Bayesian approach for centrality determination in MPD-FXT (Xe+W at T =2.5 AGeV)

Idrisov Dim, Parfenov Peter, Fedor Guber

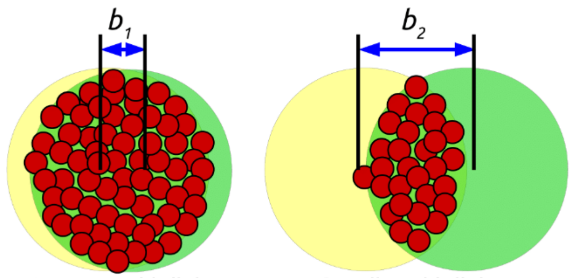
INR RAS, Moscow, Russia

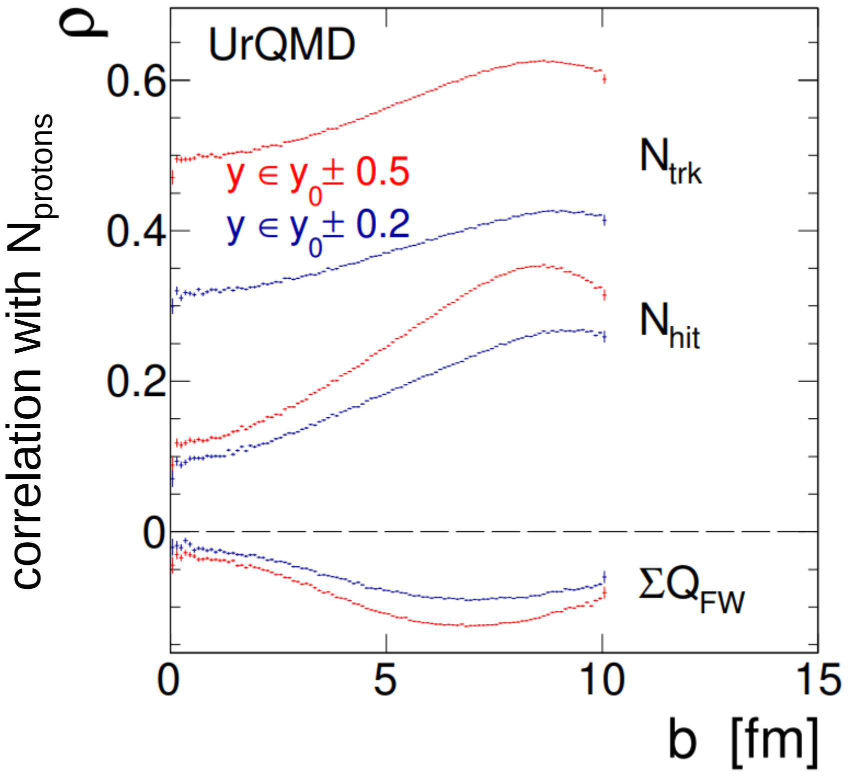
08/04/25

Centrality

- Evolution of matter produced in heavy-ion collisions depend on its initial geometry
- Centrality procedure maps initial geometry parameters with measurable quantities (multiplicity or energy of the spectators)
- **This allows comparison of the future MPD results with the data from other experiments (STAR BES, NA49/NA61 scans) and theoretical models**

$$c(b) = \frac{\int_0^b \frac{d\sigma}{db'} db'}{\int_0^\infty \frac{d\sigma}{db'} db'} = \frac{1}{\sigma_{A-A}} \int_0^b \frac{d\sigma}{db'} db'$$

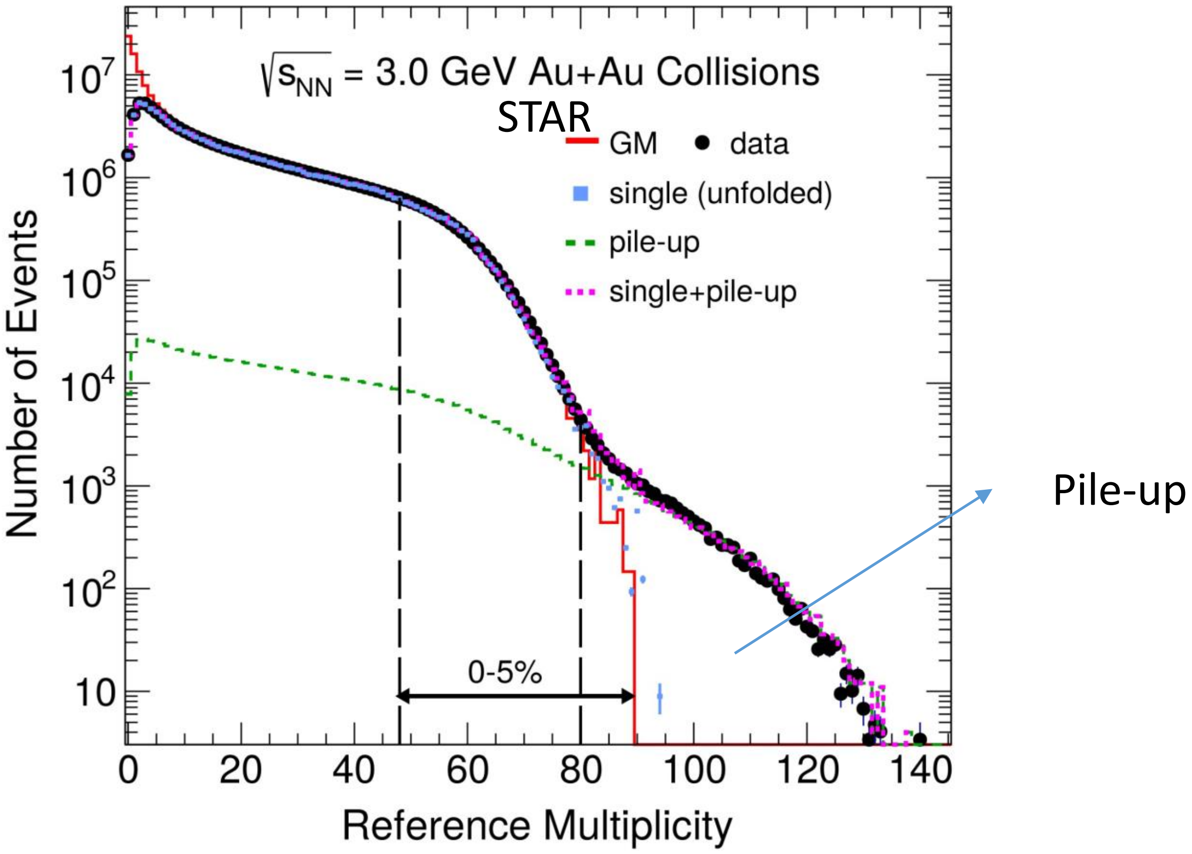




HADES; Phys.Rev.C 102 (2020) 2, 024914

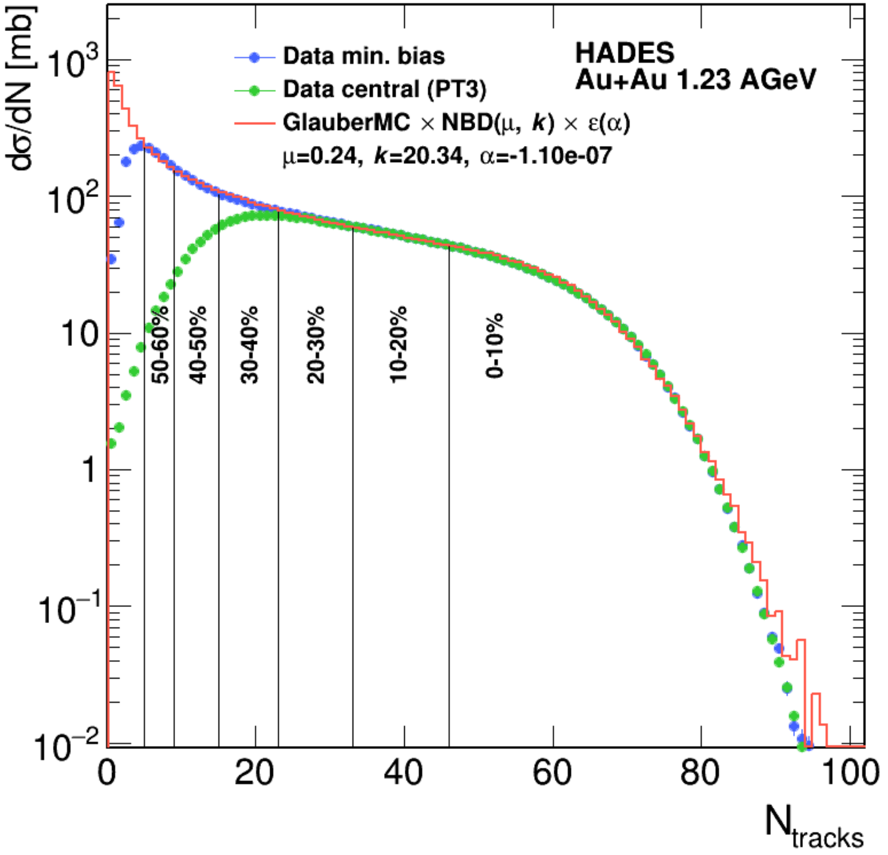
- A number of produced protons is stronger correlated with the number of produced particles (track & RPC+TOF hits) than with the total charge of spectator fragments (FW)
- to suppress self-correlation biases, it is necessary to use spectators fragments for centrality estimation

Centrality determination in the FIX-target experiments



Reference multiplicity distributions (black markers) in the kinematic acceptance within −0.5 < y < 0 and 0.4 < pT < 2.0 GeV/c ,
GM (red histogram), and single and pile-up contributions from unfolding.

https://arxiv.org/abs/2112.00240

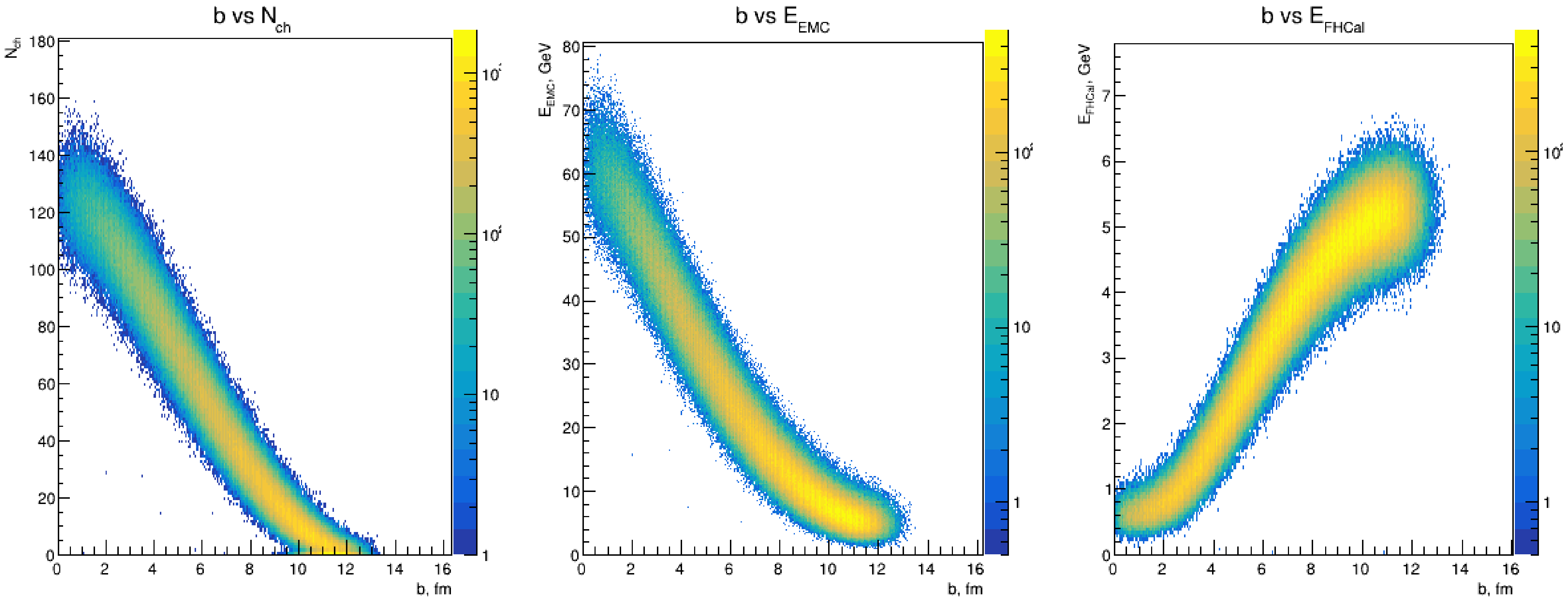


The cross section as a function of Ntracks for minimum bias (blue symbols) and central (PT3 trigger, green symbols) data in comparison with a fit using the Glauber MC model (red histogram).

https://arxiv.org/abs/1712.07993

Centrality determination in MPD

UrQMD → GEANT4 → Reconstruction → Centrality determination



Relation between impact parameter and observables

The Bayesian inversion method (Γ-fit): multiplicity based

- The fluctuation kernel Gamma distr.:

$$\boxed{P(M \mid c_b) = \frac{1}{\Gamma(k(c_b))\theta^2} M^{k(c_b)-1} e^{-M/\theta}}$$

$$c_b = \int_0^b P(b')db'$$ – centrality based on impact parameter

$$\boxed{\theta = \frac{D(M)}{\langle M \rangle}, \quad k = \frac{\langle M \rangle}{\theta}}$$

$\langle M \rangle, D(M)$ – average and variance of Multiplicty

$$\boxed{P(M) = \int_0^1 P(M \mid c_b) dc_b}$$

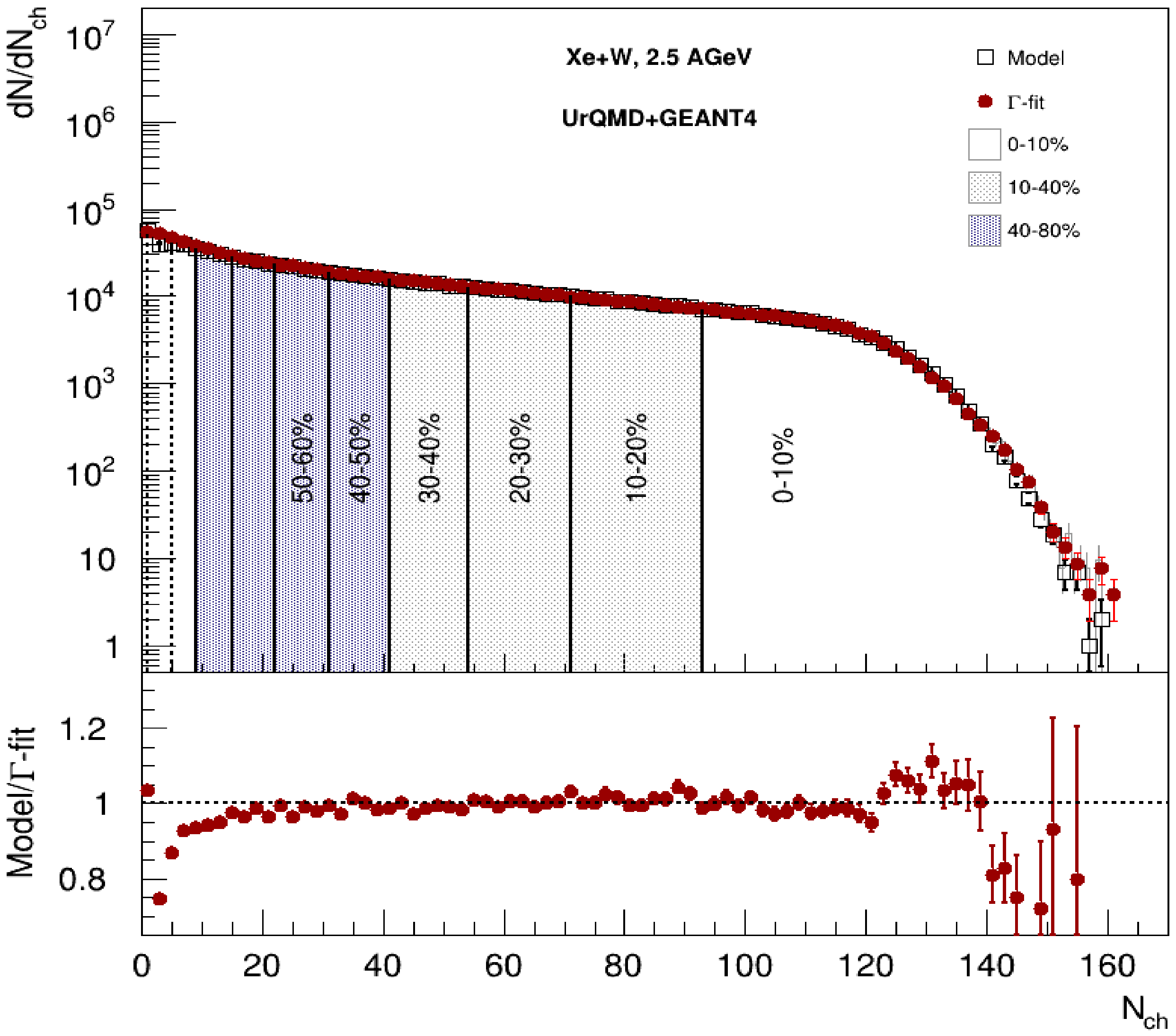
$$\langle M \rangle = m_1 \cdot \langle M' \rangle$$

$$D(M) = m_1^2 \cdot D(M') + m_1 \cdot m_2 \langle M' \rangle$$

$\langle M'(c_b) \rangle$, $D(M'(c_b))$ – average value and var. of energy/mult. from the rec. model data

- can be approximated by polynomials and exponential polynomial

Multiplicty-based Γ-fit: MPD-FXT Xe+W



Track selection:

- Nhit>10
- 0.5<η<2
- 2>Pt>0.2GeV/c

Good agreement with fit

The Bayesian inversion method (Γ-fit): 2D approach

- The fluctuation kernel for observables at fixed impact parameter can be describe by 2D Gamma distr.:

$$\boxed{P(E,E_F \mid c_b) = G_{2D}(E,E_F,\langle E\rangle,\langle E_F\rangle,D(E),D(E_F),R)}$$

$$c_b = \int_0^b P(b')db'$$ – centrality based on impact parameter

$\langle E\rangle, D(E)$ – average value and variance of energy in EMC

$\langle E_F\rangle, D(E_F)$ – average value and variance of energy in FHCal

$R(E,E_F)$ – Pirson correlation coefficient

$$\boxed{R(E,E_F) = \frac{\varepsilon_1^2 m_1^2}{\varepsilon_2 m_2} R(E',E_F')}$$

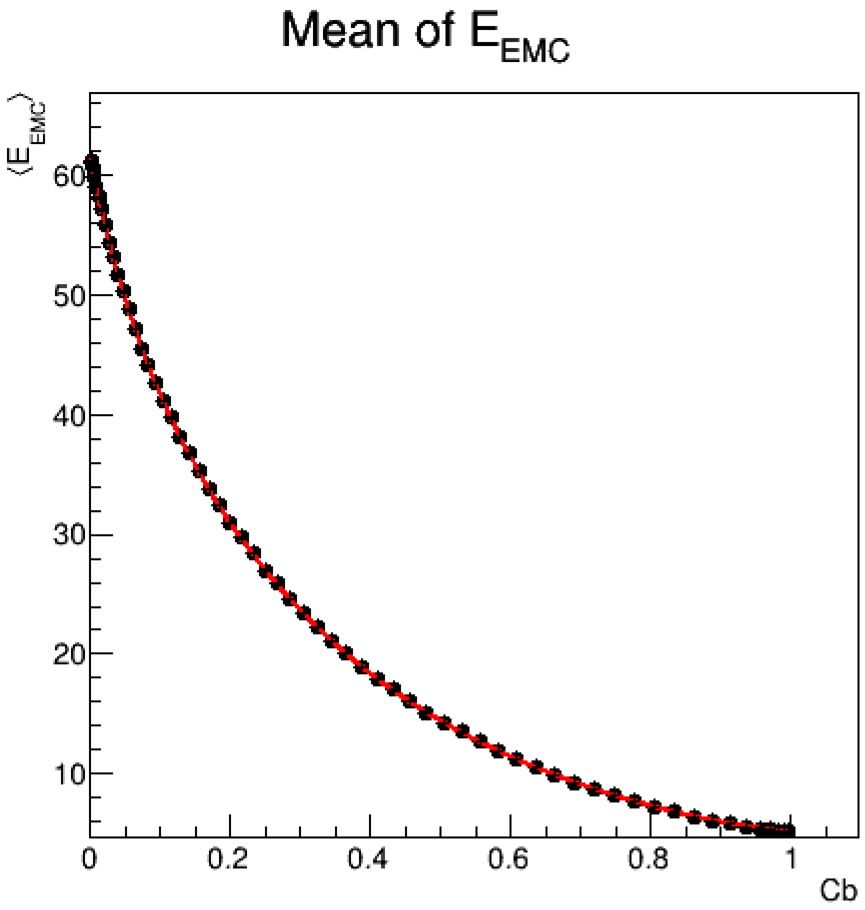
$\varepsilon_1,\varepsilon_2,m_1,m_2$ - fit parameters

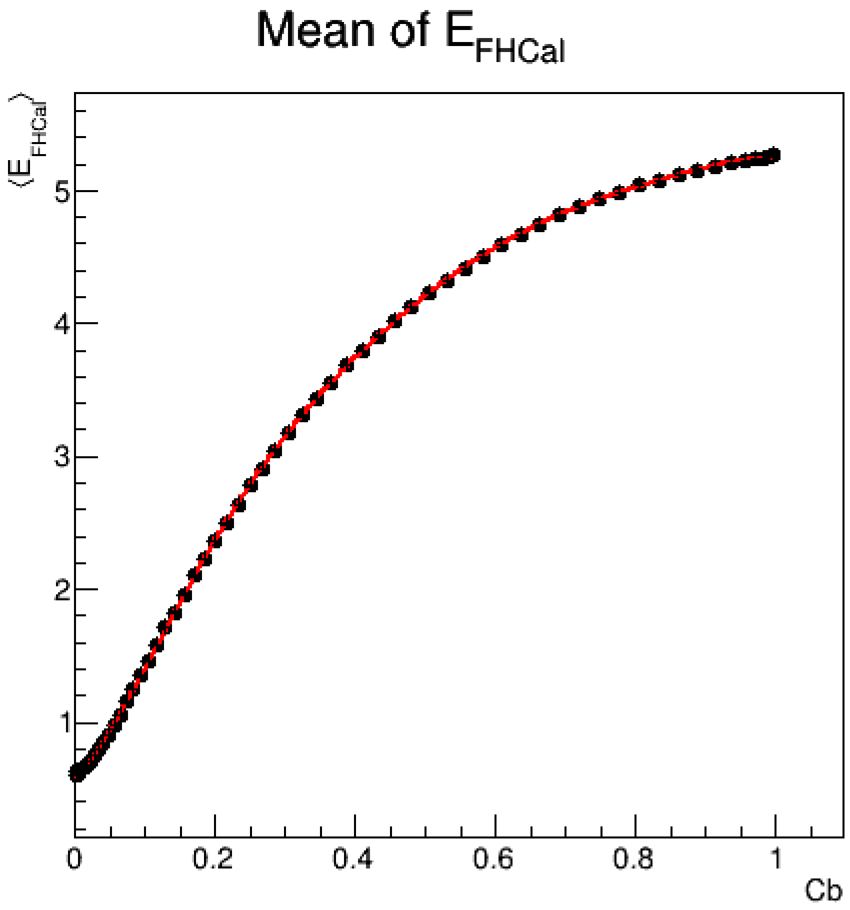
$\langle E'(c_b)\rangle$, $D(E'(c_b))$ – average value and var. of energy[EMC/FHCal] from the model

$$\boxed{\begin{array}{l}\langle E\rangle = \varepsilon_1\langle E'(c_b)\rangle, \quad D(E) = \varepsilon_2 D(E'(c_b)) \\ \langle E_F\rangle = m_1\langle E_F'(c_b)\rangle, \quad D(E_F) = m_2 D(E_F'(c_b))\end{array}}$$

$\langle E'(c_b)\rangle, \quad D(E'(c_b))$ - can be approximated by a polynomial or any other smooth function

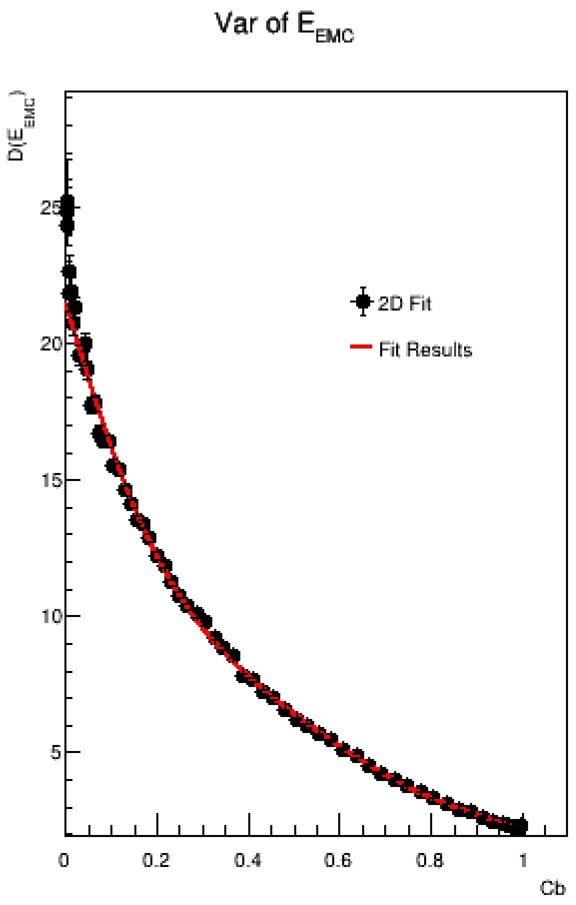
Dependence of the average value of energy in EMC and FHCal on centrality

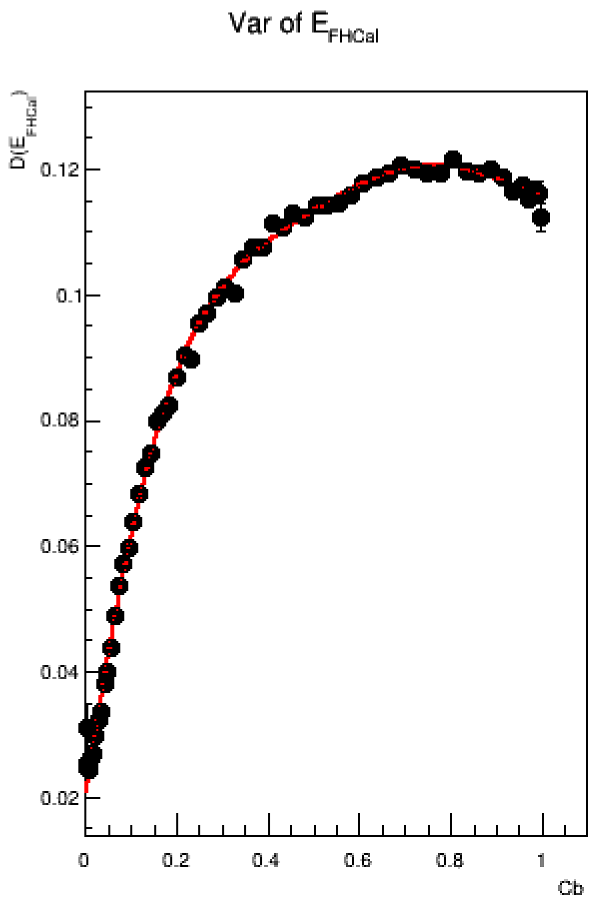


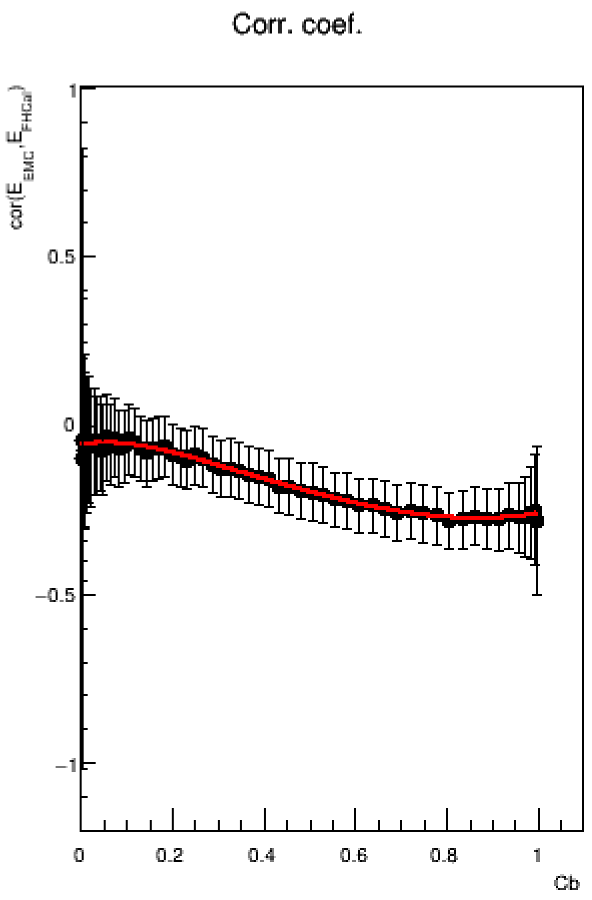


Mean values for E_{EMC} and E_{FHCal} from UrQMD and parametrization are in a good agreement over all c_b

Dependence of the variance of observables on centrality







Variance for E_{EMC} and E_{FHCal} from UrQMD and parametrization are in a good agreement over all c_b

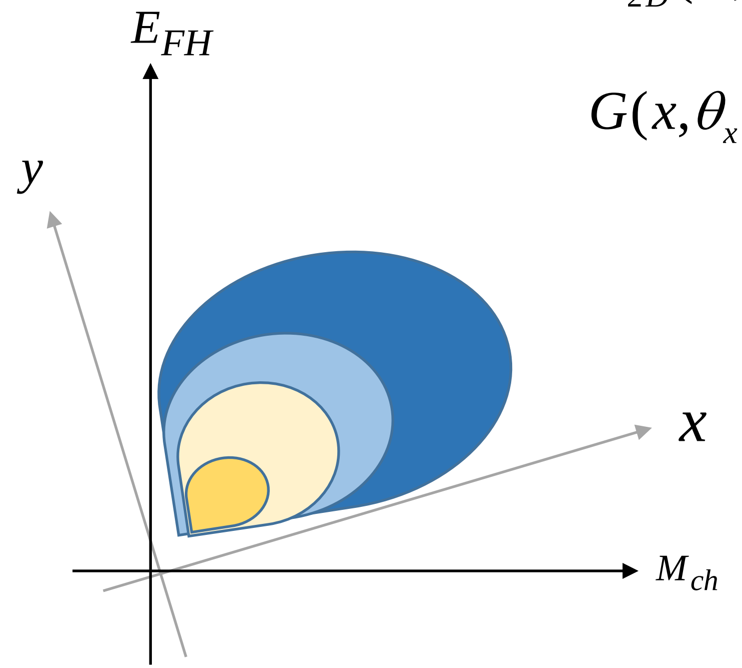
2D Gamma distribution

It is possible to find such a rotation angle of the system that $\mathrm{cov}(x,y)=0$

Then the two-dimensional distribution in the new coordinate system will be

$$G_{2D}(E,E_F,\langle E\rangle,\langle E_F\rangle,D(E),D(E_F),R)=G(x,\theta_x,k_x)\cdot G(y,\theta_y,k_y)$$

$$G(x,\theta_x,k_x)\cdot G(y,\theta_y,k_y)=\frac{(x)^{k_x(c_b)-1}e^{-x/\theta_x}}{\Gamma(k_x(c_b))\theta_x^2}\cdot\frac{(y)^{k_y(c_b)-1}e^{-y/\theta_y}}{\Gamma(k_y(c_b))\theta_y^2}$$



$$\theta_x=\frac{D(x)}{\langle x\rangle},\quad k_x=\frac{\langle x\rangle^2}{D(x)},\ \theta_y=\frac{D(y)}{\langle y\rangle},\quad k_y=\frac{\langle y\rangle^2}{D(y)}$$

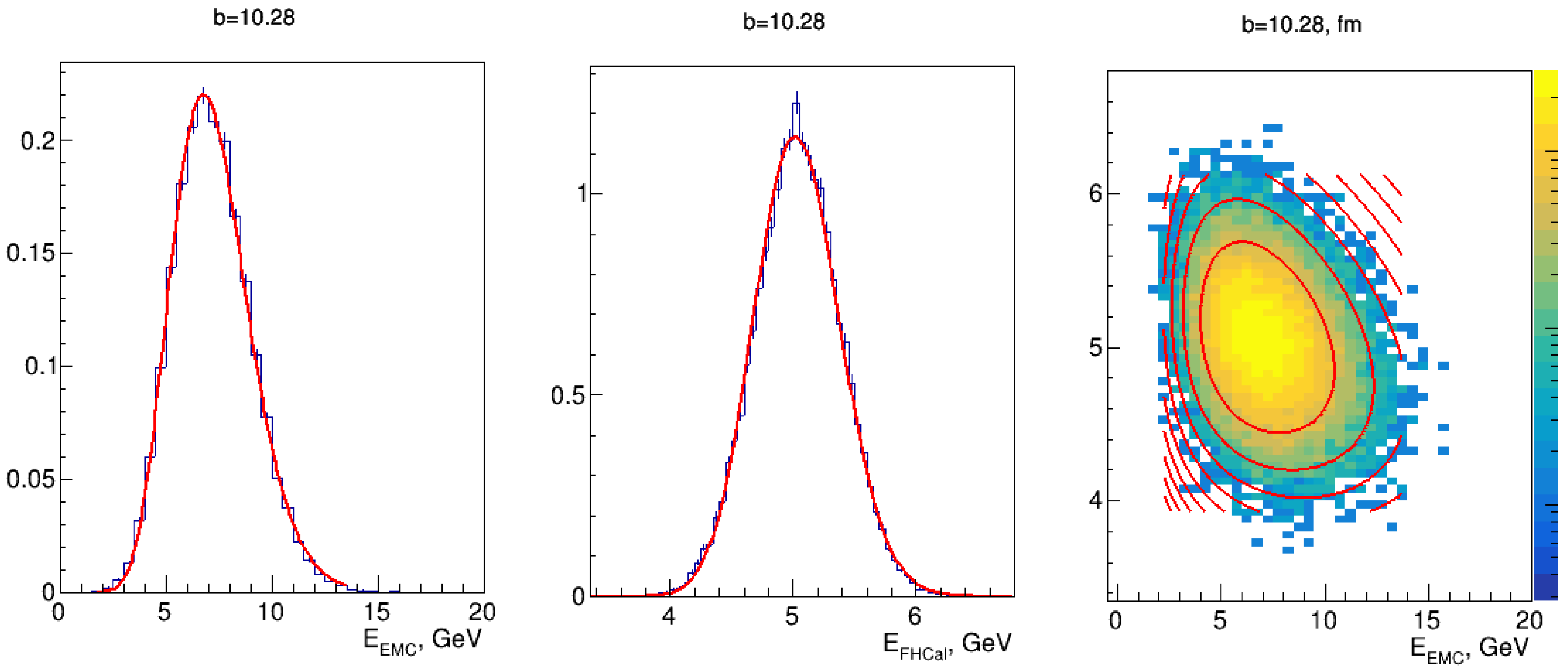
$$\alpha=\arctan\left(\frac{2\sqrt{D(E)D(E_F)}R(E,E_F)}{D(E)-D(E_F)}\right)$$

mean value and variance in the new coordinate system

$$\langle x\rangle=\cos(\alpha)\langle E\rangle+\sin(\alpha)\langle E_F\rangle \qquad D(x)=D(E)\cos(\alpha)^2+R\sqrt{D(E)D(\mathrm{E}_F)}\sin(2\alpha)+D(\mathrm{E}_F)\sin(\alpha)^2$$

$$\langle y\rangle=-\sin(\alpha)\langle E\rangle+\cos(\alpha)\langle E_F\rangle \qquad D(y)=D(E)\sin(\alpha)^2-R\sqrt{D(E)D(\mathrm{E}_F)}\sin(2\alpha)+D(\mathrm{E}_F\cos(\alpha)^2$$

The fluctuation of observables at fixed impact parameter

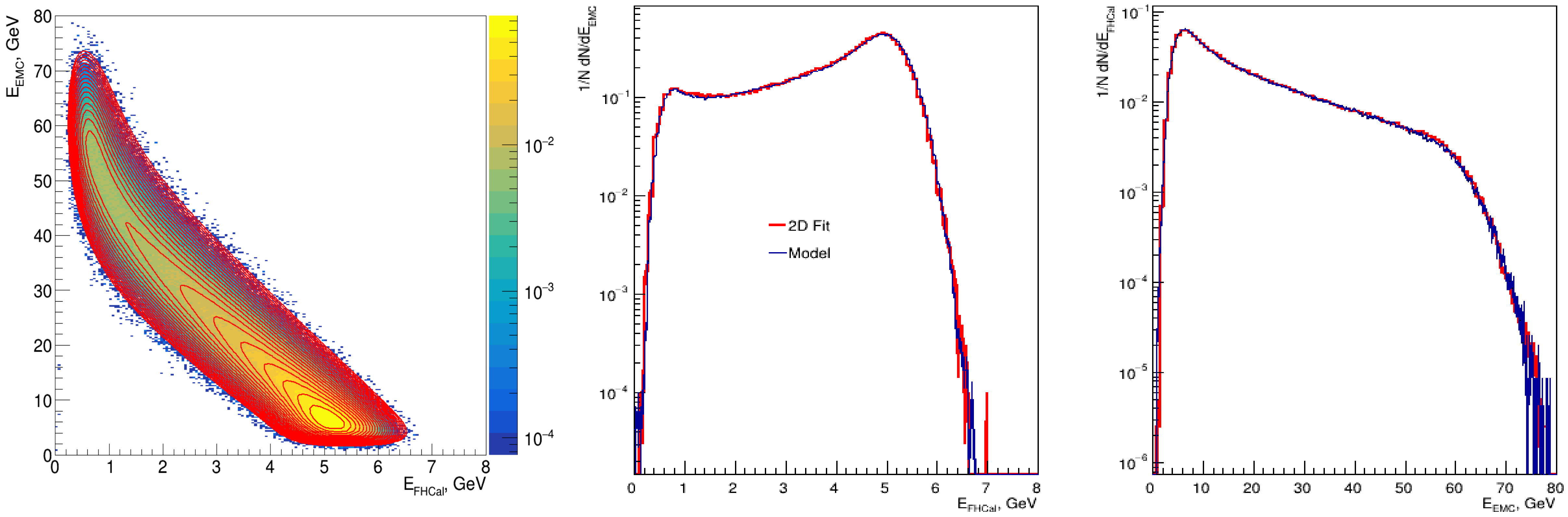


The distribution of observables at a fixed impact parameter is well described by the 2D gamma distribution

- Find probability of b for fixed range of observables using Bayes' theorem:

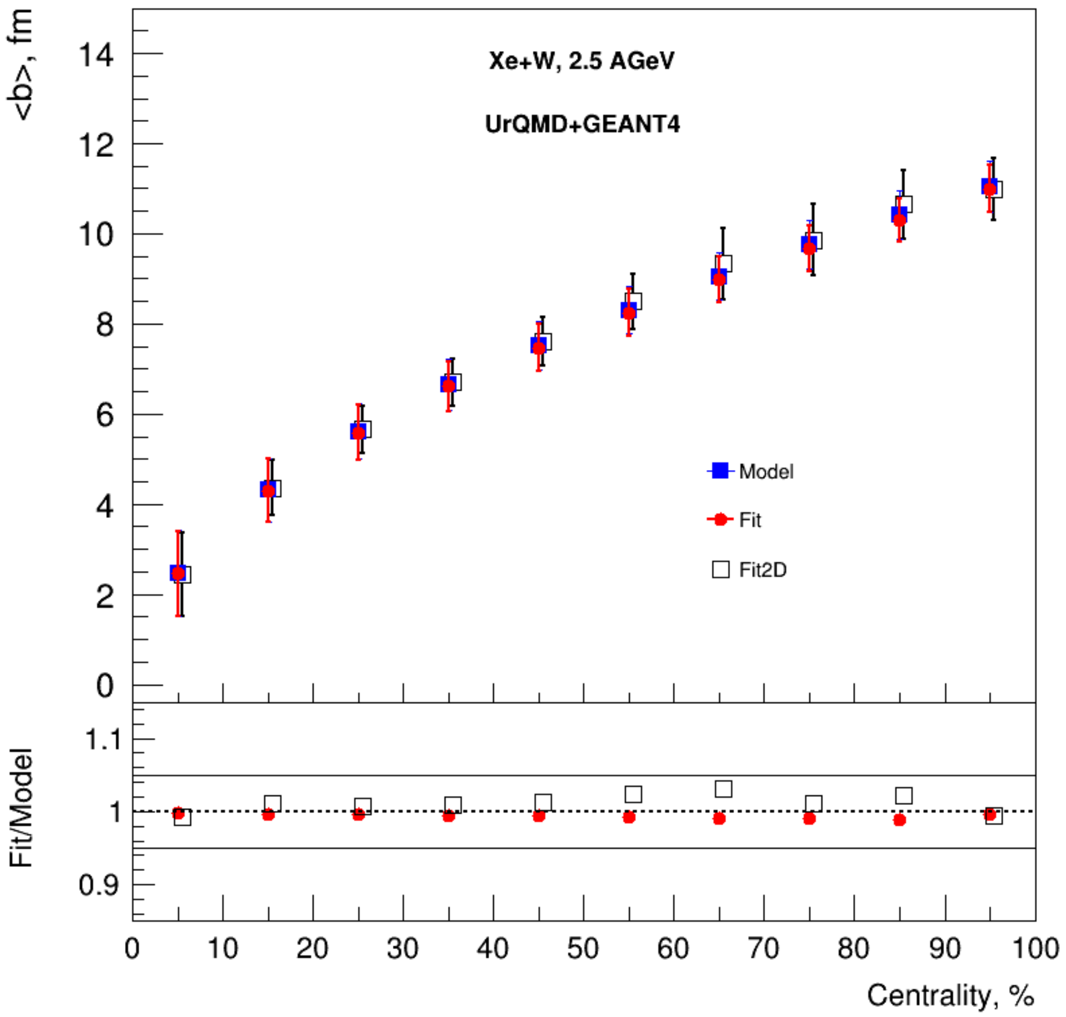
$$P(b \mid E_1 < E < E_2, \mathrm{E}_F^1 < E_F < E_F^2) = P(b) \frac{\int\limits_{E_1}^{E_2} \int\limits_{M_1}^{M_2} P(E, E_F \mid c_b) dE_F dE}{\int\limits_{E_1}^{E_2} \int\limits_{E_F^1}^{E_F^2} \int\limits_{0}^{1} P(E, E_F \mid c_b) dE_F dE dc_b}$$

2D Bayesian approach: results



Good agreement between fit and data.

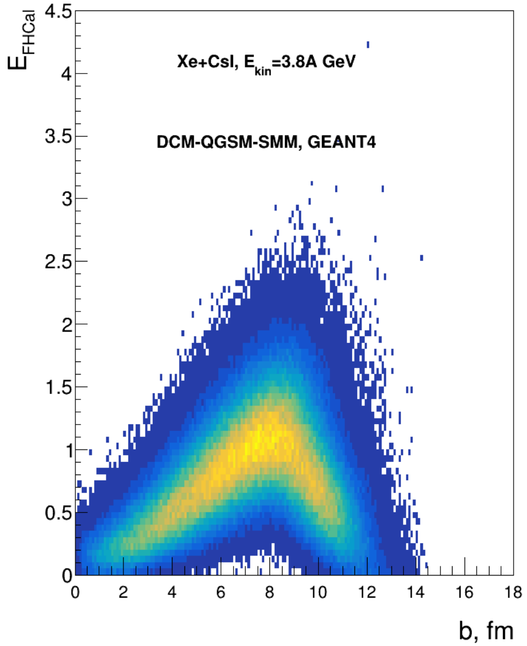
Comparison centrality determination methods

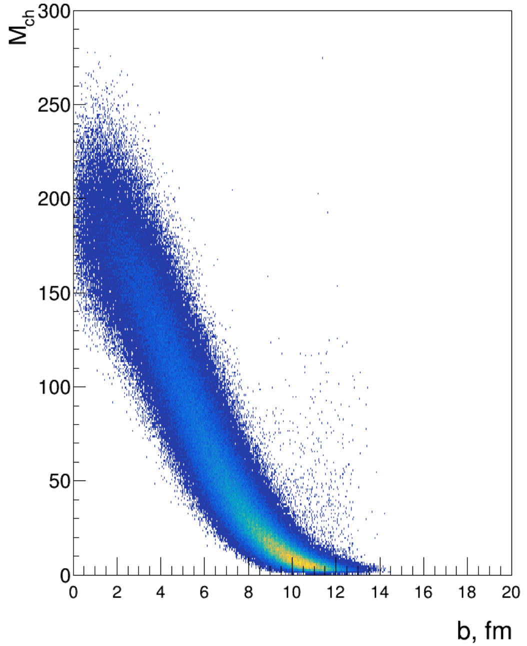


There is agreement within 5%.

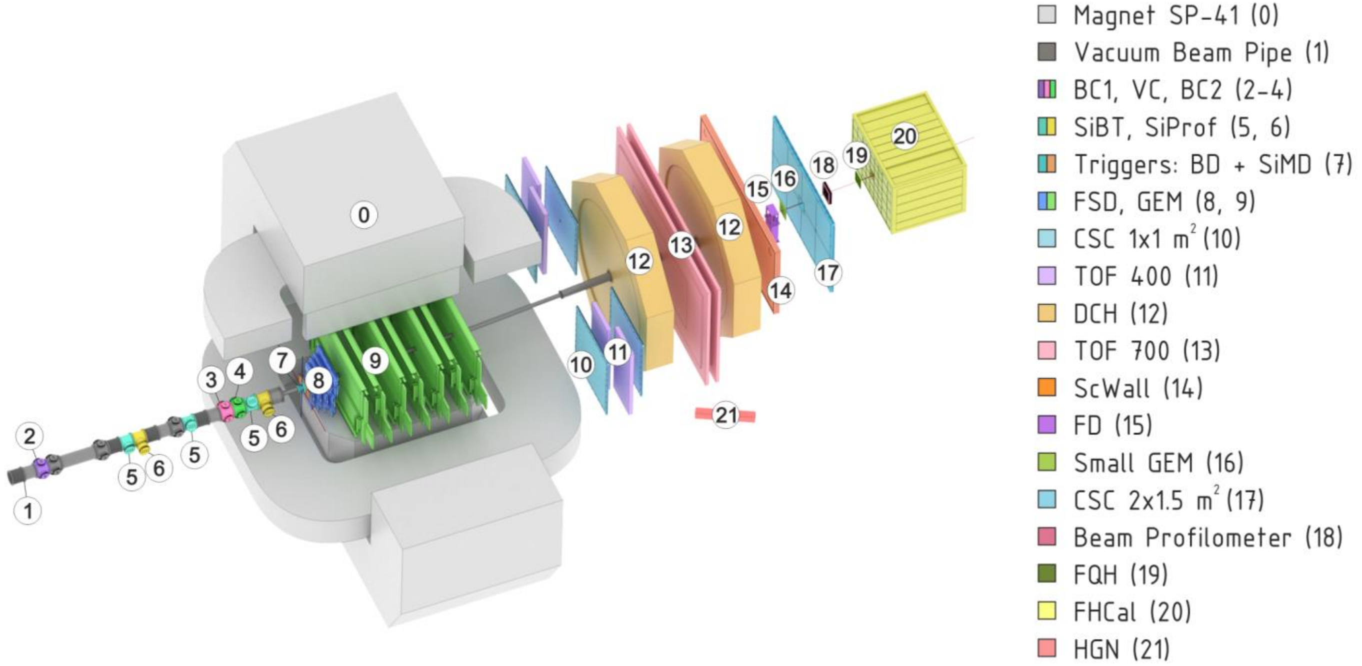
Centrality determination in BM@N

DCM-QGSM-SMM → GEANT4 → Reconstruction → Centrality determination



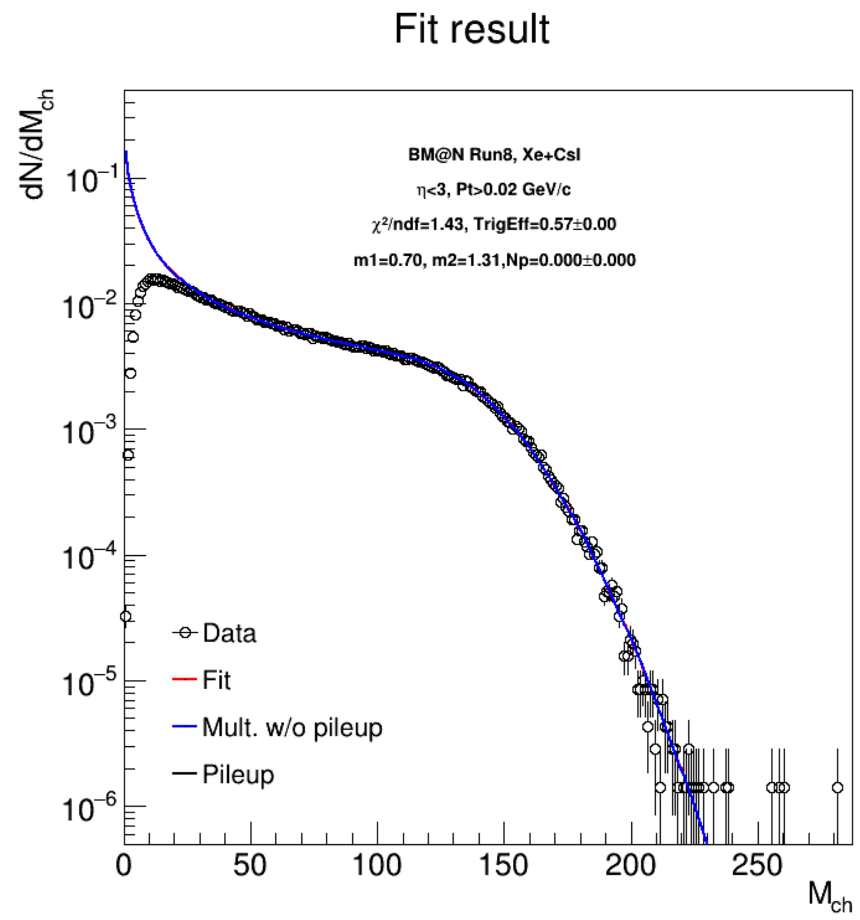


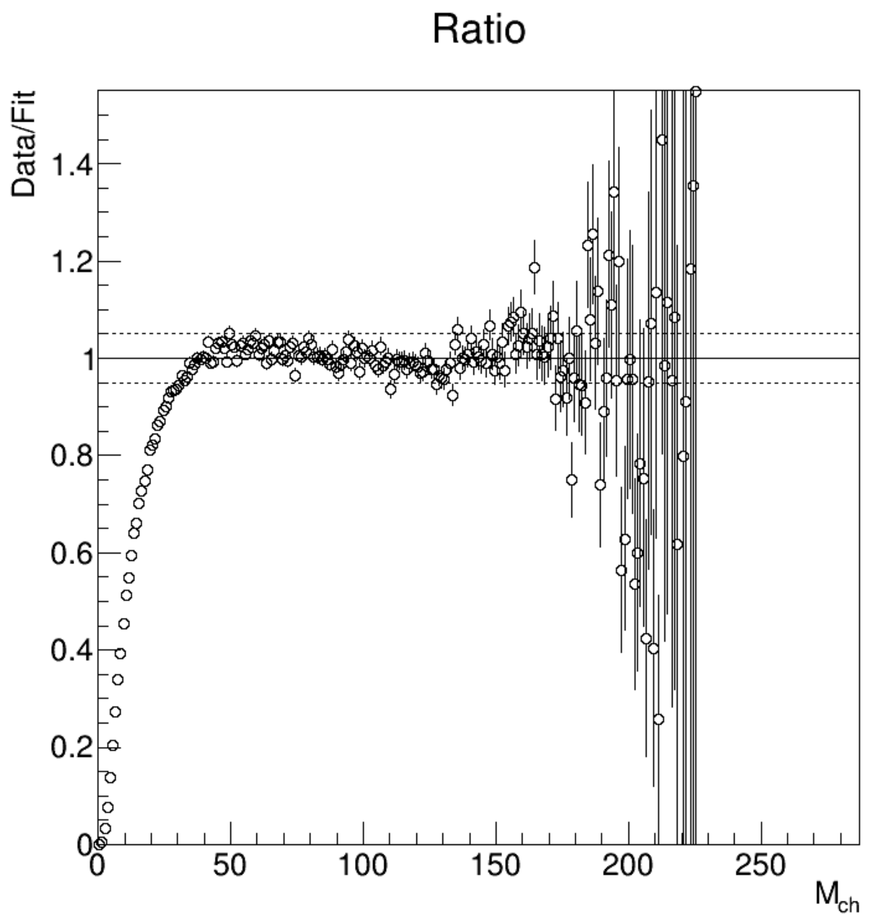
Dependence of energy in FHCal and track multiplicity
on the impact parameter



BM@N setup overview

Multiplicty-based Γ-fit: BM@N Xe+CsI



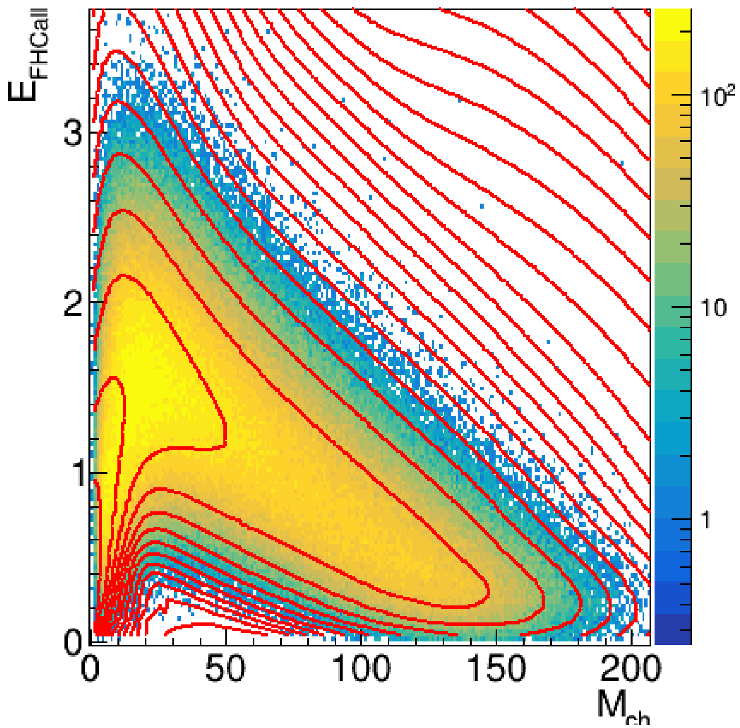


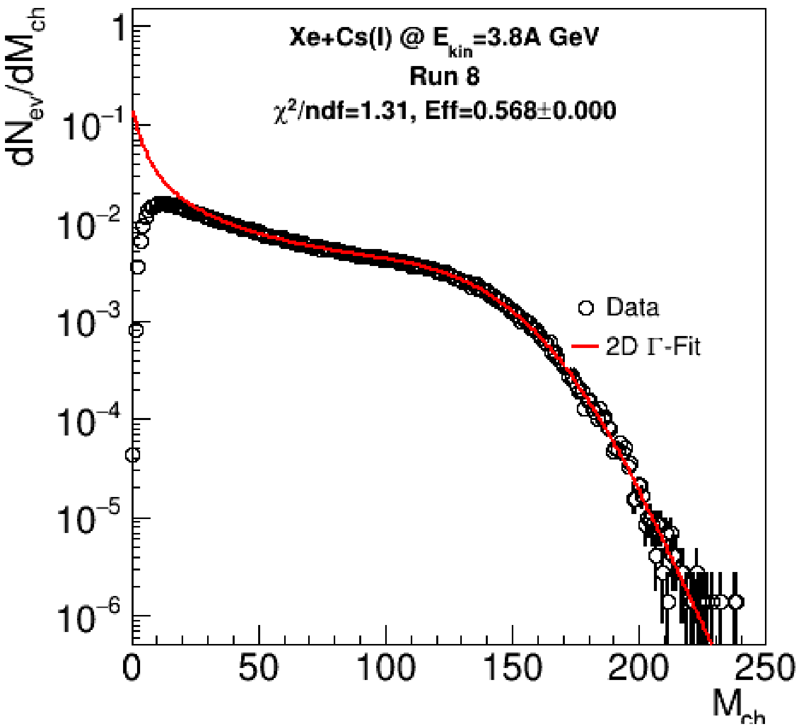
Vertex Cuts: CCT2, N_{vtxTr}>1,|$V_{x,y}$ –(0.3,0.14)|<1 cm, |V_z -0.07|< 0.2 cm

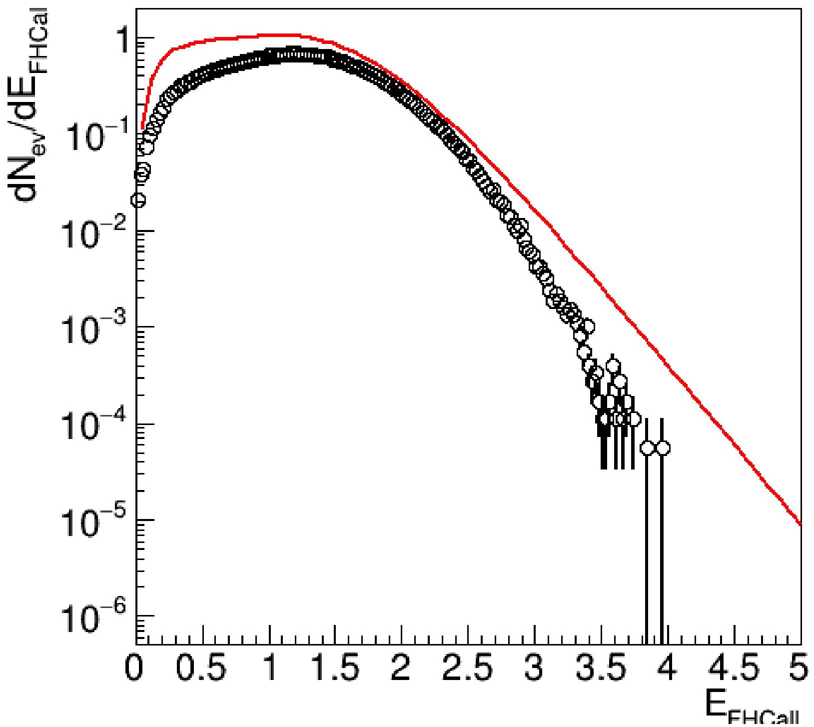
Good agreement with data

Track selection: Nhit>4, η<3, Pt>0.05 GeV/c

2D Bayesian approach: BM@N Xe+CsI

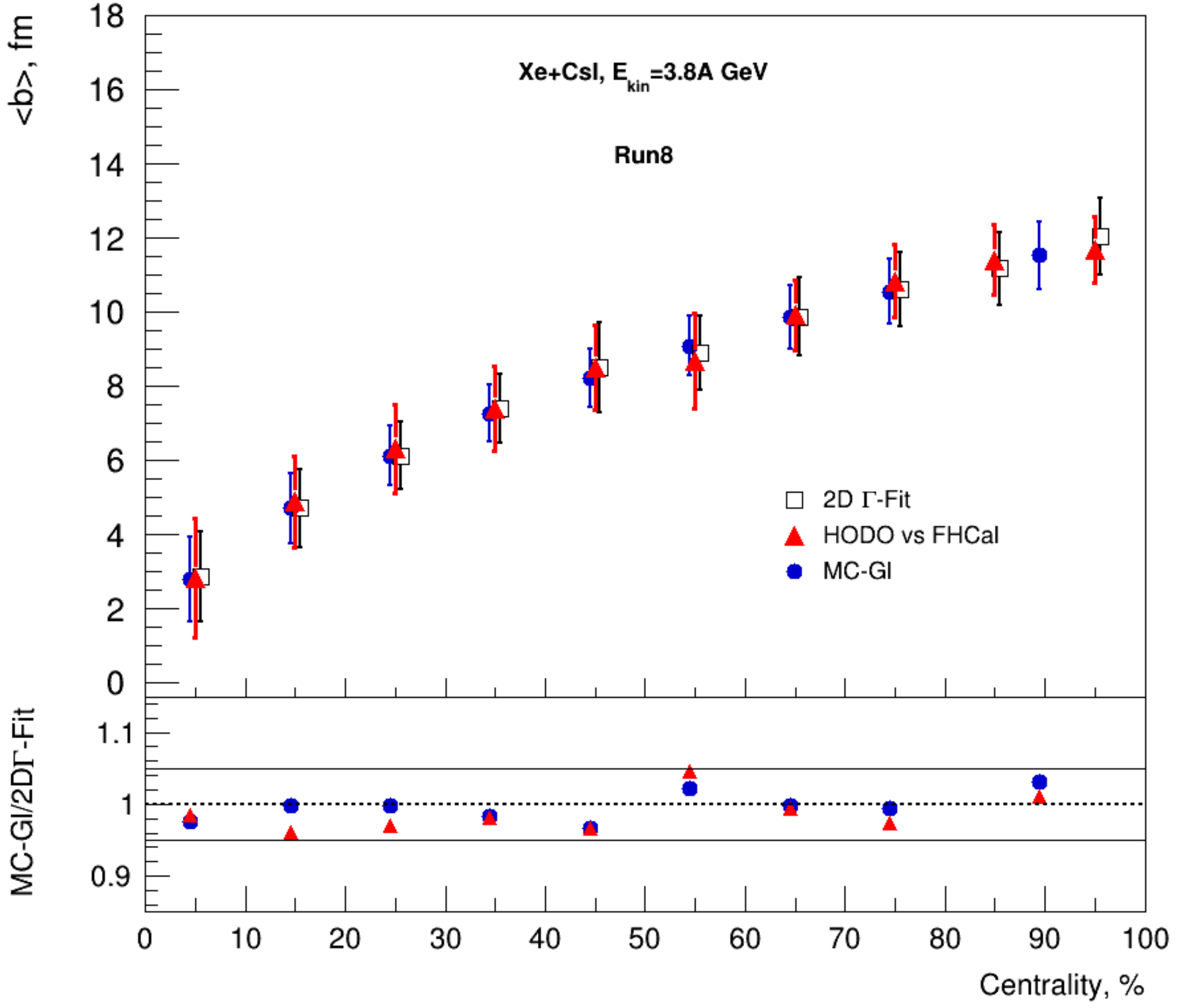


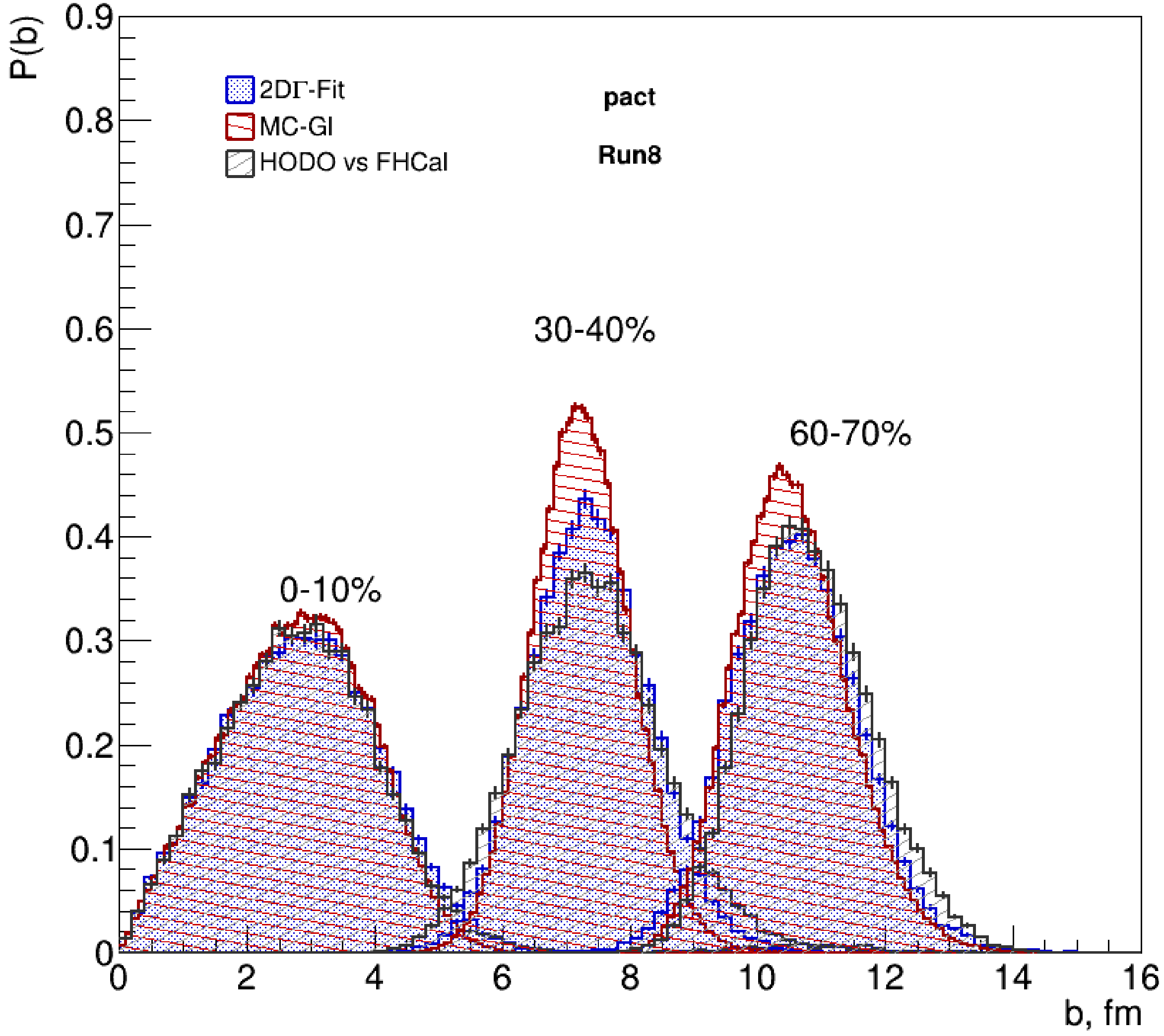




The fit function qualitatively reproduces the multiplicity-energy correlation from FHCal

Comparison with MC-Glauber fit : BM@N





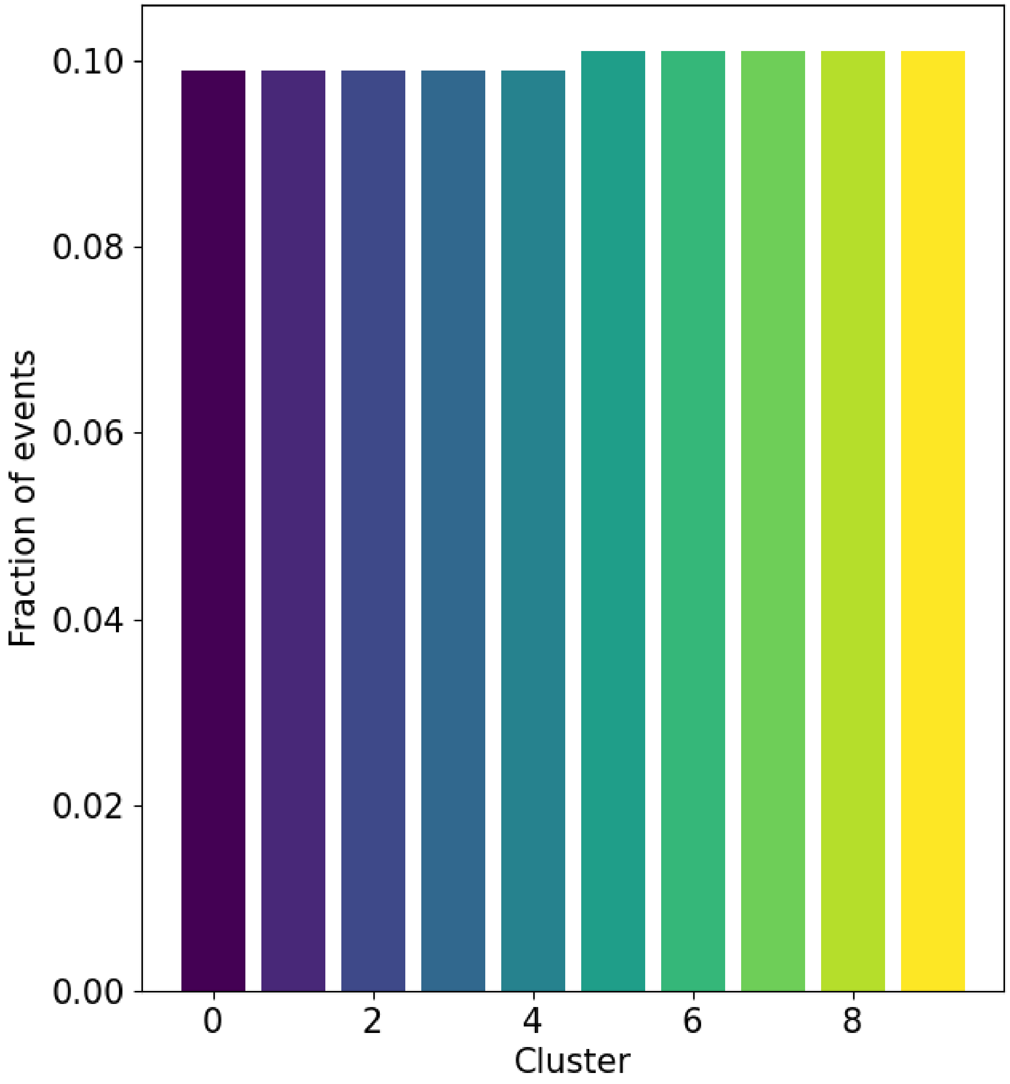
There is agreement within 5%.

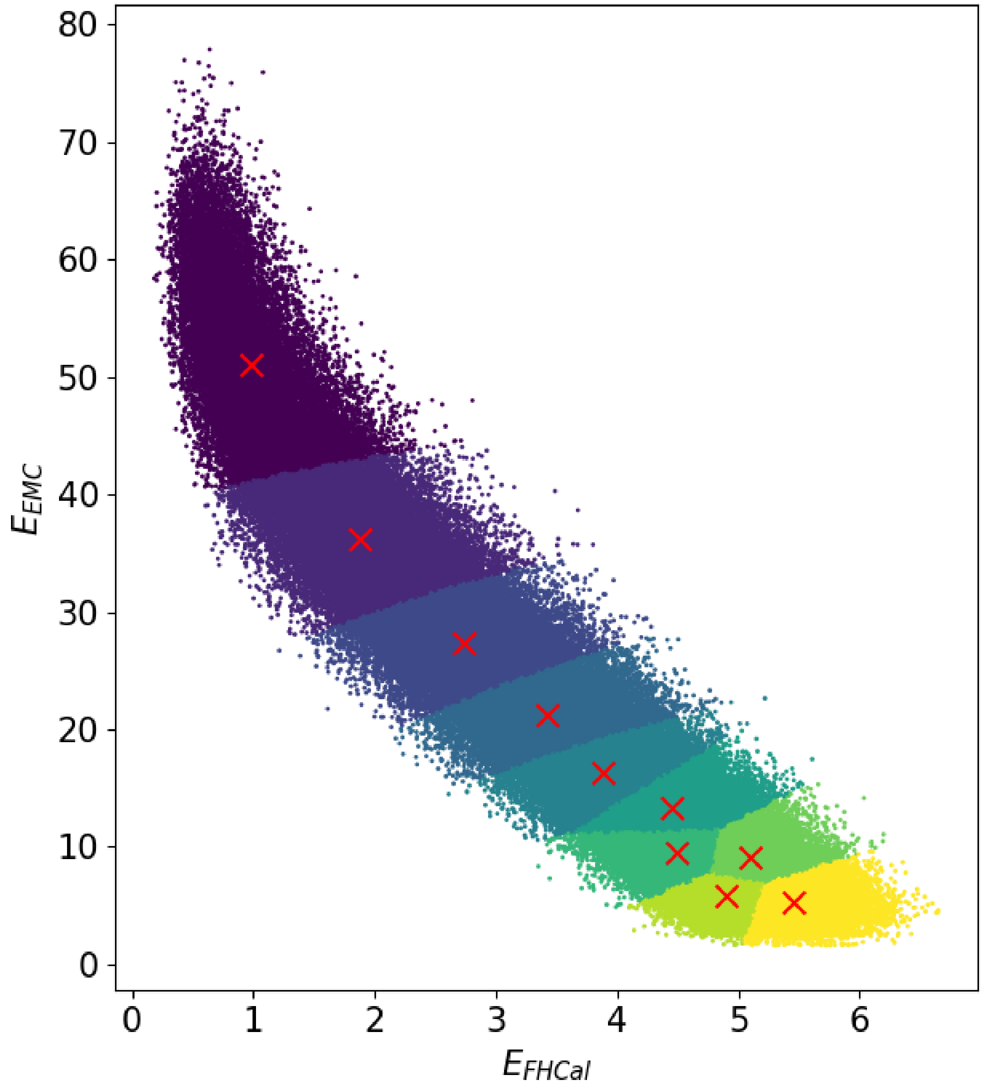
Summary and outlook

- The Bayesian inversion method reproduce observables for fixed-target mode at MPD:
    - Multiplicity-based and 2D approaches using E_{EMC} and E_{FHCal} show consistent results with model data
- The proposed method was applied to the data from BM@N experiment:
    - Multiplicity-based and 2D approaches using Q^2_{Hodo} and E_{FHCal} describe experimental data reasonably well
- To do:
    - Systematic study of different models with and without realistic fragmentation
    - Study of the spectator contribution in the centrality determination based on multiplicity

Thank you for your attention!

Centrality classes



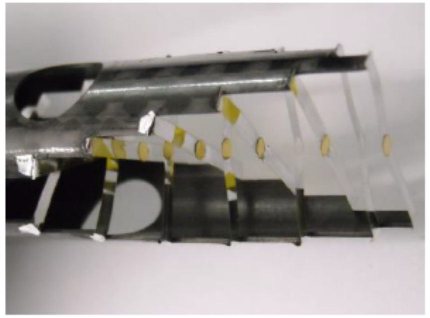


Centrality classes were obtained using the k-means algorithm

Event cleaning in HADES

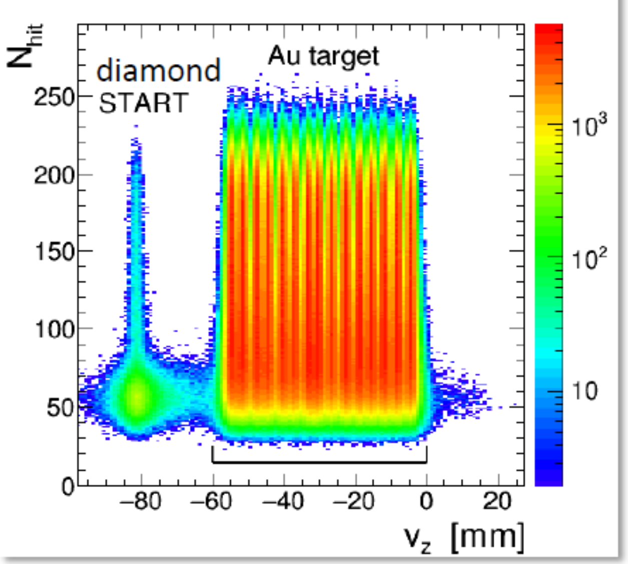
Segmented gold target:

- ^{197}Au material
- 15 discs of Ø = 2.2 mm mounted on kapton strips
- Δz = 3.6 mm
- 2.0% interaction prob.

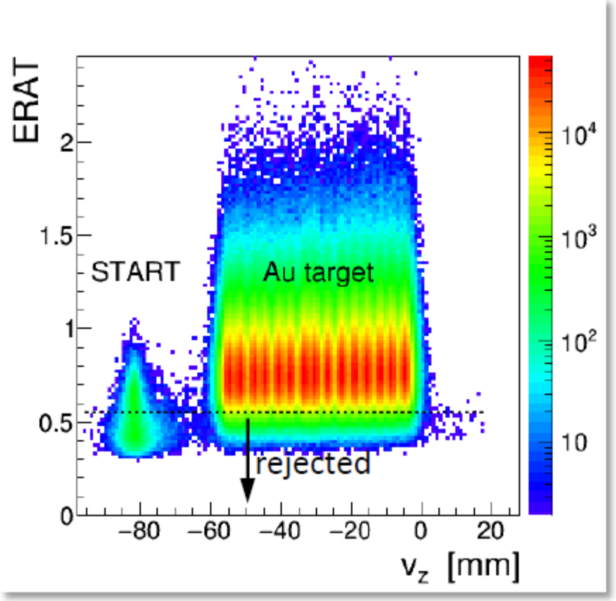


Kindler et al., NIM A 655 (2011) 95

Event vertex cut on target region



Remove Au+C bkgd on the kapton with a cut on $ERAT = \sum E_t / \sum E_l$



beam direction

http://indico.oris.mephi.ru/event/221/session/1/contribution/1/material/slides/0.pdf

Reconstruction of *b*

- Normalized multiplicity distribution P(N_{ch})

$$P(N_{ch}) = \int_0^1 P(N_{ch}|c_b)dc_b$$

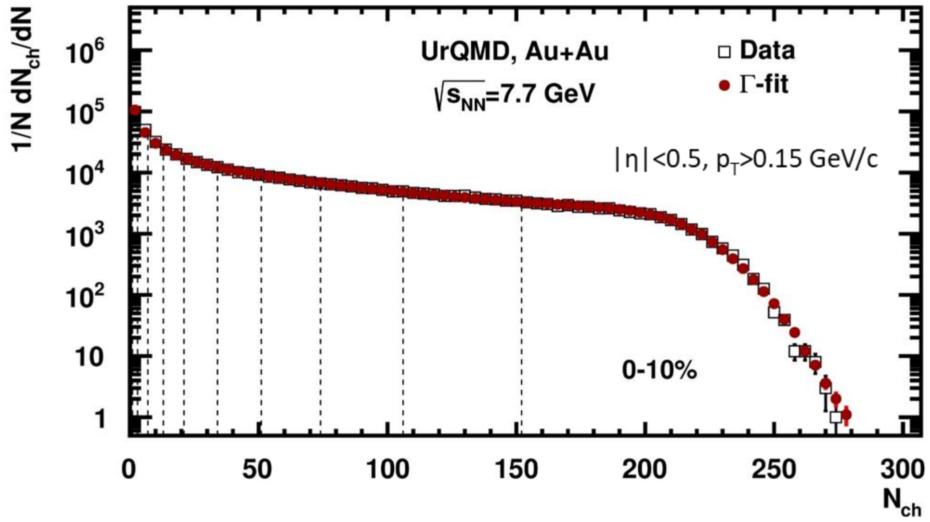
- Find probability of *b* for fixed range of N_{ch} using Bayes' theorem:

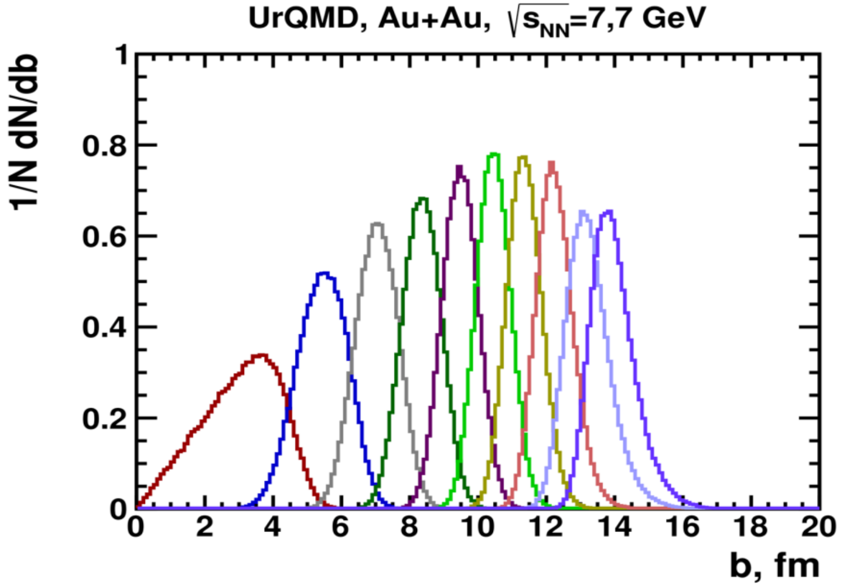
$$P(b|n_1 < N_{ch} < n_2) = P(b)\frac{\int_{n_1}^{n_2} P\,(N_{ch}|b)dN_{ch}}{\int_{n_1}^{n_2} P\,(N_{ch})dN_{ch}}$$

- **The Bayesian inversion method consists of 2 steps:**

–Fit normalized multiplicity distribution with P(N_{ch})

–Construct P(*b*|N_{ch}) using Bayes' theorem with parameters from the fit





R. Rogly, G. Giacalone and J. Y. Ollitrault, Phys.Rev. C98 (2018) no.2, 024902

Implementation in MPD: https://github.com/Dim23/GammaFit