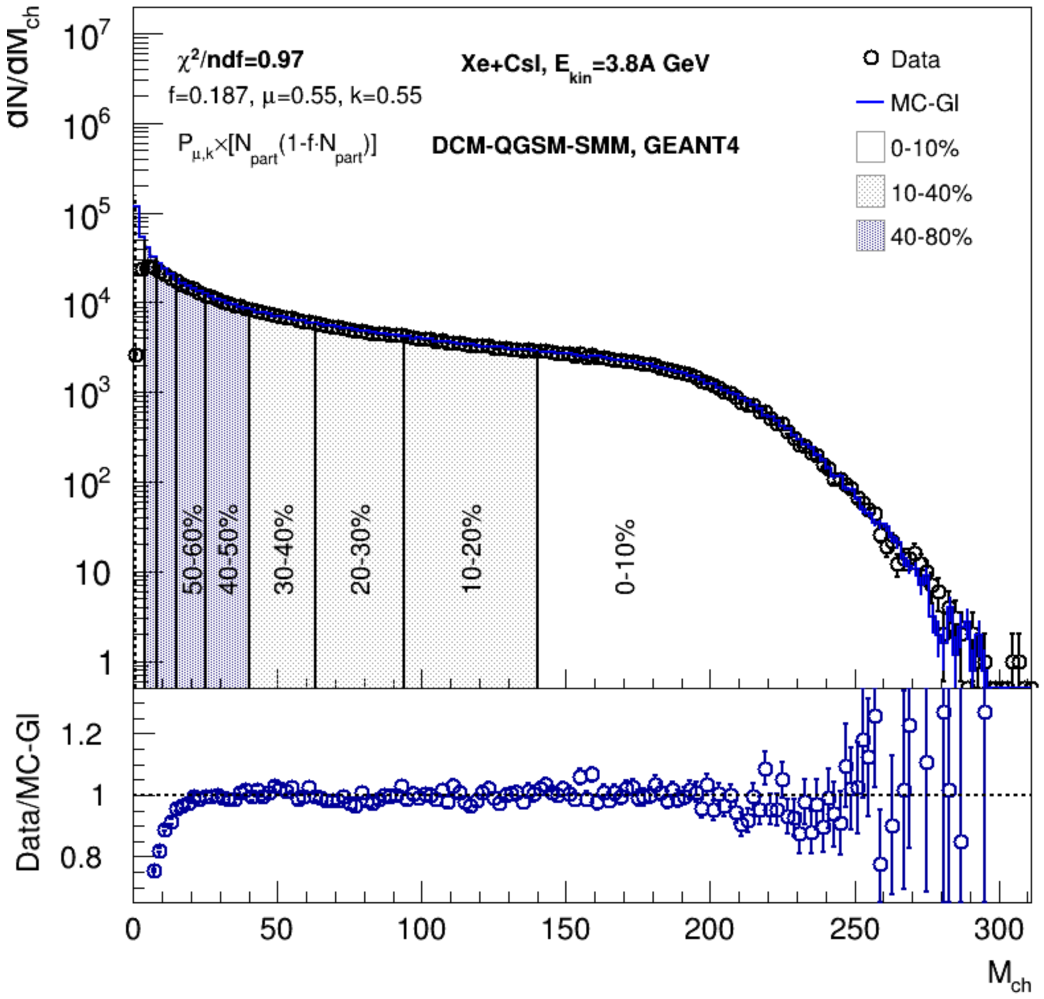
- **f** – the fraction of multiplicity from the soft component
- **μ** – mean multiplicity value
- **k** – width of the multiplicity distribution, can be connected to the fluctuations

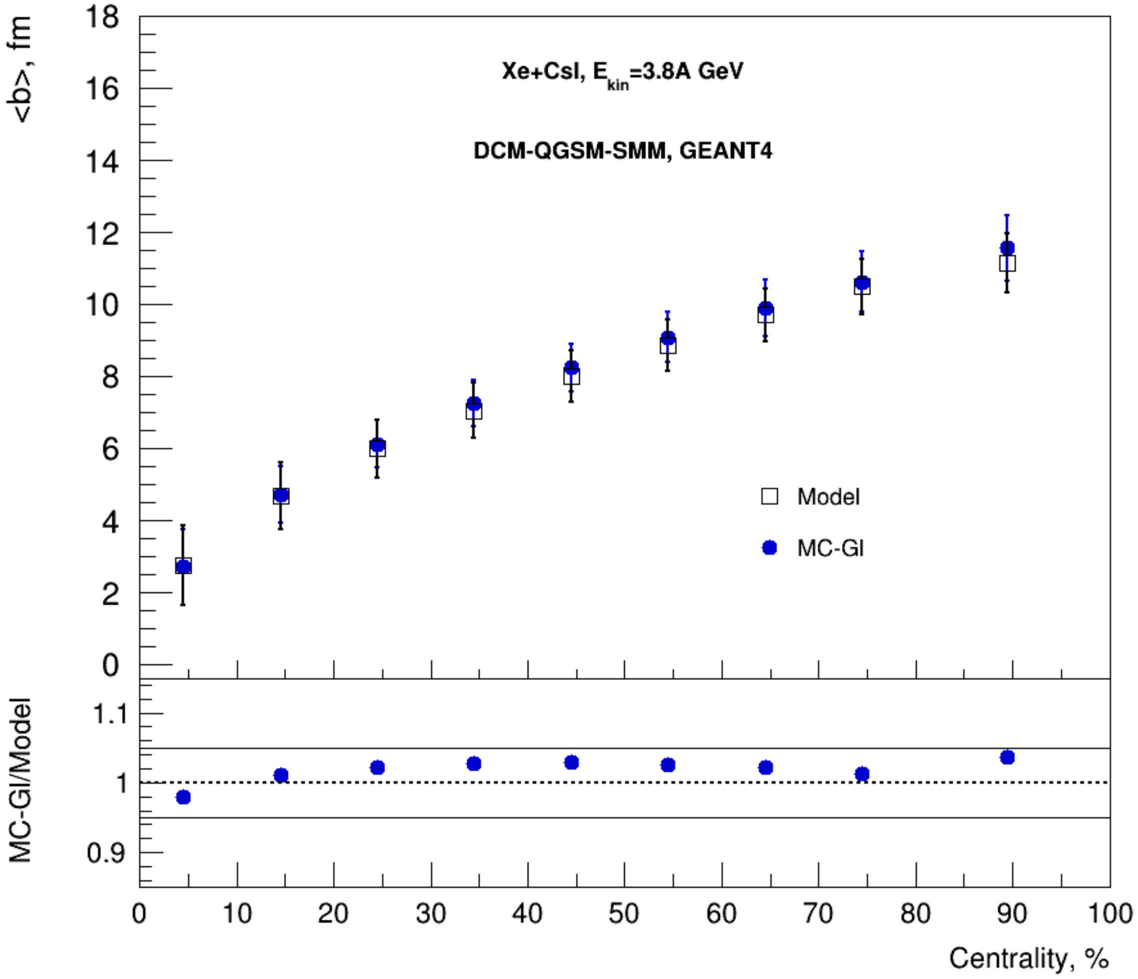
Implemantation for MPD: https://github.com/FlowNICA/CentralityFramework

P. Parfenov, et al., Particles. 2021; 4(2):275-287

Конфуз матрицы

Centrality determination for reconstructed data

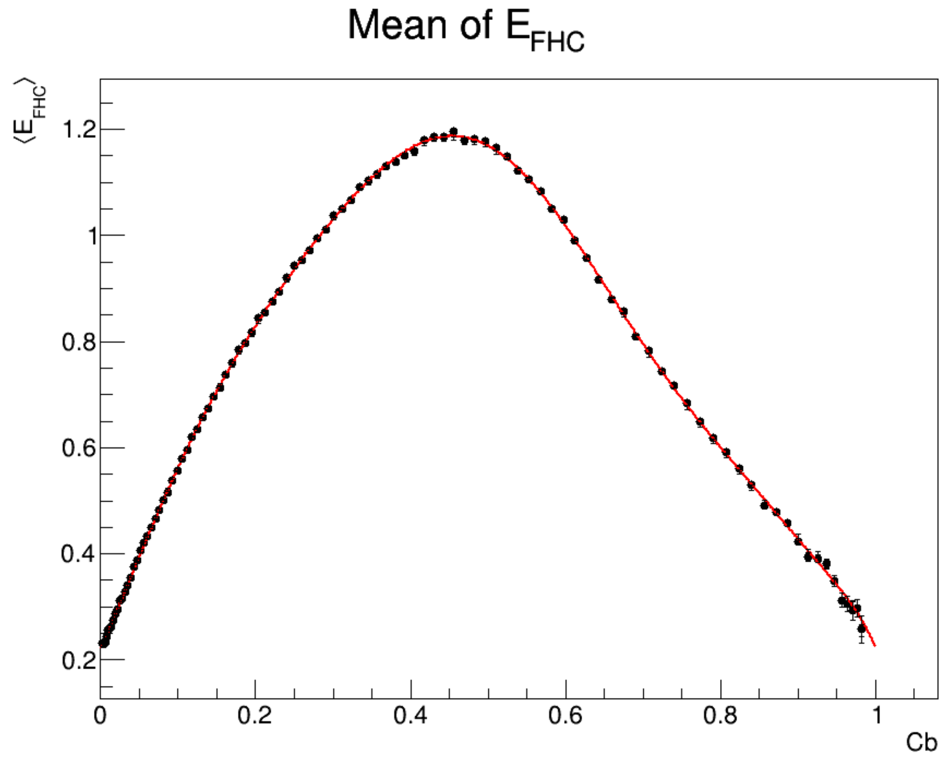


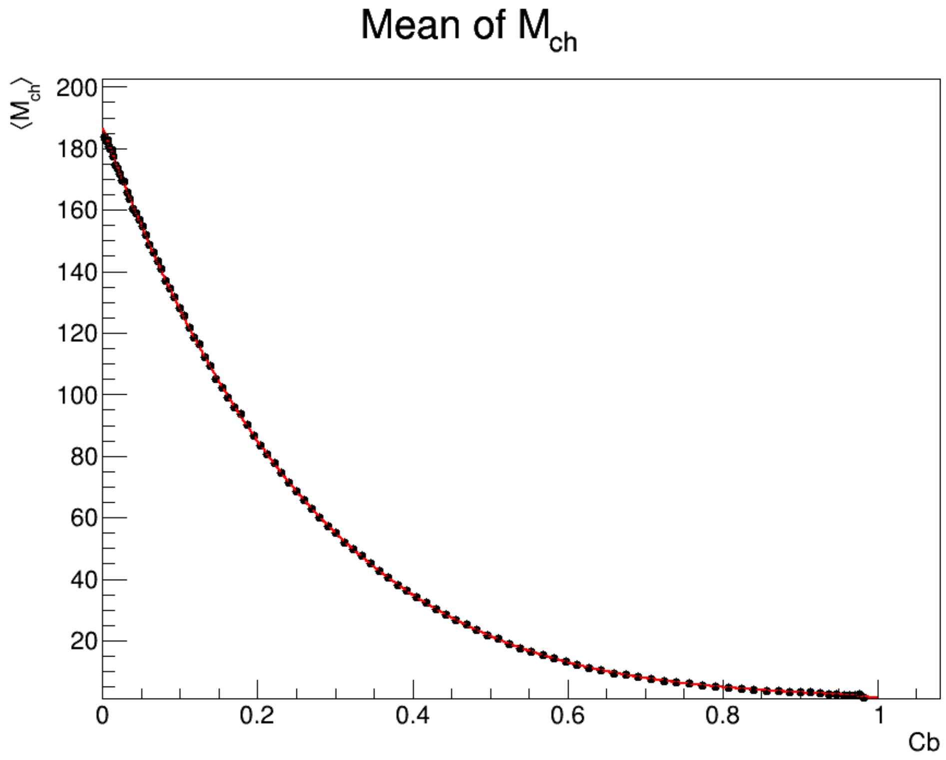


Good agreement with data

Thank you for your attention!

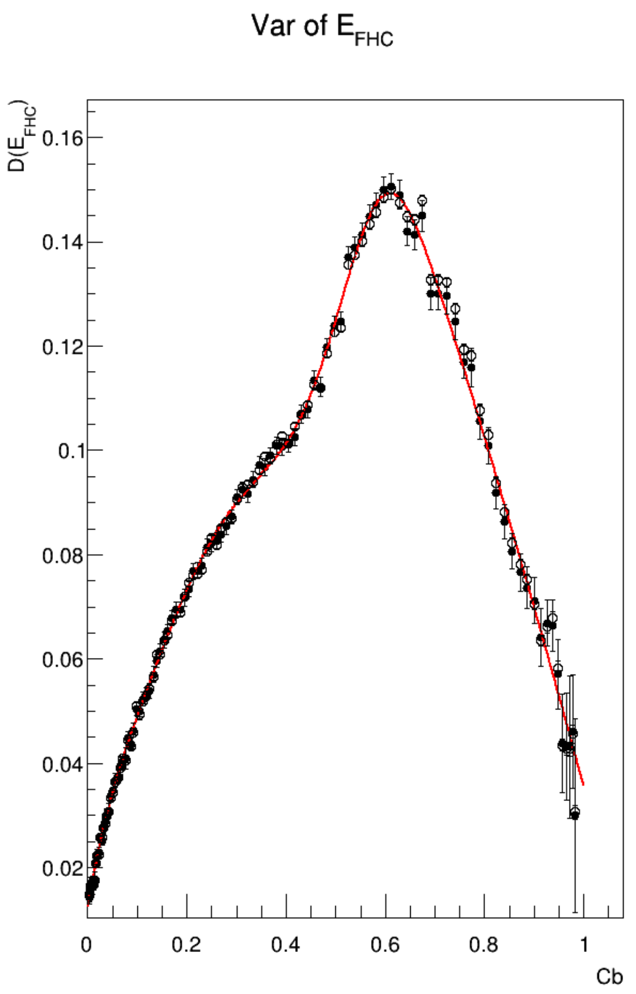
Dependence of the average value of multiplicity and energy on centrality

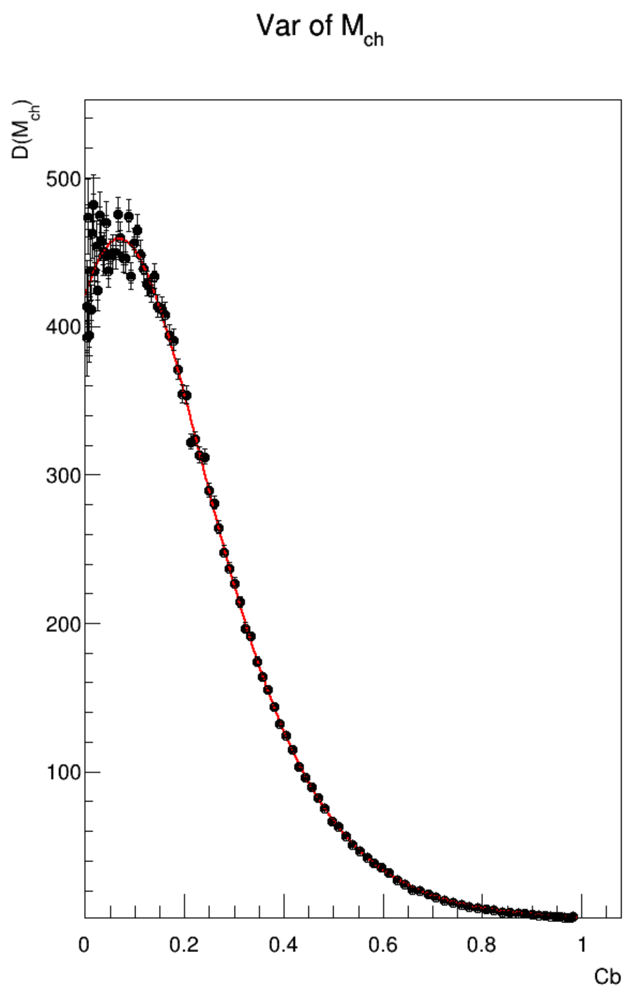


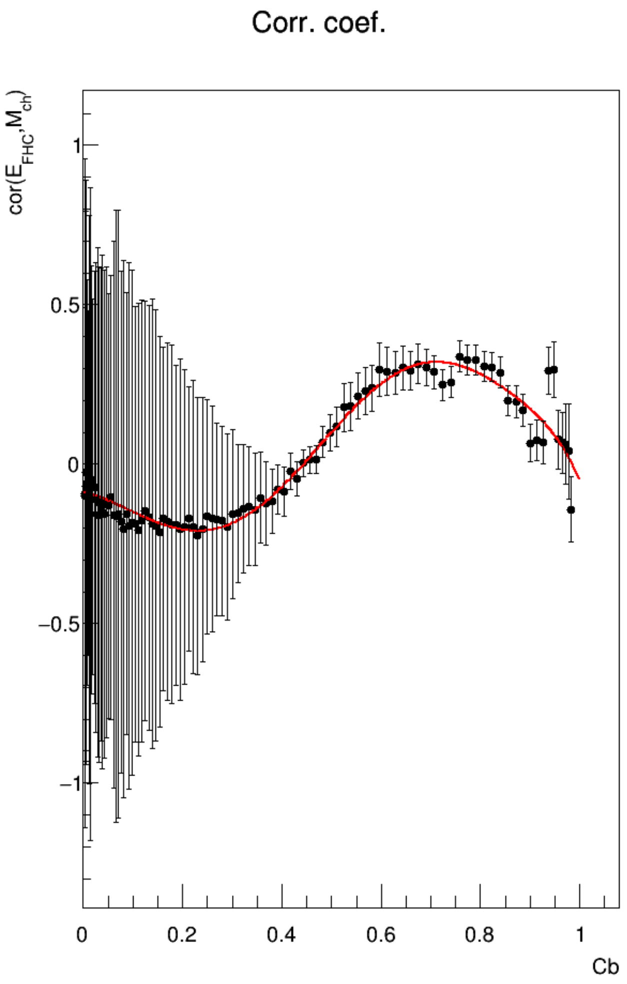


Good fit quality

Dependence of the variance of multiplicity and energy on centrality



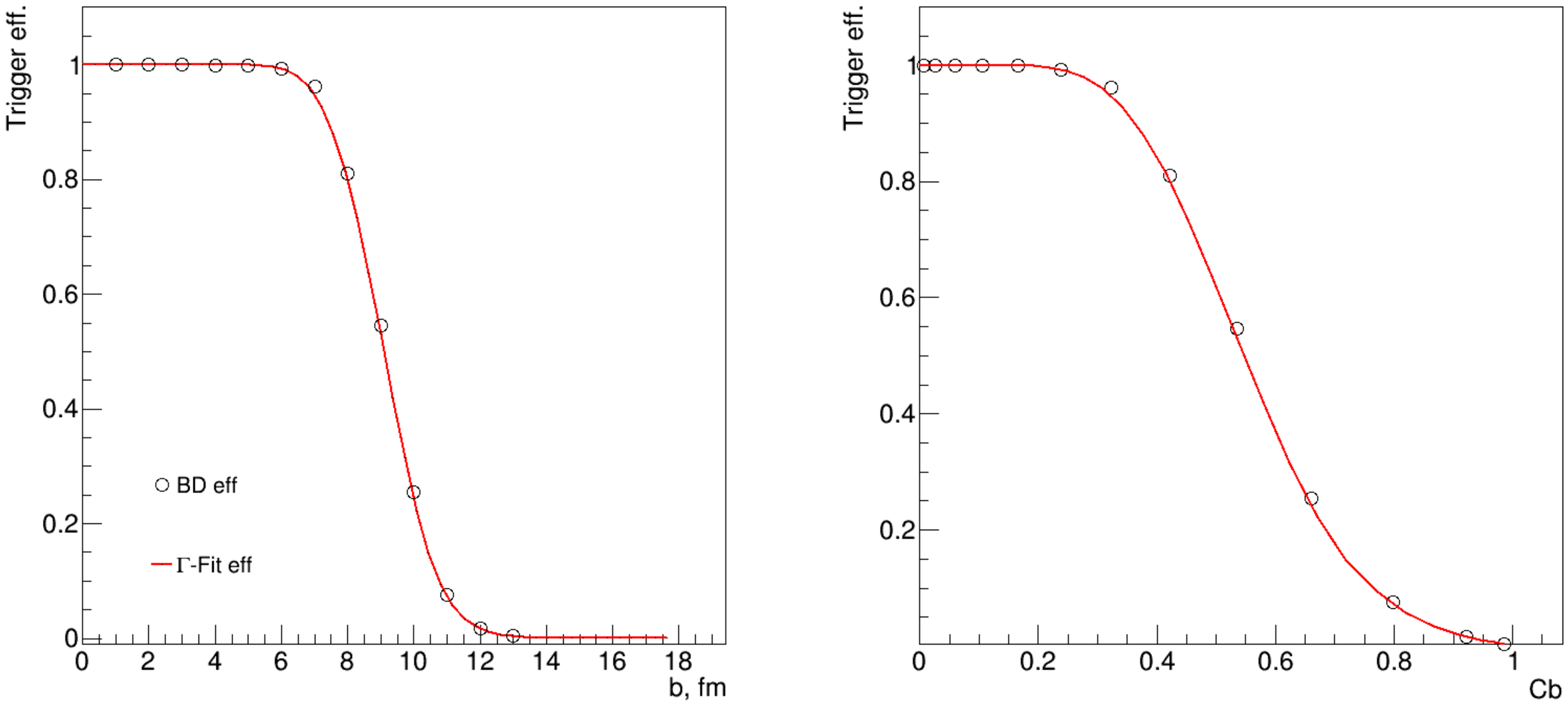




Good fit quality

The total efficiency of event registration

$$P_{eff}(b) = \int_0^{M_{max}} P_{eff}(\mathrm{M}) P(\mathrm{M} \mid \mathrm{b}) dM$$



The trigger efficiency obtained from the Bayesian approach is consistent with the results, obtained on the basis of simulations

https://indico.jinr.ru/event/4762/contributions/28478/attachments/20298/35273/lashmanov_report_BM@N-meeting_Oct-2024.pdf

Corrections for efficiency and pileup

- **Correction for efficiency of normalized multiplicity distribution P(M)**

$$P(M) = \frac{dN}{dM} / N^{ev}_{ideal} = \frac{N^{ev}_{raw}}{N^{ev}_{ideal}} \cdot \frac{1}{N^{ev}_{raw}} \frac{dN_r}{dM} = \frac{1}{K} \cdot Norm.Histogr$$

$$Eff = \frac{N^{ev}_{raw}}{N^{ev}_{ideal}} = \frac{1}{K}$$ integral efficiency

- **Fit function for multiplicity distribution P(M)**

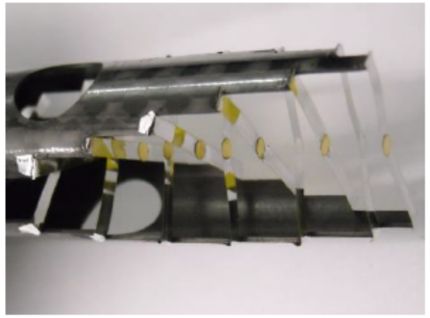
$$F(M) = K \cdot P_{total}(M), \;\; P_{total}(M) = N_p \cdot P_{pu}(M) + (1 - N_p) \cdot P(M)$$

μ, f, k, K, N_p- fit parameters, $F(M) -$ fit function, corrected for efficiency and pileup

Event cleaning in HADES

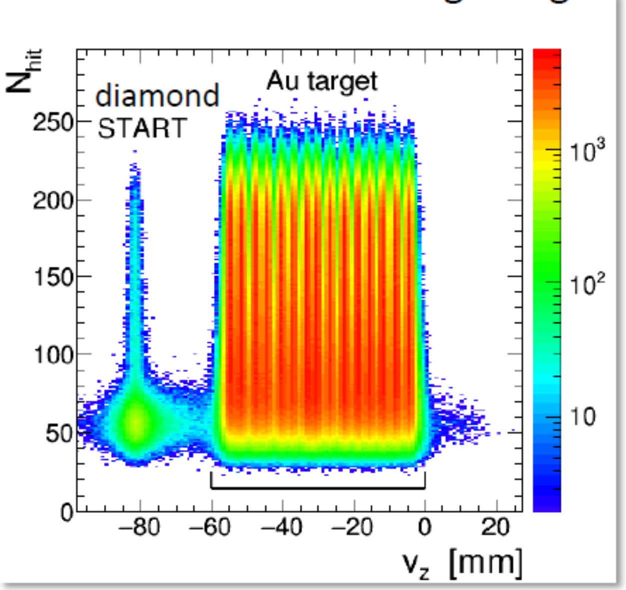
Segmented gold target:

- ^{197}Au material
- 15 discs of Ø = 2.2 mm mounted on kapton strips
- Δz = 3.6 mm
- 2.0% interaction prob.

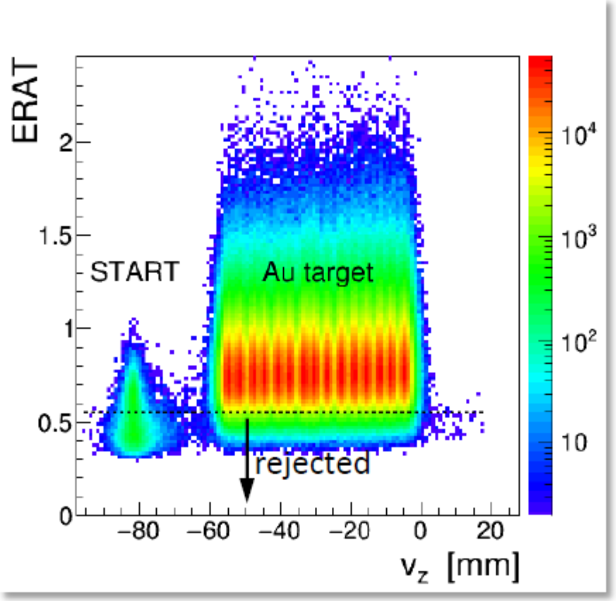


Kindler et al., NIM A 655 (2011) 95

Event vertex cut on target region



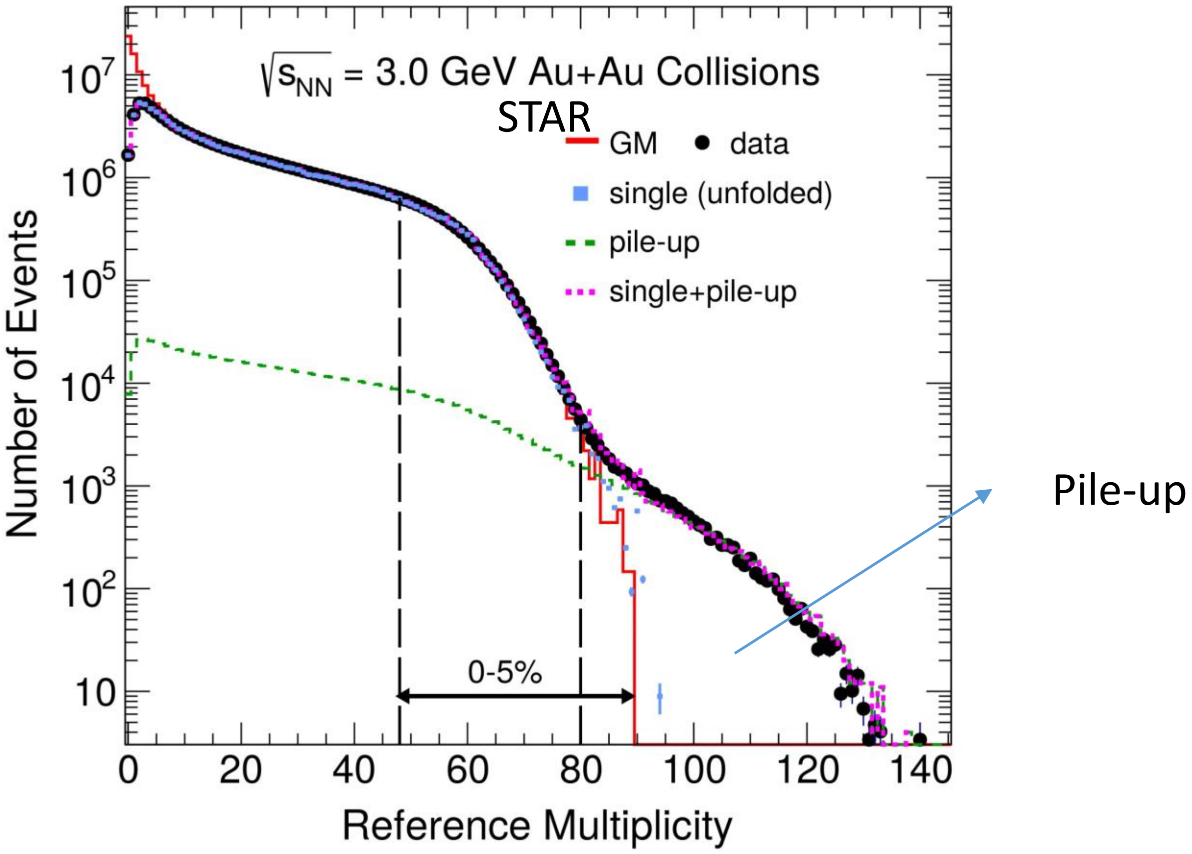
Remove Au+C bkgd on the kapton with a cut on $ERAT = \sum E_t / \sum E_l$



beam direction

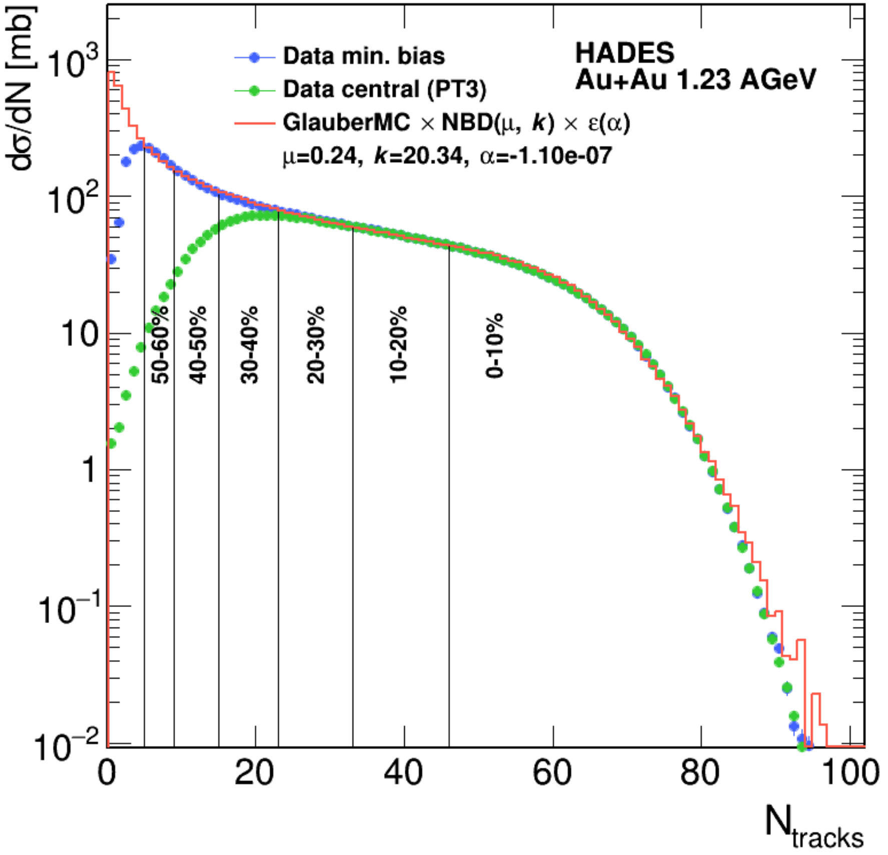
http://indico.oris.mephi.ru/event/221/session/1/contribution/1/material/slides/0.pdf

Centrality determination in the FIX-target experiments



Reference multiplicity distributions (black markers) in the kinematic acceptance within −0.5 < y < 0 and 0.4 < pT < 2.0 GeV/c , GM (red histogram), and single and pile-up contributions from unfolding.

https://arxiv.org/abs/2112.00240



The cross section as a function of Ntracks for minimum bias (blue symbols) and central (PT3 trigger, green symbols) data in comparison with a fit using the Glauber MC model (red histogram).

https://arxiv.org/abs/1712.07993

Reconstruction of *b*

- Normalized multiplicity distribution $P(N_{ch})$

$$P(N_{ch}) = \int_0^1 P(N_{ch}|c_b)dc_b$$

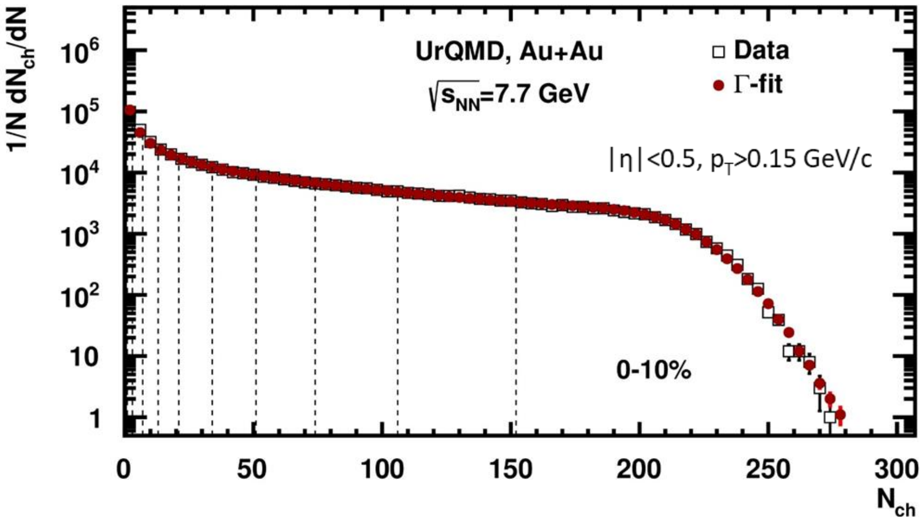
- Find probability of *b* for fixed range of N_{ch} using Bayes' theorem:

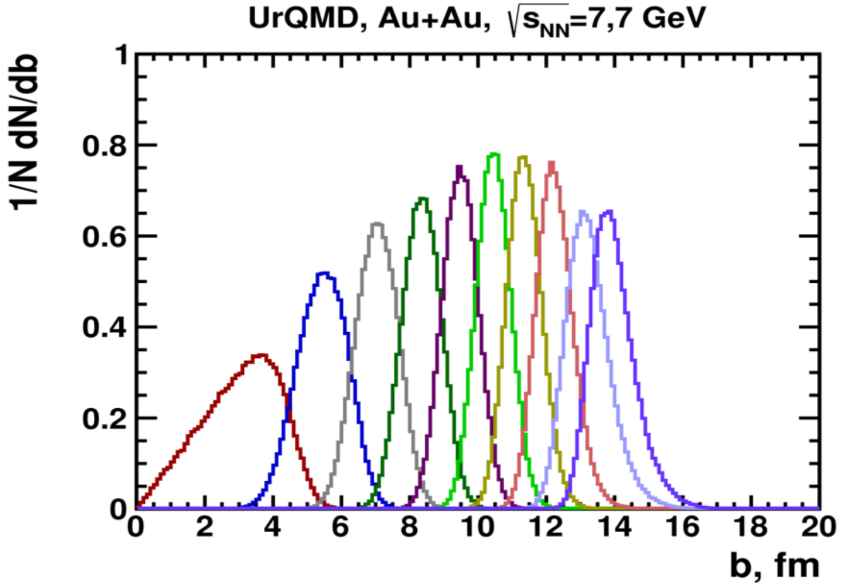
$$P(b|n_1 < N_{ch} < n_2) = P(b)\frac{\int_{n_1}^{n_2} P\,(N_{ch}|b)dN_{ch}}{\int_{n_1}^{n_2} P\,(N_{ch})dN_{ch}}$$

- **The Bayesian inversion method consists of 2 steps:**

–Fit normalized multiplicity distribution with $P(N_{ch})$

–Construct $P(b|N_{ch})$ using Bayes' theorem with parameters from the fit





R. Rogly, G. Giacalone and J. Y. Ollitrault, Phys.Rev. C98 (2018) no.2, 024902

Implementation in MPD: https://github.com/Dim23/GammaFit