





# Proton and net-proton High-Order Cumulants: can we do better with MPD at NICA?

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MPD Cross-PWG , March 18, 2025 INDICO: <u>https://indico.jinr.ru/event/5294/</u> ZOOM:

https://cern.zoom.us/j/61007466545?pwd=ZXR1WWhxMIQwK1hEUHJUQ3RsYnIPQT09

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#### 1. Some definitions

- 2. Hot topic: net-proton fluctuations
- 3. Can we do better with MPD at NICA?

## **Event-by-event fluctuations of conserved quantities**

- > Net electric charge Q,
- Net strangeness S,
- > Net baryon number B,

... are sensitive to the degrees of freedom that are active in the system.

- The moments (Variance (σ<sup>2</sup>), Skewness(S), Kurtosis(k)) of distributions of conserved quantities, such as netbaryon, net-charge and net-strangeness, are predicted to be sensitive to the correlation length of hot dense matter created in the collisions [1], i.e. sensitive to the QCD critical endpoint of the first order phase transition between quark-gluon plasma and hadron gas
- > Net proton number (as a proxy to net-baryon) :  $N_{p-\bar{p}} = N_p N_{\bar{p}} = \Delta N_p$
- > We will use N below to represent the net-proton number  $N_{p-\bar{p}}$  in one event

[1] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).

## Cumulants and correlation length ξ of hot dense matter

#### > Correlation length $\xi$ of hot dense matter[1]

- --- the cubic central moment of multiplicity  $\langle (\delta N)^3 \rangle \sim \xi^{4.5}$
- --- the quartic cumulant <(δN)<sup>4</sup>>~ ξ<sup>7</sup>

> Correlation length  $\xi$  will diverge (reach the maximum value) at the critical point[2]

[1] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
 {2} M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys.Rev. D 60, 114028 (1999) [arXiv:hep-ph/9903292].

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# Fluctuations of conserved quantities and

### susceptibilities

Fluctuations of **conserved quantities (N** or  $Q_i$  in this slide) relate directly to the susceptibilities  $\chi_i$ , which are quantities that can be calculated for thermodynamic systems, e.g. in lattice QCD. Susceptibilities are defined as derivatives of the pressure with respect to the chemical potential. Deviations  $\delta Q_i$  are related to susceptibilities  $\xi_n^{Q_i}$  by[1]:

$$\langle (\delta Q_i)^n \rangle = T^n \frac{\partial^n}{\partial \mu_i^n} \ln Z(T, \mu_i) = V T^3 \chi_n^{Q_i},$$

Here -- Q<sub>i</sub> is a conserved charge of interest,

T the temperature,  $\mu_i$  the corresponding chemical

potential and Z the partition function [1].

 Therefore the experimental values (in the Left) could be used to define thermodynamic model parameters (in the Right)
 It is tempting in such simplified approach to use the ratios

of cumulants in order to get rid of volume dependence

[1] Hannah Elfner and Berndt Müller 2023, J. Phys. G: Nucl. Part. Phys. 50 103001

#### Hot topic in 2023: Proton and net-proton High-Order Cumulants



Left: collision energy dependence of the ratios of cumulants C4/C2, for net proton (red circles) from 0%–40% and 50-60% Au + Au collisions at RHIC ([2]).

Fluctuation of conserved quantities were predicted to be sensitive -- near CEP, to the correlation length [3]

[1] Xiaofeng Luo, Nu Xu, NUCL SCI TECH (2017) 28:112 DOI 10.1007/s41365-017-0257-0;

[2] B.E.Aboona et al. \*STAR Collaboration), "Beam Energy Dependence of Fifth- and Sixth-Order Net-Proton Number
Fluctuations in Au+Au Collisions at RHIC", Phys. Rev. Lett. 130, 082301,(2023) <u>https://doi.org/10.1103/PhysRevLett.130.082301</u>
[3] C. Athanasiou, K. Rajagopal, M. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).doi:10.1103/PhysRevD.82.074008

#### In 2021: Non-monotonic energy dependence of net-proton number fluctuations[1]

The first evidence of a nonmonotonic variation in kurtosis times variance of the net-proton number (proxy for net-baryon number) distribution as a function of  $Vs_{NN}$  with 3.1s significance, for central Au+A collisions measured using the STAR detector at RHIC [1]



FIG. 4. So (1) and ko<sup>2</sup> (2) as a function of collision energy for net-proton distributions measured in Au+Au collisions. The results are shown for central (0-5%, filled circles) and peripheral (70-80%, open squares) collisions within 0.4 < pT (GeV/c) < 2.0 and jyj < 0.5 [1].

- Higher moments of distributions of conserved quantities (N) are more intersting due to their stronger dependence on the correlation length
- Centrality bin width correction (CBWC) is applied[3]

[1] J. Adam et al. (STAR Collaboration), Phys. Rev. Lett. 126, 092301 (2021).

[2] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).

[3] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013).

#### In 2021: Non-monotonic energy dependence of net-proton number fluctuations[1]



$$C_1 = M, \quad C_2 = \sigma^2, \quad C_3 = S\sigma^3, \ C_4 = \kappa\sigma^4$$

Events

Normalized Number of

[1] Xiaofeng Luo, Nu Xu, NUCL SCI TECH (2017), 28:112, DOI 10.1007/s41365-017-0257-0;
[2] (STAR Collaboration), PHYSICAL REVIEW LETTERS, 126, 092301 (2021)
[3] ] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013)



- FIG. 1. Event-by-event netproton number distributions for head-on (0%–5% central) Au +Au collisions for nine  $Vs_{NN}$ values measured by STAR [2].
- Non-gaussian shape of netbaryon distribution is expected near the CEP (M.Stepanov -2008)

FIG. 2. Cumulants (Cn) of the netproton distributions for central (0%–5%) and peripheral (70%–80%) Au +Au collisions as a function of collision energy[2].

The cumulants are corrected for the Centrality Bin Width Effect[3]

### Non-Gaussian fluctuations near the QCD critical point -

- "Higher, non-Gaussian, moments of the fluctuations are significantly more sensitive to the proximity of the critical point than the commonly employed measures based on quadratic moment" [1]
- "The fluctuations of the net proton number is a good proxy to the baryon number fluctuations, whose magnitude, proportional to the baryon number susceptibility, must diverge at the critical point"

[1] M.A.Stepanov,"Non-Gaussian fluctuations near the QCD critical point", 2008, <u>http://arxiv.org/abs/0809.3450v1</u>
 Phys. Rev. Lett. **102**, 032301 – Published 20 January, 2009
 DOI: <u>https://doi.org/10.1103/PhysRevLett.102.032301</u>

# Example of proton multiplicity distributions from three collision centralities [1].



Example of proton multiplicity distributions from three collision centralities. These distributions are not corrected for detector efficiency and pileup effects.[1]

Non-Gaussian shape of proton multiplicity distributions
 -- both in 0-5% and 0-40% centrality classes,

could be due to the volume fluctuation effects

To suppress the initial-volume fluctuation effects on cumulants for a given centrality, Centrality bin width correction [2] is performed in [1]

[1] STAR Collaboration, PHYSICAL REVIEW LETTERS 130, 082301 (2023)[2] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013).

### Problems of using $\delta N = N - \langle N \rangle$



V is a strongly fluctuating quantity, see , for example, impact parameter distribution for 0-5% class in MC simulations for Au+Au collisions at VS<sub>NN</sub> =7.7 GeV (Fig.44 from ref.[1]).

Event-by-event fluctuations of impact parameter b in the given centrality class selected for analysis will produce trivial volume fluctuations

[1]The MPD Collaboration, Status and initial physics performance studies of the MPD experiment at NICA, Eur. Phys. J. A (2022) 58:140, https://doi.org/10.1140/epja/s10050\_022\_00750\_6

#### **Volume dependence and volume fluctuations**

DEFINITIONS of cen cumulants [1]	tral moments and
the proton multiplicity in a given event	Ν
the average over all events	<>
Mean:	$M = \langle N \rangle = C_1,$
Deviation,	$\delta N = N - \langle N \rangle$
Variance	$\sigma^2 = <(\delta N)^2 > = C_2,$
skewness	S =< $(\delta N)^3$ >/ $\sigma^3$ = = $C_3/C_2^{3/2}$ ,
kurtosis	$k = <(\delta N)^4 > /\sigma^4 - 3 = = C_4 / C_2^2$
$C_1 = M,  C_2 = 0$	$\sigma^2$ , $C_3 = S\sigma^3$ ,

$$\chi_1^B = \frac{1}{VT^3} \langle N_B \rangle \; ,$$

$$\chi_2^B = \frac{1}{VT^3} \langle (\delta N_B)^2 \rangle \; ,$$

$$\chi_3^B = \frac{1}{VT^3} \left\langle (\delta N_B)^3 \right\rangle \,,$$

$$\chi_4^B = \frac{1}{VT^3} \left( \left\langle (\delta N_B)^4 \right\rangle - 3 \left\langle (\delta N_B)^2 \right\rangle^2 \right),$$

 $\rightarrow$  baryon number N<sub>B</sub> distribution can be measured via the net proton distribution

$$\langle \cdot \cdot \cdot \rangle$$
 -- the ensemble average

$$\delta N_{B} = N_{B} - \langle N_{B} \rangle$$

Volume V is fixed here (as well as T)!

Ratios of cumulants are used to reduce volume dependence:  $C_2/C_1 = \sigma^2/M, C_3/C_2 = S\sigma, \text{ and } C_4/C_2 = \kappa\sigma^2.$ 

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 $C_4 = \kappa \sigma^4$ 

#### Volume dependence and volume fluctuations ?

[1] -Xiaofeng Luo (STAR Collab.), Probing the QCD Critical Point with Higher Moments of Net-proton Multiplicity Distributions, arXiv:1106.2926v1, J. Phys.: Conf. Ser. 316, 012003 (2011), DOI:https://doi.org/10.1088/1742-6596/316/1/012003; Skokov, 1205.4756v2.pdf arXiv:1205.4756;

[2] S. Ejiri, F. Karsch, and K. Redlich, Hadronic fluctuations at the QCD phase transition, Phys. Lett. B 633, 275 (2006).

[3] W. J. Fu, X. Luo, J. M. Pawlowski, F. Rennecke, R. Wen, and S. Yin, Hyper-order baryon number fluctuations at finite temperature and density, Phys. Rev. D 104, 094047 (2021).

# Problems of using $\delta N = N - \langle N \rangle$

$$\sigma^{2} = \langle (\delta N)^{2} \rangle$$
skewness:  $S = \langle (\delta N)^{3} \rangle / \sigma^{3} = C_{3} / C_{2}^{3/2}$ 
kurtosis
$$k = \langle (\delta N)^{4} \rangle / \sigma^{4} - 3 = C_{4} / C_{2}^{2}$$

- The 2<sup>nd</sup>, 3<sup>rd</sup> or the 4<sup>th</sup> power of <δN> gives more weight
- Volume (V) dependence of baryon number susceptibilities.
- V is also a strongly fluctuating quantity
- "Therefore, we conclude that fluctuations of conserved charges in heavy ion collisions can provide robust probes of the chiral phase boundary if a good control of volume fluctuations can be achieved." (see in [1]).

[1] V.Skokov, B. Friman and K. Redlich, Volume fluctuations and higher order cumulants of the net baryon number, arXiv:1205.4756;

# Centrality Bin Width Correction procedure (CBWC) [1]

 $\sigma = \frac{\sum_{r} n_r \sigma_r}{\sum n_r} = \sum_{r} \omega_r \sigma_r$  $S = \frac{\sum_{r} n_r S_r}{\sum n_r} = \sum_{r} \omega_r S_r$  $\kappa = \frac{\sum_{r} n_r \kappa_r}{\sum n_r} = \sum_{r} \omega_r \kappa_r$  Selection of multiplicity class from multiplicity distribution Of chatged particles measured in some pseudorapidity interaval



Here  $n_r$  is the number of events in r<sup>th</sup> multiplicity for centrality determination, the  $\sigma_r$ ,  $S_r$  and  $k_r$  represent the standard deviation, skewness and kurtosis of particle number distributions at r<sup>th</sup> multiplicity. The corresponding weight for the r<sup>th</sup> multiplicity is  $\omega_r = n_r / \Sigma n_r$  [1] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013).

# Centrality Bin Width Correction procedure (CBWC) [1]



Here n<sub>r</sub> is the number of events in r<sup>th</sup> multiplicity bin

[1] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013).
 [2] MPD Collaboration<sup>^</sup> "MPD physics performance studies in Bi+Bi collisions at V1sNN = 9.2 GeV", to be published

EXAMPLE:

----The reconstructed TPC (black) and MCG modeled (red) multiplicity distributions for Bi+Bi collisions at  $Vs_{NN} = 9.2$  GeV [2] ----Blue arrows – multiplicity class boundaries

----Red arrow—some r<sup>th</sup> multiplicity bin

---- followed by calculations of :

 $n_r, \sigma_r, S_r \text{ and } k_r$ 

S<0 Negative Skew

for the net-proton  $r^{th}$  distribution in the  $r^{th}$  multiplicity bin ---- and then – by calculation of the average values of  $\sigma$ , *S* and *k* 

# Some examples of CBWC results in UrQMD model obtained in [1]



FIG. 3. The centrality dependence of the moments products ( $k\sigma^2$ ) of net-proton multiplicity distributions for Au+Au collisions at  $\sqrt{s_{NN}}$ =7.7, 11.5, 19.6, 27, 39, 62.4, 200GeV in UrQMD model. See in [1].

#### CBWC works and it is more important for the central collisions

[1] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013); arXiv:1302.2332v2

# Effects of different pseudorapidity intervals [1]



FIG. 7, See in [1]. The energy dependence of the moments products (S $\sigma$ , k $\sigma^2$ ) of net-proton multiplicity distributions for Au+Au collisions at Vs<sub>NN</sub>=7.7, 11.5, 19.6, 27, 39, 62.4, 200 GeV in UrQMD model with different centrality definition. See in [1].

> Centrality definition is using here different pseudorapidity intervals  $|\eta|$ : <0.5, <1.0, <1.5, <2.0

But we have to avoid in MPD such application as in [1] due to the fact that here different types of particle-emitting sources are mixed in case of a wide pseudorapidity region

[1] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013); arXiv:1302.2332v2

## Auto-correlation Effect (ACE)in UrQMD model [1]



FIG. 12. FIG. 12. (Color online) The centrality dependence of the moments products  $k\sigma^2$  of net-proton multiplicity distributions for Au+Au collisions at  $Vs_{NN}$ =7.7, 11.5, 19.6, 27, 39, 62.4, 200 GeV in UrQMD model with two type of centrality definition. See in [1].

-- all charged particles

- Auto-correlation Effect on kσ<sup>2</sup> is shown here as a result of using the same particles for centrality definitions and for moment analysis!
- > This should be avoided by the MPD analysis

[1] X. Luo, J. Xu, B. Mohanty, and N. Xu, J. Phys. G 40, 105104 (2013); arXiv:1302.2332v2

# Multiplicity and N<sub>part</sub> in MC Glauber model

- $\chi_1^B = \frac{1}{VT^3} \langle N_B \rangle ,$   $\chi_2^B = \frac{1}{VT^3} \langle (\delta N_B)^2 \rangle ,$   $\chi_3^B = \frac{1}{VT^3} \langle (\delta N_B)^3 \rangle ,$  $\chi_4^B = \frac{1}{VT^3} \left( \langle (\delta N_B)^4 \rangle - 3 \langle (\delta N_B)^2 \rangle^2 \right) ,$
- For the class of selected events, the mean volume V and temperature T are supposed here to be fixed in [1] during the CBWC procedure [2]



But narrow class in multiplicity, e.g. of 1% width,
 does not mean narrow distribution neither in the impact parameter b [3] nor in Npart[4]

Volume V is not fixed in the event-by-event study! This will skew distribution of N<sub>part</sub> and N<sub>B</sub>

W. J. Fu, X. Luo, J. M. Pawlowski, F. Rennecke, R. Wen, and S. Yin, Hyper-order baryon number fluctuations at finite temperature and density Phys. Rev. D 104, 094047 (2021).
 Bin width effect -Xiaofeng Luo (STAR Collab.), Probing the QCD Critical Point with Higher Moments of Net-proton Multiplicity Distributions, arXiv:1106.2926v1, J. Phys.: Conf. Ser. 316, 012003 (2011), DOI: https://doi.org/10.1088/1742-6596/316/1/012003
 see Fig.44 in the paoer: The MPD Collaboration, Status and initial physics performance studies of the MPD experiment at NICA, Eur. Phys. J. A (2022) 58:140, https://doi.org/10.1140/epja/s10050\_022\_00750\_6
 T. A. Drozhzhova, V. N. Kovalenko, A. Yu. Seryakov, G. A. Feofilov, Physics of Atomic Nuclei, September 2016, Volume 79, Issue 5, pp 737–748

# Fluctuations of conserved quantities and Volume fluctuations[1]

Consider a fixed volume V, where the net baryon number B fluctuates with the probability distribution P (B, V). The n-th order moments of the net baryon number are then defined by [1]:

The first four reduced cumulants in ref.[1] are:

where 
$$\delta B = B - \bar{B}$$
 and  $\bar{B} = \langle B \rangle_V$ . (1)  

$$\kappa_1(T,\mu) = \frac{1}{V} \langle B \rangle_V, \qquad (1)$$

$$\kappa_1(T,\mu) = \frac{1}{V} \langle (\delta B)^2 \rangle_V, \qquad (2)$$

$$\kappa_3(T,\mu) = \frac{1}{V} \langle (\delta B)^3 \rangle_V, \qquad (2)$$

[1] V. Skokov, 1, B. Friman, 2 and K. Redlich 3, "Volume fluctuations and higher order cumulants of the net baryon number". arXiv:1205.4756v2

# Proposal-1 for MPD: New Bin Width Correction procedure with the account of Volume fluctuations (CBWC –V)

Ratios of cumulants  $C_2/C_1 = \sigma^2/M$ ,  $C_3/C_2 = S\sigma$ , and  $C_4/C_2 = \kappa\sigma^2$  were used to reduce the volume dependence. However, the average values of  $\sigma$ , S and k are calculated assuming the fixed value of volume V in all events!.

We propose to use the reduced cumulants, similar to [1], but on the event-by-event basis, following the new CBWC-V procedure with V<sup>r</sup> in each r<sup>th</sup> multiplicity bin:

 $k = <(\delta N)^4 > /\sigma^4 - 3 = = C_4 / C_2^2 \qquad k_r = <(\delta N^r)^4 > /\sigma_r^4 - 3$ 

➢ We assume that for any r<sup>th</sup> multiplicity bin the relevant mean volume V<sup>r</sup> is proportional to the mean number of participants <N<sup>r</sup><sub>part</sub> > : V<sup>r</sup> = <N<sup>r</sup><sub>part</sub> >V<sub>0</sub> Here a volume factor V<sub>0</sub> = 2.83 fm<sup>3</sup> (see in [1]).

> Thus we obtain the reduced deviation  $\delta N^r = N^r/V^r - \langle N^r/V^r \rangle$  for the relevant distribution of conserved quantity  $N^r$ 

 [1] V. Skokov, B. Friman and K. Redlich, "Volume fluctuations and higher order cumulants of the net baryon number". arXiv:1205.4756v2

## Proposal-2 for MPD: careful centrality selection





The reconstructed TPC (black) and MCG modeled (red) multiplicity distributions for Bi+Bi collisions at  $\sqrt{s_{NN}} = 9.2$  GeV [1]

Mean impact parameter for 10% centrality intervals for Bi+Bi collisions at  $\lor$  s<sub>NN</sub> = 9.2 GeV, modeled with the UrQMD, PHSD, DCM-QGSM-SMM, and PHQMD in [1]

- The width of the class of centrality of A+A collisions is directly related to the unavoidable volume fluctuations [2]: We should select much narrow central classes of multiplicity for the analysis of net-proton fluctuations [2]
- > Another approach to measure spectator fragments with Hadron Calorimeter in parallel with multiplicity
- > Application of ML to select narrow centrality classes in ToF technique with high timing resolution [3]

[1] MPD Collaboration, "MPD physics performance studies in Bi+Bi collisions at V1sNN = 9.2 GeV", to be published
 [2] T. A. Drozhzhova, V. N. Kovalenko, A. Yu. Seryakov, G. A. Feofilov, <u>Physics of Atomic Nuclei</u>, 2016, V 79, <u>Issue 5</u>, pp 737–748
 [3] Galaktionov K., Rudnev V., Valiev F., Physics of Particles and Nuclei, v54, N3, (2023), 446-448, DOI: <u>10.1134/s1063779623030152</u> 22

#### Proposal-3 for MPD: We have to avoid Auto-correlation Effects

➤We have to avoid Auto-correlation Effects in the future analysis of net-proton fluctuations

--- we should not use the same particles for centrality definitions and for moment analysis!

--- we should not use the same pseudorapidity intervals for centrality definitions and for moment analysis!

#### Multi-Purpose Detector (MPD) at the 1<sup>st</sup> stage of operation[1]

Strong competition with HADES, NA61/SHINE, STAR BES, and CBM!





[1] MPD Collaboration, "MPD physics performance studies in Bi+Bi collisions at V1sNN = 9.2 GeV", to be published

MPD -- a  $4\pi$  spectrometer for detecting charged hadrons, electrons and photons in A+A collisions at high luminosity. *FXT and colliding modes* 

- *Wide rapidity coverage, excellent PID capabilities*
- Tracking at midrapidity (TPC)
- > ToF, FFD, ECal and FHCal
- > The system-size scan could be also a competitive advantage for MPD!

Beams expected at Stage-1: Xe+Xe/Bi+Bi at  $\sqrt{s_{NN=}}9.02$  GeV Or in the fixed target mode: Xe/Bi+W at  $\sqrt{s_{NN=}}3.0$  GeV

#### MPD physics performance study:

#### reconstruction of protons and anti-protons



charged hadrons as a function of  $p_{\tau}$  [1]

momentum spectra for p and anti-p for midcentral (|y| < 0.5) Bi+Bi collisions at  $Vs_{NN} = 9.2$  GeV in different centrality intervals.

Different techniques are available at MPD for the PID hadron measurements Measurements with contamination corrections span over a wider pT range Important to study the proton and net-proton High-Order Cumulants !

[1] MPD Collaboration, "MPD physics performance studies in Bi+Bi collisions at V1sNN = 9.2 GeV", to be published

#### Summary

- Strong competition of MPD with HADES, NA61/SHINE, STAR BES, and CBM!
- **>** Benefits of the MPD:
  - -- Wide rapidity coverage,
  - --- Different techniques are available for PID hadron measurements
  - -- Tracking at midrapidity (TPC)
  - -- ToF, FFD, ECal and FHCal
  - --- The system-size scan could be a competitive advantage in search of CEP by the MPD!

New procedure is proposed for Bin Width Correction with the account of Volume fluctuations
 Centrality selection criteria are being intensively studied in order to minimize trivial volume fluctuations and the auto-correlation effects in the future analysis of net-proton fluctuations in A+A collisions

Thank you for your attention!