Confluence of Quantum Matter, Topology and Strong correlation

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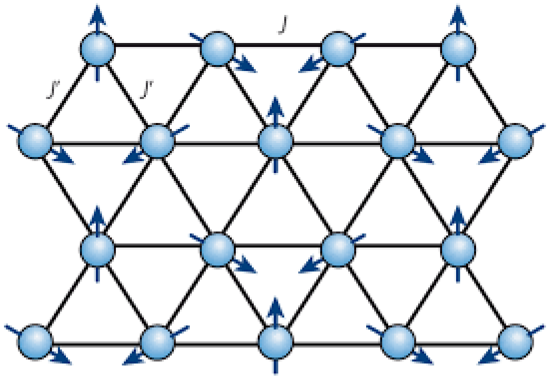
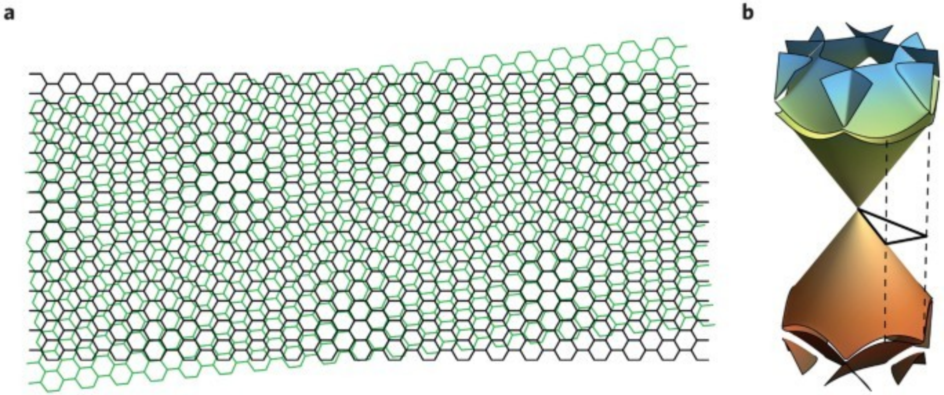
PART — I

Some definitions and backgrounds

Quantum matter

- The properties of materials can only be explained by the laws of quantum mechanics.
  - Moire Lattices
  - Low dimensional magnetism





Strong correlations

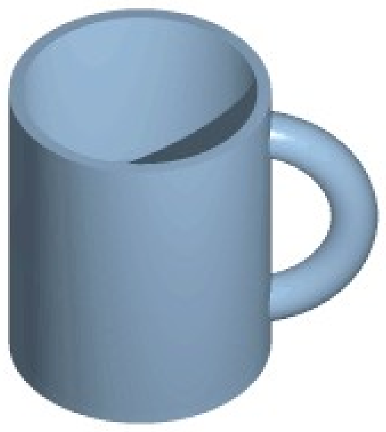
- The Coulomb repulsion between electrons can not be neglected
  - Mott insulators
  - Nagaoka Ferromagnetism

Topology

- Study of relation between Hamiltonians:
  - If one Hamiltonian can be deformed to other smoothly*, then they are topologically same.

*By smoothly we mean without changing the topological invariant.

Intuition of topological invariant

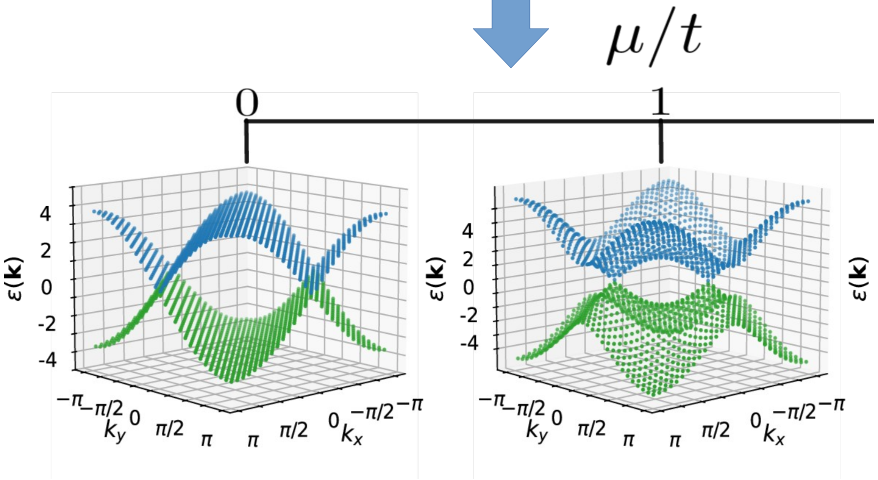


Topological invariant is number of holes in the cup and doughnut.

Topology in condensed matter

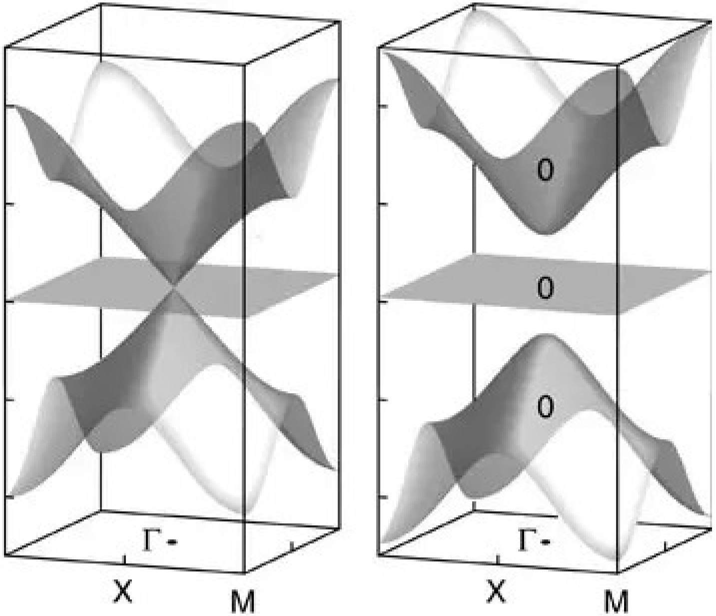
Surface (of cup and doughnut) in real space

Surface (of Hamiltonian) in momentum space



Topological invariant in condensed matter

Number of holes ⟷ Chern number



Chern number

Chern number

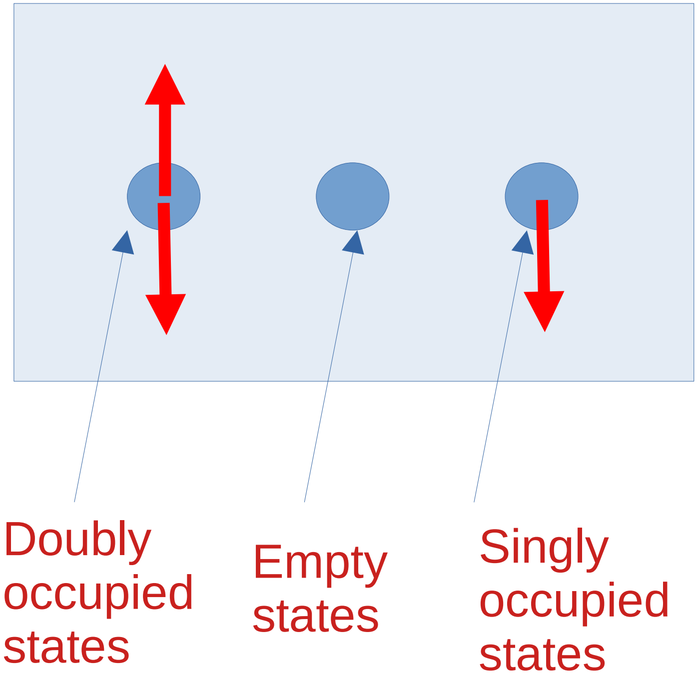
Momentum space (dk)

$$2\pi W = \iint_{\mathrm{BZ}} \mathbf{\Omega}(\mathbf{k}) \cdot d\mathbf{S}\,,$$

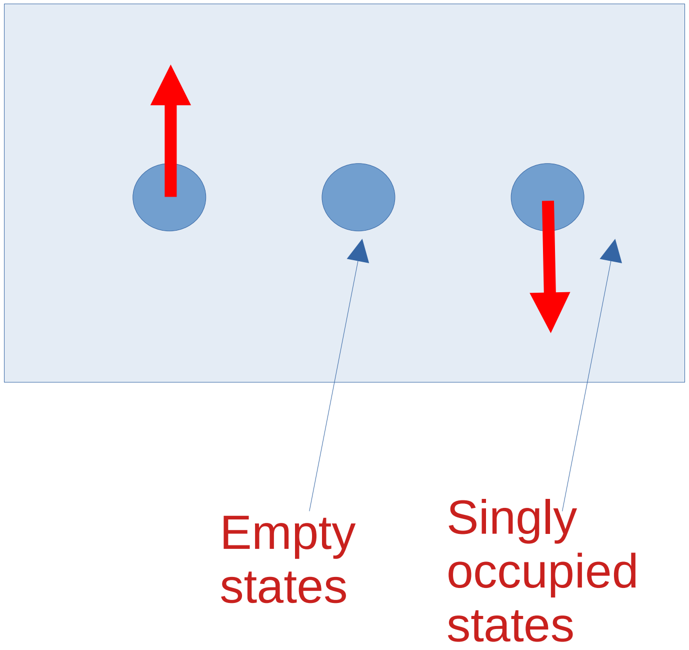
$$\mathbf{\Omega}(\mathbf{k}) = i\left[\left\langle \frac{\partial \psi(\mathbf{k})}{\partial k_x} \middle| \frac{\partial\, \psi(\mathbf{k})}{\partial k_y} \right\rangle - \left\langle \frac{\partial \psi(\mathbf{k})}{\partial k_y} \middle| \frac{\partial\, \psi(\mathbf{k})}{\partial k_x} \right\rangle\right]$$

The method **fails** for strong e-e correlation

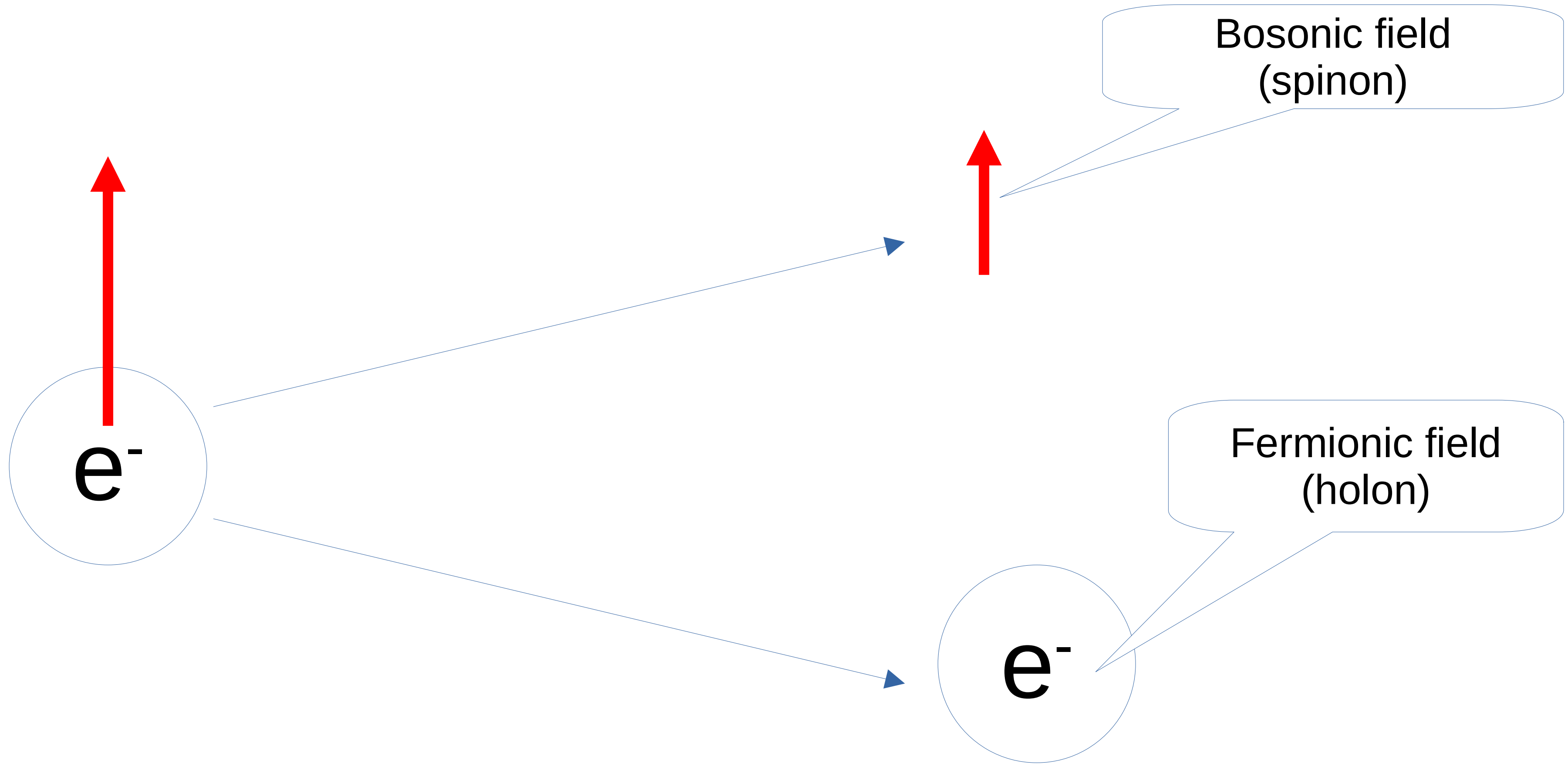
Weak correlation



Strong correlation



Fractionalization of electrons



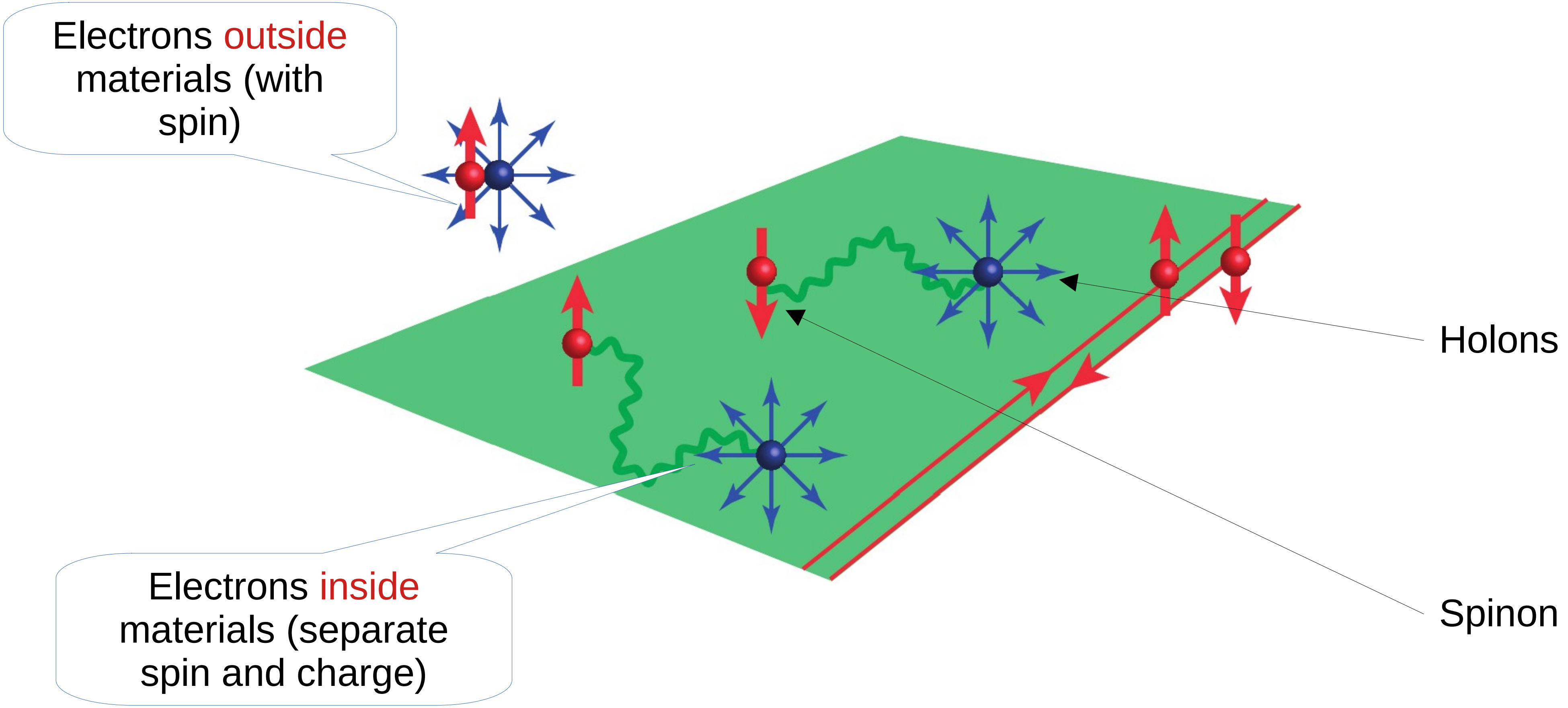
Quantum Matter

Topology

Strong Correlation

Fractionalized Topological Quantum Matter

Schematic of fractionalized topological insulators



Numerical methods

- DMRG and its variants
- Lanczos method
- Monte-Carlo and its variants
- DFT+U

1) Avella, A., and Ferdinando M., Strongly correlated systems: numerical methods. Vol. 176. Springer Science & Business Media, 2013.
2) Sénéchal, D., Tremblay, A. M., & Bourbonnais, C. (2004). Theoretical methods for strongly correlated electrons. Springer Science & Business Media.

Analytical methods

- Renormalization group techniques
- Functional derivatives, mean field, self-consistent methods
- Slave particle and extensions

1) Avella, A., and Ferdinando M., Strongly correlated systems: numerical methods. Vol. 171. Springer Science & Business Media, 2011.
2) Sénéchal, D., Tremblay, A. M., & Bourbonnais, C. (2004). Theoretical methods for strongly correlated electrons. Springer Science & Business Media.

Drawback of all the analytical method is one need to externally/explicitly project out the doubly occupied states.

$$\hat{a}_{i,\uparrow}(1-\hat{n}_{i,\downarrow})$$

$$\hat{a}_{i,\downarrow}(1-\hat{n}_{i,\uparrow})$$

Constrained for doubly occupied states

SU(2|1) Covariant operator

$$|z,\xi\rangle = \left[\frac{\exp\left(zX^{\downarrow\uparrow} + \xi X^{0\uparrow}\right)}{\sqrt{1+\bar{z}z+\bar{\xi}\xi}}\right]|\uparrow\rangle = \frac{|\uparrow\rangle + z\,|\downarrow\rangle + \xi\,|0\rangle}{\sqrt{1+\bar{z}z+\bar{\xi}\xi}}.$$

Bosonic field (spinon)

Fermionic field (holon)

Transformation of Hubbard operator to SU(2|1) Covariant

$$X_{\text{cov}} := \langle z, \xi | X | z, \xi \rangle.$$

$$X_{\text{cov}}^{0\downarrow} = -\frac{z\bar{\xi}}{1+|z|^2}, \qquad X_{\text{cov}}^{\downarrow 0} = -\frac{\bar{z}\xi}{1+|z|^2},$$

$$X_{\text{cov}}^{0\uparrow} = -\frac{\bar{\xi}}{1+|z|^2}, \qquad X_{\text{cov}}^{\uparrow 0} = -\frac{\xi}{1+|z|^2}.$$

PART-II

Application to strongly correlated one-dimensional Topological superconductors

Topological superconductors

K. K. Kesharpu, E. A. Kochetov, and A. Ferraz, **Physical Review B** 109, 115140 (2024).

K. K. Kesharpu, E. A. Kochetov, and A. Ferraz**, Physical Review B** 111, 115153 (2025)

Concentrate on this

Topological Insulators

K. K. Kesharpu, **Physical Review B** 109 (20), 205120 (2024).

K. K. Kesharpu, E. A. Kochetov, and A. Ferraz, **Physical Review B** 107, 155146 (2023).

Moire Lattices

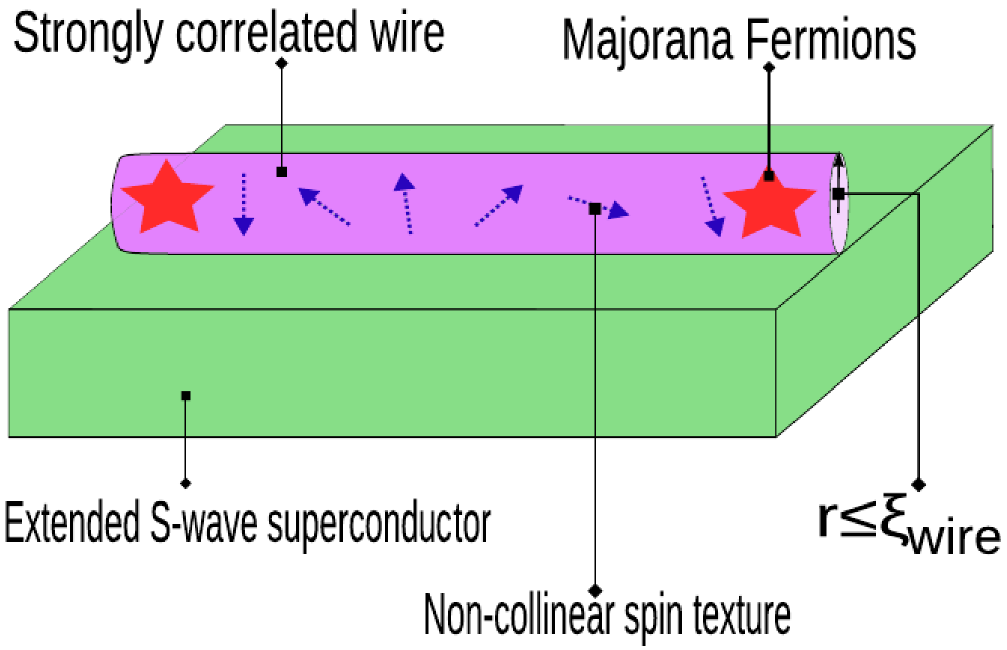
K. K. Kesharpu, **Physical Review B** 109 (20), 158147 (2024).

K. K. Kesharpu, E. A. Kochetov, and A. Ferraz, arxiv:2503.07022 (Sub. to **Physical Review B**)

The Hamiltonian of the wire

The Hamitonian of the wire contains four terms:

1. The **kinetic energy** term
2. The **superconducting** term with **extended s-wave** order parameters
3. The **Rashba spin orbit** term
4. The **chemical potential** term



The Total Hamiltonian

$$
\begin{aligned}
H_{\mathrm{eff}} = & - t \sum_i X_i^{\sigma 0} X_{i+1}^{0 \sigma} + X_i^{\sigma' 0} X_{i+1}^{0 \sigma'} \\
& - \imath \alpha \sum_i X_i^{\sigma' 0} X_{i+1}^{0 \sigma} - X_i^{\sigma 0} X_{i+1}^{0 \sigma'} \\
& + \Delta \sum_i X_i^{\sigma 0} X_{i+1}^{\sigma' 0} - X_i^{\sigma' 0} X_{i+1}^{\sigma 0} + \mathrm{H.c.} \\
& + \mu \sum_i X_i^{00} .
\end{aligned}
$$

Hamiltonian in terms of the Coherent state symbols

$$H_{\text{eff}}(z,\xi) = -\,t\sum_i \xi_i\bar{\xi}_{i+1}a_{i,i+1} + \imath\alpha\sum_i \xi_i\bar{\xi}_{i+1}\alpha^*_{i,i+1} - \Delta\sum_i \xi_i\xi_{i+1}\Delta^*_{i,i+1} + \text{H.c.} + \mu\sum_i \bar{\xi}_i\xi_i\,,$$

Hamiltonian in terms of the Coherent state symbols

$$H_{\text{eff}}(z,\xi) = -t\sum_i \xi_i\bar{\xi}_{i+1}a_{i,i+1} + \imath\alpha\sum_i \xi_i\bar{\xi}_{i+1}\alpha^*_{i,i+1} - \Delta\sum_i \xi_i\xi_{i+1}\Delta^*_{i,i+1} + \text{H.c.} + \mu\sum_i \bar{\xi}_i\xi_i,$$

$$a_{i,i+1} \equiv \frac{1+\bar{z}_i z_{i+1}}{\sqrt{\left(1+|z_i|^2\right)\left(1+|z_{i+1}|^2\right)}},$$

$$\alpha^*_{i,i+1} \equiv \left[\frac{z_i-\bar{z}_{i+1}}{\sqrt{\left(1+|z_i|^2\right)\left(1+|z_{i+1}|^2\right)}}\right]^*,$$

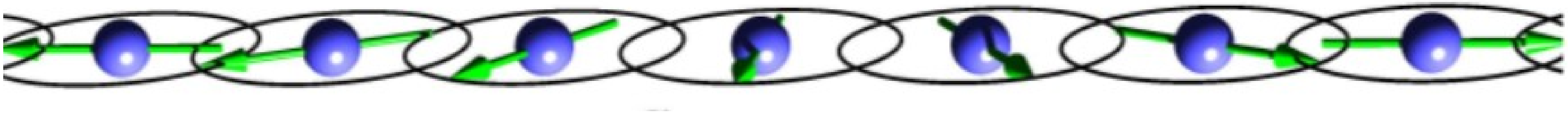
$$\Delta^*_{i,i+1} \equiv \left[\frac{z_i-z_{i+1}}{\sqrt{\left(1+|z_i|^2\right)\left(1+|z_{i+1}|^2\right)}}\right]^*.$$

z : Even Grassman param./ spin dof

ξ : Odd complex Grassman param./ charge dof

Spin spiral (helical) texture

$$\vec{S}_i = (S_i^x, S_i^y, S_i^z) = \frac{1}{2}(\cos\theta_i, \sin\theta_i, 0)\,.$$



Hamiltonian in momentum space

$$H = \sum_k \Psi_k^\dagger H_k \Psi_k; \quad H_k = \begin{bmatrix} M_k & \Delta_k \\ -\Delta_k^\dagger & -M_k^\dagger \end{bmatrix}; \qquad (12)$$

$$\Psi_k^\dagger = \left(c_{1,k}^\dagger, \ldots, c_{N,k}^\dagger, c_{1,-k}, c_{N,-k}\right).$$

Here H_k is $2N \times 2N$ matrix. M_k and Δ_k are $N \times N$ matrices; for $i \in (1, N-1)$ the elements are $M_k^{i,i+1} = M_k^{i+1,i} = -\tilde{t}_i$, $M_k^{i,i} = \mu/2$, $\Delta_k^{i,i+1} = -\Delta_k^{i+1,i} = \Delta \sin(\theta/2)$; for $i = N$ the elements are $M_k^{N,N} = \mu/2$, $M_k^{N,1} = -\tilde{t}_N e^{-\imath Nk}$, $M_k^{1,N} = -\tilde{t}_N e^{+\imath Nk}$, $\Delta_k^{N,1} = \Delta \sin(\theta/2)\, e^{-\imath Nk}$, $\Delta_k^{1,N} = -\Delta \sin(\theta/2)\, e^{+\imath Nk}$. The k lie in the first BZ, $k \in (-\pi/N, \pi/N)$.

Belongs to BDI class TS

- The time reversal, particle hole symmetry and chiral symmetry is conserved.

$$UH_kU^\dagger = \begin{bmatrix} 0 & A_k \\ A_k^\dagger & 0 \end{bmatrix}; \quad A_k = M_k + \Delta_k .$$

It is a Z2 topological invariant system

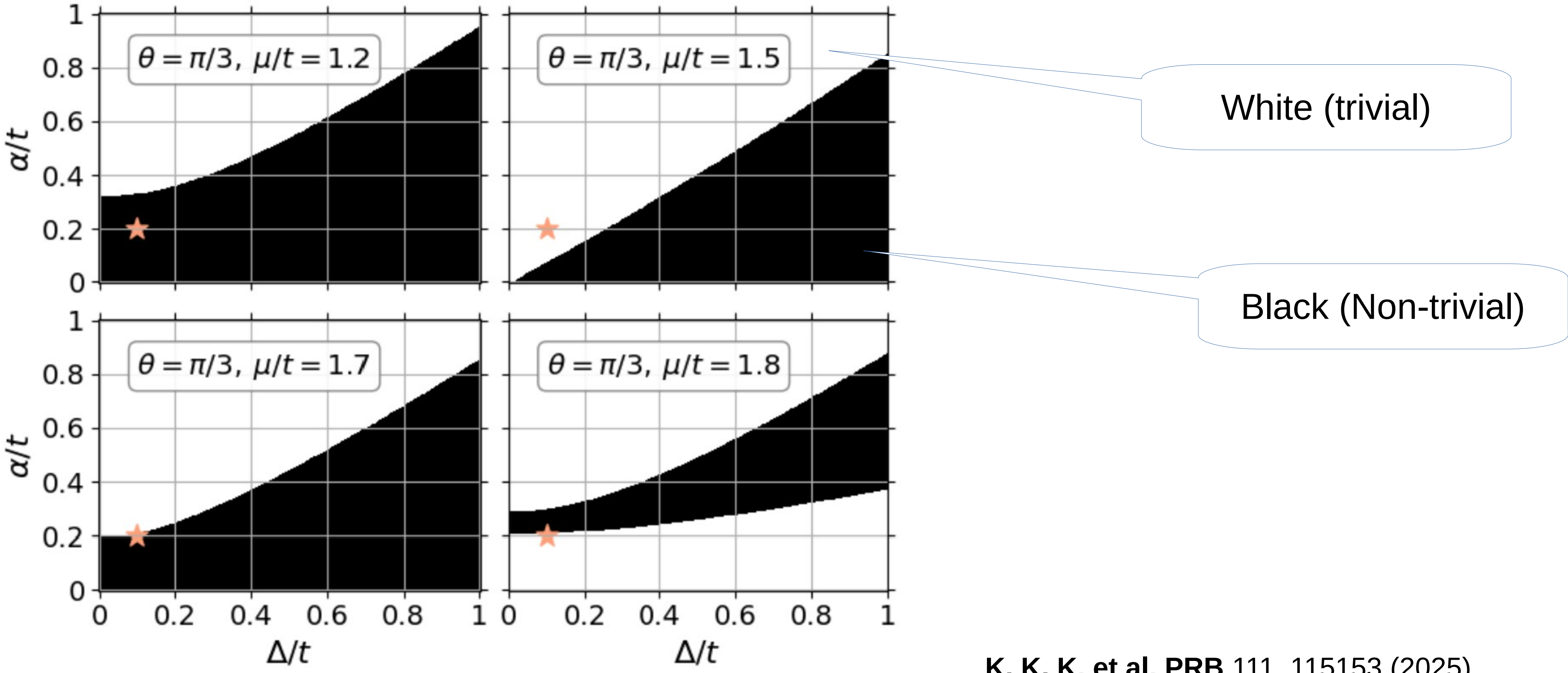
- W gives the no of majorana modes at the end of the wire. Related to the Z topological invariant.

- Q gives the parity of Majorana modes at the end of the wire. It is related to the Z2 topological invariant.

$$W = \frac{-i}{\pi} \int_{k=0}^{k=\pi/N} dz_k / z_k; \quad z_k = \mathrm{Det}\,(A_k)\,/|\mathrm{Det}\,(A_k)|\,. \quad (14)$$

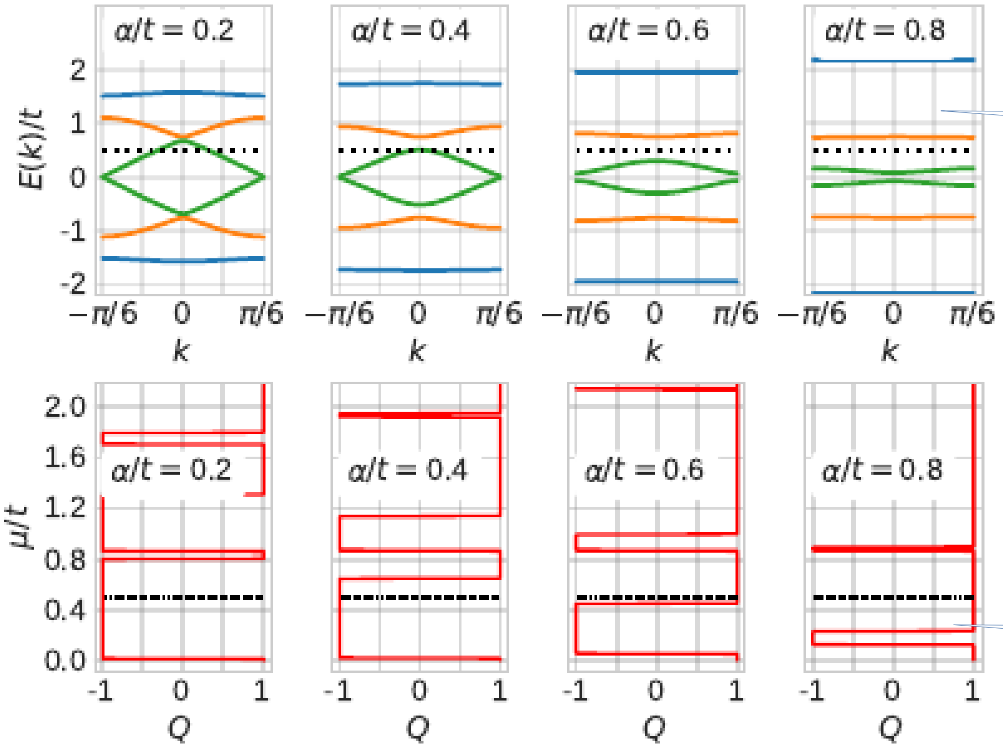
$$Q = \mathrm{sgn}\left[\frac{\mathrm{Det}\,\{A_{k=\pi/N}\}}{\mathrm{Det}\,\{A_{k=0}\}}\right] = (-1)^W.$$

Dependence of the topolgoical invariant on SC gap and RSOC



K. K. K, et al, PRB 111, 115153 (2025)

Explanation through the Energy spectrum

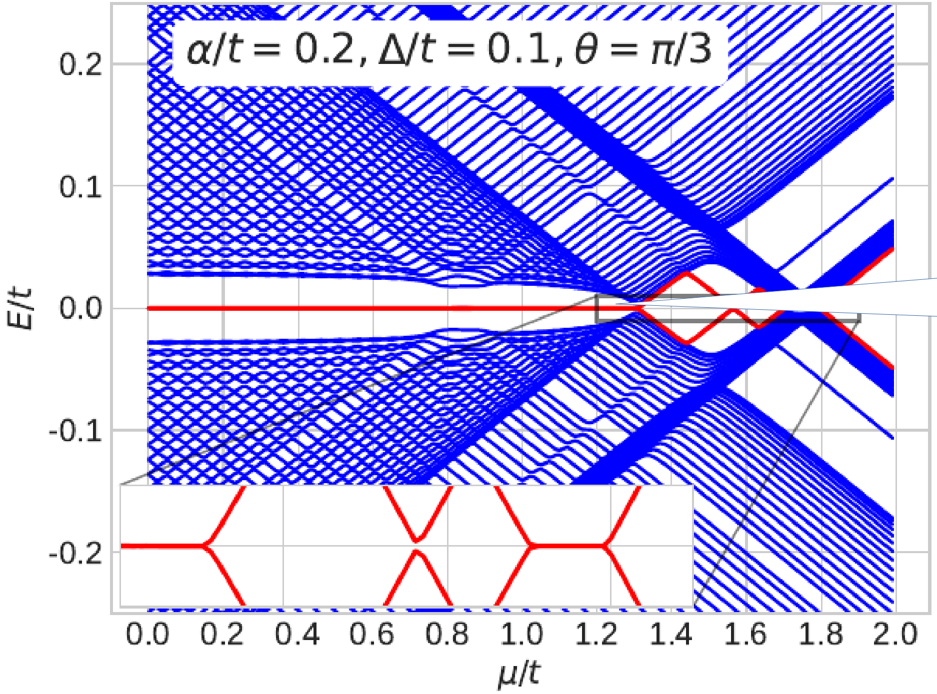


Energy spectrum

Topological invariants

K. K. K, et al, PRB 111, 115153 (2025)

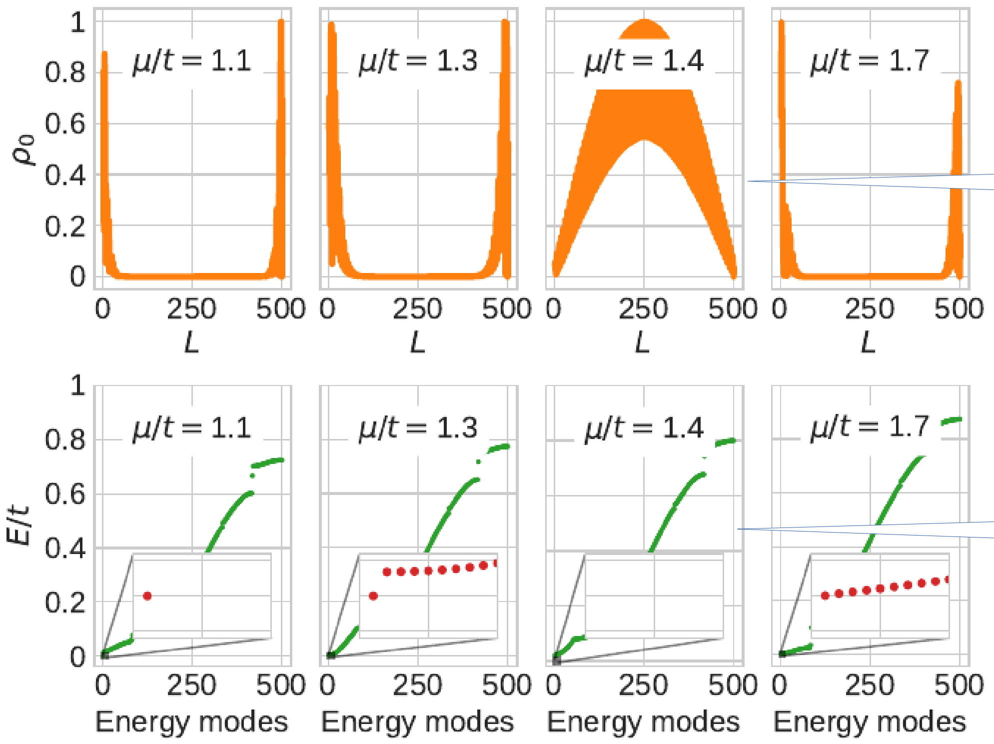
Numerical analysis (Diagnolization of Ham)



Red line (zero energy Majorana states)

FIG. 3. Energy spectrum found by diagonalizing Eq. (11) in Majorana basis under open boundary condition for a chain of length $L = 100.$, $\theta = \pi/3$, $\Delta/t = 0.1$ and $\alpha/t = 0.2$; these values of the parameters are same as the marked (red, star) position in Fig. 1 for $\pi/3$. (inset) Zoomed portion of energy spectrum around zero energy level from $\mu/t = 1.2$ to 1.9.

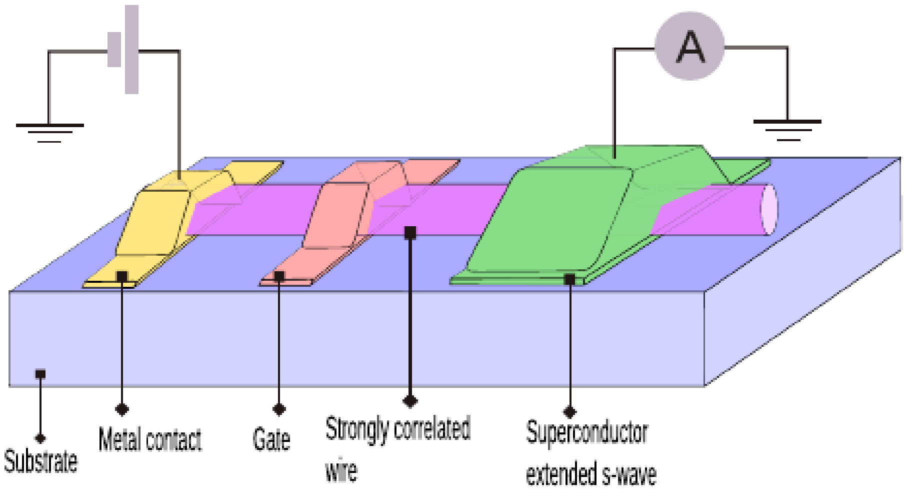
Numerical analysis (LDOS)



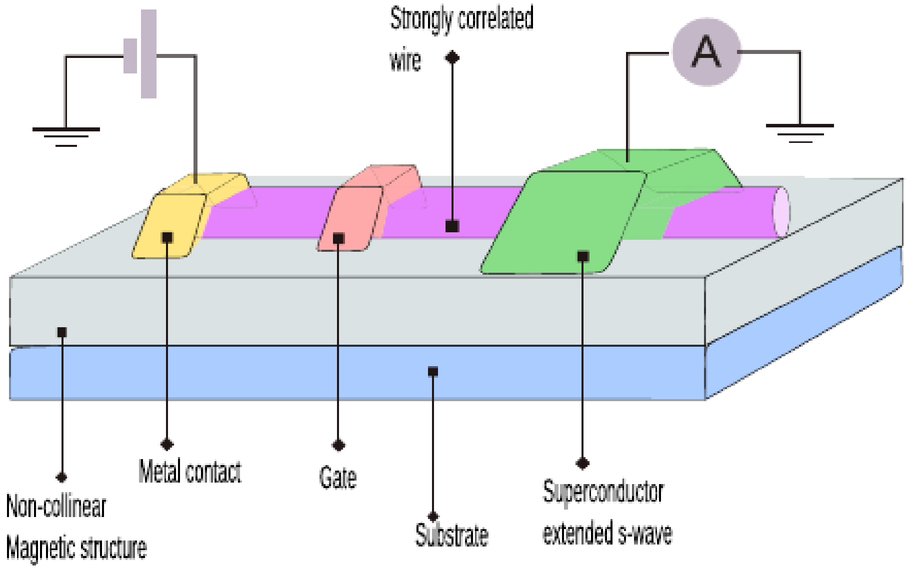
LDOS

Zero energy modes

Device proposals



(a)



Conclusions

1. Presented a possible way to treat strongly correlated Hamiltonian using su(2|1) coherent state path integral method.
2. Two ingredients are necessary for method to work:
    1. Strong correlation should project out doubly occupied states
    2. A lattice Hamiltonian with necessary effects
3. Method has been successfully applied to the topological insulators, 1D topological superconductors, altermagnets, Moire lattices, 2D topological nodal superconductors

Thank you for your attention

- **K. K. Kesharpu,** E. A. Kochetov, and A. Ferraz**, Physical Review B** 111, 115153 (2025)
- **K. K. Kesharpu**, **Physical Review B** 109 (20), 205120 (2024).
- **K. K. Kesharpu**, E. A. Kochetov, and A. Ferraz, **Physical Review B** 109, 115140 (2024).
- **K. K. Kesharpu**, E. A. Kochetov, and A. Ferraz, **Physical Review B** 107, 155146 (2023).
- **K. K. Kesharpu**, E. A. Kochetov, and A. Ferraz, **Physical Review B** 109, 03323 (2024)
- **K. K. Kesharpu**, E. A. Kochetov, and A. Ferraz, arxiv:2503.07022 (Sub. to **Physical Review B**)

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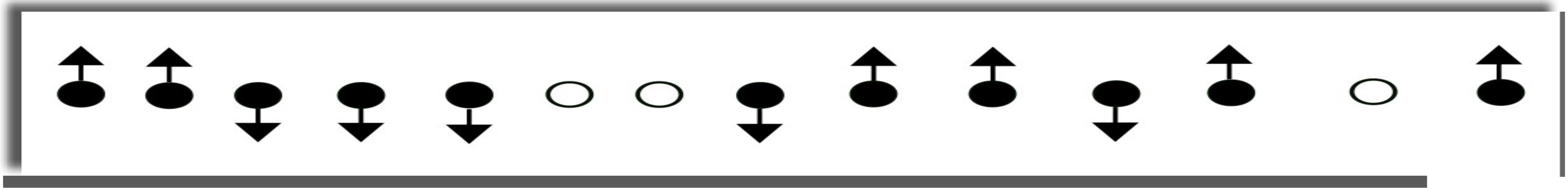
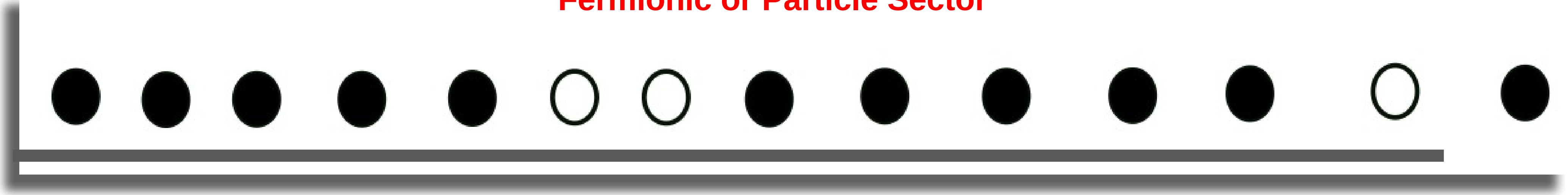
Appendix

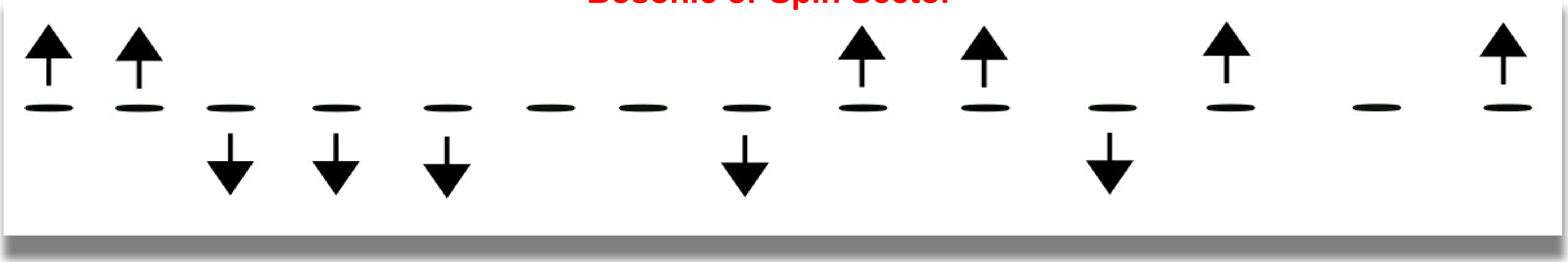
Fractionalization of the electrons



Fermionic or Particle Sector



Bosonic or Spin Sector



Strategy to solve strongly correlated systems

Numerical Methods

- DMRG
- QMC
  - Variational Field
  - Auxilary Field
  - Constrained Path
- ...

Field theoretical methods

- RG Flow
- Diagramatic techniques
- Bosonization of sytems
- ...

Other methods

- Mean field self consistent
- Diagramatic peseudo particle
- Slave boson
- Two particle self consitent
- ...

The local constraint should be imposed explicitly $Q = \sum_{\sigma} c_{i,\sigma} \leq 1$

A good review on these methods: Sénéchal, D., Tremblay, A. M., & Bourbonnais, C. (2004). Theoretical methods for strongly correlated electrons. Springer Science & Business Media.

Method with implicit local constraint

$$H = -\sum_{ij\sigma}\left(t_{ij} + \frac{3J}{4}\delta_{ij}\right)c^{\dagger}_{i\sigma}c_{j\sigma} + J\sum_{i}\hat{\mathbf{S}}_i \cdot (c^{\dagger}_{i\sigma}\vec{\sigma}_{\sigma\sigma'}c_{i\sigma'}).$$

At $J \to \infty$ limit the KLM model is equivalent to $U \to \infty$ Hubbard model

$$H_{U=\infty} = -\sum_{ij\sigma} t_{ij}\tilde{c}^{\dagger}_{i\sigma}\tilde{c}_{j\sigma}.$$

Here $\tilde{c}_{i\sigma}$ are the Gutzwiller projected electronic operators.

I. Ivantsov, A. Ferraz, and E. Kochetov, Ann. Phys. 441, 168859 (2022).

Gutzwiller projected operators can be identified with Hubbard operators

$$H \to H = \mathcal{P} H \mathcal{P}, \quad \mathcal{P} = \prod_i \mathcal{P}_i,$$

$$\mathcal{P}_i = 1 - n_{i\sigma} n_{i-\sigma},$$

Defn. of Gutzwiller operators

$$\mathcal{P}_i c_{i\sigma} \mathcal{P}_i =: \tilde{c}_{i\sigma} = c_{i\sigma}(1 - n_{i-\sigma}).$$

Gutzwiller projected elect. Operators.

$$X^{0\sigma} = \left(X^{\sigma 0}\right)^{\dagger} = |0\rangle\langle\sigma|, \quad \sigma = \uparrow, \downarrow.$$

Gutzwiller projected operators can be written in terms of Hubbard X operators

The Bos./Ferm. X Operators are closed under su(2|1) relation

The important point is that the fermionic Hubbard operators $X^{0\sigma} = (X^{\sigma 0})^{\dagger}$ along with the bosonic ones, $X^{\sigma\sigma'}$, X^{00}, are closed under commutation/anticommutation relations into the *superalgebra* $su(2|1)$ [9]. The $su(2|1)$ superalgebra can be thought of as the simplest possible extension of the conventional spin $su(2)$ algebra to incorporate fermionic degrees of freedom. Namely, the bosonic sector of the $su(2|1)$ consists of three bosonic superspin operators,

$$Q^{+} = X^{\uparrow\downarrow}, \qquad Q^{-} = X^{\downarrow\uparrow}, \qquad Q^{z} = \frac{1}{2}(X^{\uparrow\uparrow} - X^{\downarrow\downarrow}) \tag{3}$$

closed into $su(2)$, and a bosonic operator X^{00} that generates a $u(1)$ factor of the maximal even subalgebra $su(2) \times u(1)$ of $su(2|1)$. The fermionic sector is constructed out of four operators $X^{\sigma 0}, X^{0\sigma}$ that transform in a spinor representation of $su(2)$. We do not intend here to discuss a

P. B. Wiegmann, Phys. Rev. Lett. 60, 821 (1988).

su(2|1) coherent state symbols of X operators

Bosonic superspin operators

Fermionic superspin operators

$$|z,\xi\rangle = (1+\bar{z}z+\bar{\xi}\xi)^{-1/2}\exp\left(zX^{\downarrow\uparrow}+\xi X^{0\uparrow}\right)|\uparrow\rangle$$
$$= (1+\bar{z}z+\bar{\xi}\xi)^{-1/2}(|\uparrow\rangle + z|\downarrow\rangle + \xi|0\rangle),$$

z : Complex even Grassman number

ξ : Odd complex Grassman number

su(2|1) coherent state symbols of X operators

$$X_{cs} = \langle z, \xi | X | z, \xi \rangle,$$

su(2|1) coherent state symbols of X operators

The CS symbols of the X operators, $X_{cs} = \langle z, \xi | X | z, \xi \rangle$, read

$$X_{cs}^{0\downarrow} = -\frac{z\bar{\xi}}{1+|z|^2}, \quad X_{cs}^{\downarrow 0} = -\frac{\bar{z}\xi}{1+|z|^2},$$

$$X_{cs}^{0\uparrow} = -\frac{\bar{\xi}}{1+|z|^2}, \quad X_{cs}^{\uparrow 0} = -\frac{\xi}{1+|z|^2},$$

$$Q_{cs}^{+} = X_{cs}^{\uparrow\downarrow} = \frac{z}{1+|z|^2}\left(1 - \frac{\bar{\xi}\xi}{1+|z|^2}\right),$$

$$Q_{cs}^{-} = X_{cs}^{\downarrow\uparrow} = \frac{\bar{z}}{1+|z|^2}\left(1 - \frac{\bar{\xi}\xi}{1+|z|^2}\right),$$

$$Q_{cs}^{z} = \frac{1}{2}(X_{cs}^{\uparrow\uparrow} - X_{cs}^{\downarrow\downarrow}) = \frac{1}{2}\frac{1-|z|^2}{1+|z|^2}\left(1 - \frac{\bar{\xi}\xi}{1+|z|^2}\right). \qquad (3)$$

There is a one-to-one correspondence between the $su(2|1)$ generators and their CS symbols [5].

$$X_{cs} = \langle z, \xi | X | z, \xi \rangle,$$

z :A complex number

The bosonic field

ξ :A Grassman paramter

The fermionic field

The partition function

$$Z = \int D\mu(z,\xi)\, e^{S}$$

$$D\mu(z,\xi) = \prod_{i,t} \frac{d\bar{z}_i(t)dz_i(t)}{2\pi i(1+|z_i|^2)^2}\, d\bar{\xi}_i(t)d\xi_i(t)$$

$$S = \sum_i \int_0^\beta \left(ia_i^{(0)} - \bar{\xi}_i\left(\partial_t + ia_i^{(0)}\right)\xi_i\right)dt - \int_0^\beta H\, dt$$

The Action

$$S = \sum_i \int_0^\beta \left(i a_i^{(0)} - \bar{\xi}_i \left(\partial_t + i a_i^{(0)} \right) \xi_i \right) dt - \int_0^\beta H \, dt$$

$$i a^{(0)} = -\langle z | \partial_t | z \rangle = S \, \frac{\dot{\bar{z}} z - \bar{z} \dot{z}}{1 + |z|^2}, \qquad H = -t \sum_{\langle ij \rangle} \bar{\xi}_i \xi_j e^{i a_{ji}} + \text{H.c.} + \mu \sum_i \bar{\xi}_i \xi_i,$$

$$a_{ij} = -i \log \langle z_i | z_j \rangle, \qquad \langle z_i | z_j \rangle = \frac{(1 + \bar{z}_i z_j)^{2S}}{(1 + |z_j|^2)^S (1 + |z_i|^2)^S}.$$

Representation of the bosonic field in terms of spin operators

$$H = -t \sum_{\langle ij \rangle} \bar{\xi}_i \xi_j e^{i a_{ji}} + \text{H.c.} + \mu \sum_i \bar{\xi}_i \xi_i,$$

$$a_{ji} = \phi_{ji} + i \chi_{ji}, \quad \bar{\phi}_{ji} = \phi_{ji}, \quad \bar{\chi}_{ji} = \chi_{ji}.$$

$$\phi_{ji} = iS \log \frac{1 + \bar{z}_i z_j}{1 + \bar{z}_j z_i} = iS \log \frac{(S + S_i^z)(S + S_j^z) + S_i^- S_j^+}{(S + S_i^z)(S + S_j^z) + S_j^- S_i^+},$$

$$\chi_{ji} = -S \log \frac{(1 + \bar{z}_i z_j)(1 + \bar{z}_j z_i)}{(1 + |z_i|^2)(1 + |z_j|^2)} = -S \log \left(\frac{\vec{S}_i \cdot \vec{S}_j}{2S^2} + \frac{1}{2} \right).$$

$$H = -t \sum_{\langle ij \rangle} \bar{\xi}_i \xi_j e^{i a_{ji}} + \text{H.c.} + \mu \sum_i \bar{\xi}_i \xi_i,$$

$$H = -t \sum_{\langle i,j \rangle} \bar{f}_i f_j e^{i\phi_{ji}} \left(\frac{\vec{S}_i \cdot \vec{S}_j}{2S^2} + \frac{1}{2} \right)^S + \mu \sum_i \bar{f}_i f_i.$$

Algorithm

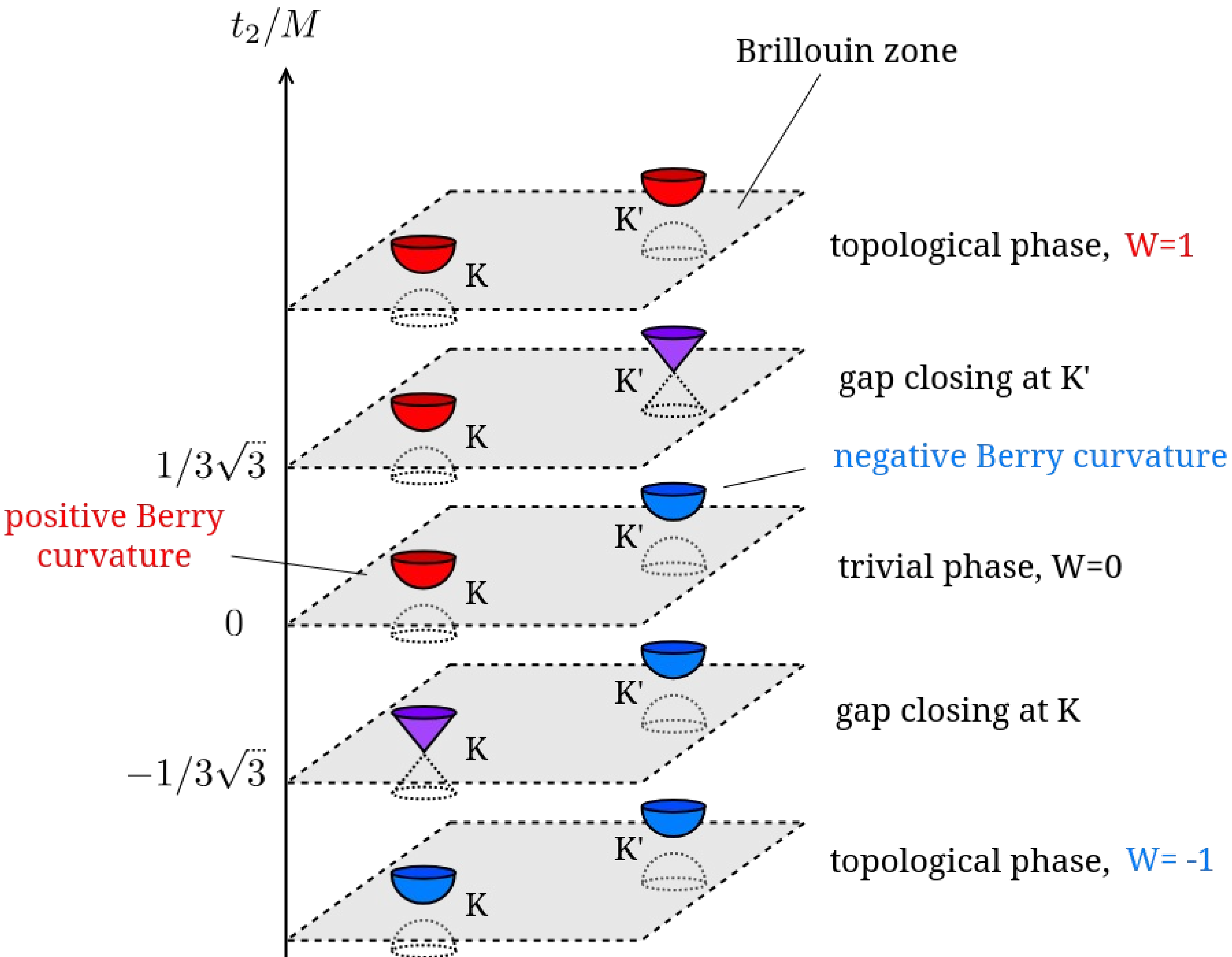
Step-1 Represent the non-interacting Hamiltonian in Gutzwiller projected operators

Step-2 Represent the Gutzwiller projected Hamiltonian in terms of Hubbard-X operators

Step-3 Represent the Hubbard-X Hamiltonian in terms of su(2|1) coherent state symbols

Step-4 Solve the Hamiltonian using su(2|1) path-integral method

Step-5 The final Hamiltonian will have independent bosonic and fermionic dof



t_2/M
Brillouin zone
K'
K
topological phase, W=1
gap closing at K'
$1/3\sqrt{3}$
negative Berry curvature
positive Berry curvature
trivial phase, W=0
0
gap closing at K
$-1/3\sqrt{3}$
topological phase, W= -1