Phase diagram of nuclear matter.
Collective effects and phase transitions in nuclear systems

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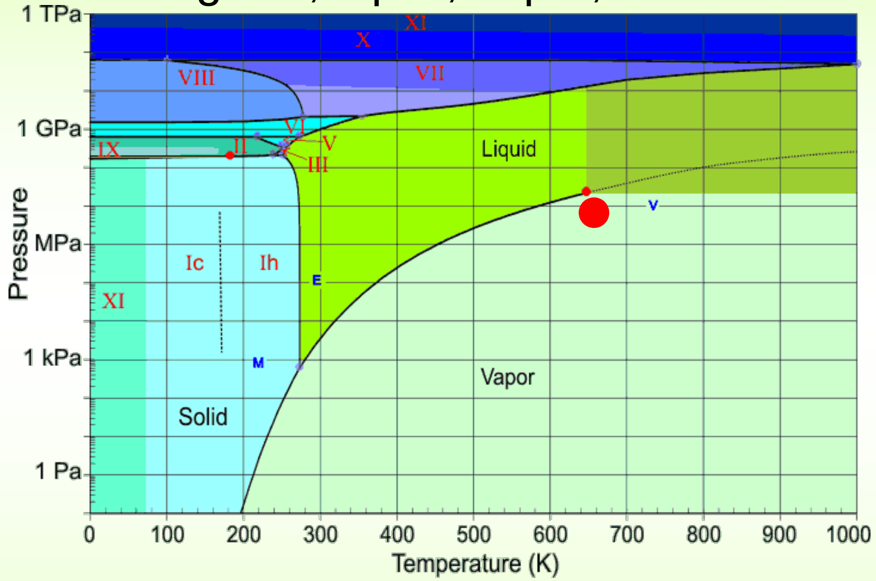
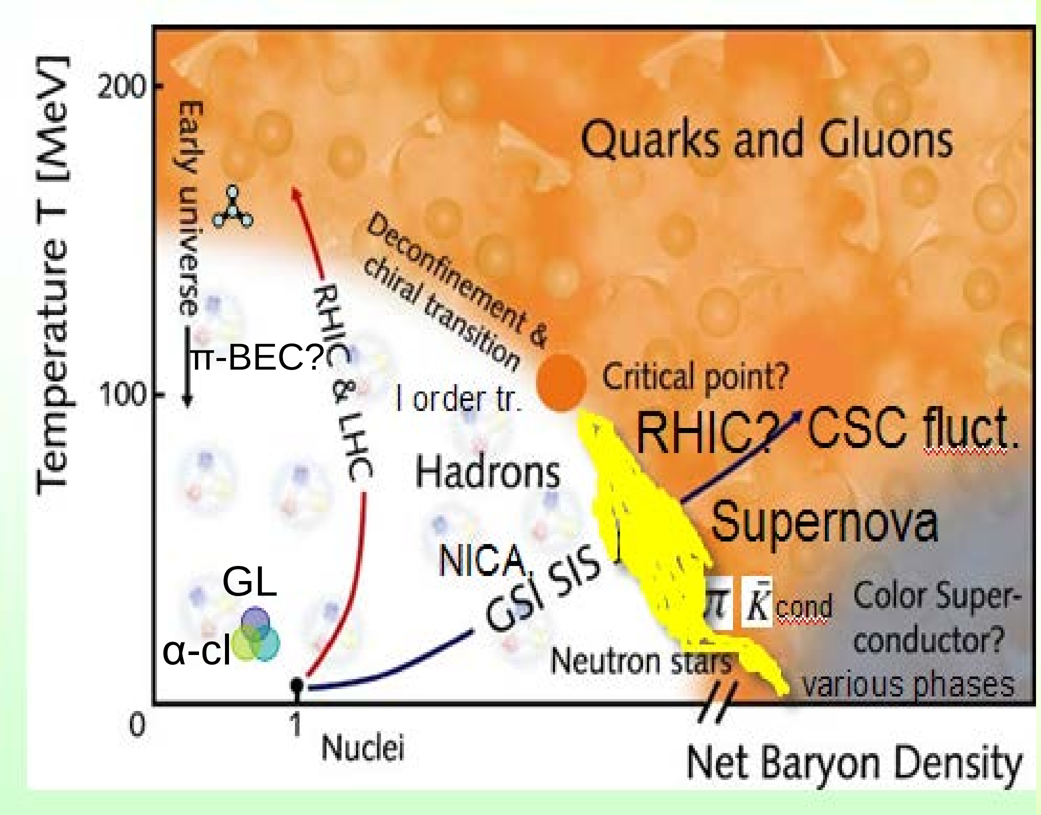
Phase Diagrams
Water and Nuclear Matter

various possible phases

Variety of phases: 12 crystalline, 3 glass, liquid, vapor, CEP

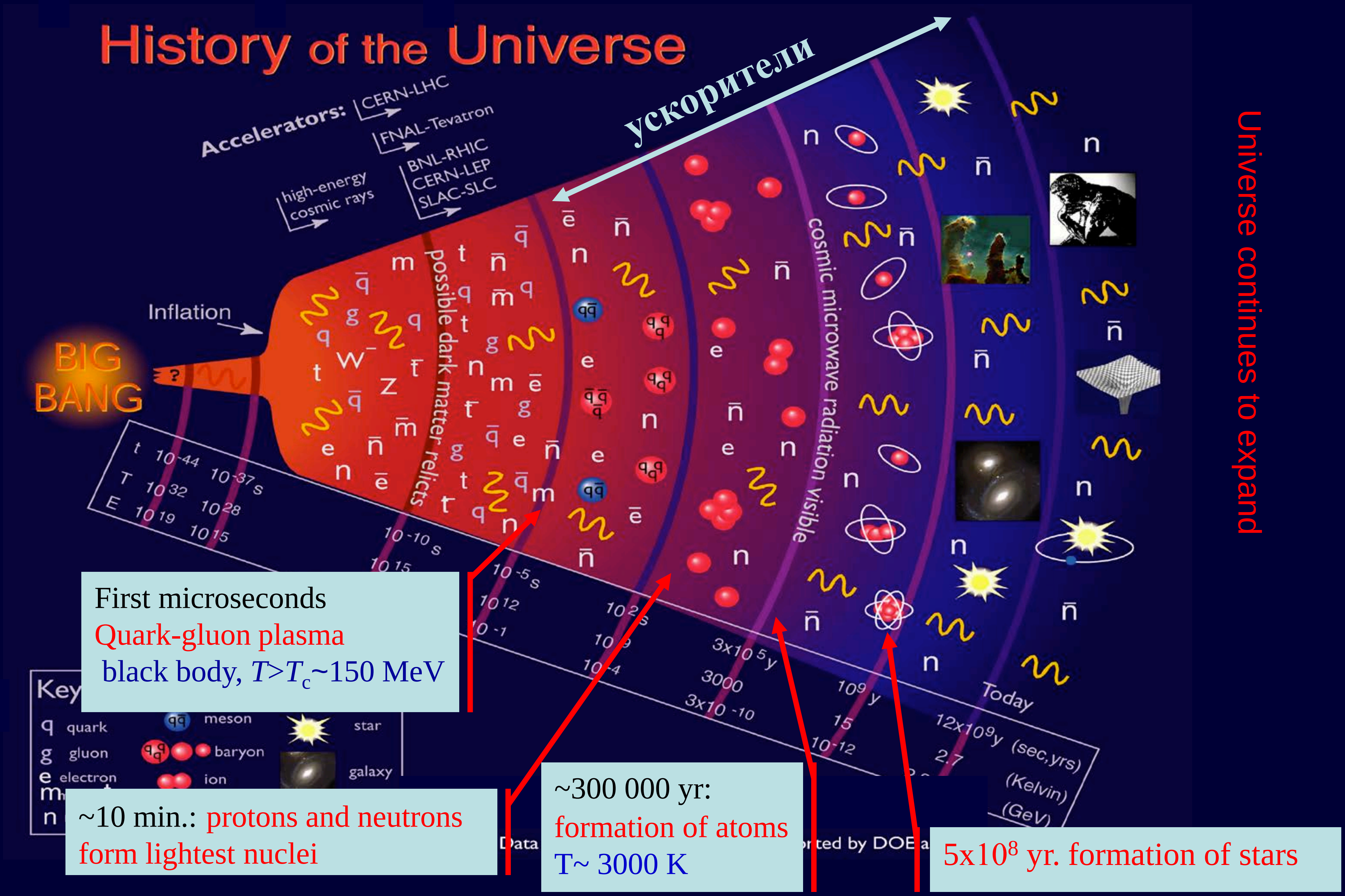


Chapline et al. (2007)



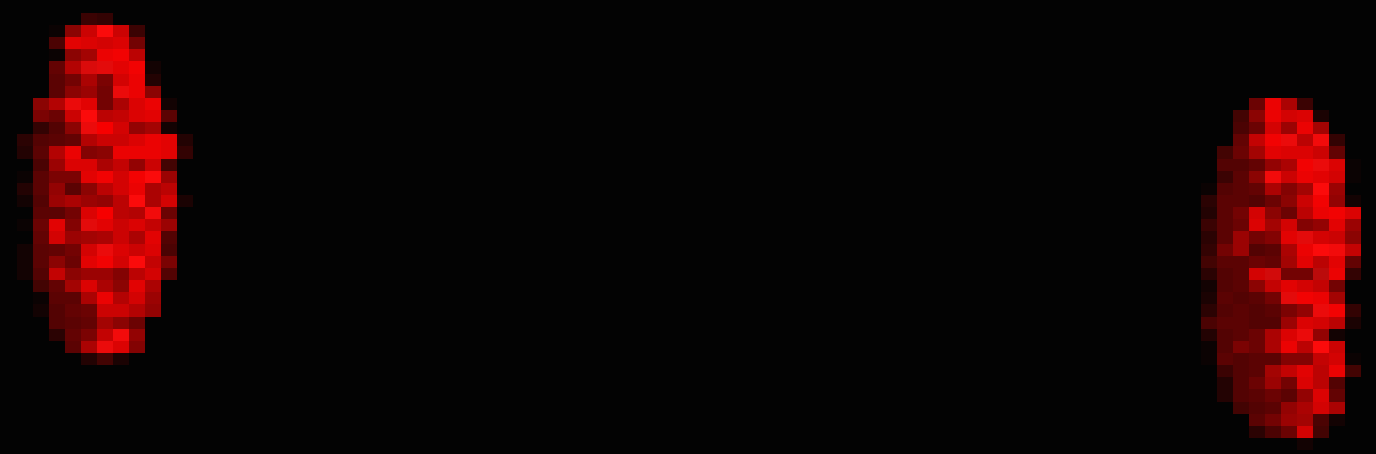
High T (T_c>150MeV), low baryon density $n < n_0 \approx 2.7 \cdot 10^{14}$ g/cm^3: deconfinement ph. tr.- crossover in *early Universe, in HIC at RHIC, LHC. With* growing n, decreasing *T*–CEP I order phase tr.- **challenge for NICA to seek it,** mixed QH pasta phase; Low T, low n: *HIC:* α-cl, nuclear GL ph.tr.; Low T, $n \approx n_0$: *nuclei:* NN pairing; *π/K atoms & hyper-nuclei*, Low T, high density*: Star mergers*,***SN,NS*:** nuclear pasta, BB-pairing, π,K,ρ^- condensates; *hybrid stars:* CSC

My aim is passing through Phase Diagram show connection between different phenomena in nuclear systems



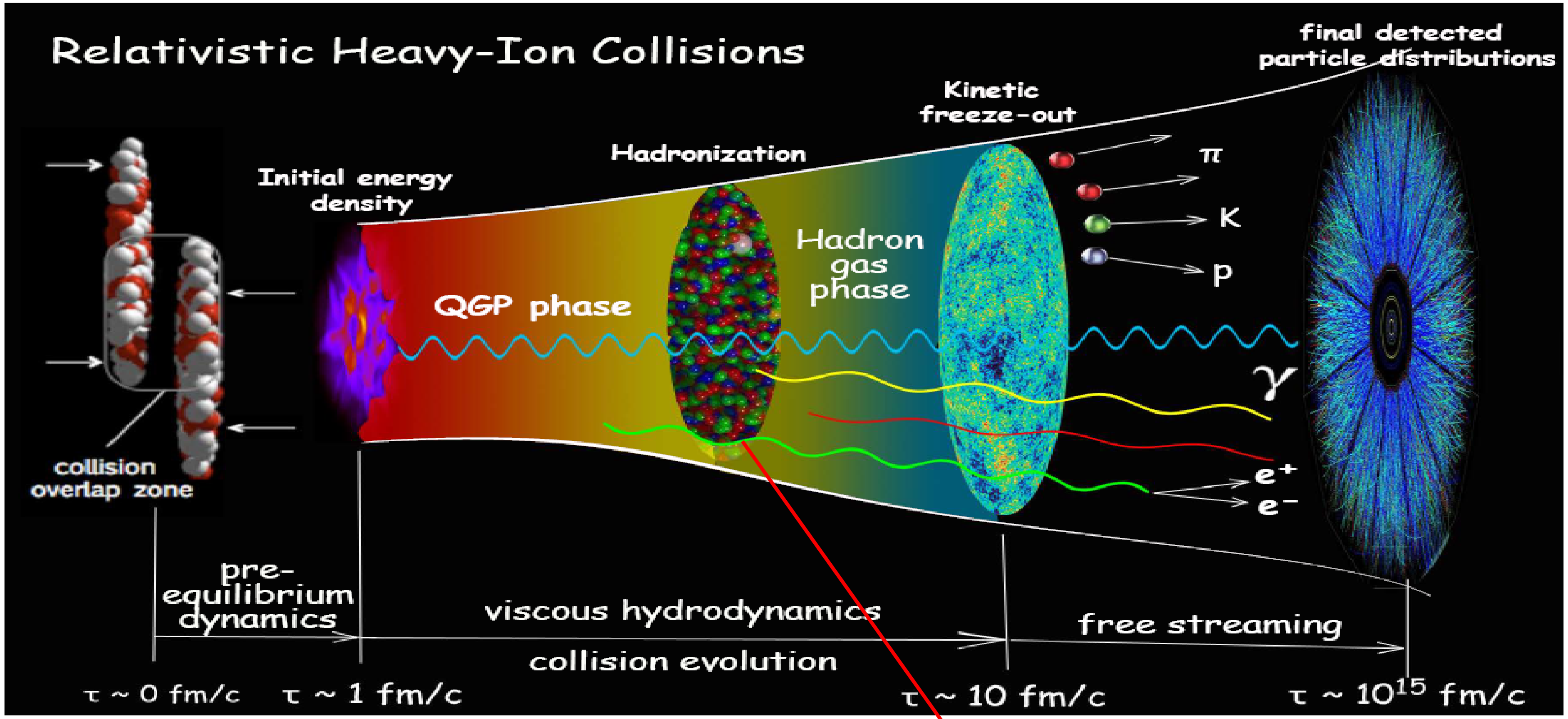
Universe continues to expand

Collision of Galaxies



Collision of U+U at 23 AGeV
(UrQMD transport code , Frankfurt Univ.)

Evolution of fireball created in RHIC



From Taranenko talk

"Tomography of fireball": *n(T(t)),* various particle species measure different stages of nuclear fireball

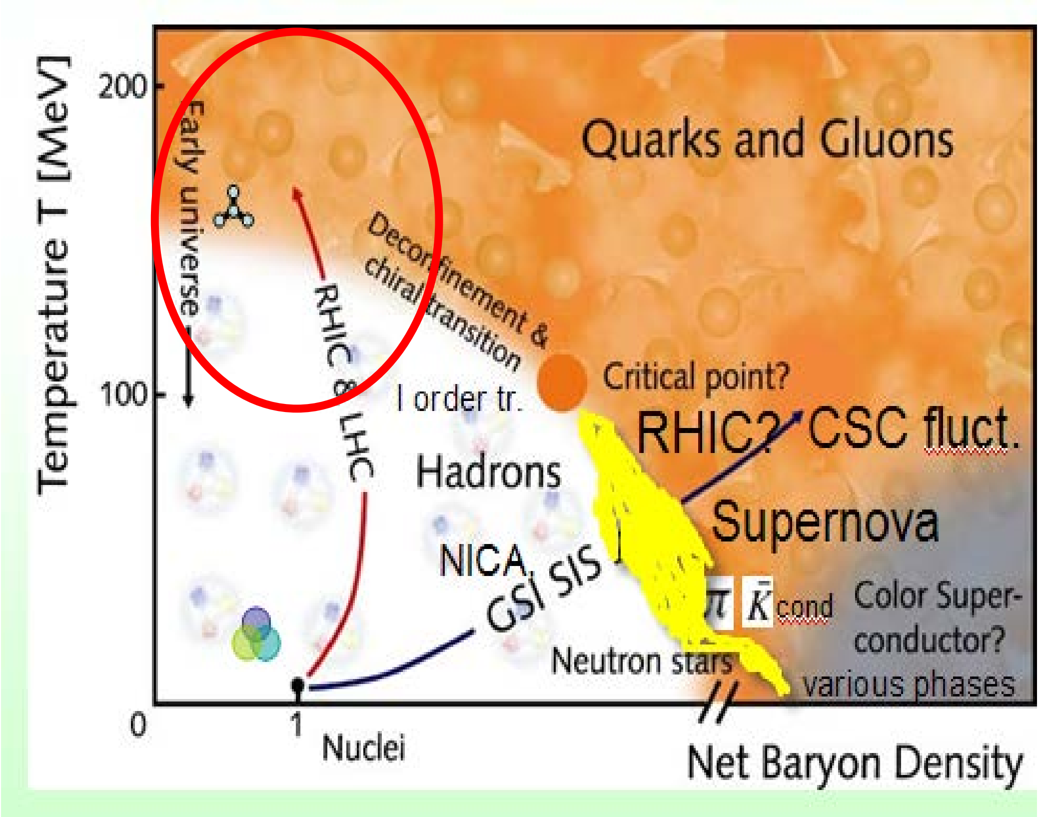
Long path-length particles (ee$^+$, μμ$^+$, photons) measure initial stage,

K$^+$, π with k<1.5 m_π fly from intermediate stage,

Short path-length particles (π with k>1.5 m_π, N, K$^-$) measure late "freeze out" stage

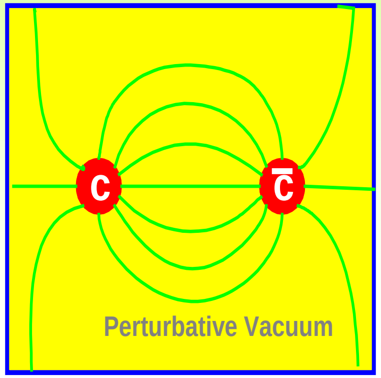
High *T*, rather low baryon density (baryon chemical potential)

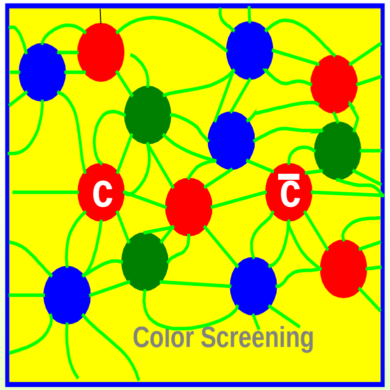
Hot QGP in early Universe and at LHC, crossover transition to hadron hot liquid

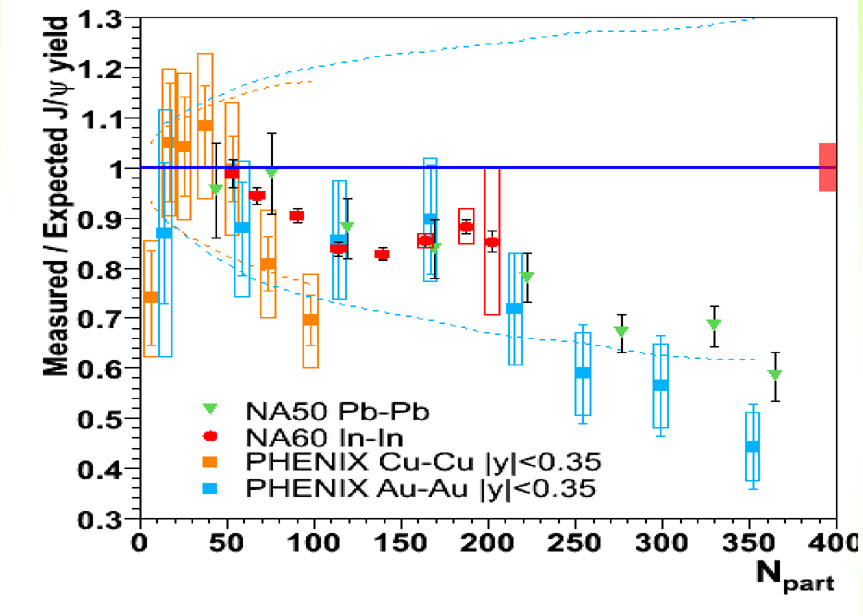


Why we say about QG state in ultrarelativistic HIC?

J/ψ suppression

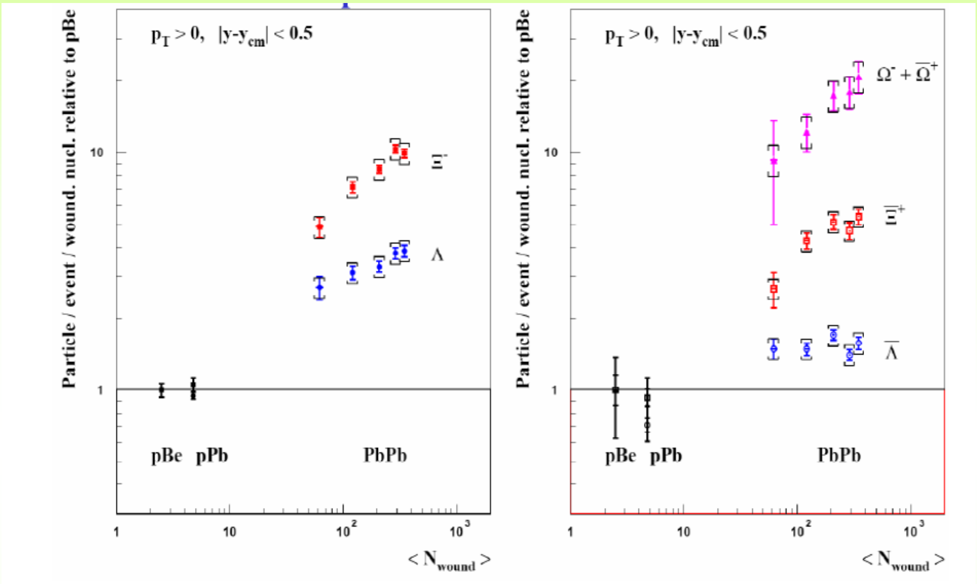






Dissociation of bounded cĉ pair in QGP

Strangeness enhancement

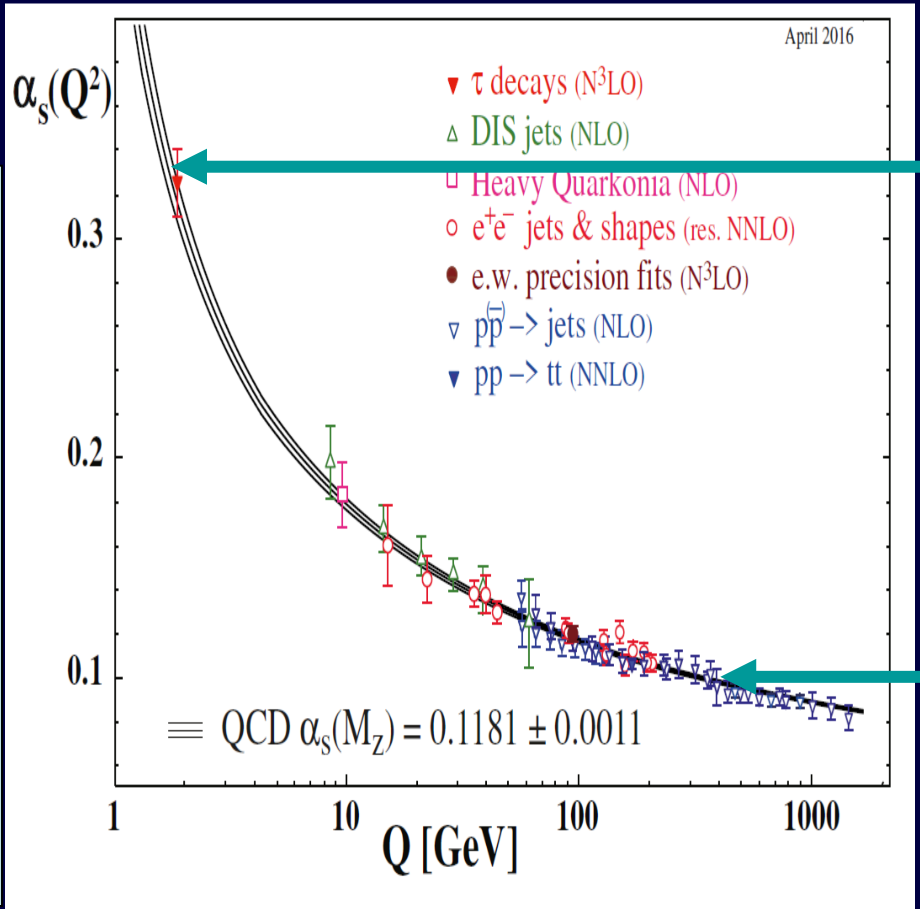


when $T > m_s$

Interaction of energetic parton with quark medium: yield of particles at large $p_\perp$ is suppressed

Strong interaction running coupling constant

Strong coupling const



energy →

← distance

confinement

At large distance;

Quantum number – ***color.***

Asymptotic freedom

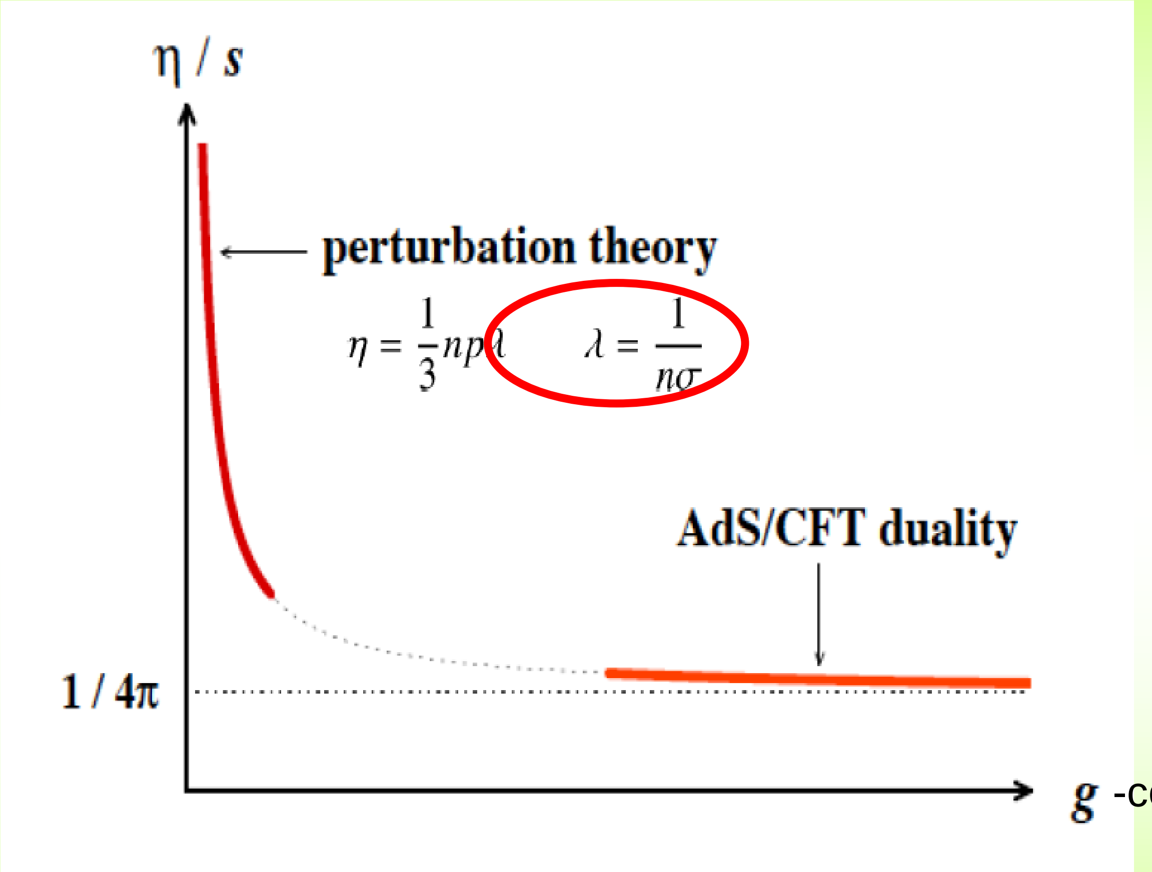
at small distance

(r<<L~0.2 fm, g →0)

Free quarks of small mass

Tanabashi et al 2018

Ratio of shear viscosity to entropy density



η/s [ADS/CFT]=$1/4\pi$ absolute minimum

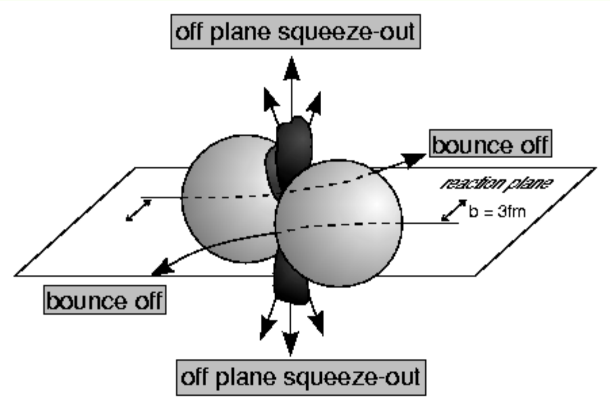
maps QGP to black hole formation in ADS_5 space, cf. arXiv 2211.16265

-coupling constant

pQGP or sQGP? Crossover region: nonideal hydro

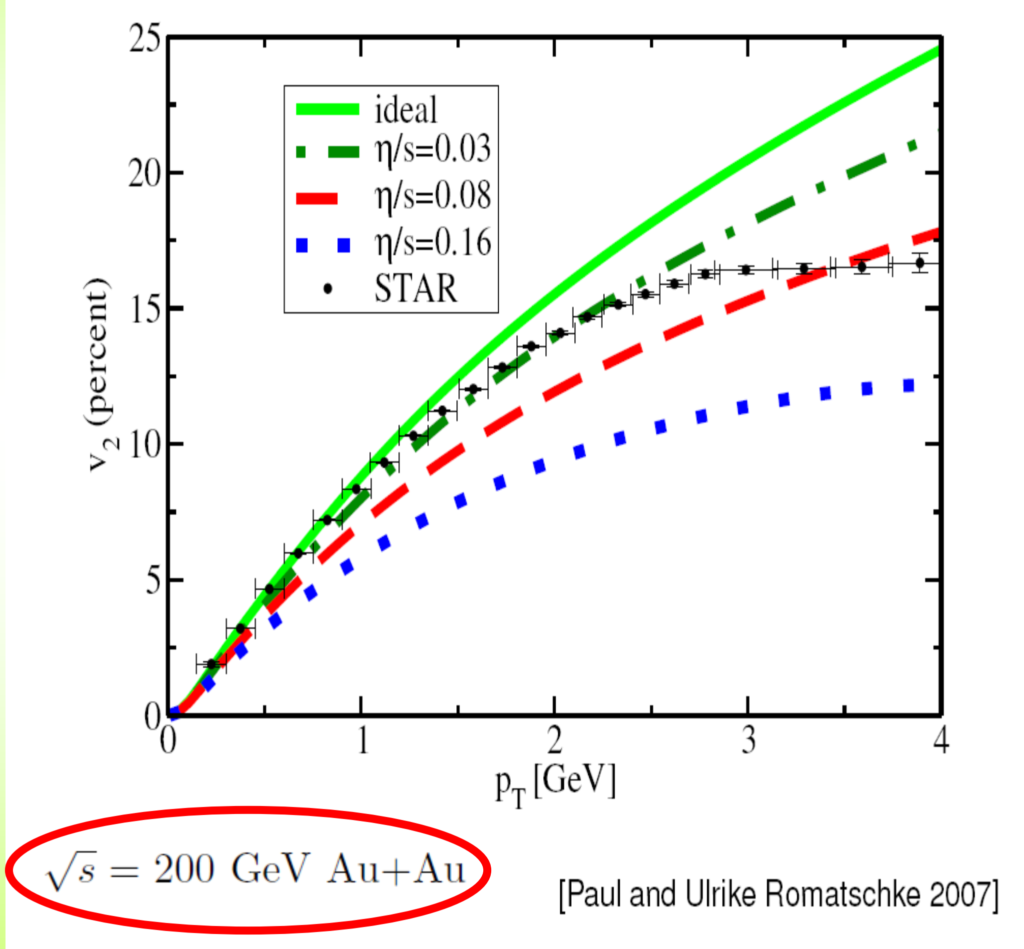
η/s – ratio of shear viscosity to entropy density

Matter is smashed in perp. direction
→ collective nucleon flow

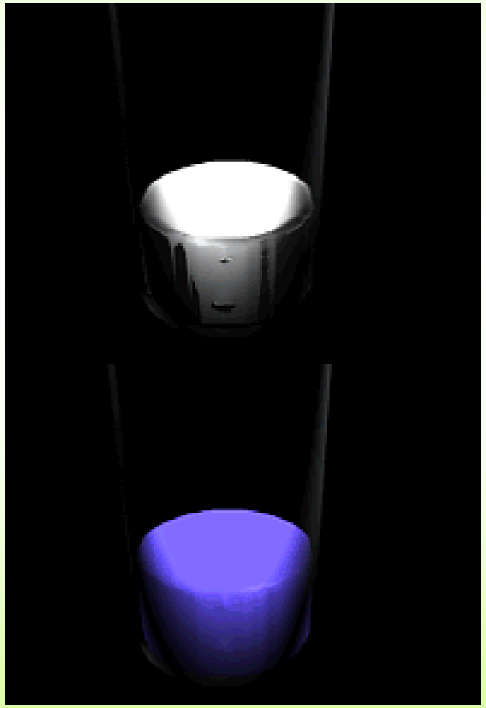


From Taranenko talk

coefficients v_n quantify anisotropic flow



Role of viscosity

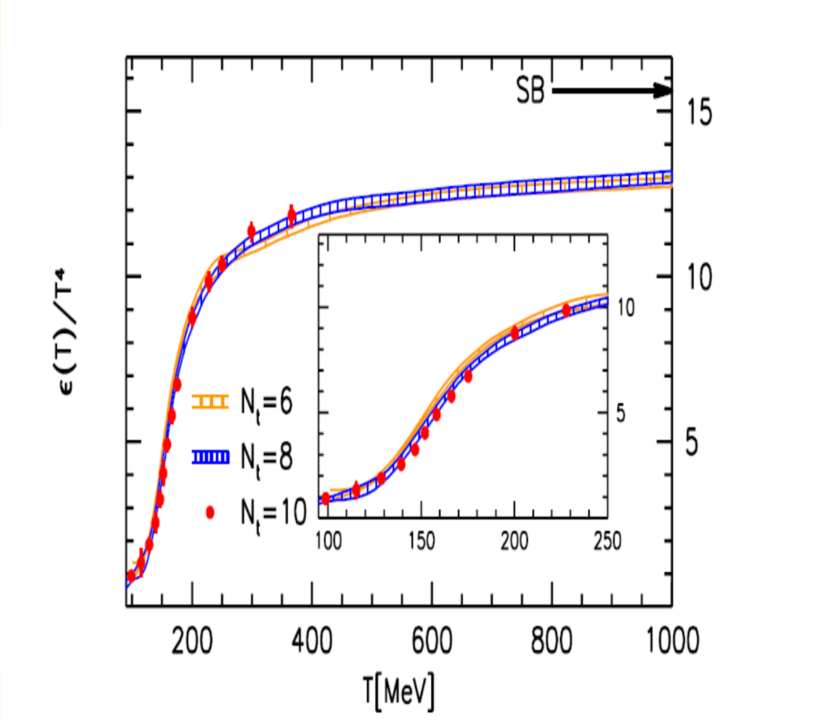


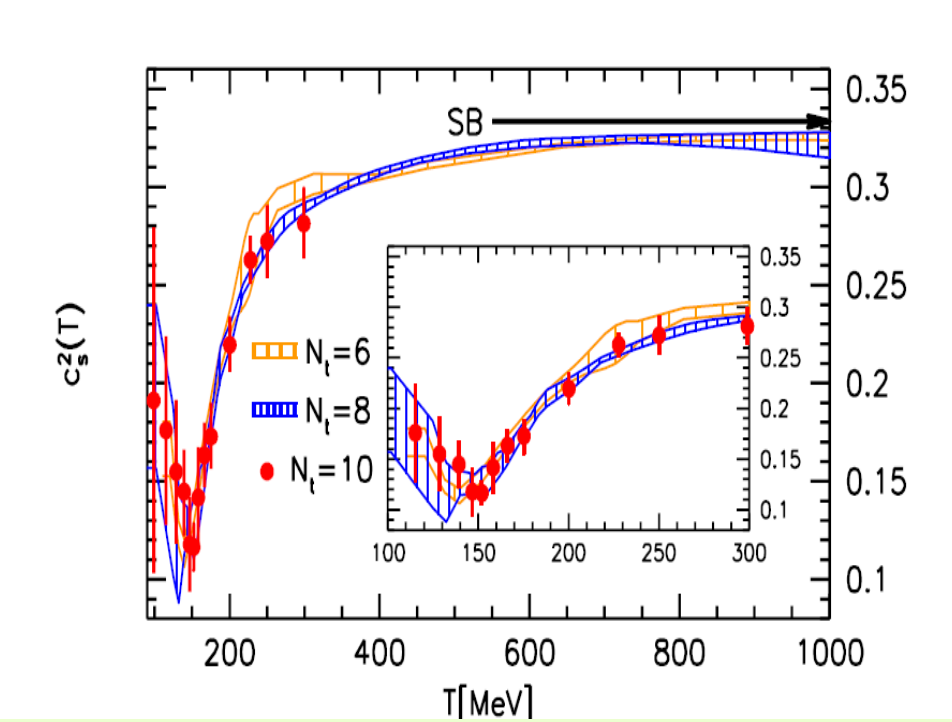
A small viscosity is required ! → **Almost ideal QGP liquid**

QCD on lattice

(numerical experiment on computers)

μ_B=0 - crossover

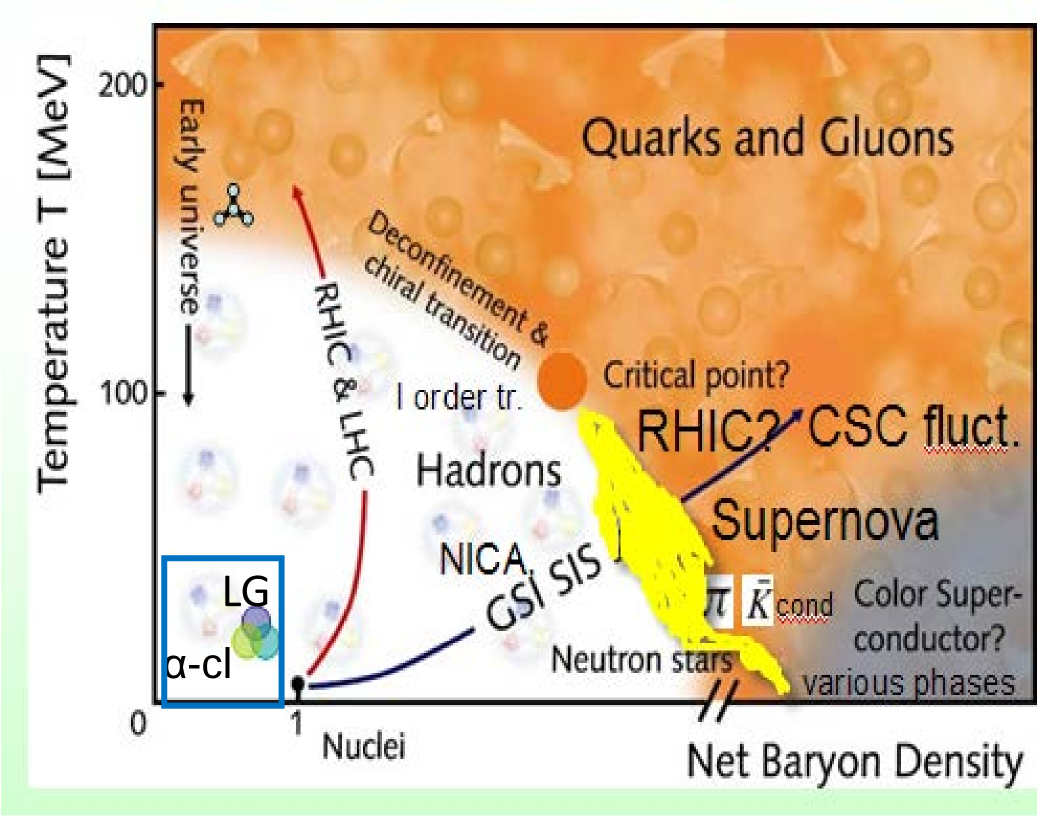




Black-body ideal gas limit is not reached ⇒ **strongly interacting QGP– liquid, not gas!**

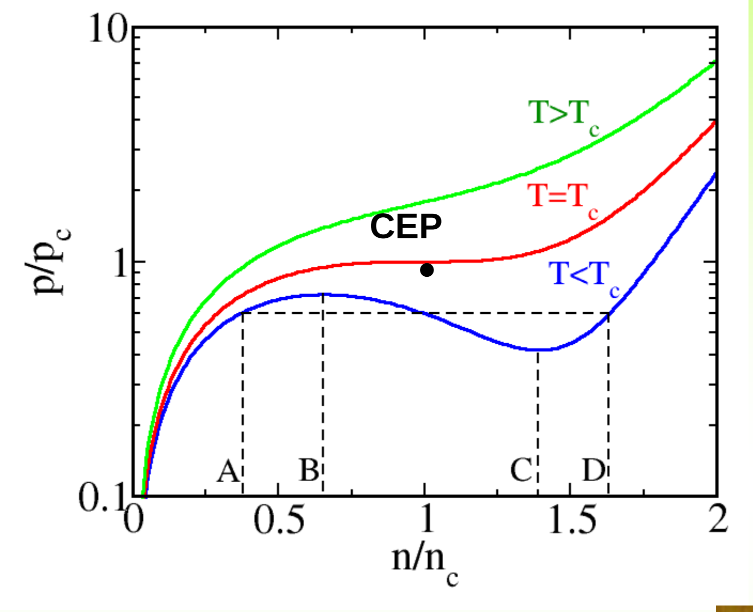
State of pQGP is expected only for n_B>$40n_0$ or for T> 3 T_c *not reachable even at LHC*

Low T, rather low baryon density (large baryon chemical potential)

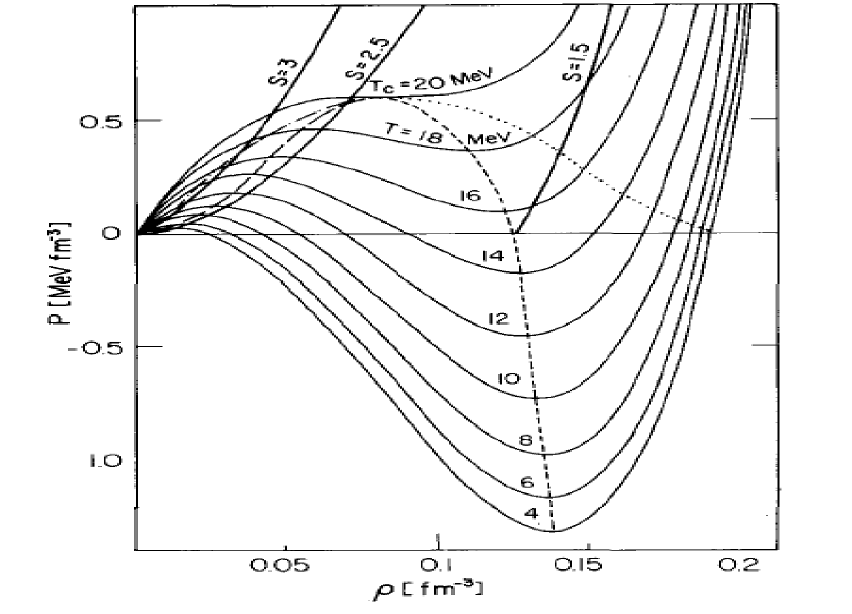


Gas (nucleons)-Liquid (nuclear clusters) I-order phase transition

EoSs for $n<n_0$ show van der Waals-like shape (expanding fireball :*S(t)*≈const, *T, n* decreasing till freeze out)



N=Z



In spinodal region

$$K = n\frac{d^2E}{dn^2} = \frac{2}{3}\epsilon_F(1+f_0) < 0$$

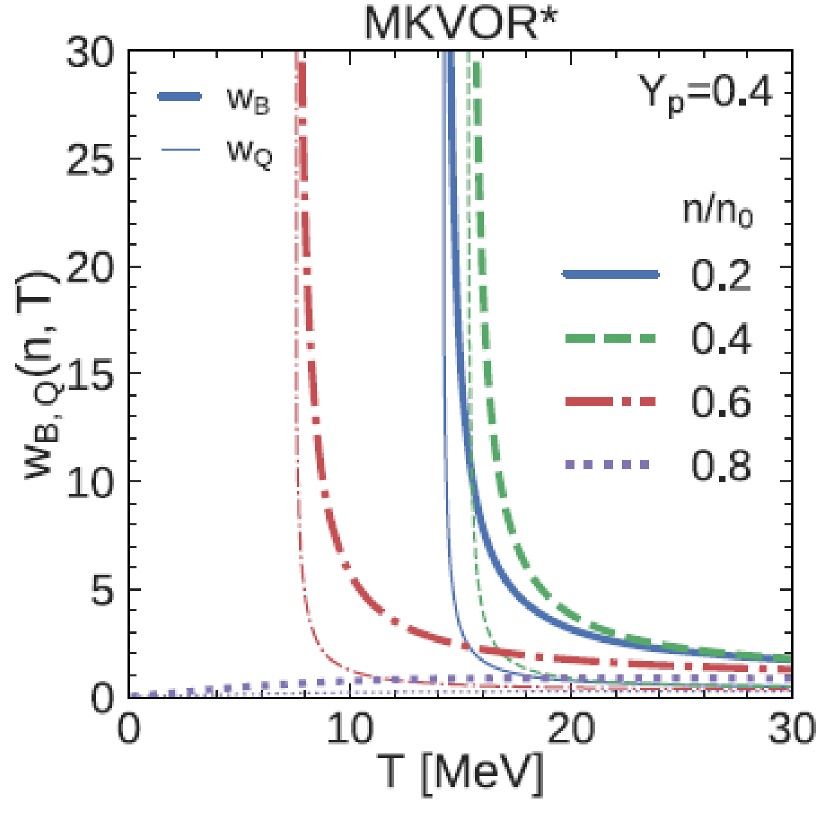
Scalar LM parameter $f_0(n)$<-1-
Pomeranchuk instability
In Fermi liquids

Normalized variance of the baryon number

Maslov, D.V. 2019

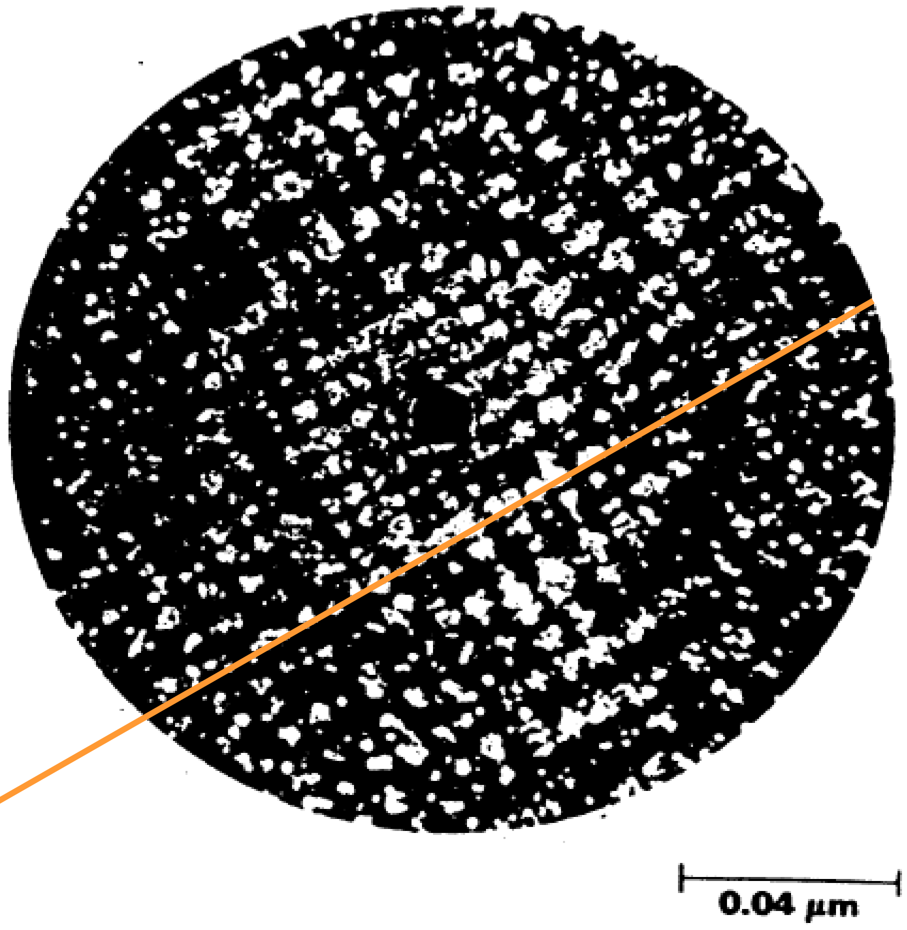
$$w = \frac{\overline{(\Delta N)^2}}{N} = \frac{T}{n}\left(\frac{\partial n}{\partial \mu_B}\right)_T$$

diverges on ITS line



Effects of crossing of the ITS line m.b. easier to observe than the vicinity of the critical point (due to a critical slowing down effect in latter case, since correlation time $t \sim 1/|T_c - T|^{1/2}$)

Spinodal region in alloys presents structured phase with fluctuations at k ≠0



Looks like a curved spine

Fig. 9 Field-ion micrograph of spinodally decomposed Fe-25Be (at.%) alloy aged 20 min at 400 °C (750 °F). The axis of the needle-like specimen is [001]. The iron-rich phase images brightly because of the different contrast mechanism operating in the field-ion microscope. 375 000×. (M.K. Miller)

GL phase transition in HIC at low energies

Schulz, Munchow,Roepke, Schmidt 1982, see Review Borderie 2019 (ALADIN,INDRA)

N=Z

Region with C_p <0

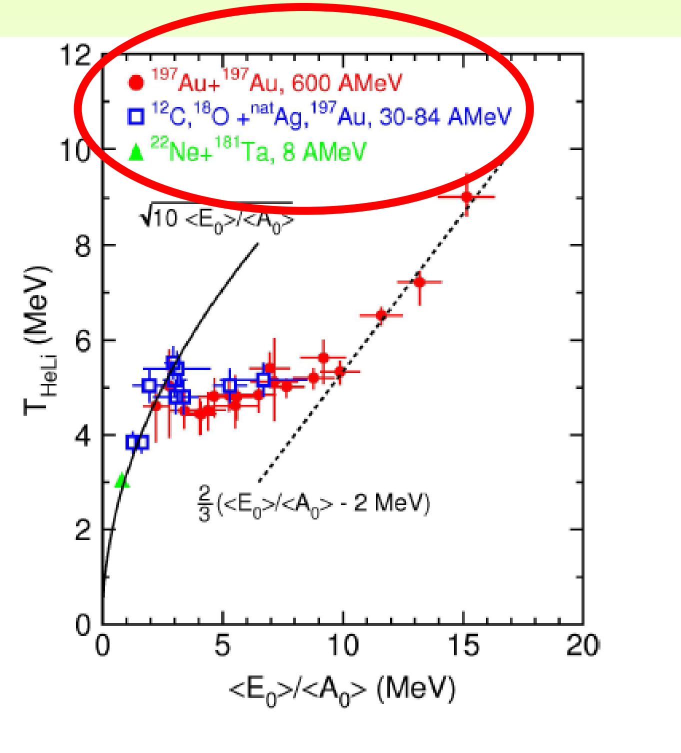
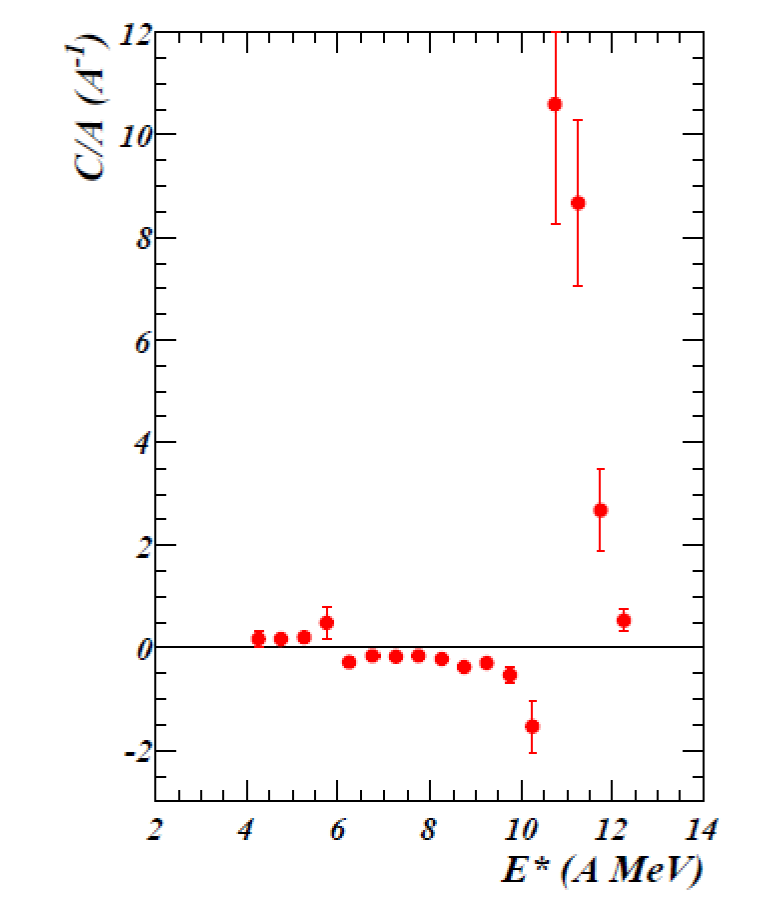


Fig. 27. Caloric curve of nuclei determined by the dependence of the isotope temperature on the excitation energy per nucleon (see text). From [165].

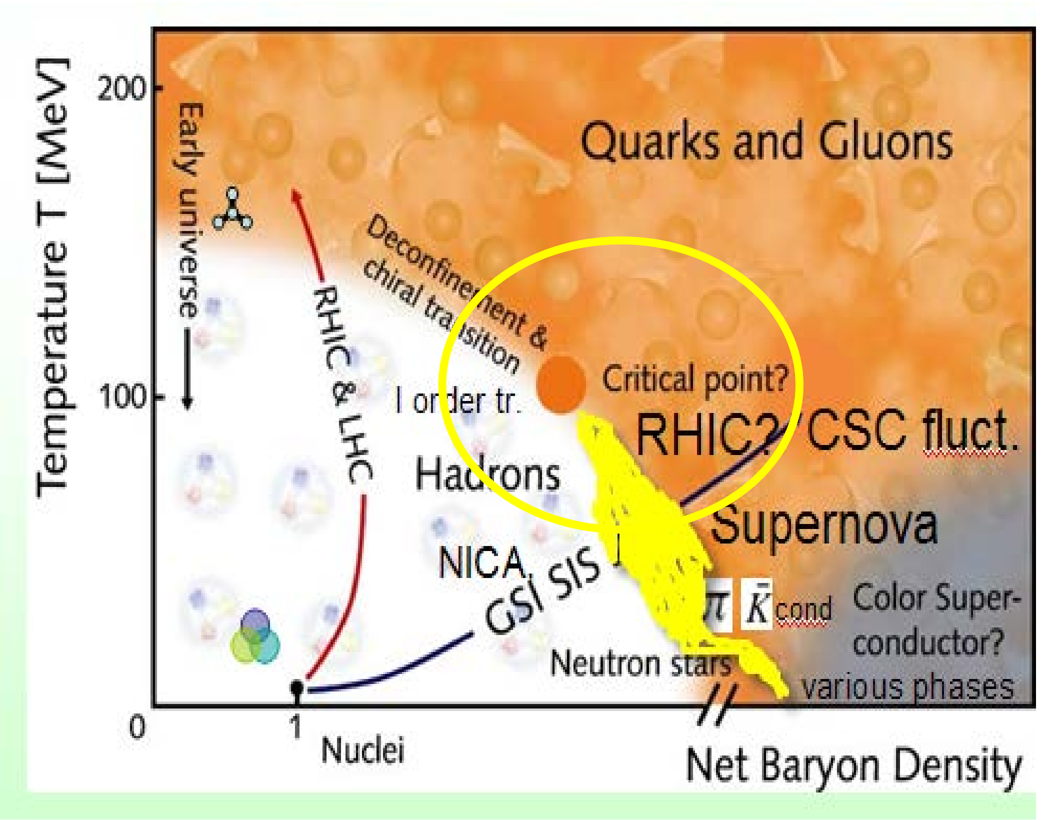


Negative heat capacity which signs a first order phase transition for finite systems is observed

Pochodzalla et al (1995)

Rather large *T*, large baryon density (baryon chemical potential larger than *T*),

typical NICA energies



Hydrodynamics of spinodal instability at QH transition near CEP

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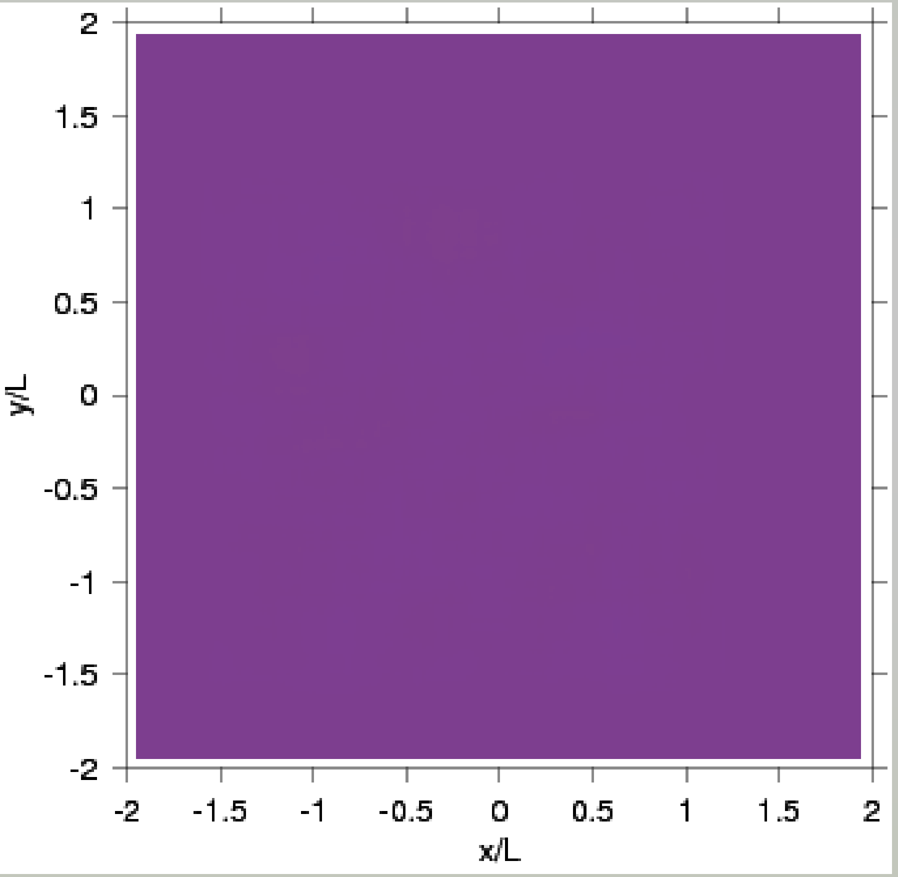
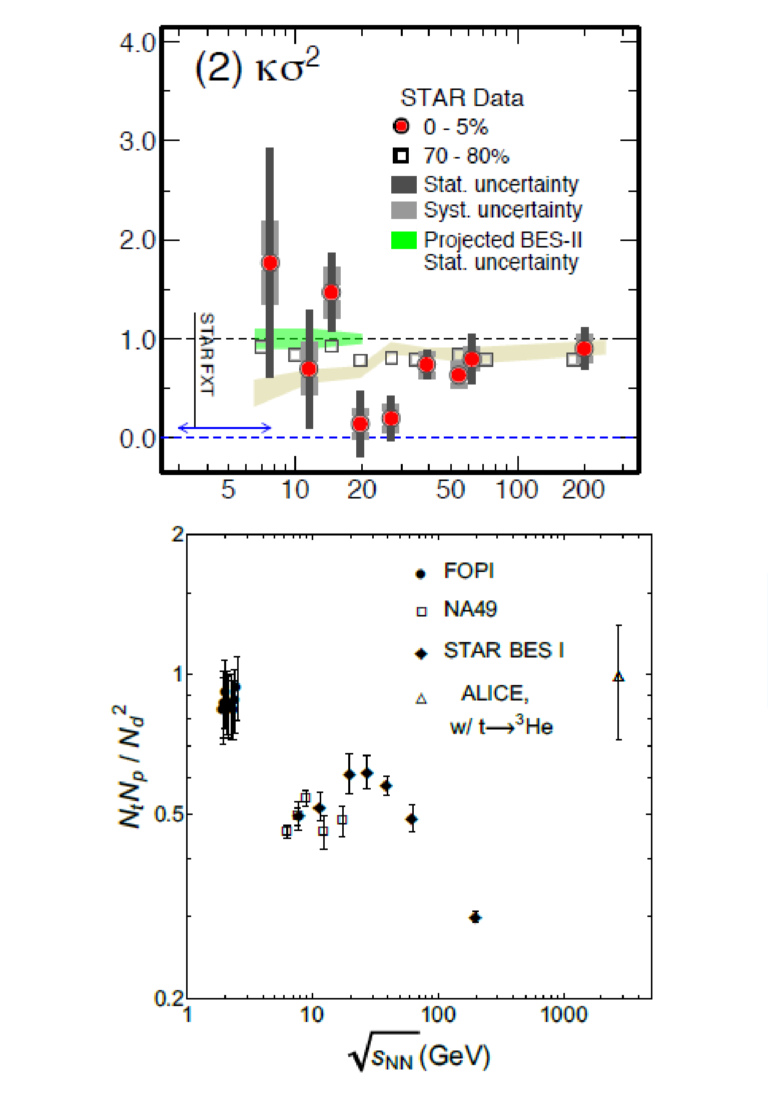
Blue – hadrons,
Red – quarks.



Skokov, D.V. 2009

Quark-hadron CEP, measurements



From Shuryak 2024

First evidence of non-monotonic variation in kurtosis of net proton number in central Au+Au collisions at RHIC -- indication to CEP (?)

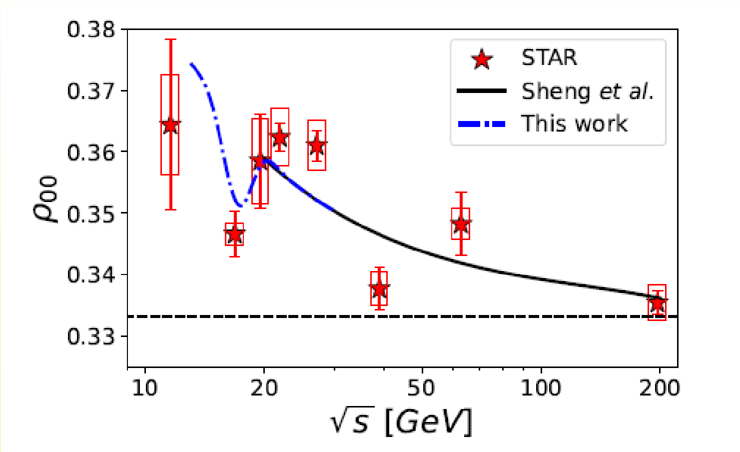
In Boltzmann gas would be $N_p \propto e^{\mu/T}, \; N_d \propto e^{2\mu/T}, \; N_t \propto e^{3\mu/T}$

$$\mathcal{O}_{tpd} \equiv \frac{N_t N_p}{N_d^2} \simeq 0.29(1 + \Delta n)$$

density fluctuations $\Delta n = \langle (\delta n)^2 \rangle / \langle n \rangle^2$

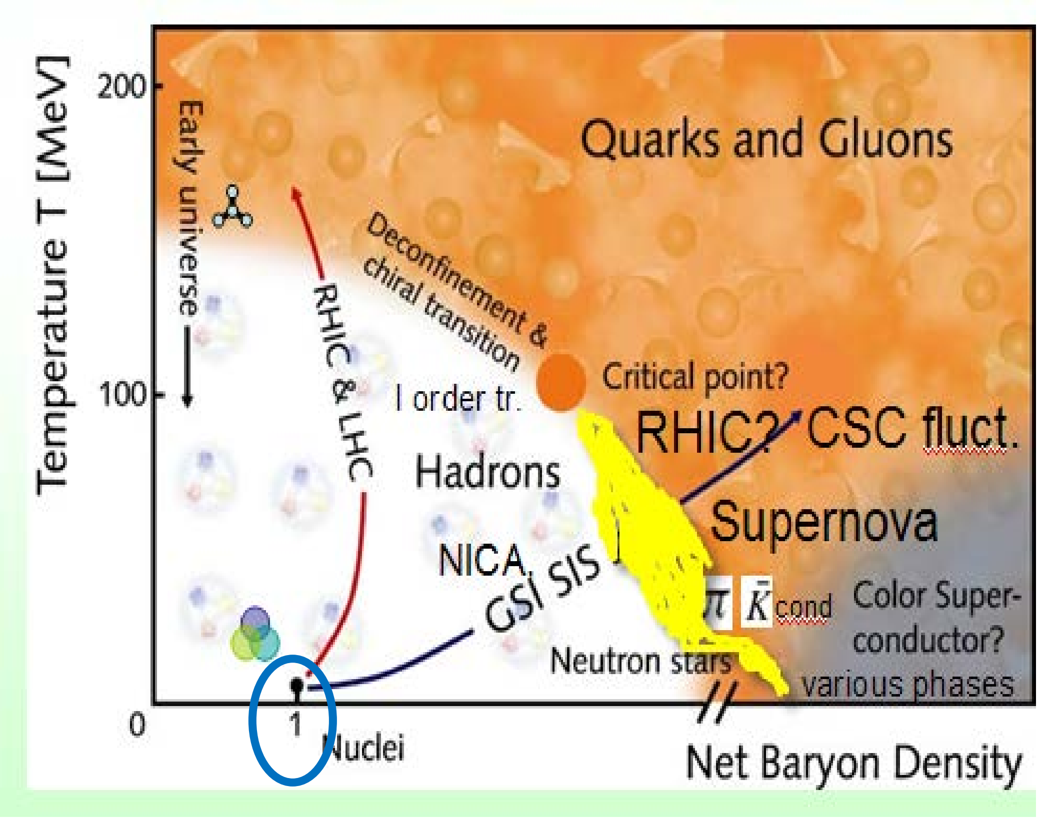
Similar irregularity for $\mathcal{O}_{\alpha p ^3\mathrm{He} d} \equiv \frac{N_\alpha N_p}{N_{^3\mathrm{He}} N_d}$

Variance of quark spin



Preliminary Wilks 2024,
see Chen et arXiv .2410.20704

Phase transitions in excited nuclei



Cooper pairing in nuclei

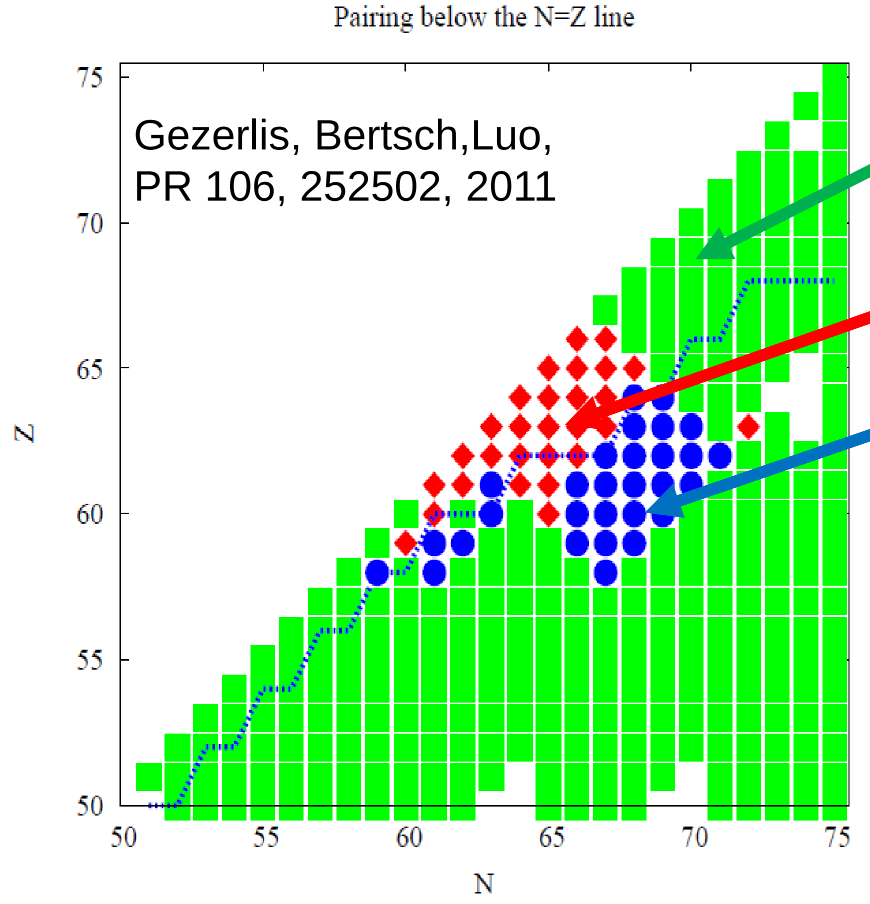


FIG. 1: (color online) Chart of nuclides with $Z \leq N$ for neutron numbers from 50 to 75. Blank squares denote nuclei that exhibit practically no pairing ($E_{corr} < 0.5$ MeV), green squares signify the case where the pairing condensate is mostly spin-singlet, red diamonds are used for the nuclei that exhibit spin-triplet pairing, while blue circles denote nuclei for which the pairing is a mixture of spin-singlet and spin-triplet. The blue dashed line is the proton-drip line from Ref. [16].

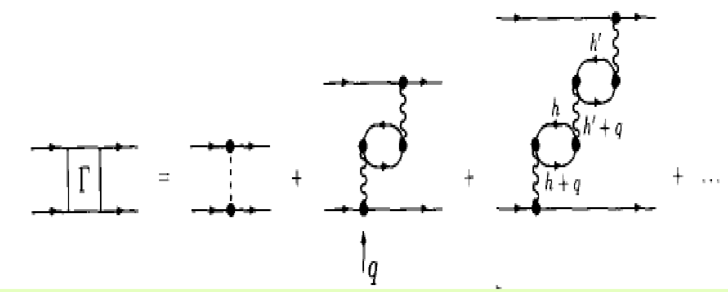
Singlet pairing of nn, pp
(np-exp. Cederwall et al, Nature 2011)

Triplet nn pairing

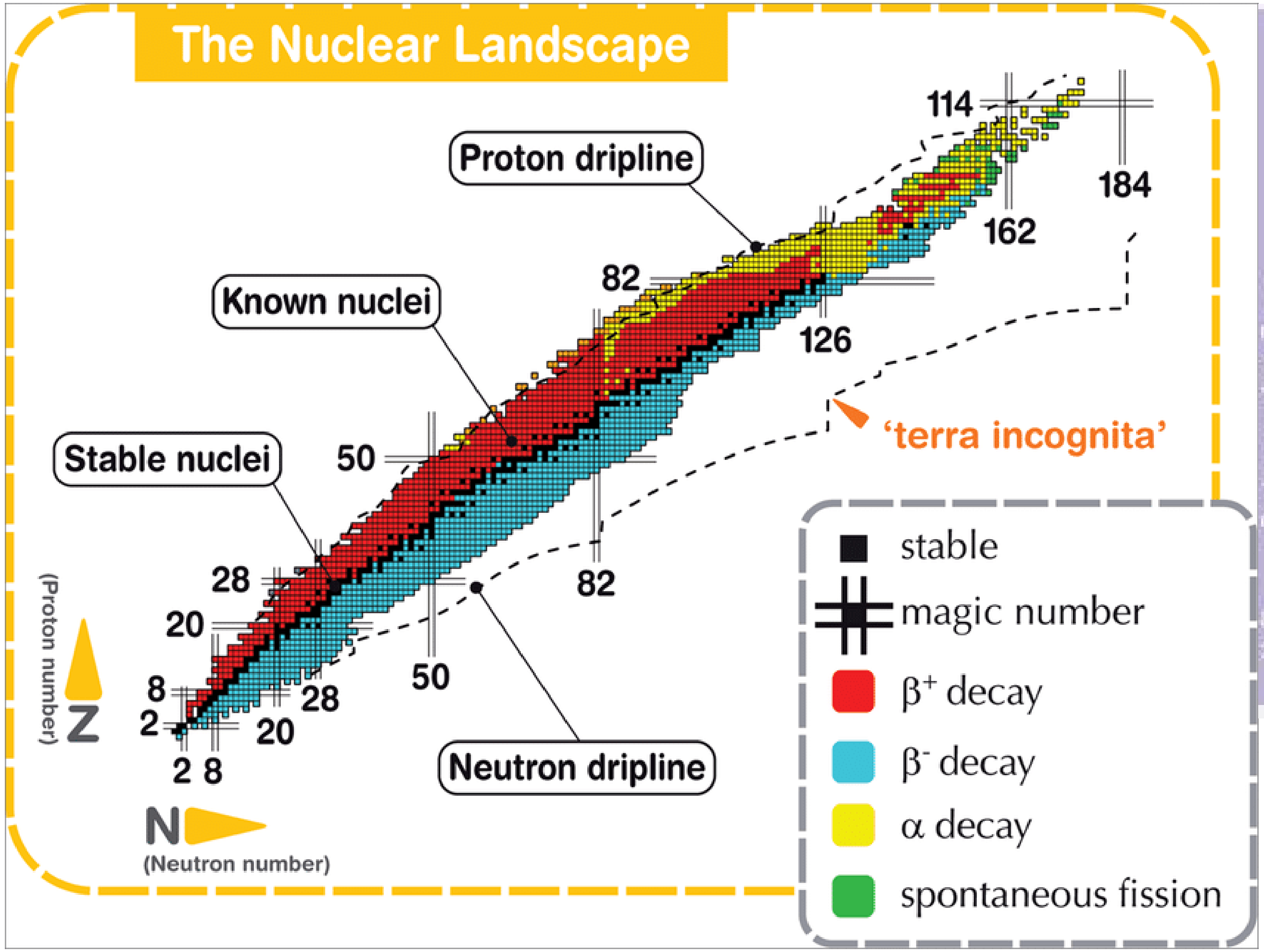
Mixture of singlet and triplet pairing

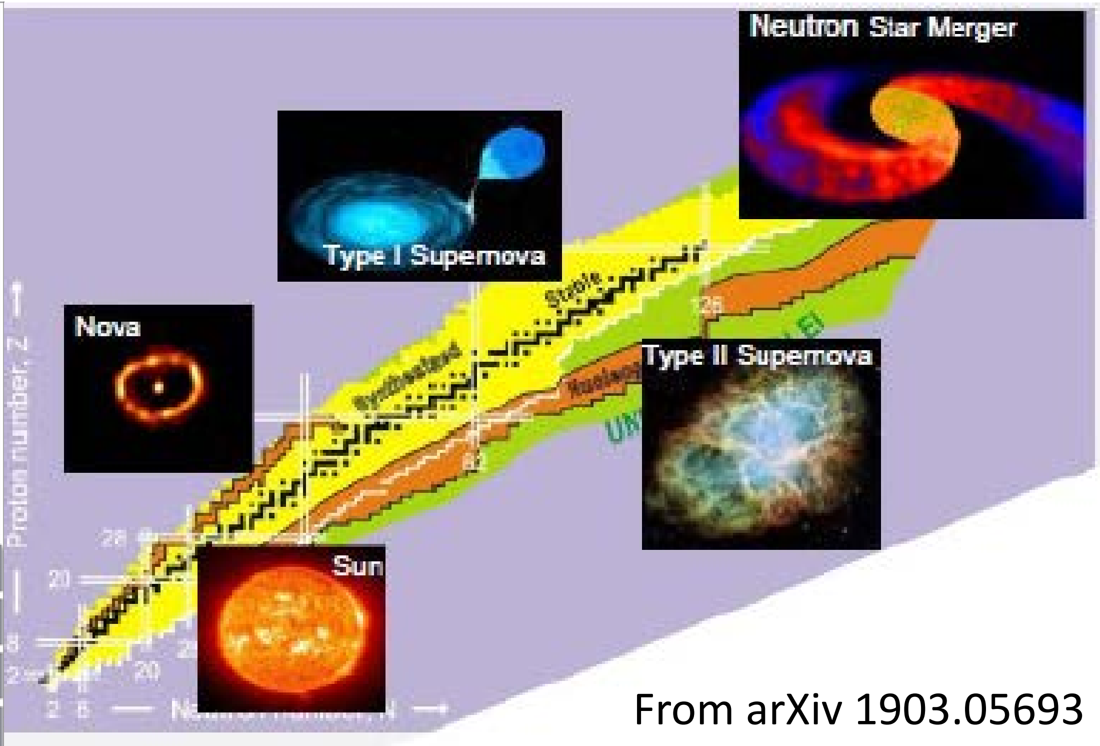
Fulde–Ferrell–Larkin–Ovchinnikov **periodic Δ(r) for N ≠Z (not yet found in nuclei)**

In-medium effects are important



Pairing influences moments of inertia of nuclei (data)

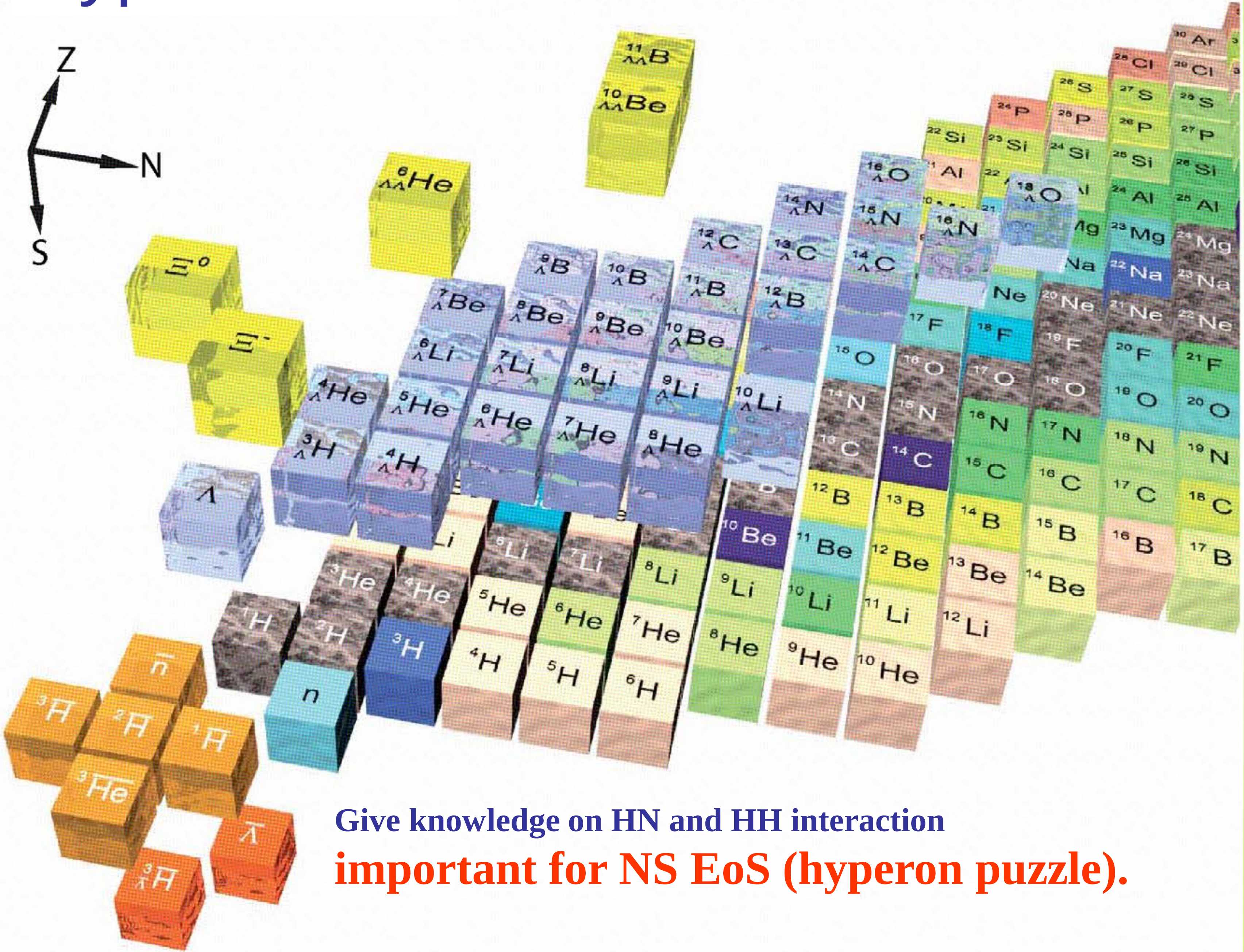




From arXiv 1903.05693

Only about ~1/3 of possible states is identified. Important Info for astrophysics and HIC.
Renaissance of low-energy nuclear physics.

Hyper-nuclei

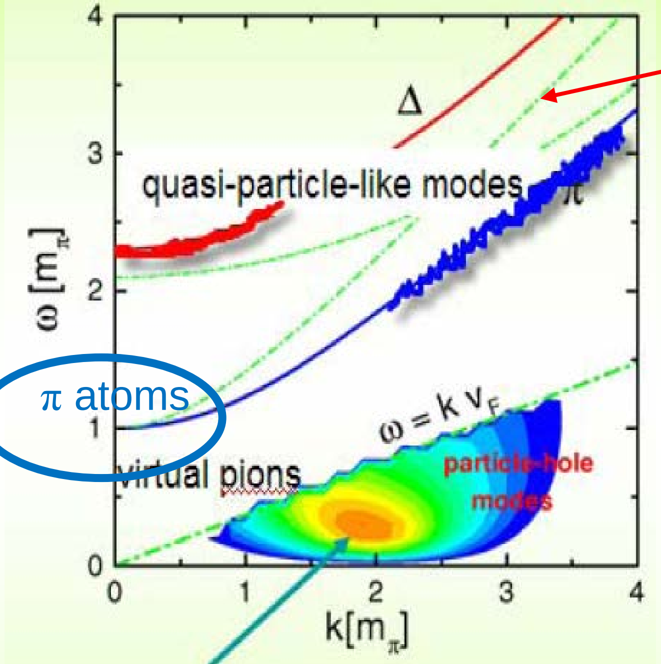


Give knowledge on HN and HH interaction

important for NS EoS (hyperon puzzle).

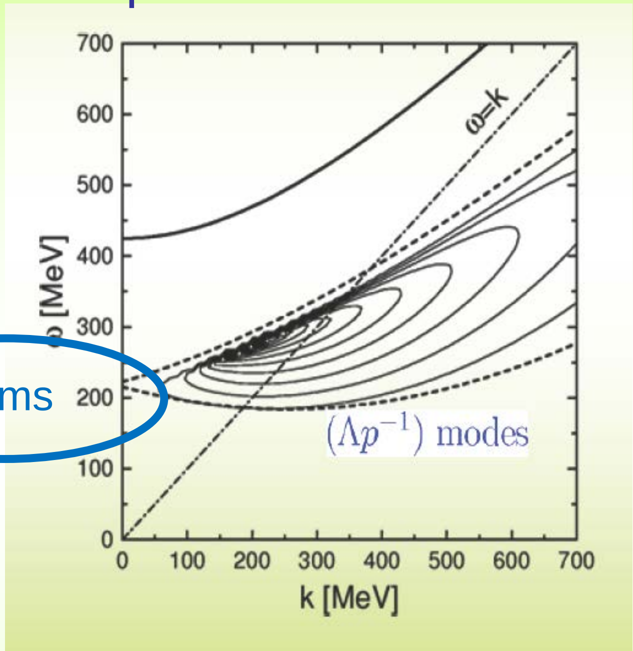
Pion and kaon in nuclear matter are complicated collective Bose excitations:

Pion spectrum for N=Z and for π^0 at N>> Z at $n \sim n_0$



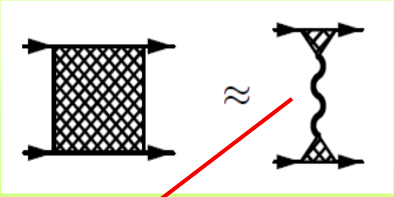
Migdal et al. 1990

Spectrum of antikaons at $n \sim n_0$



Already for *$n < n_0$*, appears minimum in complex *ω* plane and it decreases with *growing n.*

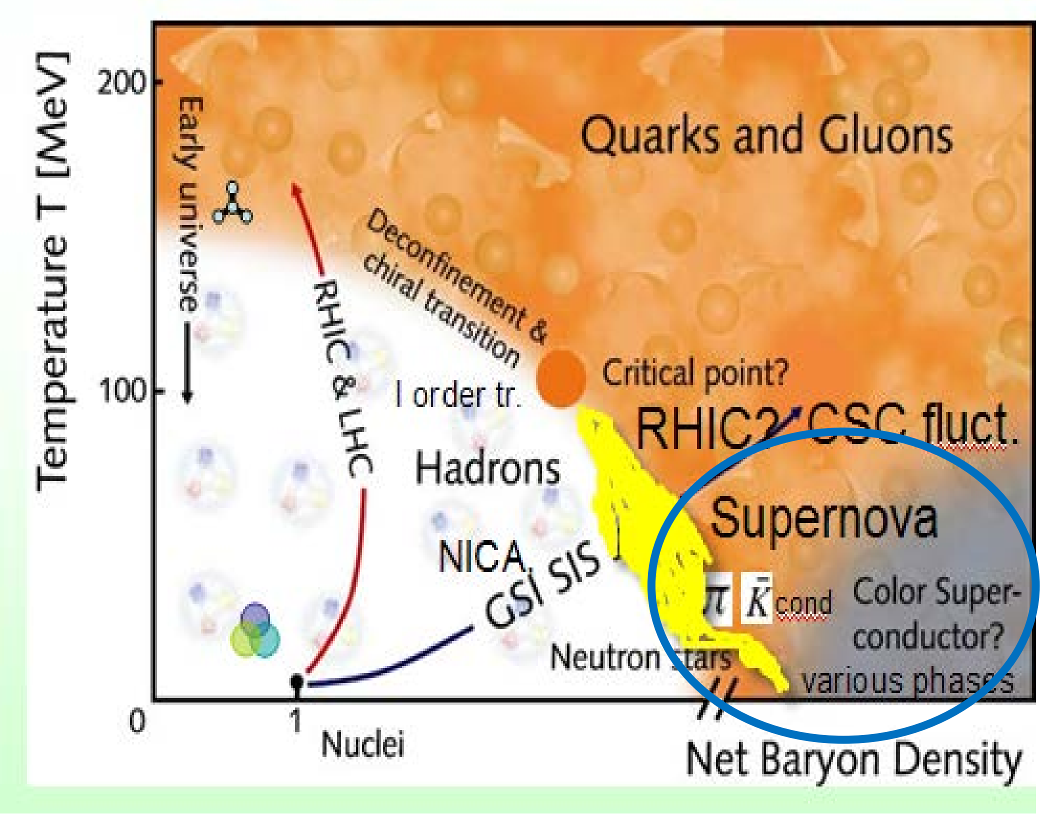
NN amplitude shows strong increase with density for *$n > n_0$* owing to in-medium pion exchange



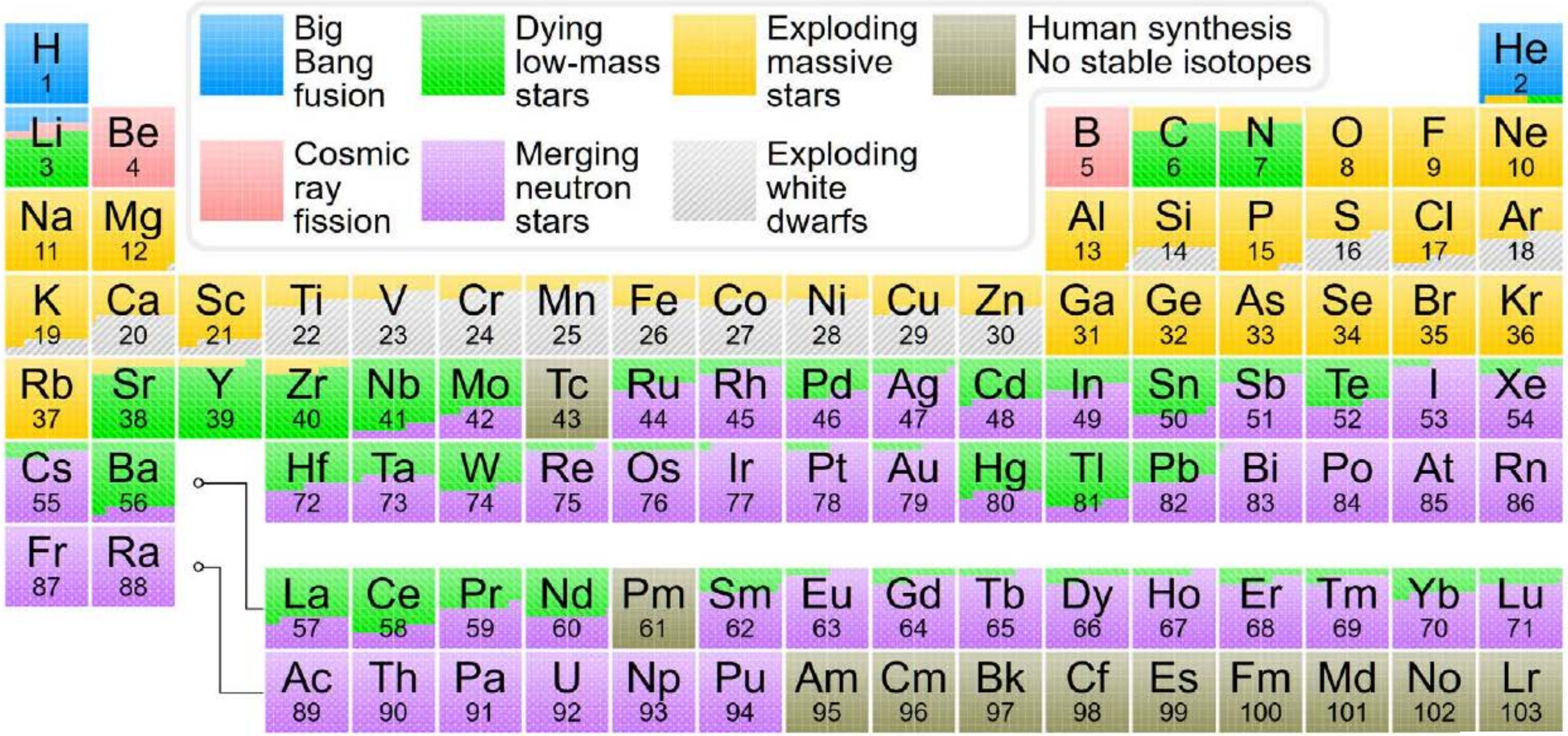
For *$n > n_c \sim (2\text{--}3) n_0$*, *$\omega^2 < 0$* instability occurs & p-wave (k≠0) pion condensation

Pion and similarly antikaon condensation may appear in dense NS interior

Mergers, supernovas, neutron stars, hybrid stars



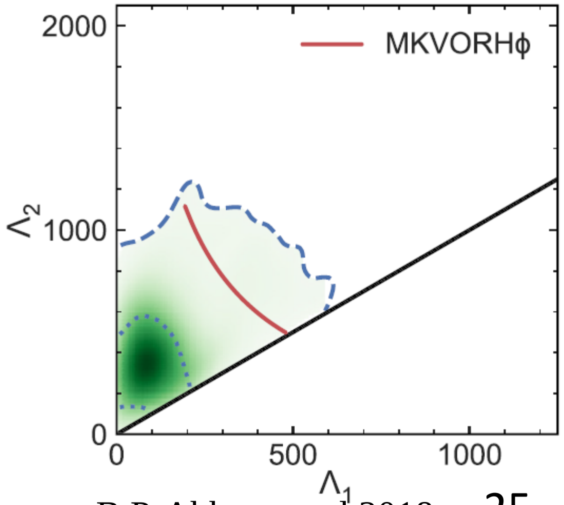
Heavy elements in natural conditions are produced in mergers of NSs (our parents)



Gravitational waves from neutron star mergers (LIGO/Virgo)

GW170817

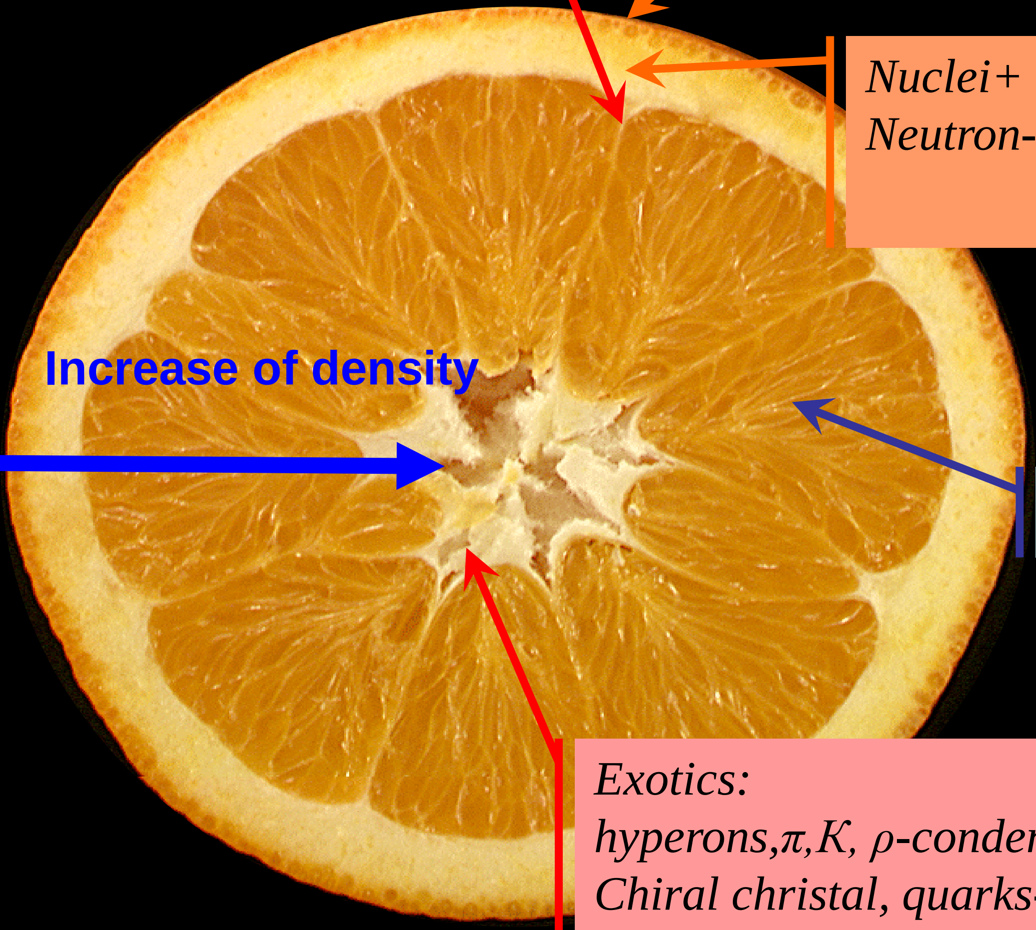
Tidal deformabilities



B.P. Abbott, et al 2018

Neutron star schematically

R~10km
M~(0.4-2) M_{sol}

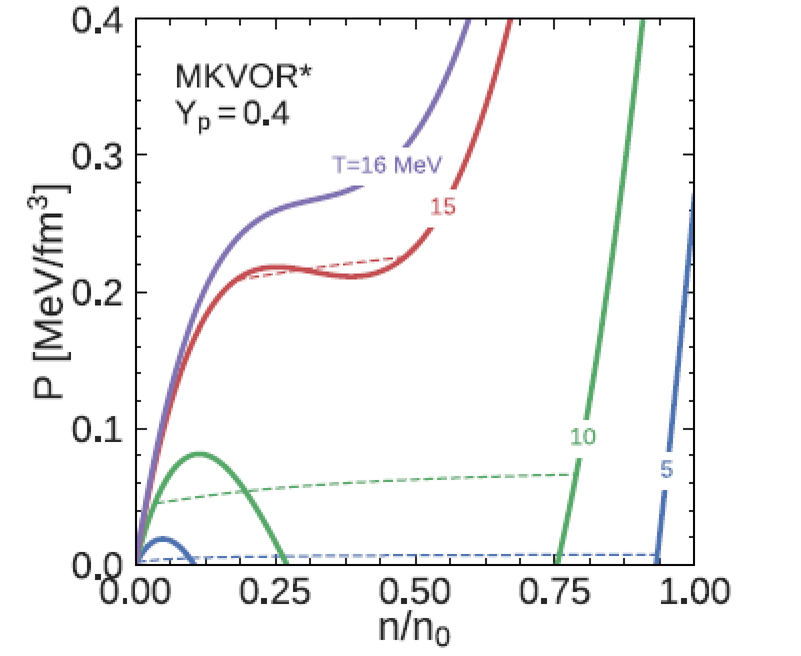


Magnetic field-

Up to ~10^{15} Гс in magnetars

Fig. E. Kolomeitsev

About peculiarity of 1-order phase transition in asymmetric nuclear matter (N≠Z)

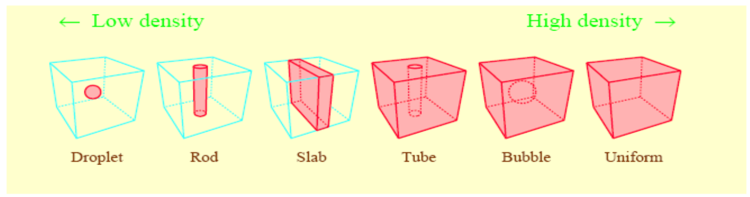


When two conserved charges: P(n) has saddle

Maxwell construction from horizontal line transforms to a curve

electric charge can be conserved globally: balance of surface tension + screening – WZ cells

Rawenhall, Pethick, Wilson 1983, Glendening 1992



Density →

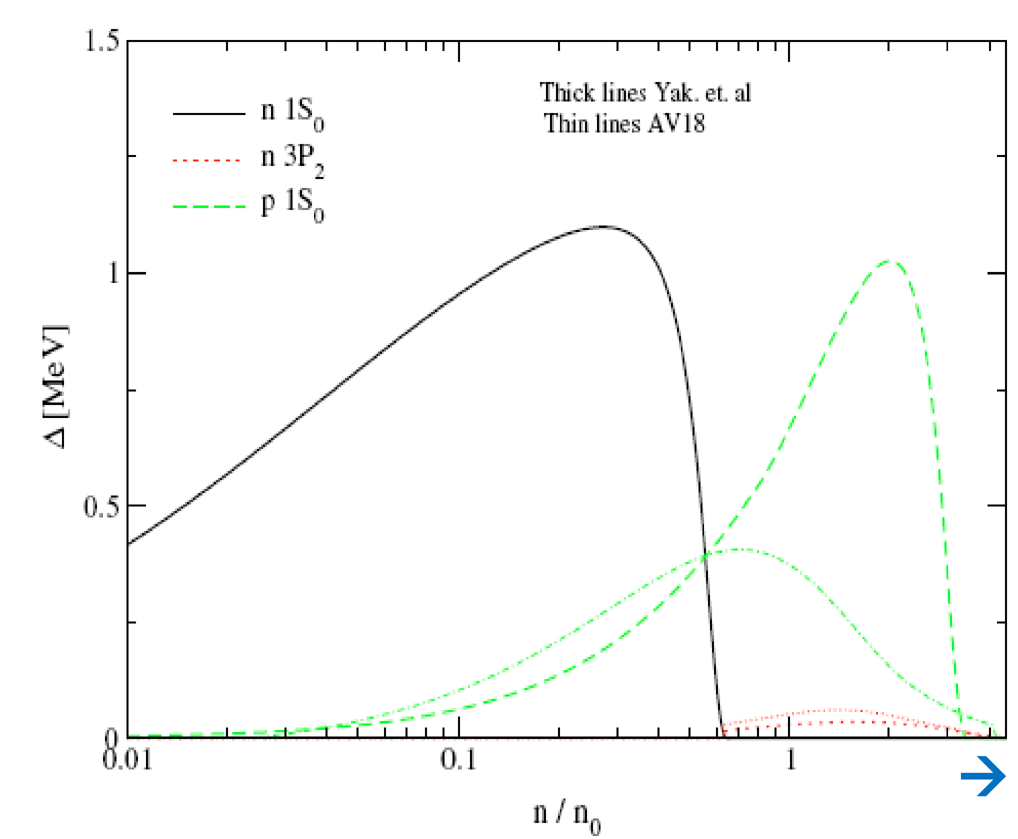
In NSs inner crust ($n<n_0$) structured nuclear pasta phase (neutron gas-nuclear liquid).

For larger density ($n>2\text{-}3\ n_0$) π cond., antikaon cond., HQ pasta phases (?)

D.V.,Yasuhira,Tatsumi 2002,T.Maruyama,…2005, Maslov, D.V. 2019

Features of pasta in HIC (?)

NN, HH pairings in NSs and qq in hybrid stars



Calcul.:1S_0-typical NN **Δ <MeV**,
For nn 3P_2 , Δ ~(0.01-0.1) MeV

→ **HH, *qq* pairing**

Cooper BB pairing → superfluidity: glitches of pulsars and post-glitch relaxation.

Cooling of compact stars is sensitive to *3P_2 nn and 1S_0 pp*,

+ hyperon-hyperon, and qq pairings in dense interiors

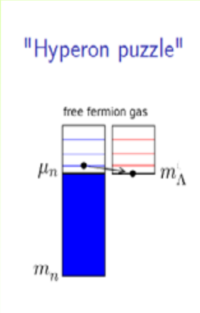
Constraints on EoS, masses of comp. stars

Bethe & Brown 1994: **$M \sim 1.4 < 1.56\ M_{sol}$**

PSRJ0740-6620 Cromartie et al. Nature 2020 **$M = 2.07 \pm 0.07\ M_{sol}$**

Data: NS masses are changed in interval ~(1-2) M_{sol}

Radii are also measured.

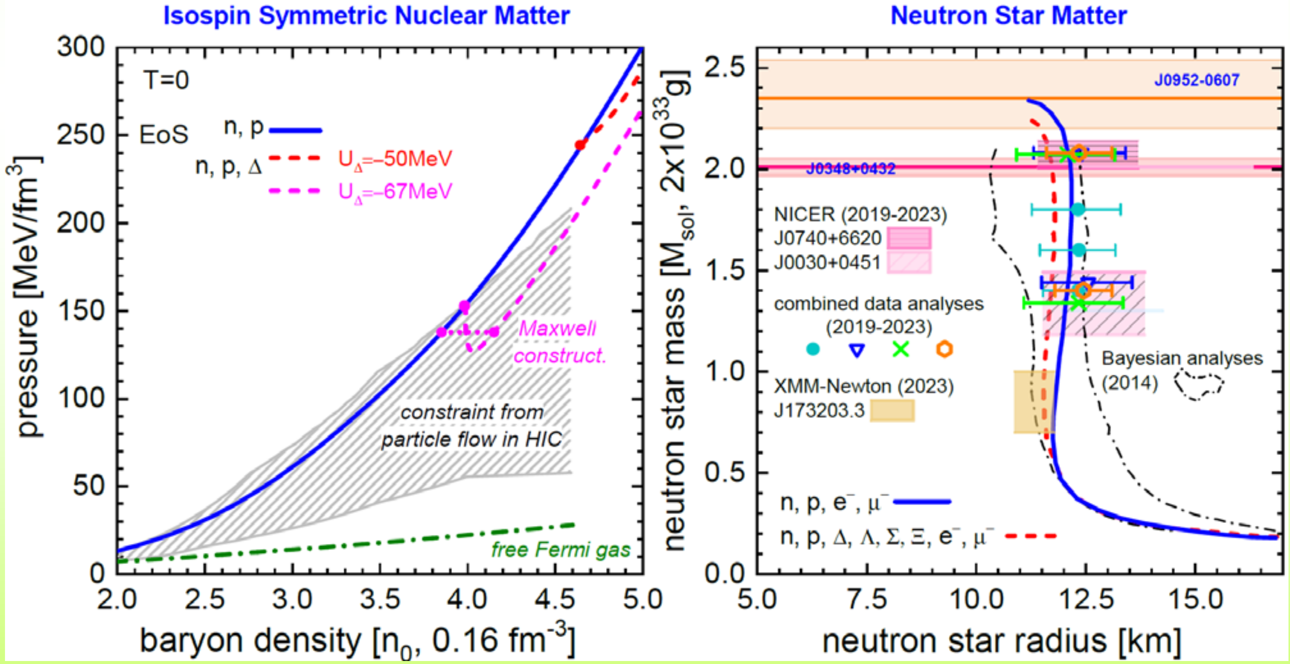


In most massive NSs n_{centr} ~6-8 n_0 . Strangeness appears in NS already for n>(2-4) n_0.

With H and Δ EoS softens (P(n) & max. NS *mass* decrease): hyperon and Δ isobar puzzles.

HIC need soft EoS, NSs need stiff EoS, Most of EoSs cannot fulfill all known constraints.

Example of appropriate EoS:



Kolomeitsev, Maslov, D.V. 2016

Appropriate EoS implies nontrivial density dependence of symmetry energy.

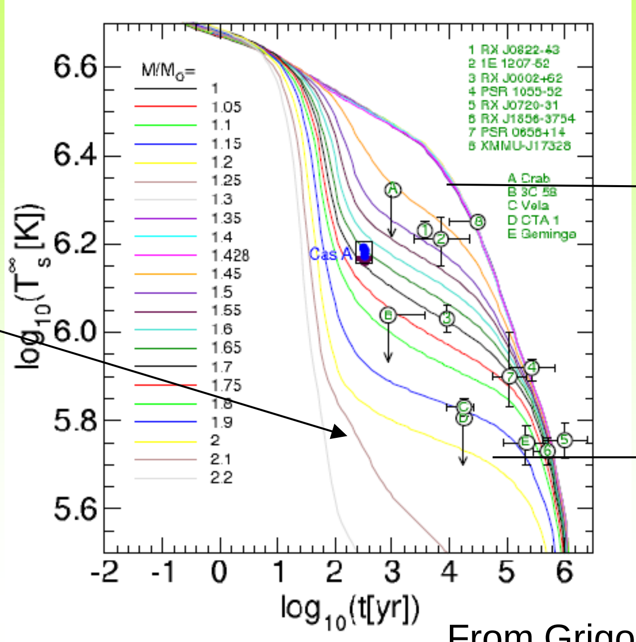
Neutrino cooling of neutron stars (similar to di-lepton production in HIC)

for 10^2s<t<10^6yr, T<1 MeV λ_ν >>R white-body radiation:

Direct Urca reactions $n \to p e \bar{\nu}$

yield too rapid cooling of pulsars

To occur DU needs >11-15% of *protons*

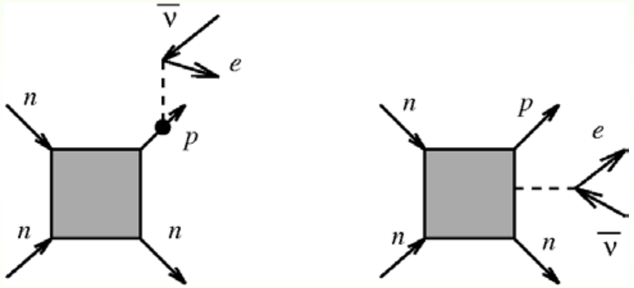


difference by 3 orders of magnitude in neutrino emissivity

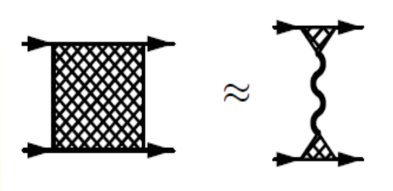
From Grigorian, D.V., Maslov 2018

Again, **a constraint on *n*-dependence of symmetry energy**

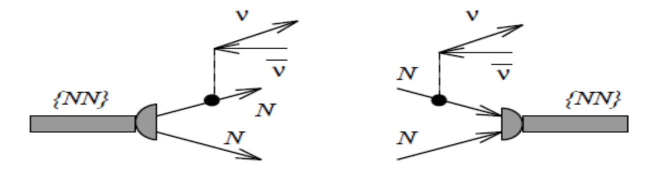
MU processes $N n \to N p e \bar{\nu}$



Necessary strong density (NS mass) dependence of NN amplitude is provided by *NN* exchange by **in-medium pion**



Pairing is also important:
PBF processes



Data are explained with strongly *n*-dependent *NN* interaction +*NN* pairing:
low mass NSs cool slowly, massive NSs-rapidly

Color superconductivity in most massive NSs (hybrid stars)

for n>~5 n_0 ($T<T_c$), M>~1.7 M_{sol}

Typical values of superconducting diquark gap: T_c ~**Δ~ 10-100 MeV** ~ 10^{11}-10^{12} K,

Order parameter- matrix:

many different phases (2SC, CFL,CSL etc) in most massive NS.

Alford, Rajagopal, and Wilczek, 1998

Due to strong interaction- broad fluctuation region at $T\sim T_c$ as in He-II

Precritical CSC fluctuations in HIC (?) at $T<1.5\,T_c$ ~100 -150 MeV

Kitazawa et al PRD2002,PTEP2022, DV PRC2004, Kerbikov PRD2020

What was discussed:

In crossover region of high T, low n, QGP is not gas but strongly interacting liquid (small viscosity).

Nuclear GL I order phase transition occurs at low-energy HICs and m.b. (?) QGP-hadron 1-order transition occurs at NICA energies (spinodal decomposition, pasta phase in NSs).

Cooper NN pairing occurs in nuclei, BB pairing in NSs, qq- in hybrid stars.

NN interaction amplitude is strongly density dependent due to in-medium pion exchange.
Neutrino cooling of NSs can be described by this with account of BB pairing.

In spite of hyperons soften EoS of NS matter it should be rather stiff since there exist massive NSs.

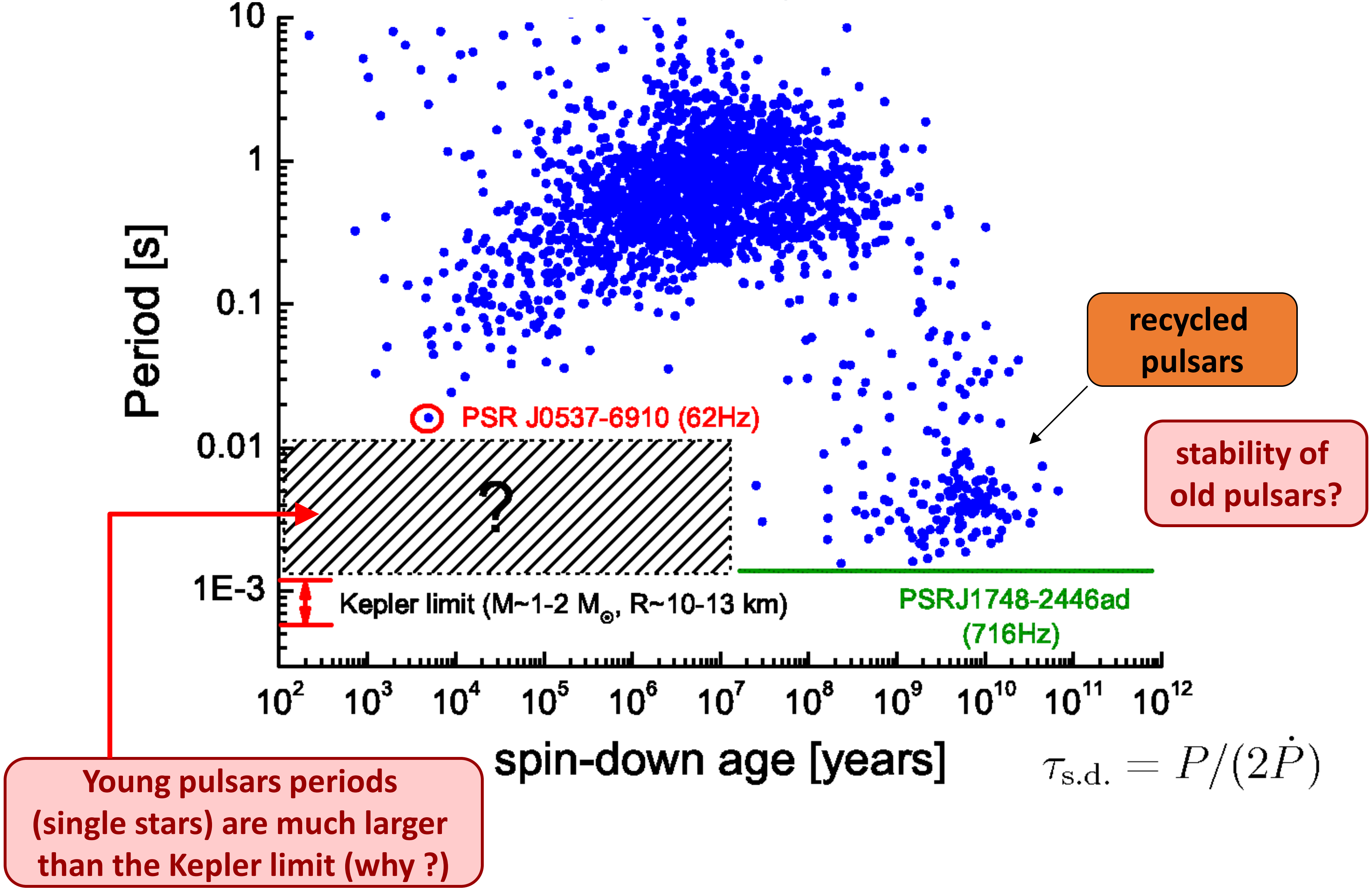
Key message–interrelation between phenomena in different domains of nuclear physics

Thank you for attention!

Supplementary Info

Pulsar period-age diagram for pulsars

- The ATNF pulsar catalogue

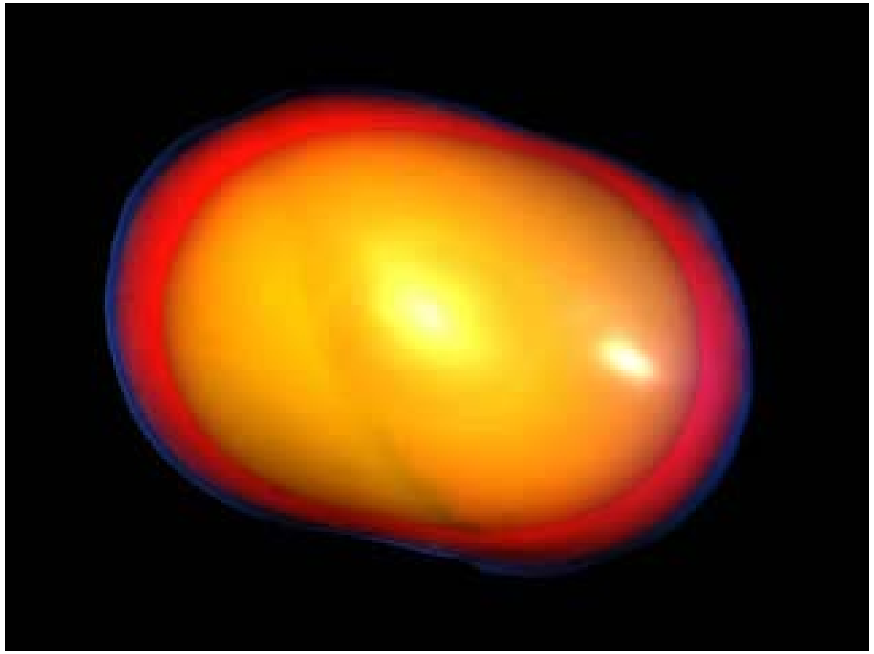


$\tau_{\mathrm{s.d.}} = P/(2\dot{P})$

Again viscosity effect!

In a dense system like a NS the Rossby waves (driven by Coriolis force) are sources of r-mode instability

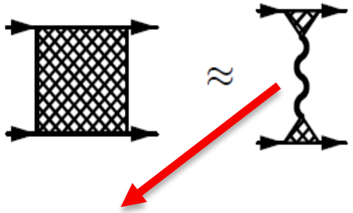
Andersson, Friedman and Morsnik 1998



r-modes can either destroy the star
or the star stops rotating

r-mode is stable if

$$\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta} > 0$$

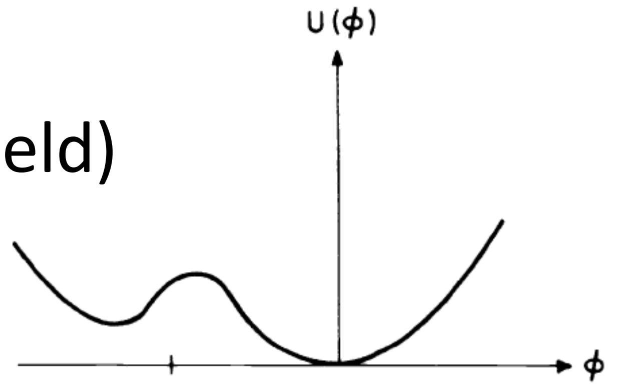


Bulk viscosity of NS matter is increased again owing to in-medium pion exchange that allows to damp r-modes and save the rotating star

Kolomeitsev, D.V. 2015, Kraav et al 2023

M.b. more exotics (?)

If energy as function of order parameter (e.g. density, or a field) had second minimum

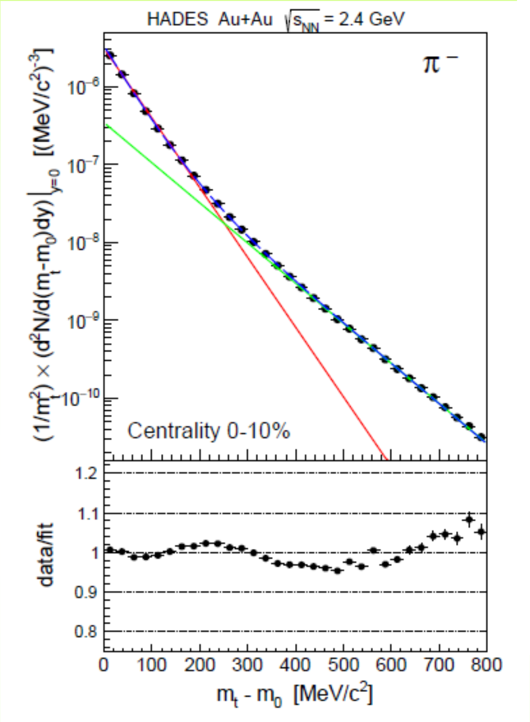


would exist stable/metastable nuggets of arbitrary size (without gravity) glued by condensates, strange quarks, and quark stars and, m.b., objects including DM.

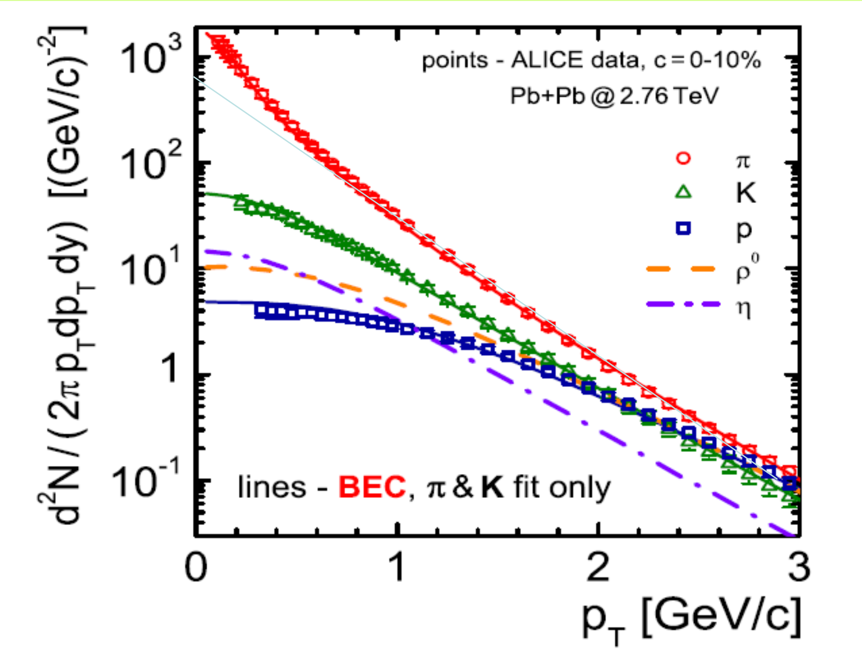
Migdal 1971, Bodmer 1971, T.D. Lee & Wick 1974, Chernoutsan, Sorokin, D.V. 1977, Witten 1984, Glashow & De Rujula 1984, …, Zhitnitsky 2025

Bose enhancement or BEC of pions for low momenta < m_π in HIC

For k **< 1.5** m_π pion path length λ~R, **mainly elastic collisions**. Pion number is approximately (dynamically!) conserved.



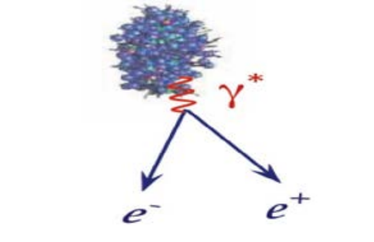
HADES 2020

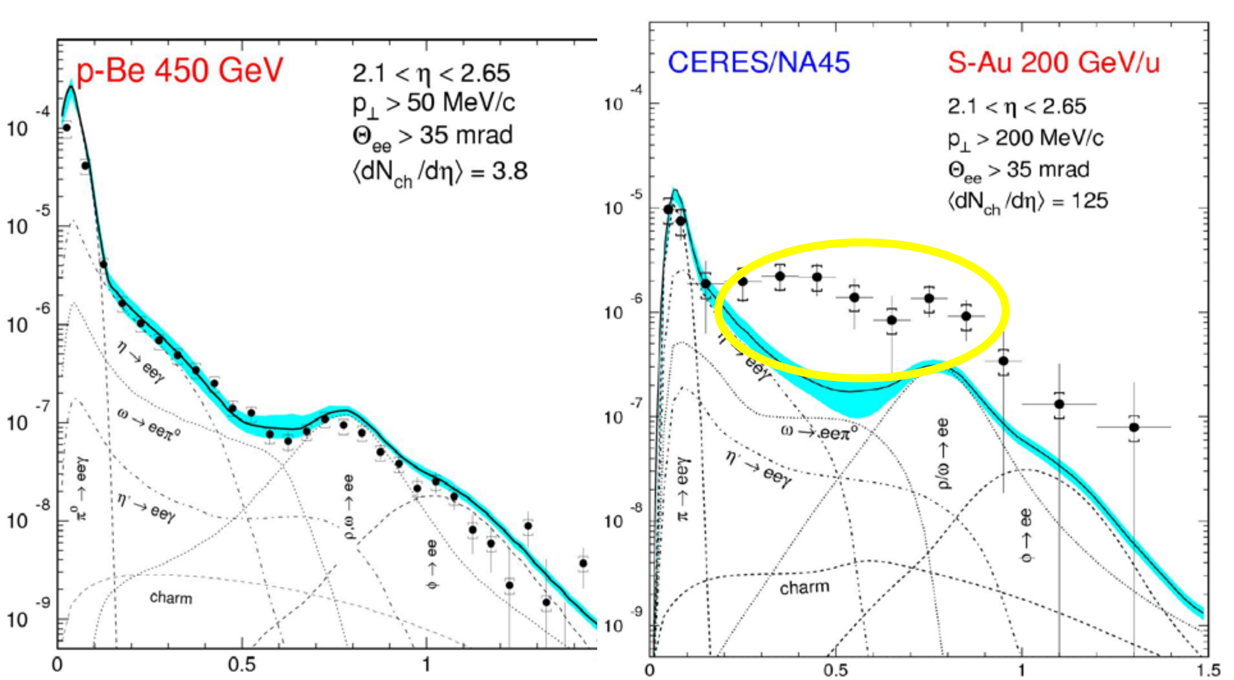


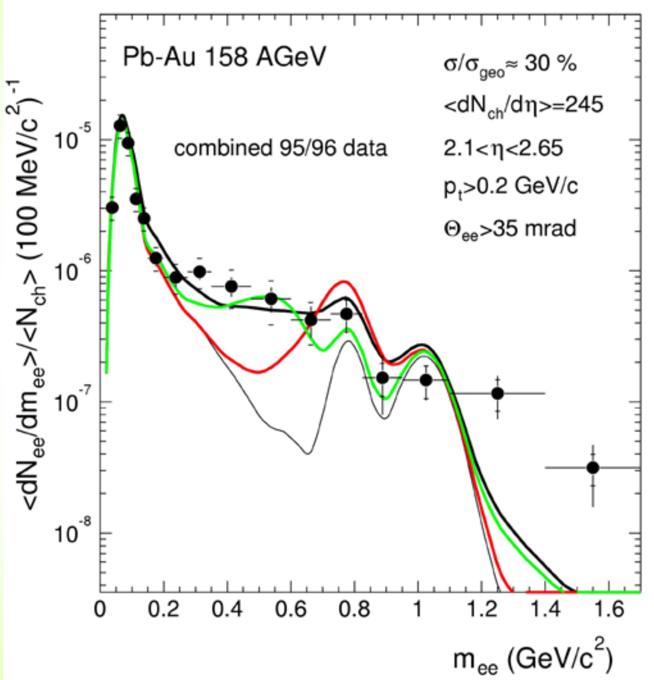
⇨ increase of n_π (k **< 1.5** m_π) and m.b. BEC at sufficiently rapid cooling of the system

D.V. 1994, Kolomeitsev, D.V. 2018.

Dilepton production in p-Be, S-Au, Pb-Au





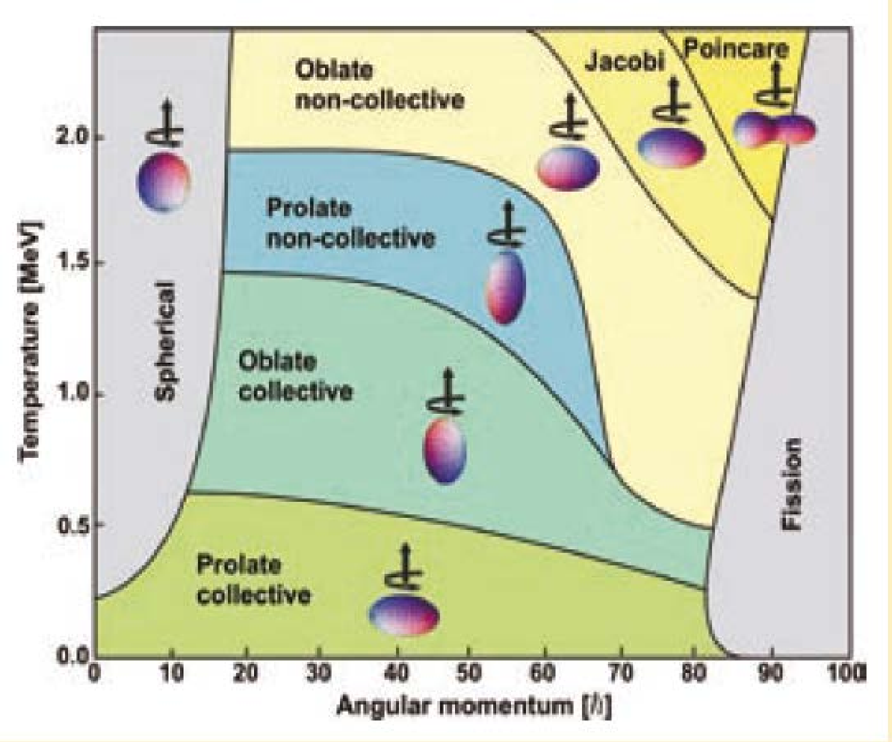


- — without participation of ρ μεσονσ
- free ρ
- with account of decrease of ρ mass
- with account of brodening of ρ σπεχτραλ φυνχτιον ιν μεδιυμ

(Super)deformed nuclei

Nuclei with high angular momentum

phase tr. with change of nucleus form



Prolate-lemon, oblate-orange, triaxial-mango, pear-like shapes arXiv 2211.06037

Ratio of nuclear ellipsoid axes 2:1:1.

Pulsars: vortex structure at

$\omega_{c1} \sim 10^{-14}$ Hz $< \omega < 10^{4}$ Hz

Rapidly spinning fireballs: in peripheral HIC

$\omega \sim 10^{21}$ -10^{22} Hz

Rotation bodies with charged constituents→strong magnetic fields

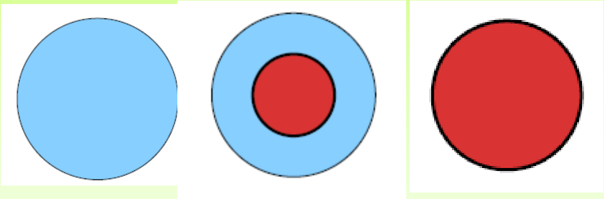
In HIC $H \sim 10^{17}$ -10^{19} G.~(0.1-10) m^2_{π}, DV, Anisimov JETP1980, Toneev et al. , IntJModPh2009.

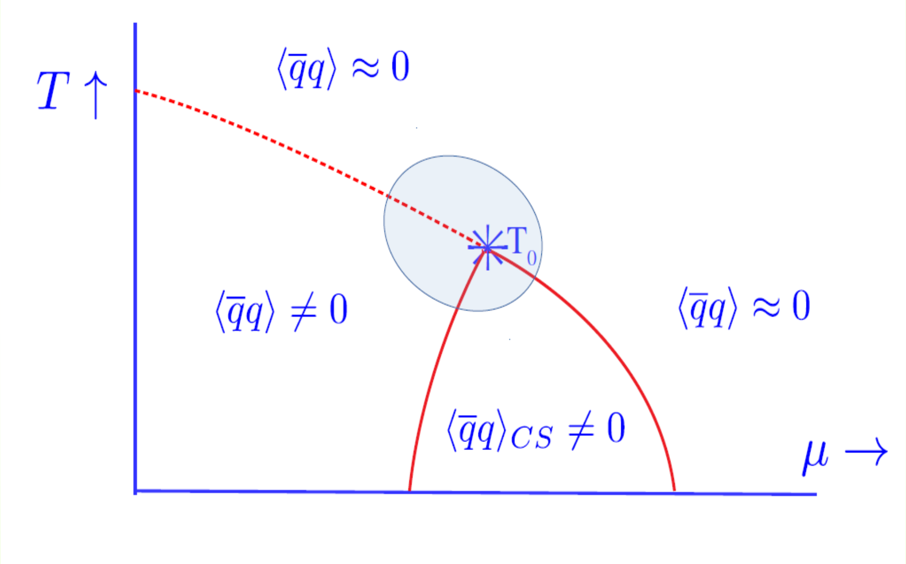
In magnetars ~10^{15} -10^{16} G at surface, m.b. up to ~10^{18} G in central regions of NS

Related problem: spin polarization and vorticity in HIC

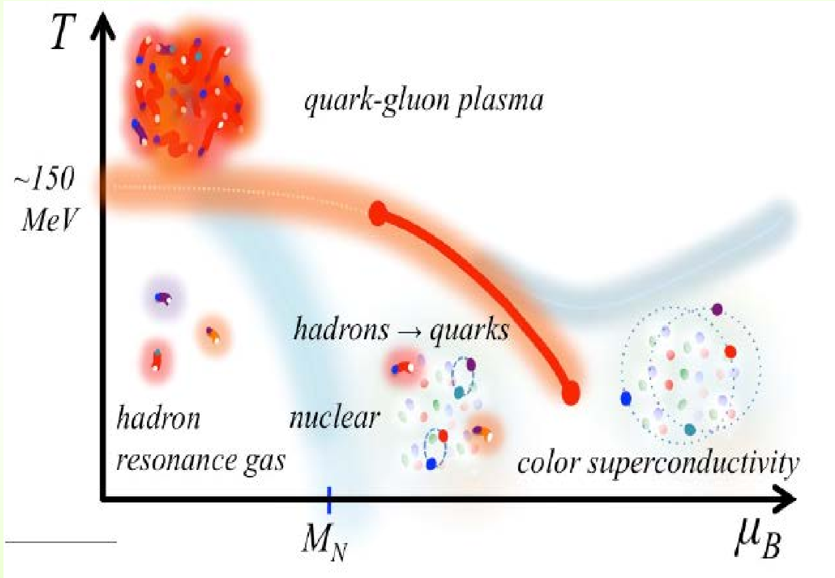
More exotics: quarkyonic crystals, similar to p-wave pion and kaon condensates and A-smectic liquid crystals

Baryon-quarkyonic-quark Fermi seas
cf. Duarte et al., 2302.04781:





Pisarski, Skokov, Tsvelik PRD 2018

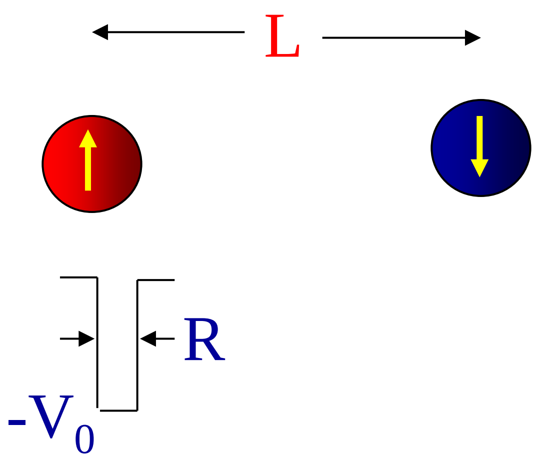


Baym et al, RepProgPh 2017: m.b. continues tr. from nucleon to quark pairing, **two CEP points (?)**

Strong interaction manifests *at all distances* in nuclear systems

In dilute neutron matter : a = –18 Fm, L ~ 5 Fm for n~0.1 n_0,
i.e. it is strongly interacting matter

For cold atoms universality



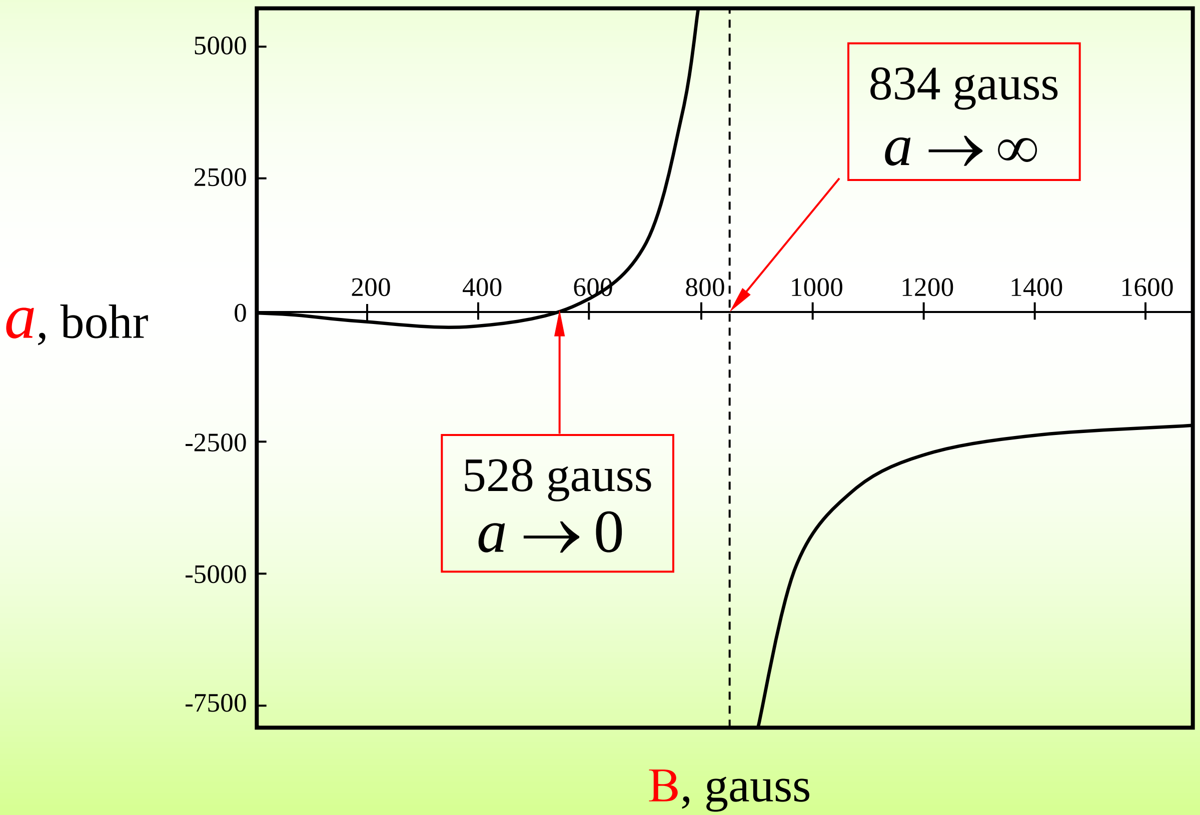
For $a\to\infty$ there is no dependence on details of paired potential: ρ и T.

$$\Rightarrow \quad \sigma \neq 4\pi a^2, \quad \sigma = 4\pi / k_F^2$$

Cold Fermi gases

Scattering lengh *a*, as function of magnetic field *B*:

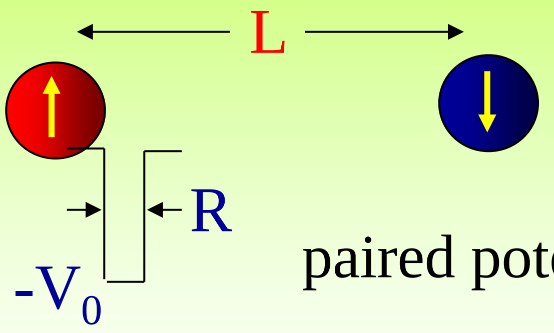
$$\lim_{k \to 0} \sigma_e = 4\pi a^2 \ .$$



From A.Turlapov talk

Varying magnetic field, one may change strength and sign of interaction between atoms

In nuclear matter we may vary only *n* and *T*



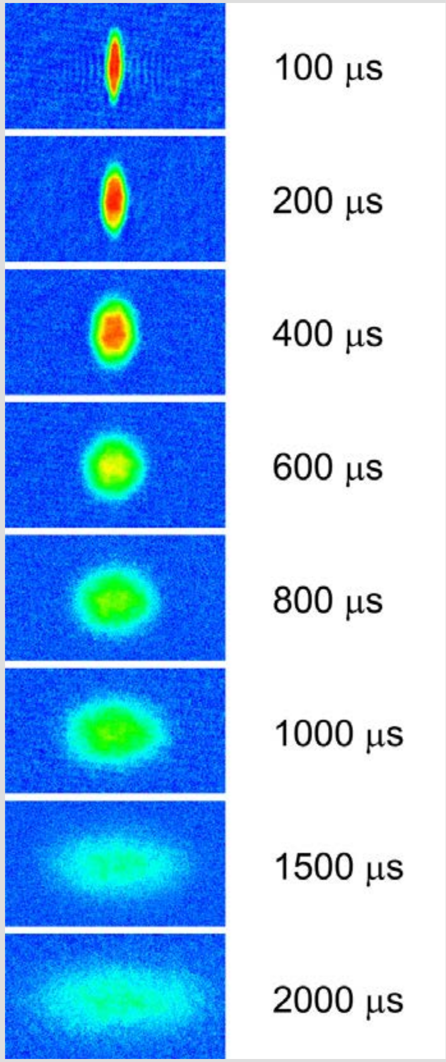
Strong interaction: $|a|>L>>R$

For $a \to \infty$ there is no dependence on details of paired potential: ρ и T.

$$\Rightarrow \quad \sigma \neq 4\pi a^2, \quad \sigma = 4\pi / k_F^2$$

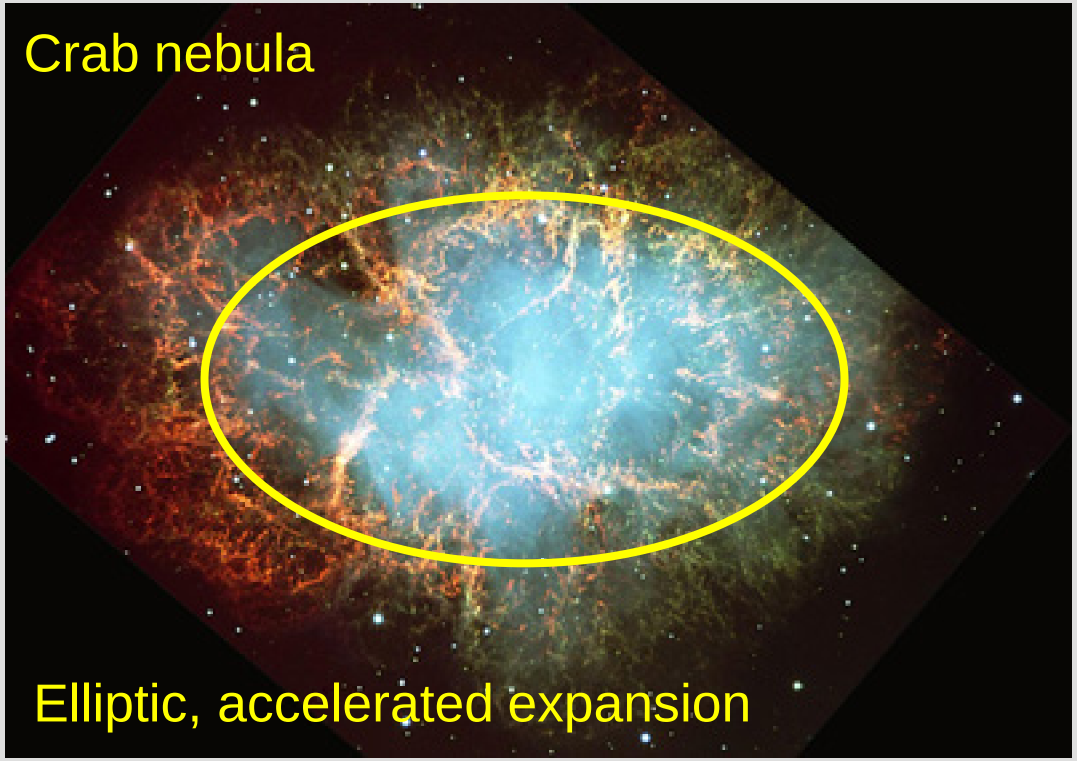
In dilute neutron matter is strongly interacting matter : a = –18 Fm, L ~ 5 Fm for n~0.1 n_0,

Superfluid and normal hydrodynamics of a strongly-interacting cold Fermi gas ($a \to \infty$)



[Duke, Science (2002)]

M. Gyulassy: "Elliptic flow is everywhere"

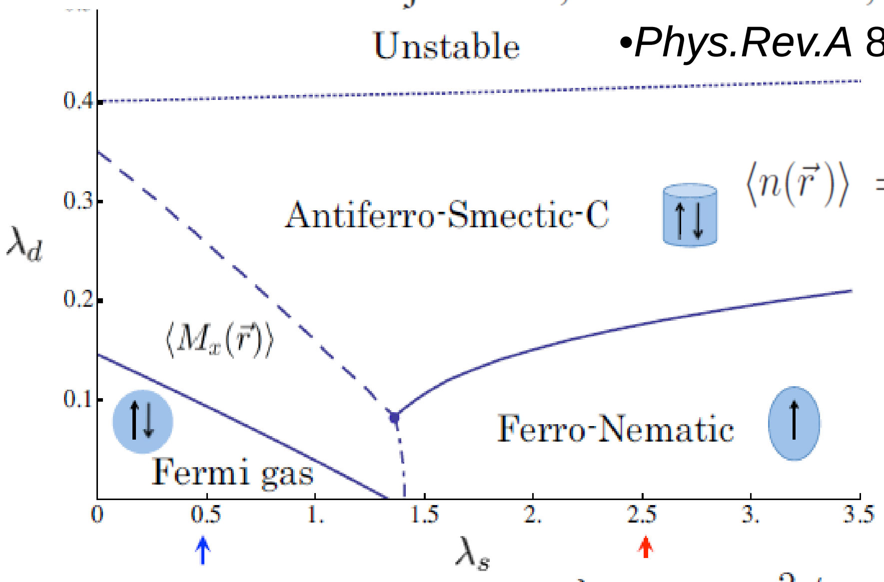


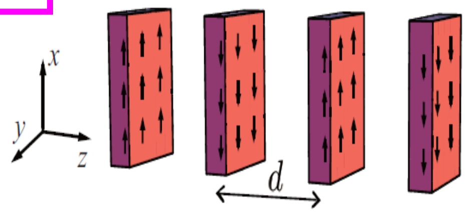
From A. Turlapov talk

Antiferrosmectic ground state of two-component dipolar Fermi gases – an analog of meson condensation in nuclear matter

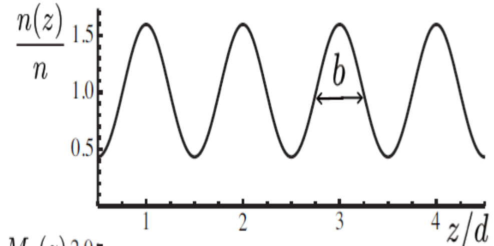
Kenji Maeda,[1] Tetsuo Hatsuda,[2,3] and Gordon Baym[4]

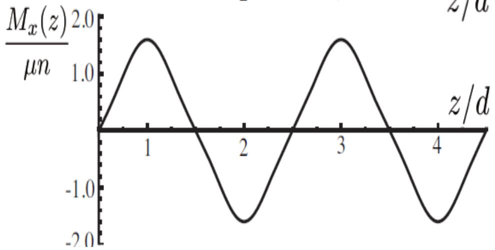
•*Phys.Rev.A* 87 (2013) 2, 021604





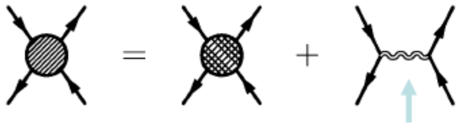
$$\langle n(\vec{r})\rangle = \frac{nd}{b\sqrt{\pi}} \sum_{\ell=-\infty}^{\infty} e^{-(z-d\ell)^2/b^2},$$





$$\lambda_d = n\mu^2/\epsilon_F, \quad \lambda_s = ng/\epsilon_F,$$

FIG. 2: Schematic phase structure of dipolar fermions as a function of λ_s and λ_d, showing the Fermi gas phase, the ferronematic phase, the onset of spatially varying magnetization and antiferrosmectic-C phase. The dashed line shows where the AFSC phase becomes favorable compared with the uniform unmagnetized interacting Fermi gas, and the dash-dot line the transition between the uniform Fermi gas and the FN phase. Beyond the upper dotted line the system becomes unstable against collapse.



analog of tensor forces, p-wave πNN inter.

$$V(\vec{r}_1,\vec{r}_2)^{ij}_{\alpha\alpha',\beta\beta'} = \frac{\mu^2}{r^3}\{\sigma^i_{\alpha\alpha'}(\delta_{ij} - 3\hat{r}_i\hat{r}_j)\sigma^j_{\beta\beta'}\} + g\delta_{\alpha\alpha'}\frac{\delta_{ij}}{3}\delta(\vec{r}_1-\vec{r}_2)\delta_{\beta\beta'},$$

analog of NN correlations

^{161}Dy and ^{163}Dy are the most magnetic fermionic atoms
novel nearly quantum degenerate dipolar Bose-Fermi gas

Similarly, for fermionic polar molecules, e.g., ^{40}K-Rb with large electric dipole moments

How describe nonequilibrium and equilibrium dilute and dense nuclear systems?

Hierarchy of time scales

- At $t \sim t_{cor} \sim 1$ Fm, decay of initial correlations (No Wick theorem, no Feynman diagrams). Growth of entropy.
- At $t \gg t_{cor}$ Eqs. for nonequilibrium Green functions at neglect of initial correlations – **are not solved.**
- At $t \sim t_{rel} \gg t_{cor}$, $t_{micro} \sim 1/E_T$ semiclassical approximation –KB kinetic equations for virtual particles (resonances) are solved in some codes within additional assumption.
- Quasiparticle approximation – **widely used.**
- At $t \gg t_{rel}$ nonideal hydro – **is used.**
- At still larger t – thermodynamical approach.

Often simplified approximation schemes are used: relativistic Boltzmann eq. (perturbation theory), ideal hydro with phenomenologically induced friction, etc.