Polarization, vorticity and shear in hadronic and heavy-ion collisions

South Africa - JINR Workshop on Theoretical and Computational Physics

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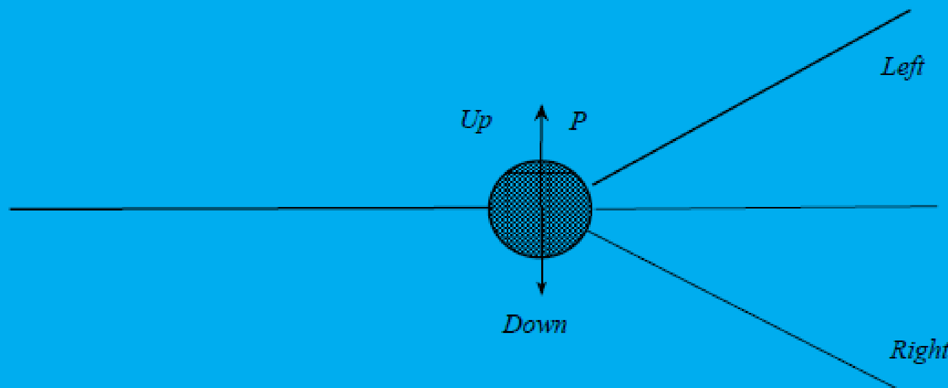
Main topics

- Polarization from quantum mechanics to hadronic reactions
- Polarization in heavy-ion collisions
- Vorticity modelling
- Shear inside hadrons

Single Spin Asymmetries

Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi\vec{N} \to \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$ $\vec{n}$ is the normal to the scattering plane.

Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n})$, $A = \frac{2Im(ab^*)}{|a|^2+|b|^2}$

Properties of SSA

The same for the case of initial or final state polarization.

Various possibilities to measure the effects: change sign of $\vec{n}$ or $\vec{P}$: left-right or up-down asymmetry.

Qualitative features of the asymmetry

Transverse momentum required (to have $\vec{n}$)

Transverse polarization (to maximize $(\vec{P}\vec{n})$)

Interference of amplitudes

IMAGINARY phase between amplitudes - absent in Born approximation

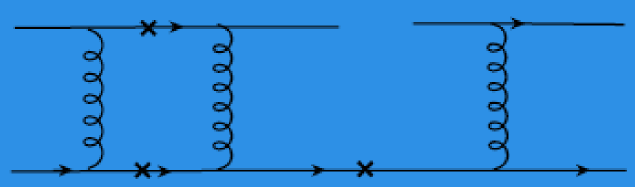
Phases and T-oddness

Clearly seen in relativistic approach:
$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$
Than: $d\sigma \sim Tr[\gamma_5 ....] \sim im\varepsilon_{sp_1p_2p_3}...$
Imaginary parts (loop amplitudes) are required to produce real observable.
$\varepsilon_{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$ each index appears once: $P-$ (compensate S) and $T-$ odd.
However: no real $T-$violation: interchange $|i> \leftrightarrow |f>$ is the nontrivial operation in the case of nonzero phases of $<f|S|i>^* = <i|S|f>$.
SSA - either T-violation or the phases.
DIS - no phases ($Q^2 < 0$)- real T-violation.

Perturbative PHASES IN QCD: Kane&Pumplin&Repko'78

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q-e$ scattering in DIS):



$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

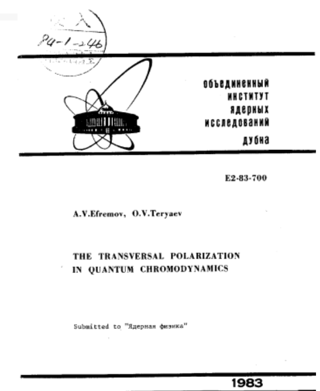
Large SSA "...contradict QCD or its applicability"

Quark mass -> twist 3

- Twist 4(EFP)->twist 3(ET)

The T -invariance, which is convenient to take into account just now, provides further simplifications. Note that in T - invariant theories the phase of hadron-parton amplitudes is fully determined by the Born approximation, because the coupling constant is real (and the cuts providing the imaginary part are absent after taking the discontinuity in M_x^2). By this reason, B^A is real and B^V is pure imaginary. (This fact also provides the absence of single asymmetries in Born approximation [15]). On the other hand, making use of translational in-

15. Christ A., Lee T. Phys.Rev., 1966, 143, p.1316.



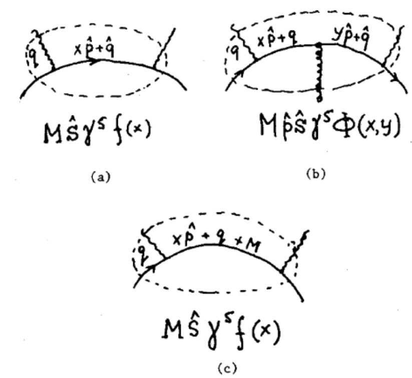


Fig.1

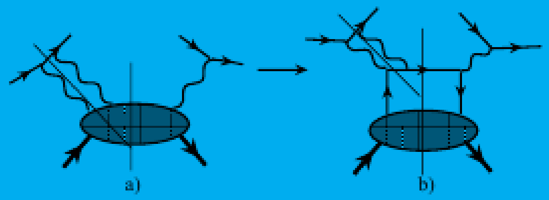
- DVCS: Anikin, Pire, OT'2000

S_T <-> Δ_T/M

T-odd effects: Low and high energies

Unpolarized leptons scattering on the transverse polarized nucleon. Completely analogous to the asymmetry in the scattering of transverse polarized neutrons on tensor polarized deuterons - modern low energy test of $T-$violation.

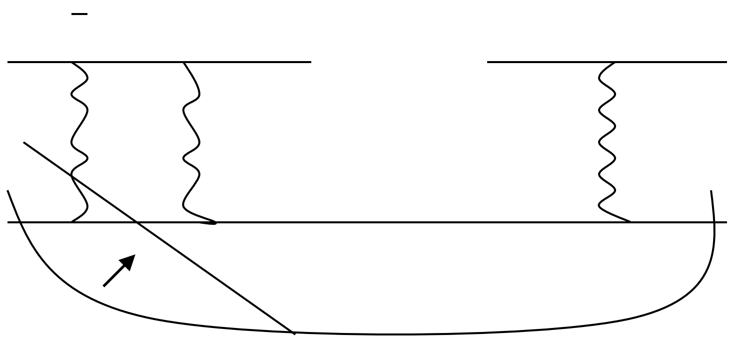
However, 2-photon exchange between lepton and nucleon - imaginary phase (due to QED -$\sim \alpha$).



What are QCD sources?

Phases from twist 3

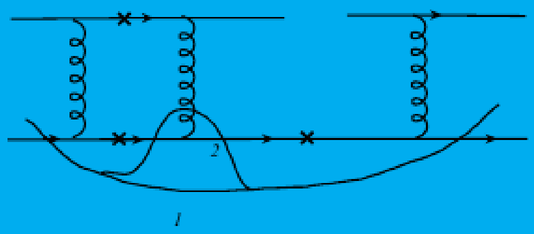
- Quarks – only from hadrons
- Various options for factorization – shift of SH separation



- New option for SSA: Instead of 1-loop twist 2 – Born twist 3: Efremov, OT (85, Fermionic poles); Qiu, Sterman (91, GLUONIC poles)

Twist 3 correlators

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop $\rightarrow$ Born diagram
At Large distances - quark distribution $\rightarrow$ quark-gluon correlator.
Physically - process proceeds in the external gluon field of the hadron.
Leads to the shift of α_S to non-perturbative domain AND
"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m p_T}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.

Sivers function

- D. Sivers: Hadron spin/quark transverse momentum correlation
- J. Collins: violates T-invariance!
- In JINR (A. Efremov, OT – understood that and did not try to introduce)
- S. Brodsky, D.-S. Hwang, I. Schmidt, J. Collins ’02 – FSI (Wilson lines)
- ~ Gluonic poles (twist 3 suppression compensated by pole)!

Λ-polarisation

- *Self-analyzing (spin-momentum couplins) in weak decay*
- *Directly related to s-quarks polarization: complementary probe of strangeness*
- *Hadronic processes*
- *Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: '87): Randomization – smearing – of the scattering plane*

Global polarization

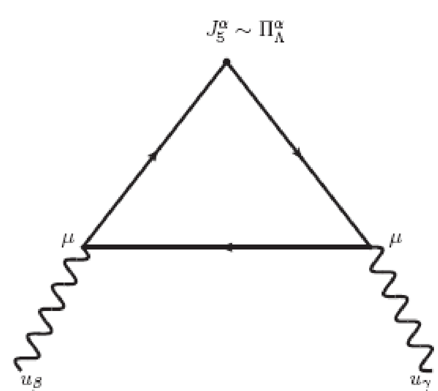
- *Global polarization normal to REACTION plane*
- *Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization*
- *Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!*
- *Due to locality of LS coupling while large orbital angular momentum is distributed*
- *How to transform rotation to spin?*

Anomalous mechanism – polarization similar to C(A)VE (talks of V. Zakharov, G. Prokhorov). LS-> classical/quantum Phases -> dissipation

- *4-Velocity is also a GAUGE FIELD (V.I. Zakharov et al):* $\mu \varrho = \mu J_0 V^0 \;\text{->}\; \mu J_\gamma V^\gamma$

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- *Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT '10)*
- *Analogous to anomalous gluon contribution to nucleon spin (Efremov,OT'88)*
- *4-velocity instead of gluon field!*



STAR, **Nature 548 (2017) 62-65**

Prediction

of polarization decrease with energy

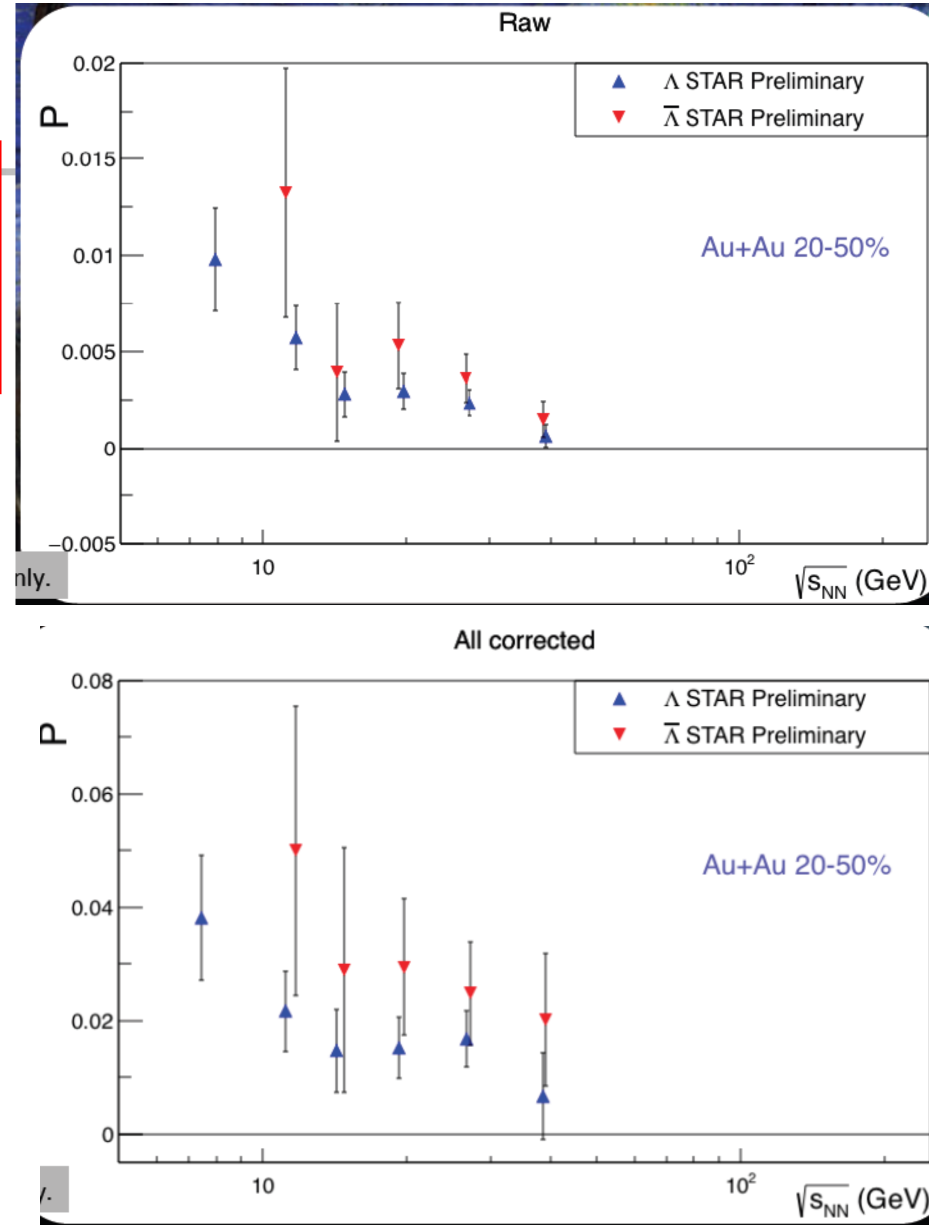
O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \tag{6}$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.



“Anomalous” mechanism

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

- Prediction of decrease with energy (due to chemical potential)

One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

- Prediction of P~1%

BAZNAT, GUDIMA, SORIN, AND TERYAEV

PHYSICAL REVIEW C **88**, 061901(R) (2013)

$$\langle P_\Lambda \rangle \sim \frac{\langle \mu^2 \rangle N_c H}{2\pi^2 \langle N_\Lambda \rangle}.$$

For numerical estimate at NICA energies, we take (see Fig. 3) $H = 30\ \mathrm{fm}^2 (c = 1)$ and, as typical values, $\langle \mu^2 \rangle = 900\ \mathrm{MeV}^2$, $\langle N_\Lambda \rangle = 15$ to get $\langle P_\Lambda \rangle \sim 0.8\%$. This value is

- Postdiction of larger polarization of antilambdas

ALEXANDER SORIN AND OLEG TERYAEV

PHYSICAL REVIEW C **95**, 011902(R) (2017)

The proportionality of the polarization to the square of the chemical potential related to C-even parity of axial current leads to the same sign of polarization of Λ and $\bar{\Lambda}$ hyperons. The smaller number of the latter should result in a larger fraction of the axial charge, corresponding to each antihyperon and to a larger absolute value of polarization. Detailed numerical sim-

TD approach to gauge anomaly (D.T. Son and P. Surowka; S.-Z. Yang, J.-H. Gao and Z.-T. Liang) -> gravitational anomaly

Gravitational chiral anomaly in hydrodynamics

*G. Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov, **Phys. Rev. Lett.** 129, 151601 (2022)*

- In heavy ion collisions (in particular, at **NICA**) a relativistic quantum liquid is formed with **extremely high vorticity ω^μ** and **acceleration a^ν**. The axial current is related to the hyperon **polarization**. **New** contributions to current and **polarization**:

KVE:
$$\boxed{j_A^\mu = \lambda_1(\omega_\nu\omega^\nu)\omega^\mu + \lambda_2(a_\nu a^\nu)\omega^\mu}$$

Transport coefficients

Connection with anomaly and gravity:

$$\boxed{\lambda_1 - \lambda_2 = 32\mathcal{N}}$$

$$\nabla_\mu j_A^\mu = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

- **The novel kinematical vortical effect (KVE)** depends only on vorticity and acceleration, but is independent of **temperature** and **chemical potential**, and is determined by a **quantum anomaly** in **curved** space.
- The effect exists even when there are **no gravitational** fields ('Cheshire cat" or "Structural stability of hydrodynamics")
- Derivation is essentially making use of the method developed (for the case of electromagnetic fields) In the paper
- Besides further exploration of the anomalous transport the collaboration in investigations of various polarization effects in hadronic and heavyi-ion collisions iwould be of great importance for us. Both the theoretical studies and modeling for experiments (in particular, at NICA and HIAF) are of interest

[27] Shi-Zheng Yang, Jian-Hua Gao, and Zuo-Tang Liang, Constraining non-dissipative transport coefficients in global equilibrium, Symmetry **14**, 948 (2022).

Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (Angular velocity ~ c/Compton wavelength)
- ~25 orders of magnitude faster than Earth’s rotation
- Differential rotation – vorticity
- P-odd :May lead to various P-odd effects
- Calculation in kinetic quark - gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes): Baznat,Gudima,Sorin,OT, PRC’13,16

Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- $L=\Sigma\ r \times p$
- Differential pseudovector characteristics – vorticity
- ω = curl v
- Pseudoscalar – helicity
- H ~ <(v curl v)>
- Maximal helicity – Beltrami chaotic flows v || curl v

Simulation in QGSM (Kinetics -> HD)

Baznat,Rogachevsky, Sorin,OT '13,15,17

$50 \times 50 \times 100$ cells $\quad dx = dy = 0.6$ fm, $dz = 0.6/\gamma$ fm

- Velocity

$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$

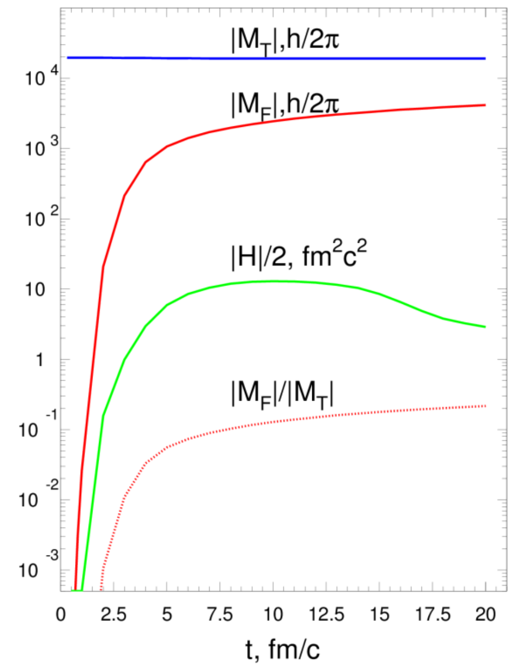
- Vorticity – from discrete partial derivatives

Vorticity studies in BLTP

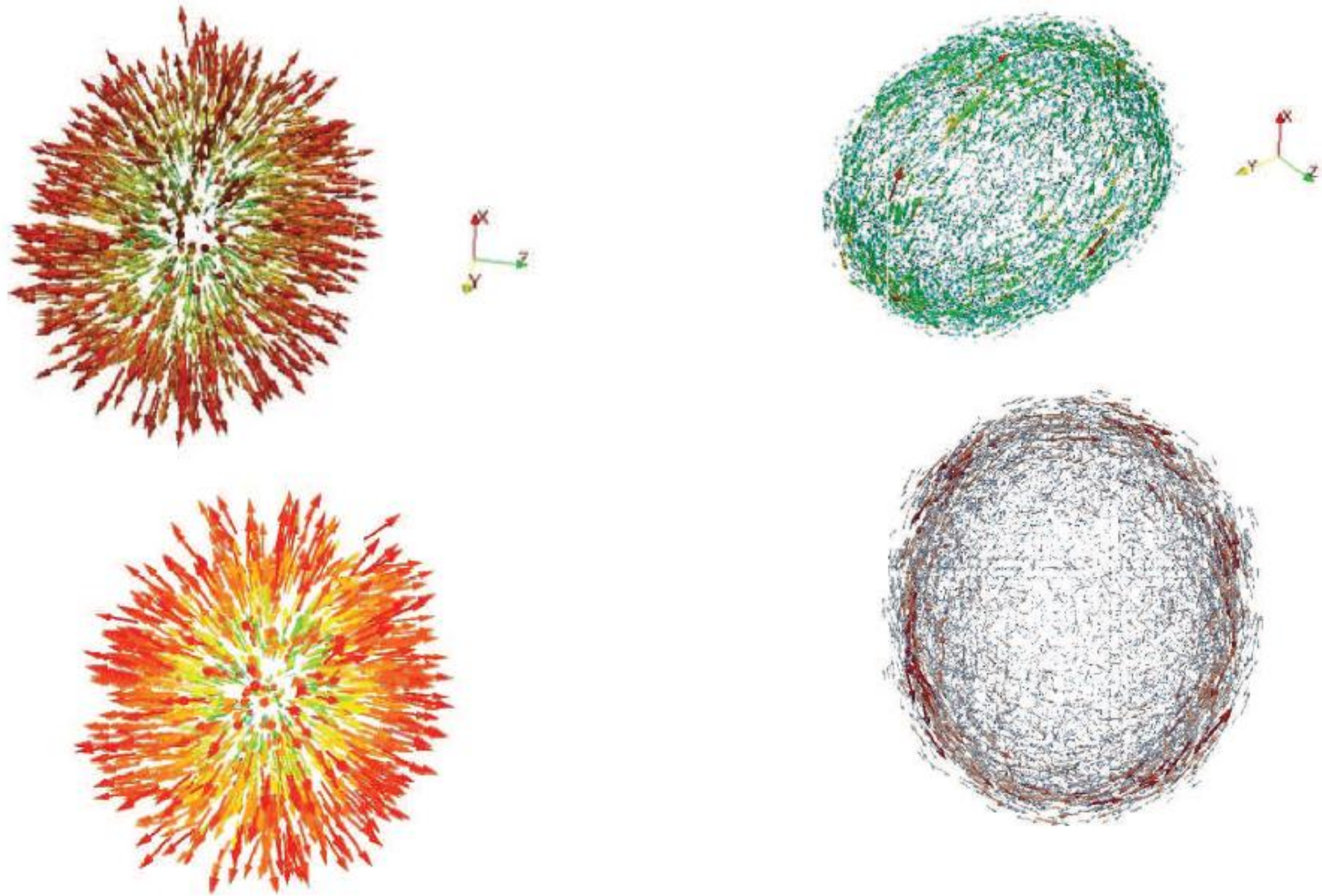
- Direct hydrodynamic approach
  Yu.B. Ivanov
- Advanced kinetic approaches PHSD
  E.Kolomeitsev, N. Tsefelnik,
  V. Voronyuk
- Effect to pion condensation
  D.N. Voskresensky
- Lattice studies of phase transitions under rotation V. Braguta, A. Roenko

Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- (~10% of total)
- Conservation of OAM with a good accuracy!

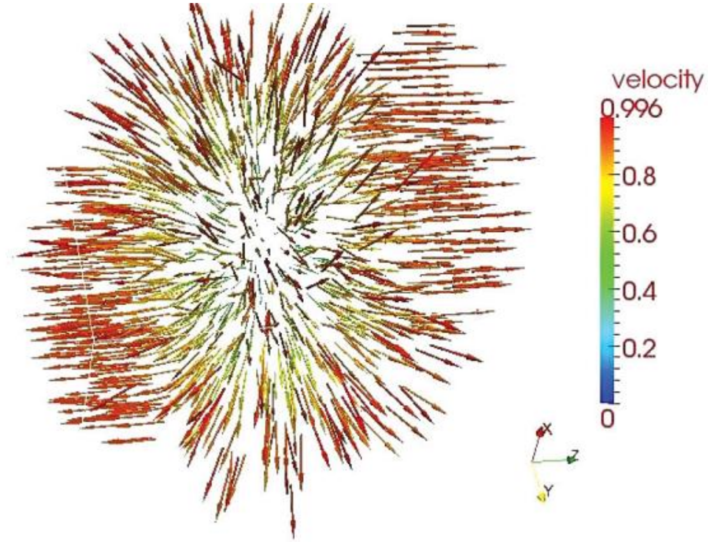


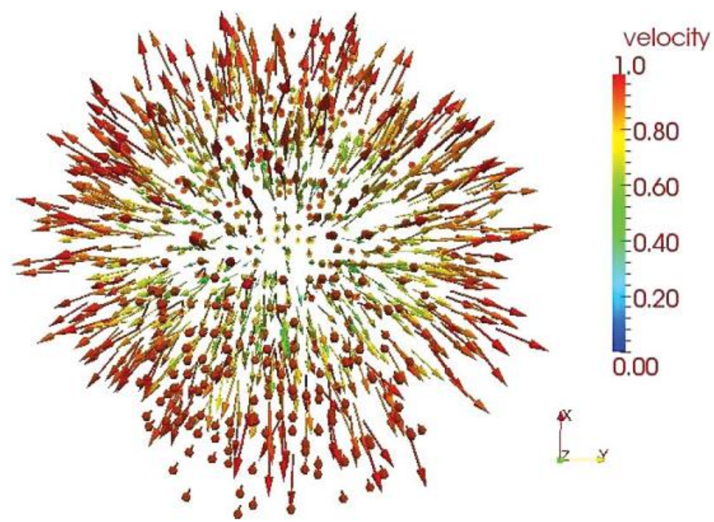
Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



Distribution of velocity ("Small Bang")

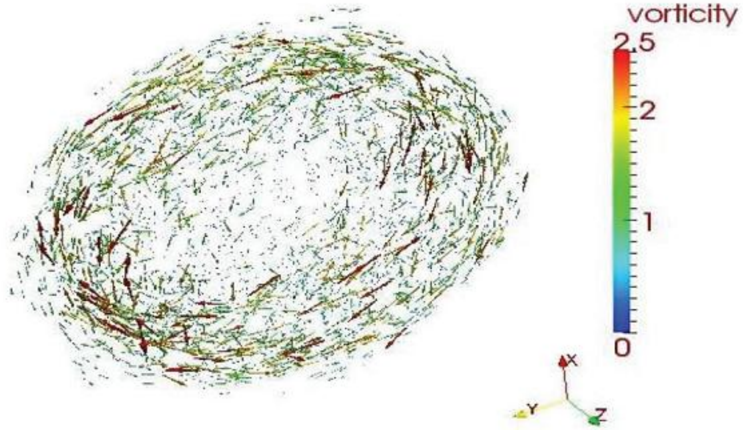
- 3D/2D projection
- z-beams direction
- x-impact paramater

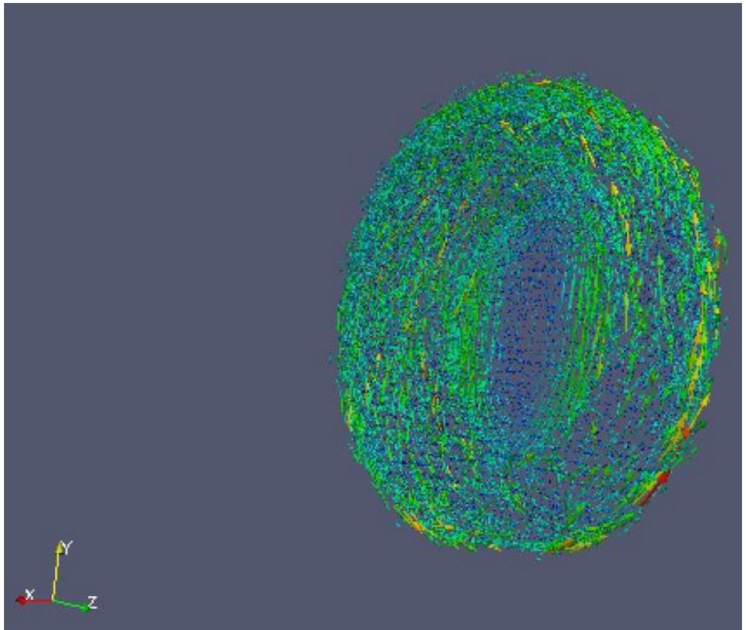


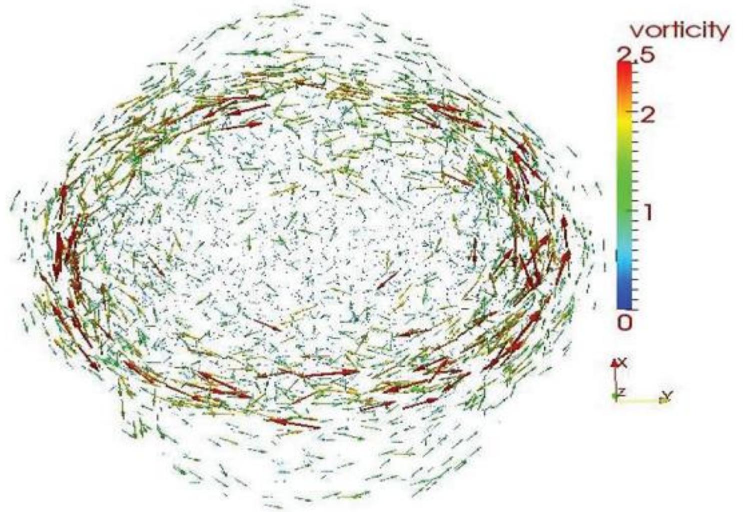


Distribution of vorticity ("small galaxies")

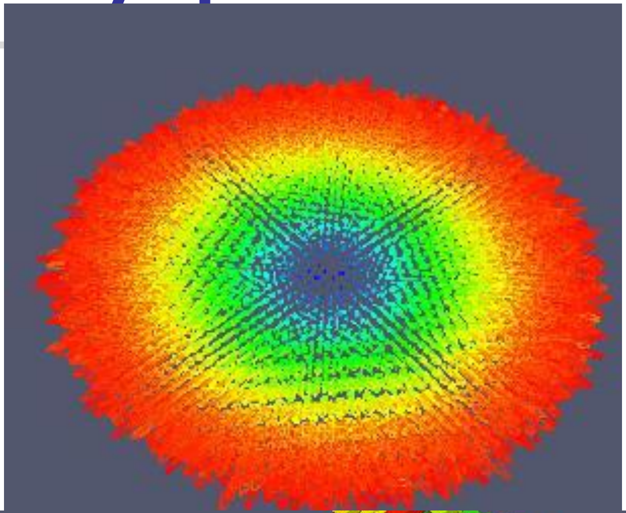
- Layer (on core - corona borderline) patterns



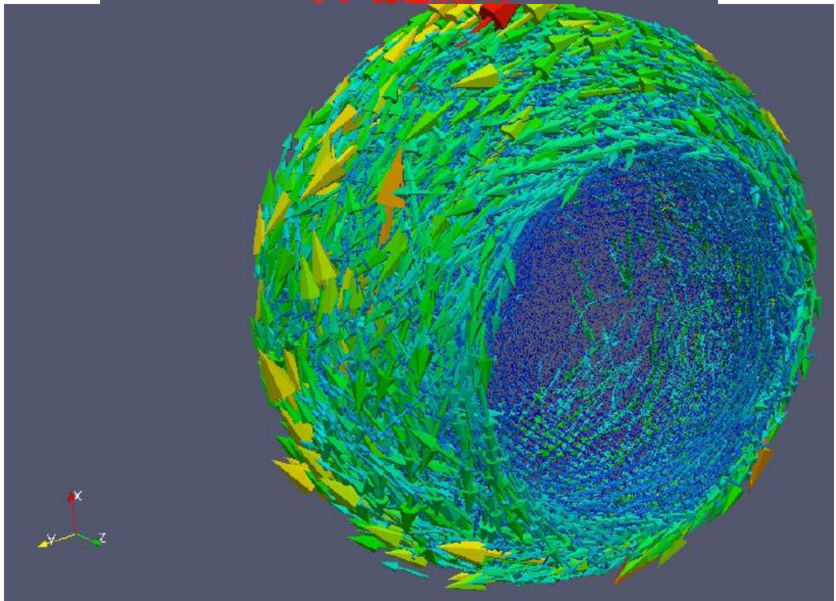




Velocity and vorticity patterns

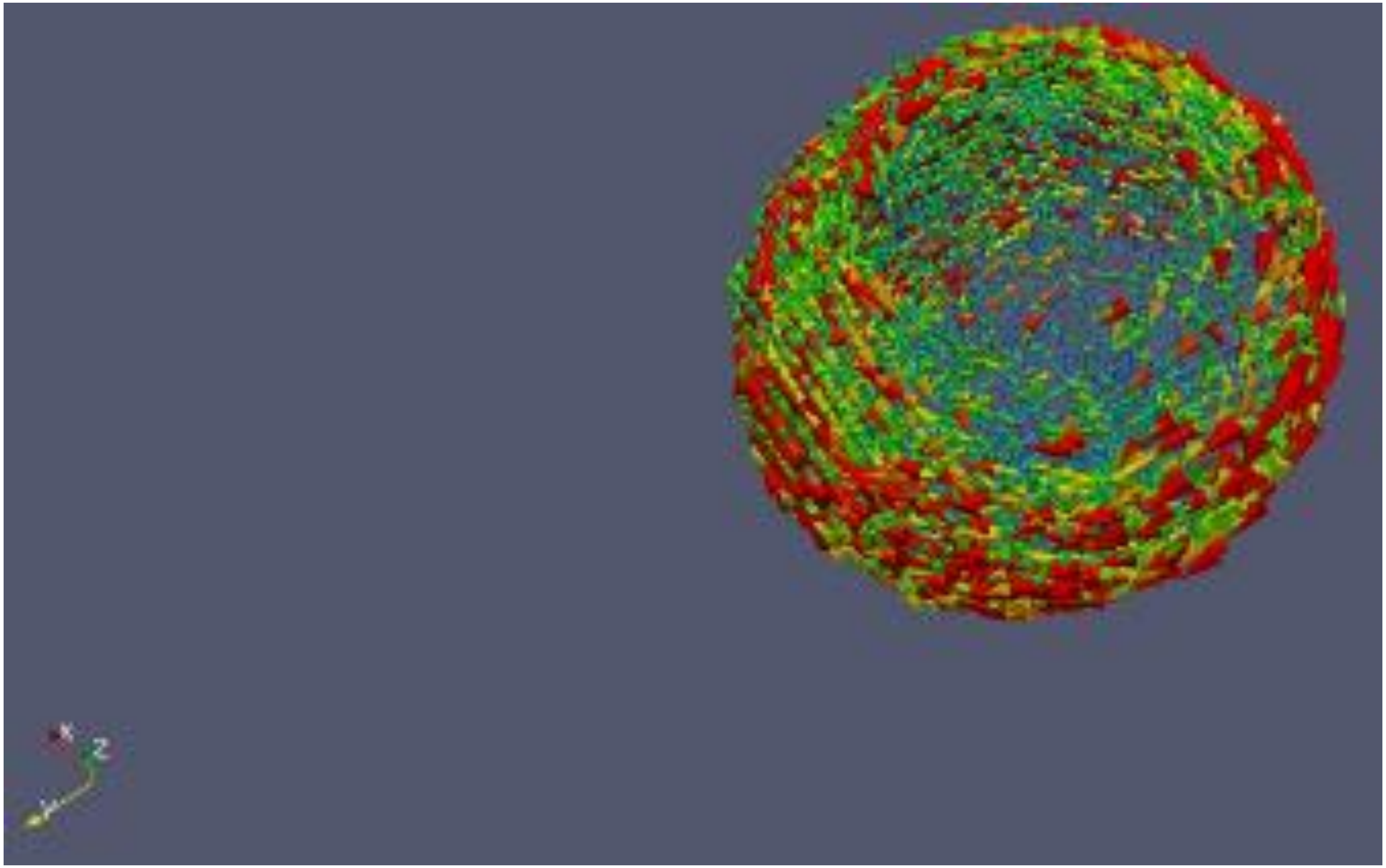


- Velocity

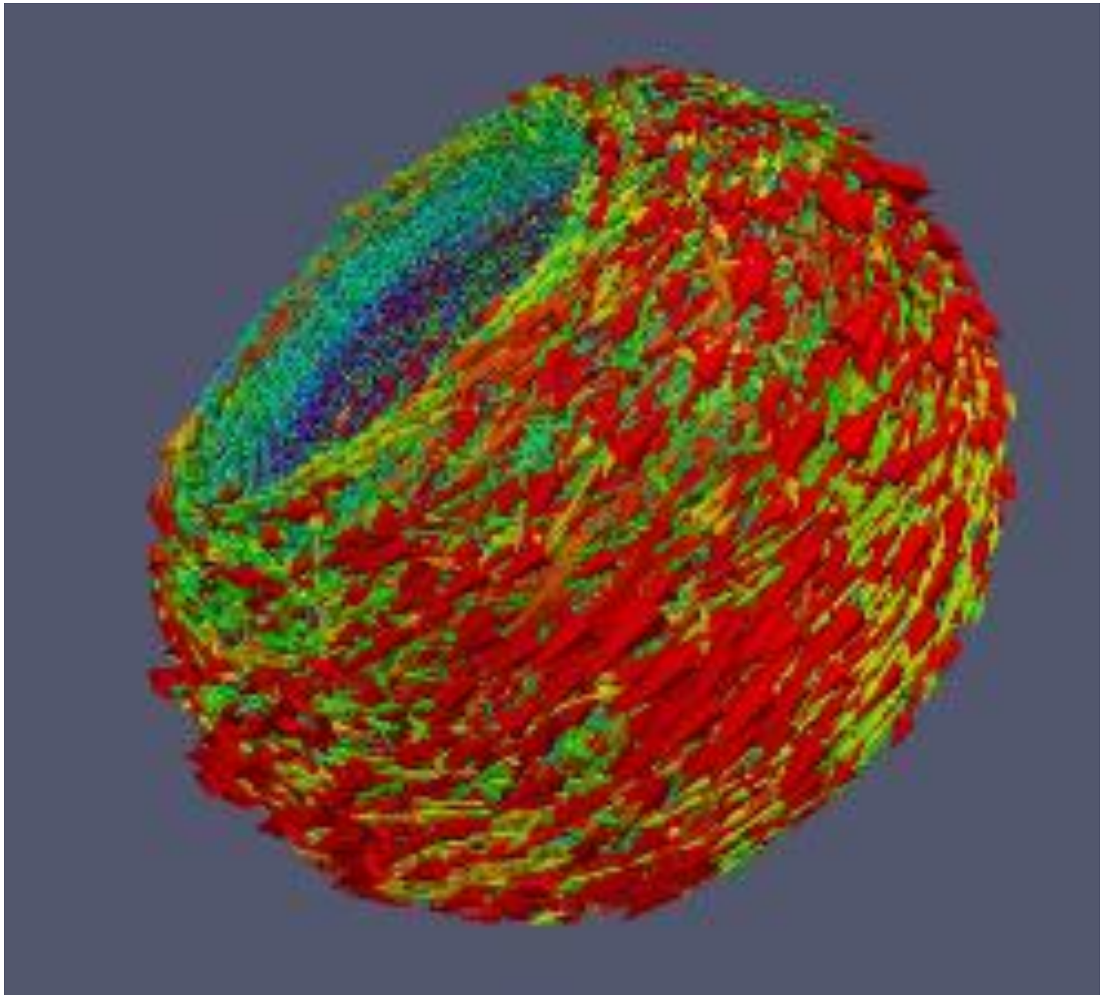


- Vorticity pattern – vortex sheets - due to L BUT cylinder symmetry!

Vortex sheet (fixed direction of L)

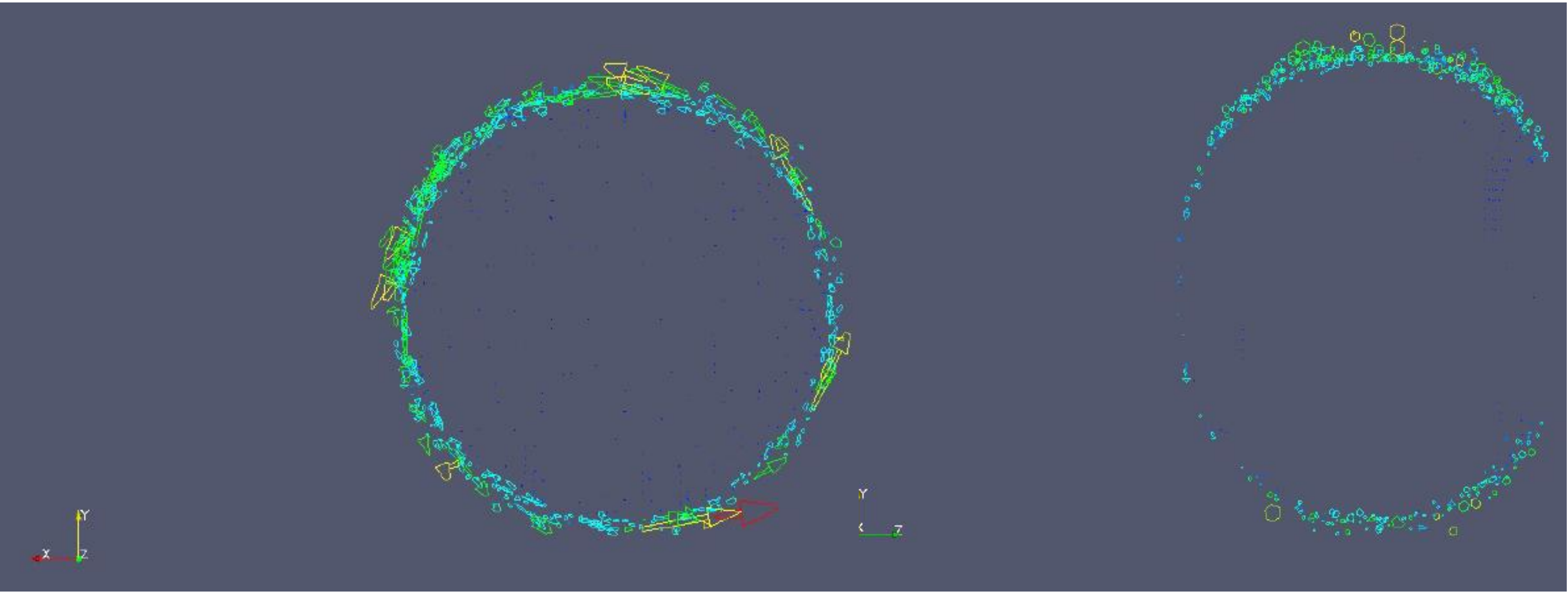


Vortex sheet （ Average over L directions ）



Sections of vorticity patterns

- Front and side views



Beltrami limit: is it close?

- Cauchy-Schwarz inequality:
- $r^2 = \langle(v \text{ curl } v)\rangle^2 / \langle(\text{curl } v)^2\rangle\langle v^2\rangle \leq 1$

- Maximal average cosine ~ 0.1

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **88**, 061901(R) (2013)

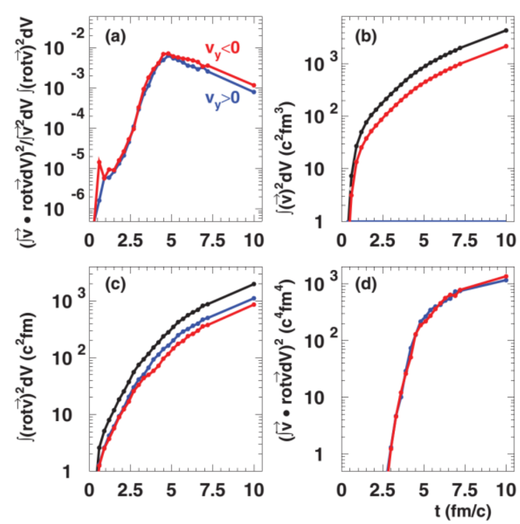
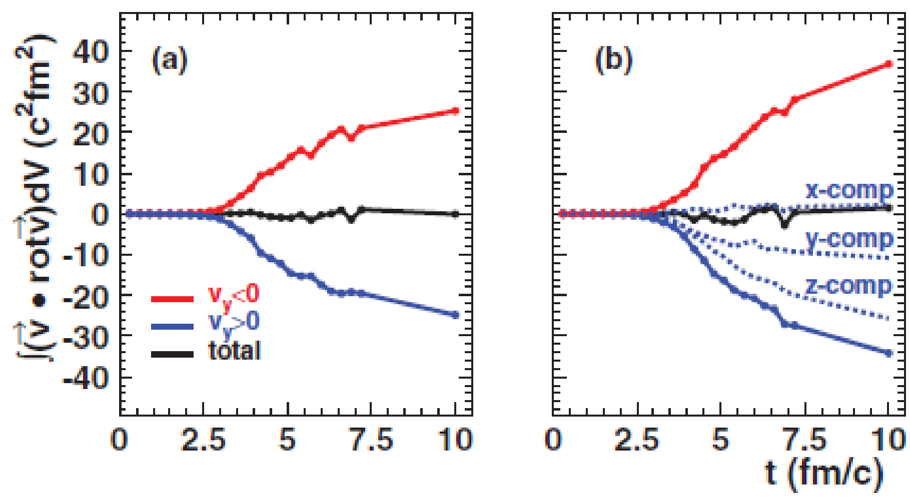


FIG. 4. (Color online) Time dependence of Cauchy-Schwarz bound for helicity in Au + Au at $s^{1/2} = 5$ GeV at impact parameter $b = 8$ fm(a); the integrated squares of velocity (b), vorticity (c), and helicity (d).

Helicity separation in QGSM **PRC88 (2013) 061901**

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane – required by boost!
- Confirmed in HSD (OT, Usubov, PR**C92 (2015) 014906**
- **zz: vorticity quadrupole structure**



Back to hadrons:

- Pressure inside protons from matrix elements of energy momentum tensors (gravitational formfactors)

LETTER

NATURE | www.nature.com/nature

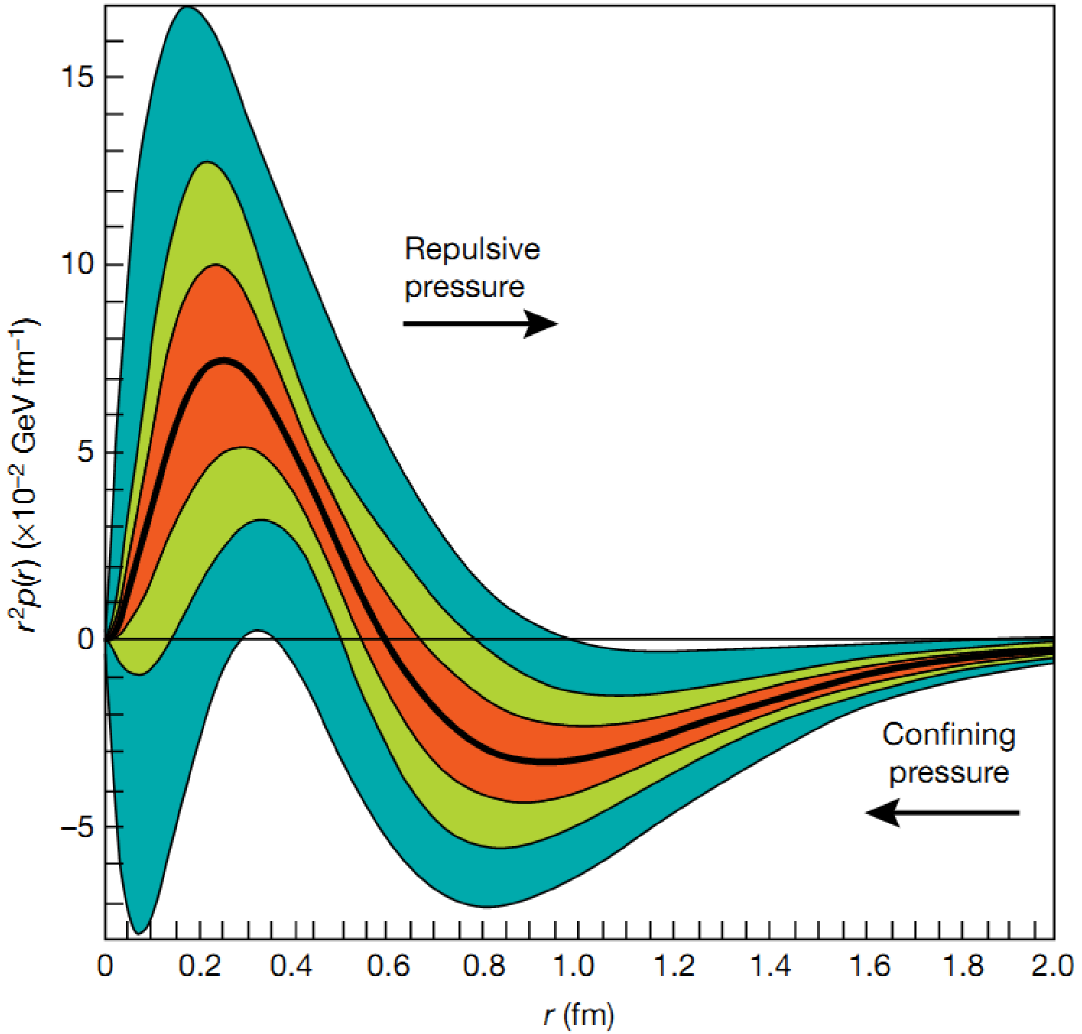
https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

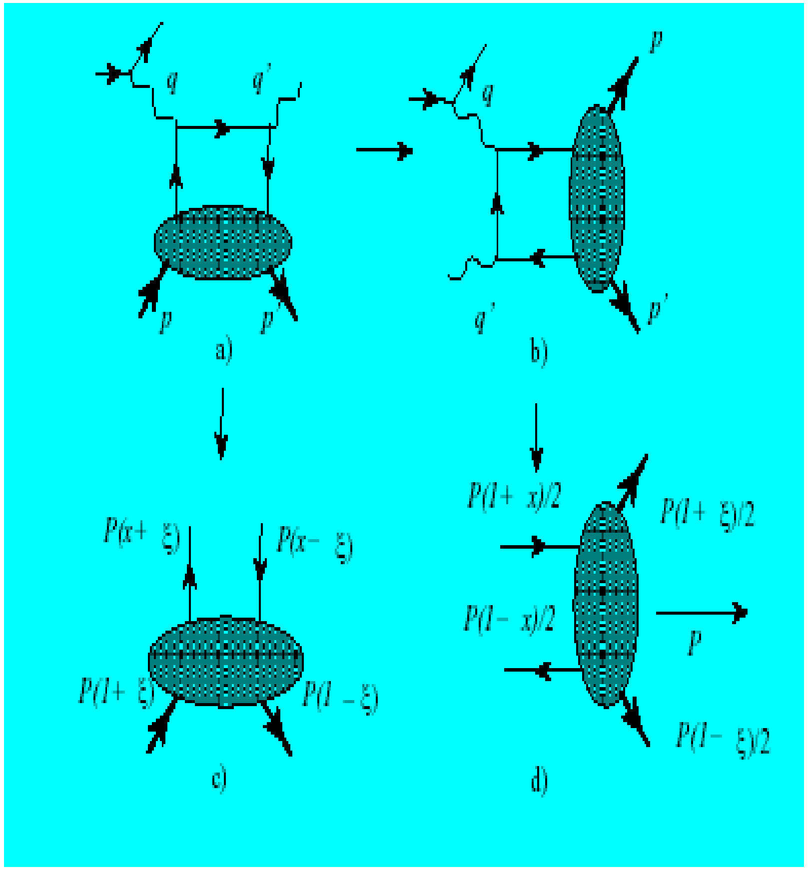
5. Teryaev, O. V. Gravitational form factors and nucleon spin structure. Front. Phys. 11, 111207 (2016)

15. Anikin, I. V. & Teryaev, O. V. Dispersion relations and QCD factorization in hard reactions. Fizika B 17, 151–158 (2008)



Road to timelike GrFFs: Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes (Diehl,Gousset, Pire,OT'98)



Gravitational FFs from Belle data on GDAs [1711.10086](1711.10086)

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

- Gravitational FFs are related to twist-2 GDAs

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\varepsilon^{\mu}(\lambda_1)\varepsilon^{\nu}(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi^q(z,\xi,W^2)$$

$$\int dz(2z-1)\Phi_q^+(z,\xi,W^2) = \frac{2}{(P^+)^2}\langle \pi^+(p_1)\pi^-(p_2)|T_q^{++}(0)|0\rangle$$

$$\langle \pi^0(p_1)\pi^0(p_2)|T^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[(sg^{\mu\nu} - P^{\mu}P^{\nu})\Theta_1 + \Delta^{\mu}\Delta^{\nu}\Theta_2\right]$$

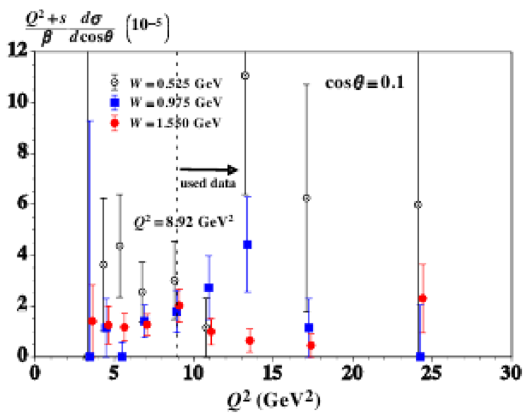
$$P = p_1 + p_2\ ,\ \Delta = p_1 - p_2$$

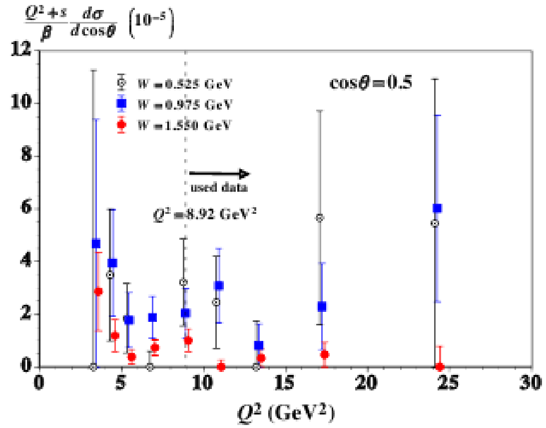
M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003

- Belle data and scaling : W=0.525,0.975, 1.55 GeV

$$\frac{(Q^2+s)d\sigma}{\beta d|\cos\theta|} \propto \left|\Phi^{\pi^0\pi^0}(z,\cos\theta,W,Q)\right|^2$$

$$\sqrt{\langle r^2\rangle_{\text{mass}}} = 0.39\,\text{fm},\ \sqrt{\langle r^2\rangle_{\text{mech}}} = 0.82\,\text{fm}$$





New EMT formfactor : "Shear viscosity" (OT'2020)

- From spherically symmetric object to fluid (EoS!)
- $T^{\mu\lambda} = (e+p)\, v^\mu v^\lambda - p\, g^{\mu\lambda}$
- $V^\mu = P^\mu/M$: correct normalization but no coordinate dependence
- Another suggestion (OT'19):
- $V^\mu = (P^\mu + a(t)\, k_T{}^\mu) /(M^2+a^2(t)\, k_T{}^2)^{1/2}$
- Viscosity: $\eta\, dv^\mu/d\, x_T{}^\lambda \sim E_\eta\, p^{[\mu} \Delta^{\lambda]}$
- NO such term in total EMT (but can be for quarks separately)
- Naïve T-oddness: phases in GPD channel from decays in TDA
- Phases <-> dissipation: polarization in pionic superfluidity model (V. I. Zakharov, OT' 17)

Timelike GrFF: Viscosity in GDA channel

- Viscosity:will correspond to Exotic $J^{PC}=1^{-+}$ meson (studied long ago without mentioning gravity: Anikin, Pire, Szymanowski,OT, Wallon'06)
- Spin: related to structure of matrix element: One index of EMT (0^{th} in rest frame) is carried by momentum and other by polarization vector - just what we need for viscosity
- NO for conserved EM: zero coupling for (G)DA!
- $\pi\eta$ pairs observation instead of $\pi\ \pi$ required
- Smallness of viscosity: related to smallness of exotic GDAs and ExEP violation?!

Estimate of viscosity

$(e+p)\, v^\mu v^\lambda \sim A\, P^\mu P^\lambda$

- $\eta\, dv^\mu/d\, x_T{}^\lambda \sim E_\eta\, p^{[\mu} \Delta^{\lambda]}$
- TD: e+p -> Ts
- η/s ($> 1/(4\pi)$)$\sim E_\eta T/AM$ (smallness due to ExEP and small coupling to exotics)
- Correct dependence on Planck constant recovered via $\Delta^\lambda \to -i\hbar\, d/d\, x_T{}^\lambda$
- Song,OT,Yoshida,2503.11316: relation in QCD factorization to structure of pseudoscalar mesons: $\eta/s \sim 0.05$

From time like FFs to HICs:properly averaged momentum correlations

- Shear : $dv^{\{\mu}/d\, x_{T\ \lambda\}} \sim \langle p_{\{i}\, p_{j\}}\rangle / \hbar$

- Vorticity: $dv^{\{[\mu}/d\, x_{T\ \lambda]} \sim \langle p_{[i}\, p_{j]}\rangle / \hbar$

- Helicity: $\langle v \text{ curl } v\rangle \sim e_{ijk}\langle p_i\, p_j\, p_k\rangle / \hbar$

Shear for deuterons: Spin 1 EMT and inclusive processes

- Forward matrix element -> density matrix
- Contains P-even term: tensor polarization $S^{\alpha\beta}$ New EMT FF
- $< P|T^{\alpha\beta}|P>= A P^{\alpha} P^{\beta} + T S^{\alpha\beta}$
- Symmetric and traceless: correspond to (average) shear forces
- Cf with spin ½ and spin 1 vector polarization : P-odd vector polarization requires another vector (q) to form vector product

SUM RULEs

- Efremov,OT'82 : zero sum rules:
- Current conservation: 1st moment: also in parton model by Close and Kumano (90)
- EMT conservation: 2nd moment (forward analog of Ji's SR: $\sum$ B=0 <=> $\sum$ T=0)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT'09
- No monopole spin-gravity coupling!

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

- Tensor polarization -coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$ (AVE,OT’9

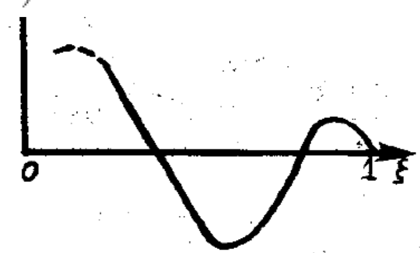
$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

=0 for ExE

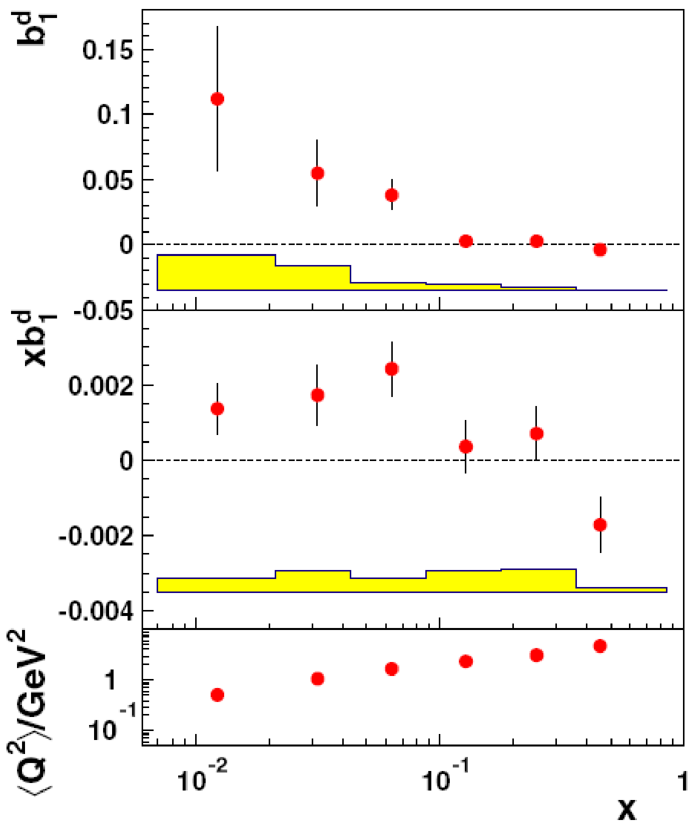


$$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$$

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

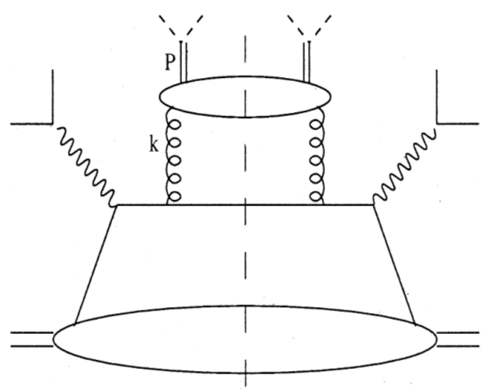
- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective tensor polarized glue << sea)



Fragmentation functions

- Tensor polarized fragmentation functions: (Szymanowski, Schaefer, OT'99)

A. Schäfer et al. / Physics Letters B 464 (1999) 94–100



- Suggestion'21: zero SRs (analogous to momentum SR) may probe the (Ex)EP for hadrons inside partons (EIC: gluons)

NICA (Nuclotron based Ion Colider fAcility)

– the flagship project in HEP of Joint Institute for Nuclear Research (JINR)

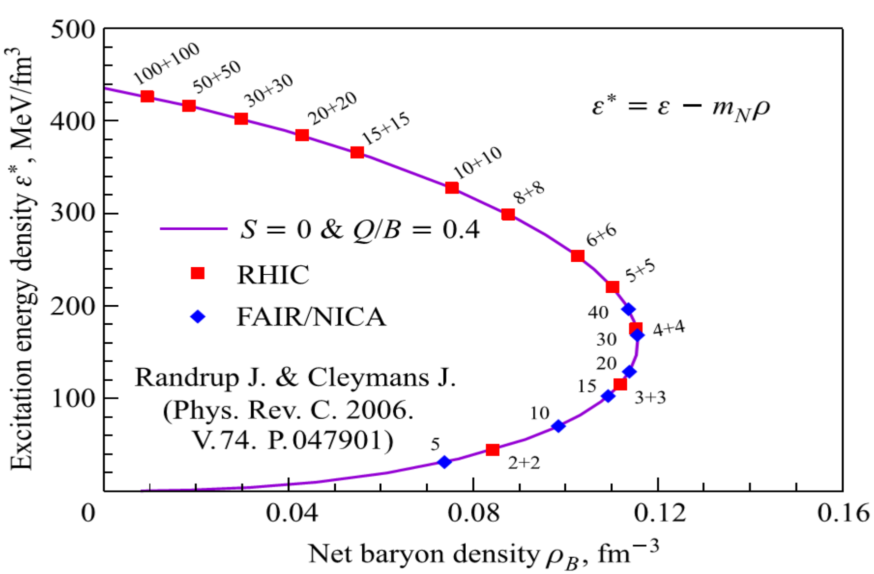
Main targets of "NICA Complex":

- ***study of hot and dense baryonic matter***
- *investigation of hadronic spin structure through various polarization phenomena*
- *development of accelerator facility for HEP @ JINR providing intensive beams of relativistic ions from p to Au*

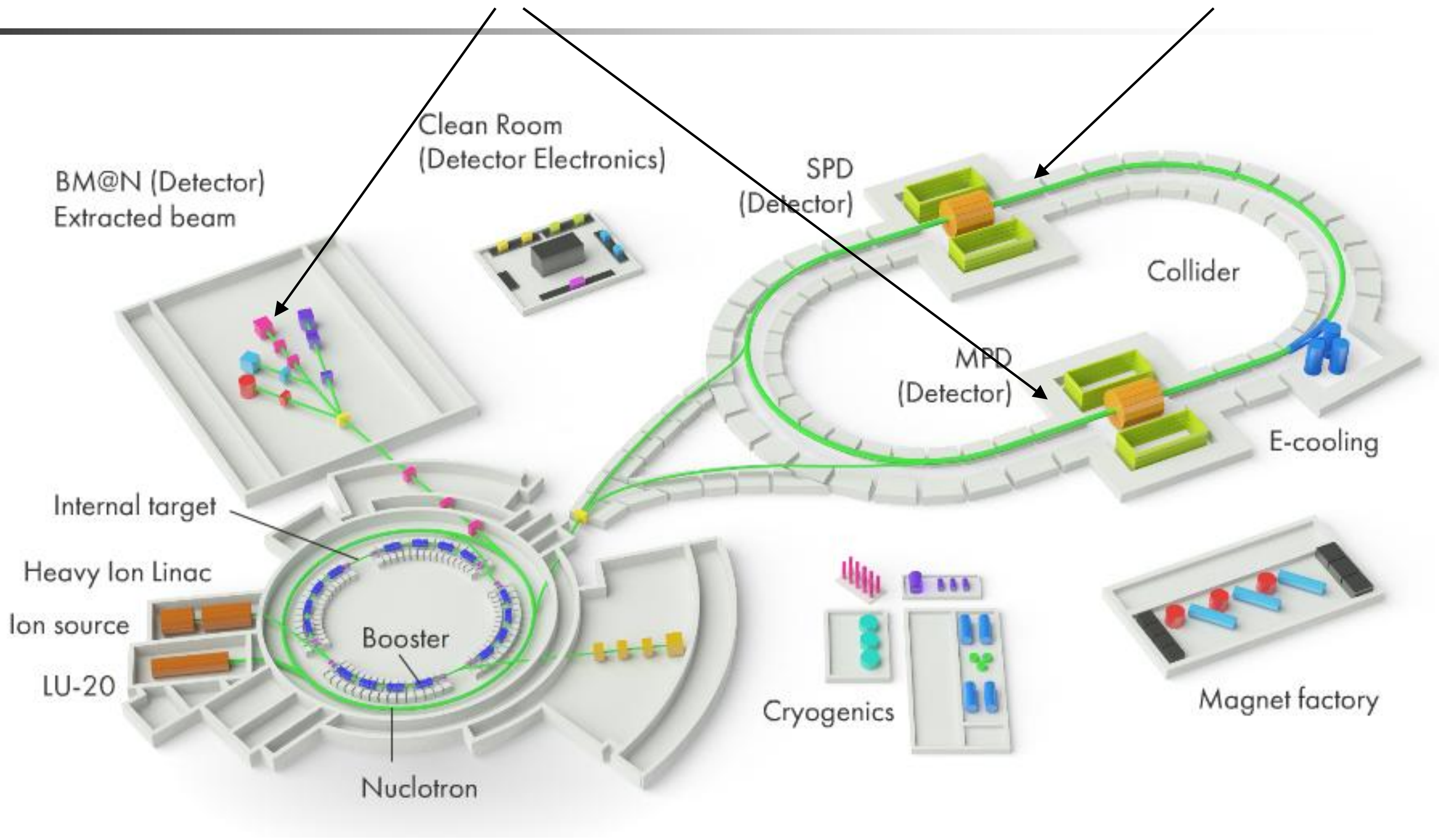
polarized protons and deuterons with energy up to

*√SNN = 11 GeV (Au 79+, **L ~ 10 32** cm -2 c -1)*

*V (p, L ~ **10 32** cm -2 c -1)*



NICA: heavy ions and hadrons



Conclusions

- Polarization vorticity and shear appear in various branches of High-Energy Physics
- Also relations to condenced matter theory, gravity…
- Studies at NICA plannes

How to add SSAs?

- NOT just sum (P<1!)
- Similar to velocities in special relativity (OT, 2204.08796 and PRC)

$\vec{P}_0 = \vec{n}\frac{2\Im(ab^*)}{|a|^2+|b|^2}$ $P_f = \frac{(|a|^2+|b|^2)P + 2\Im(ab^*)}{|a|^2+|b|^2+2P\Im(ab^*)} = \frac{P+P_0}{1+PP_0}$ $\hbar \leftrightarrow c$ $P_f^2 = \frac{(\vec{P}+\vec{P}_0)^2 - [\vec{P}\vec{P}_0]^2}{(1+(\vec{P}\vec{P}_0))^2}$

- Transition from “macro” to micro” d.o.f.
- Dominance of FSI (Polarizing FF) or ISI (Sivers) corresponds to Galilean (c->∞) and Carroll (c->0) limits (in preparartion)

CONCLUSIONS

- SSA in QCD – phases: stem from AV-1978
- To be studied at EIC, SPD,…
- HIC: QFT/Statistics/gravity

phases<-> dissipation?

Quantum<-> classical?

- “Macro” <-> “Micro” ?
- Relation to LS in Condenced matter?

Spin-gravity interactions

- How to describe hadron spin/gravity(inertia) couplings?
- Matrix elements of Energy- Momentum Tensor
- May be studied in non-gravitational experiments/theory
- Simple interpretation in comparison to EM field case

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Diistributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT ’09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \quad = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\rm Gr}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

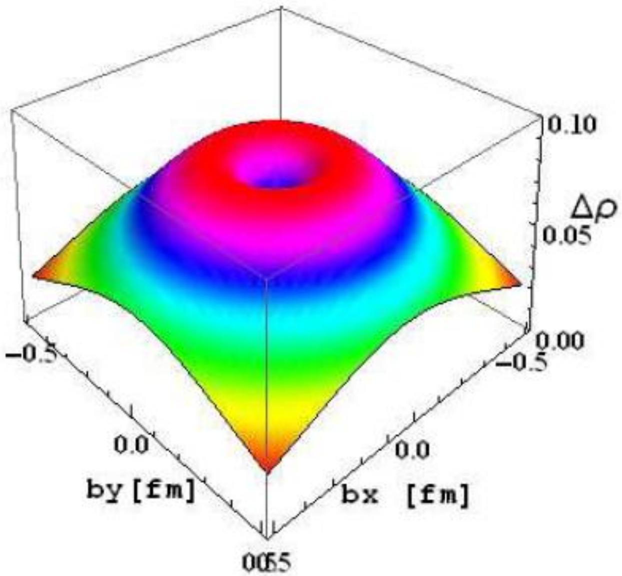


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q) \qquad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G} \langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot \vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot \vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

VOLUME 68 | 13 JANUARY 1992 | NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195

(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \dfrac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim \mathbf{P}^2$) is compensated by $1/\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99)

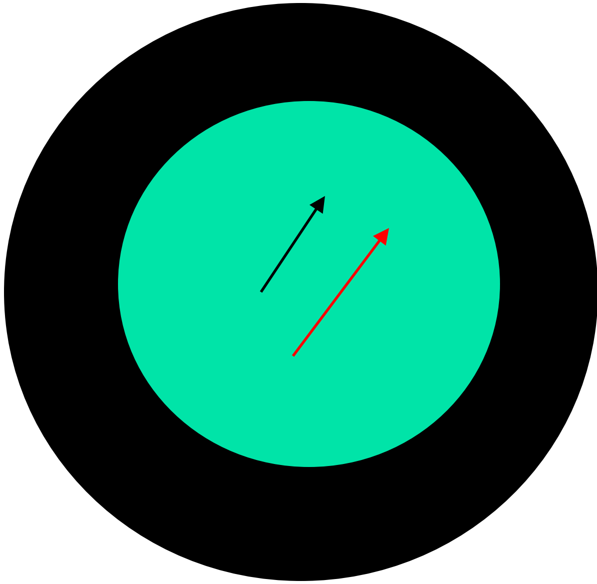
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $<P+q|T^{mn}|P-q> = P^{\{m}<P+q|J^{n\}}|P-q>/e$ up to the terms linear in q
- Special role of g=2 for any spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Torsion – acts only on spin (violates EP)

Dirac eq+FW transformation-Obukhov,Silenko,OT, **arXiv:1410.6197**

- Hermitian Dirac Hamiltonian

$$e_i^{\widehat{0}} = V\,\delta_i^0, \qquad e_i^{\widehat{a}} = W^{\widehat{a}}{}_b\left(\delta_i^b - cK^b\,\delta_i^0\right)$$

$$\mathcal{H} = \beta mc^2 V + q\Phi + \frac{c}{2}\left(\pi_b\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right) + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma} - \Upsilon\gamma_5\right),$$

$$ds^2 = V^2c^2dt^2 - \delta_{\widehat{a}\widehat{b}}W^{\widehat{a}}{}_cW^{\widehat{b}}{}_d\left(dx^c - K^c cdt\right)\left(dx^d - K^d cdt\right)$$

$$\mathcal{F}^b{}_a = VW^b{}_{\widehat{a}}, \qquad \Upsilon = V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}}, \qquad \Xi^a = \frac{V}{c}\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\left(\Gamma_{\widehat{0}\widehat{b}\widehat{c}} + \Gamma_{\widehat{b}\widehat{c}\widehat{0}} + \Gamma_{\widehat{c}\widehat{0}\widehat{b}}\right)$$

- Spin-torsion coupling

$$-\frac{\hbar cV}{4}\left(\boldsymbol{\Sigma}\cdot\check{\boldsymbol{T}} + c\gamma_5\check{T}^{\widehat{0}}\right)$$

$$\check{T}^\alpha = -\frac{1}{2}\eta^{\alpha\mu\nu\lambda}T_{\mu\nu\lambda}$$

- FW – semiclassical limit - precession

$$\Omega^{(T)} = -\frac{c}{2}\check{\boldsymbol{T}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'}, \left\{\boldsymbol{p}, \check{T}^{\hat{0}}\right\}\right\} + \frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)}, \left(\{\boldsymbol{p}^2, \check{\boldsymbol{T}}\} - \{\boldsymbol{p}, (\boldsymbol{p}\cdot\check{\boldsymbol{T}})\}\right)\right\}$$

Experimental bounds for torsion

- Magnetic field+rotation+torsion

$$H = -g_N \frac{\mu_N}{\hbar} \boldsymbol{B} \cdot \boldsymbol{s} - \boldsymbol{\omega} \cdot \boldsymbol{s} - \frac{c}{2} \check{\boldsymbol{T}} \cdot \boldsymbol{s}.$$

- Same ’92 EDM experiment

$$\frac{\hbar c}{4} |\check{T}| \cdot |\cos\Theta| < 2.2 \times 10^{-21}\,\mathrm{eV}, \qquad |\check{T}| \cdot |\cos\Theta| < 4.3 \times 10^{-14}\,\mathrm{m}^{-1}$$

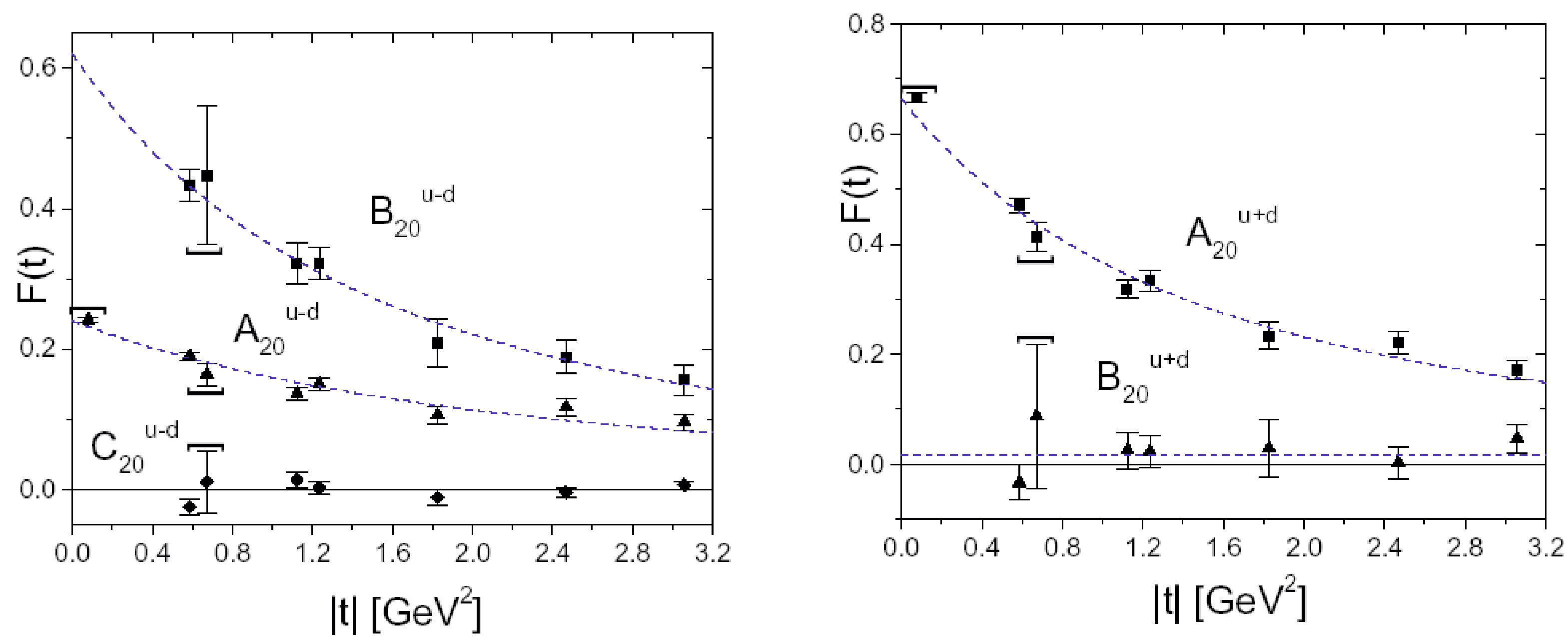
- New(based on Gemmel et al ’10)

$$\frac{\hbar c}{2} |\check{T}| \cdot |(1-\mathcal{G})\cos\Theta| < 4.1 \times 10^{-22}\,\mathrm{eV}, \qquad |\check{T}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\,\mathrm{m}^{-1}.$$

$$\mathcal{G} = g_{He}/g_{Xe}$$

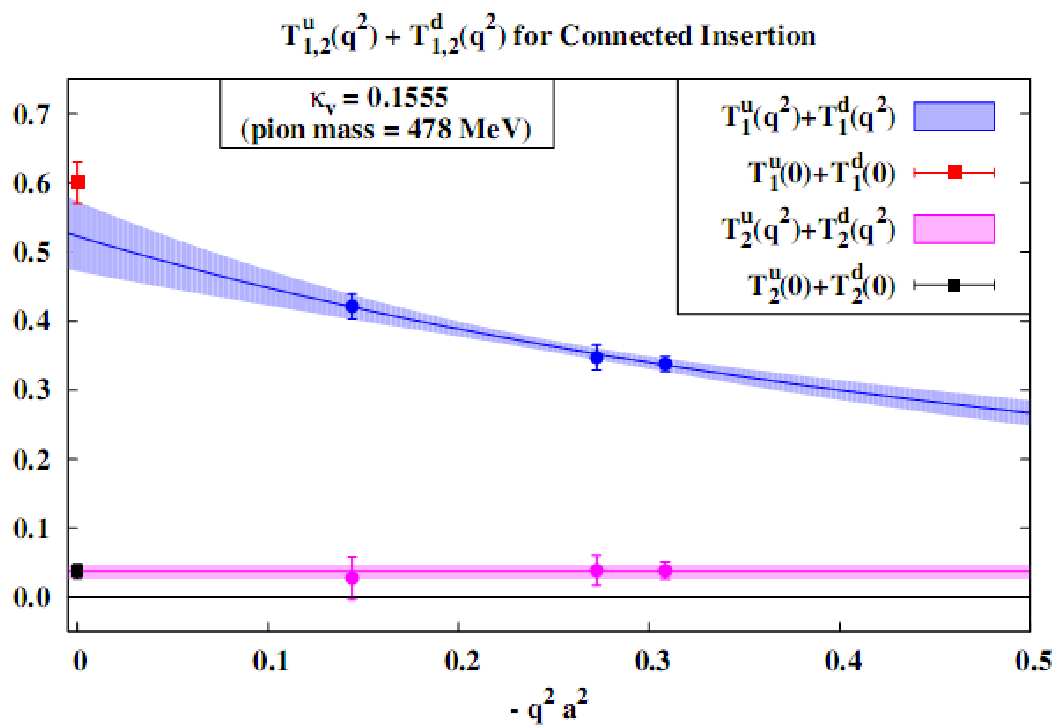
Generalization of Equivalence principle

- Various arguments: AGM $\approx$ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Recent lattice study (M. Deka et al. **arXiv:1312.4816**)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravity-proof confinement (should the hadrons survive enetering Black Hole?)?!

Conclusionss

- Rotation in heavy-ion collisions – essentially non-inertial frame
- Related P-odd effects are not numerically large (smearing) but may be observable

- BACKUP SLIDES

Sum rules for EMT (and OAM)

- First (seminal) example: X. Ji’s sum rule (’96). Gravity counterpart – OT’99
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Yes, if provide correct prescription to gluonic pole (OT’14)

Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part!
$$\sum \int \int dx_1 dx_2 \frac{T(x_1,x_2)}{x_1 - x_2 + i\varepsilon} = 0$$
- SR: $\sum \int dx T(x,x) = 0$ (but relation of gluon Sivers to twist 3 still not founs – prediction!)
- Can it be valid separately for each quark flavour: nodes (related to "sign problem")?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: If GI separation of EMT – forbidden: SR valid separately!

Another manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$\langle P,S|\bar{\psi}(0)\gamma^{\nu}D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

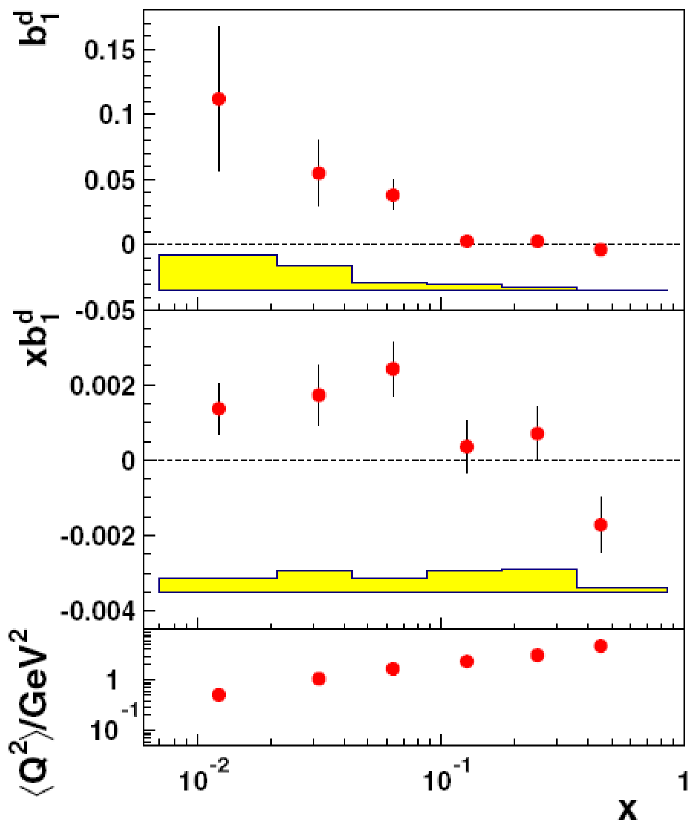
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}\delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$\sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2)$ =0 for ExEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea) – for valence: $\int_0^1 C_i^T(x)dx = 0$



CONCLUSIONS

- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studing EMT matrix element
- Torsion and EP are tested in EDM experiments
- SR's for deuteron tensor polarization- indirectly probe EP and its extension separately for quarks and gluons

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible