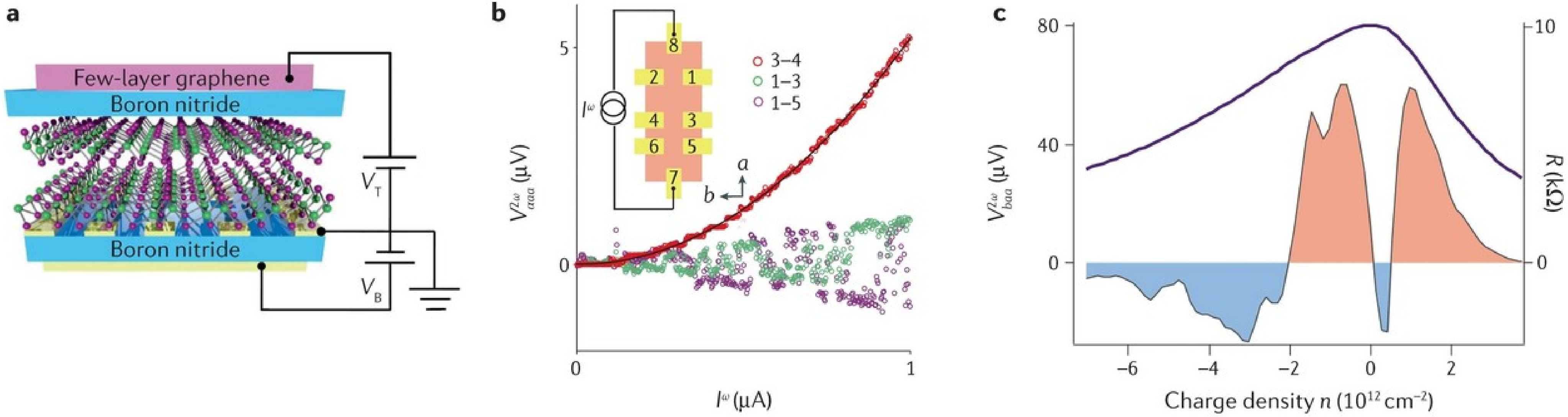
Non-Linear Hall effect

- The nonlinear Hall effect shows a tranverse voltage of $V_{2\omega}$ for an applied current of $I_{2\omega}$
- Unlike usual (linear) Hall effects time reversal symmetry is not broken in this case.



Du, Z.Z., Lu, HZ. & Xie, X.C. Nonlinear Hall effects. Nat Rev Phys 3, 744–752 (2021).

Orbital Magnetization

- Orbital magnetization is part of the magnetic moment due to orbital motion of electrons.
- In condensed matter theory it is correlated to the Berry curvatures.

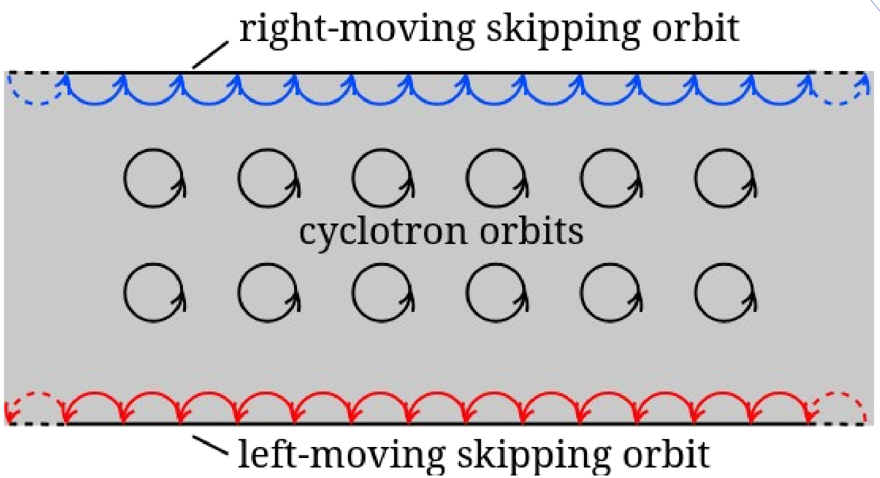
$$\mathbf{M}_{\mathrm{orb}} = \frac{e}{2\hbar c}\,\mathrm{Im}\sum_n \int \frac{d^3k}{(2\pi)^3}\, f_{n\mathbf{k}}\, \langle \partial_{\mathbf{k}} u_{n\mathbf{k}} | \times (H_{\mathbf{k}} + E_{n\mathbf{k}} - 2\mu)\, | \partial_{\mathbf{k}} u_{n\mathbf{k}} \rangle \,,$$

T. THONHAUSER, THEORY OF ORBITAL MAGNETIZATION IN SOLIDS, International Journal of Modern Physics B 2011 25:11, 1429-1458

Orbital Magnetization due to Berry curvature

- Orbital magnetization due to chiral edge states

$$\boldsymbol{M} = \frac{e}{\hbar} \sum_{a} \int_{\boldsymbol{k}} \boldsymbol{\Omega}_{a,\boldsymbol{k}} \frac{1}{\beta} \log\left[1 + e^{-\beta(E_{a,\boldsymbol{k}} - \mu)}\right],$$



$$\Omega_n(k) = -Im\langle \nabla_k u_{nk} | \times | \nabla_k u_{nk} \rangle$$

Relation between orbital magnetization and Hall current density

- The Hall current density is defined as

$$J_{H,i} = \epsilon_{ijk} \frac{\partial M_k}{\partial r_j} = \epsilon_{ijk} \frac{\partial \mu}{\partial r_j} \frac{\partial M_k}{\partial \mu} .$$

- Electric field is the gradient of chemical potential

$$\boldsymbol{E} = -\frac{1}{e} \partial_{\boldsymbol{r}} \mu,$$

Relation between orbital magnetization and Hall current density

- The Hall current density is:

$$J_{H,i} = \sigma_{ij}^{H} E_j$$

- The conductivity is:

$$\sigma_{ij}^{H} = -\frac{e^2}{\hbar} \sum_a \int_{\boldsymbol{k}} \epsilon_{ijk} \Omega_{a,\boldsymbol{k}}^{k} f\left(E_{a,\boldsymbol{k}}\right) d\boldsymbol{k},$$

Orbital magnetization due to peturbation

- For the case of magnetoelectric case, the peturbation of magnetization gives

$$M_i = \chi_{ij} E_j .$$

$$\chi_{ij} = -\frac{e^2}{\hbar} \sum_a \int_{\boldsymbol{k}} \left(\frac{\tau}{\hbar} \Omega^i_{a,\boldsymbol{k}} v^j_{a,\boldsymbol{k}} + 2\epsilon_{ikq} G^{jk}_{a,\boldsymbol{k}} v^q_{a,\boldsymbol{k}} \right) f\left(E_{a,\boldsymbol{k}}\right)$$

- Substituting petrubated magnetization back into hall conductivity we get

$$J_{H,i} = \sigma^H_{ijk} E_j E_k ,$$

$$\sigma^H_{ijk} = -\frac{e^3}{\hbar} \sum_a \int_{\boldsymbol{k}} \left[\frac{\tau}{2\hbar} \epsilon_{iqk} \partial_{k_j} \Omega^q_{a,\boldsymbol{k}} + \left(\partial_{k_i} G^{jk}_{a,\boldsymbol{k}} - \partial_{k_j} G^{ik}_{a,\boldsymbol{k}} \right) + (j \leftrightarrow k) \right] f\left(E_{a,\boldsymbol{k}}\right) . \quad (5)$$