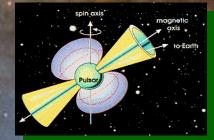
Neutron Stars as Nuclear Physics Laboratory

Evgeni Kolomeitsev

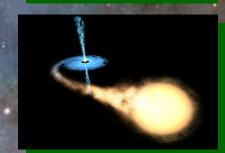
LTP JINR, Dubna, Russia

Neutron Star Zoo

>2000 neutron stars in isolated rotation-powered pulsars
~ 30 millisecond pulsars



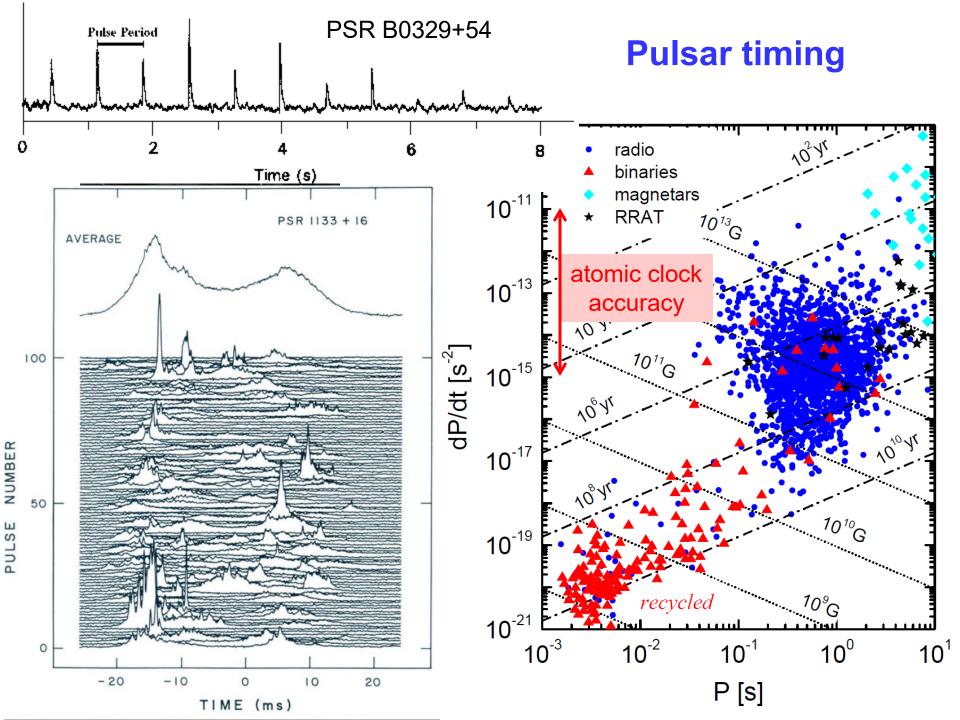
>100 neutron stars in accretion-powered X-ray binaries ~ 50 x-ray pulsar intense X-ray bursters (thermonuclear flashes)



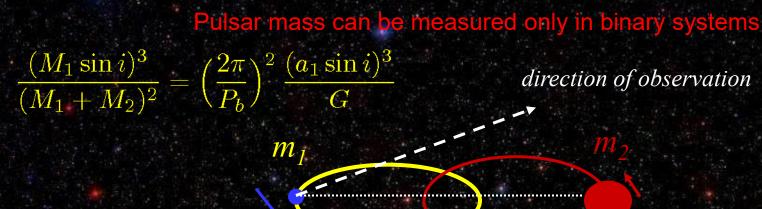
short gamma-ray bursts neutron star -- neutron star, neutron star -- black-hole mergers



soft gamma-ray repeaters – magnetars (super-strong magnetic fields)



Measuring pulsar mass



Newton gravity \longrightarrow 5 Keplerian orbital parameters: orbital period, semi-major axis length, excentricity,

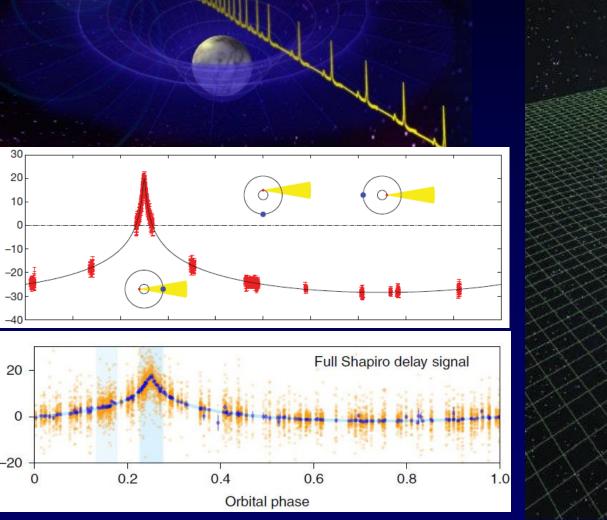
Do not determine individual masses of stars and the orbital inclination.

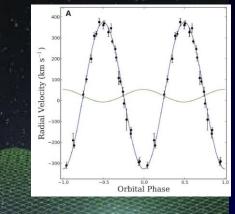
Measurement of any 2 post-Keplerian parameters allows to determine the mass of each star.

Shapiro Delay

Time signal is getting delayed when passing near massive object.

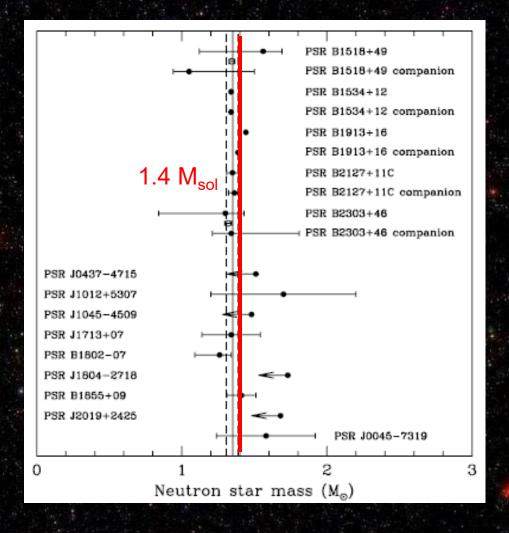
Measured Phase-Resolved Spectra of the optical counterpart. (e.g.,hydrogen Balmer lines)





Neutron star masses



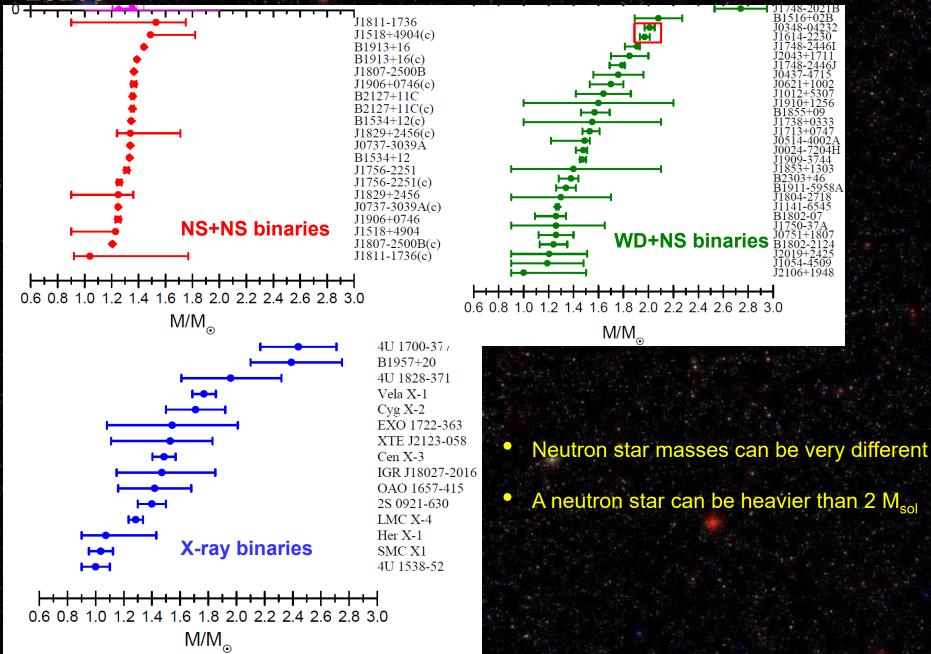


Are masses of all NS the same?

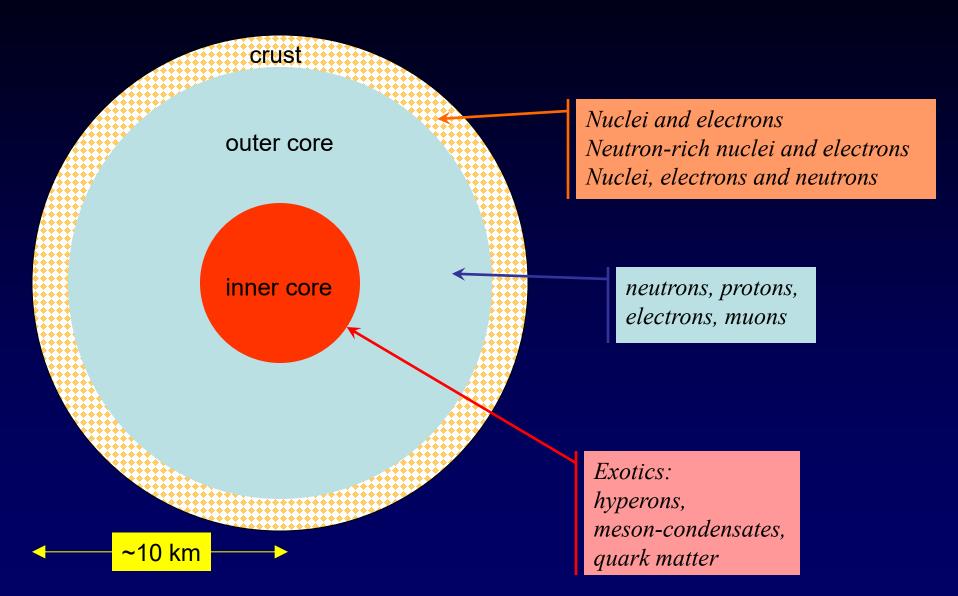
[Thorsett, Chakrabarty, ApJ 512 (1999)288]

2020s

Neutron star masses

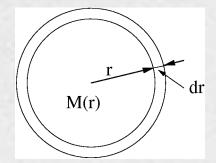


Cross section of a neutron star



Tolman-Oppenheimer-Volkov equation

Equilibrium condition for a shell in a non-rotating neutron star



OUTPUT:

 $S_{\Omega}(r) dp = dF_G$ Newton's Law $4 \pi r^2 dp = G \frac{M(r) dM}{r^2}$ $dM = 4 \pi r^2 \varepsilon(p) dr$

INPUT: equation of state (EoS)

$$\varepsilon = \varepsilon(p)$$
 or $\begin{cases} p = p(n) \\ \varepsilon = \varepsilon(n) \end{cases}$

boundary conditions: $\varepsilon(r=0) = \varepsilon_c$, M(r=0) = 0, P(r=R) = 0

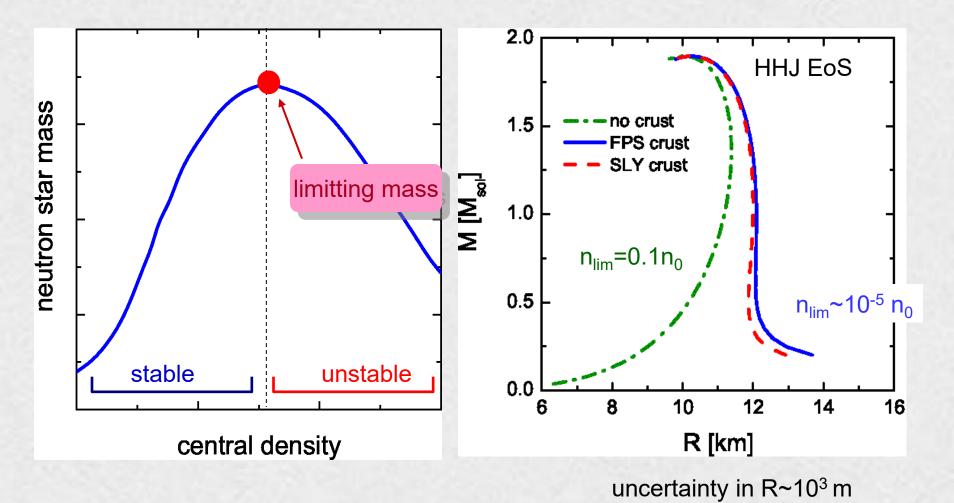
neutron star density profile, radius R and mass M

relativistic corrections

 $/c^2$

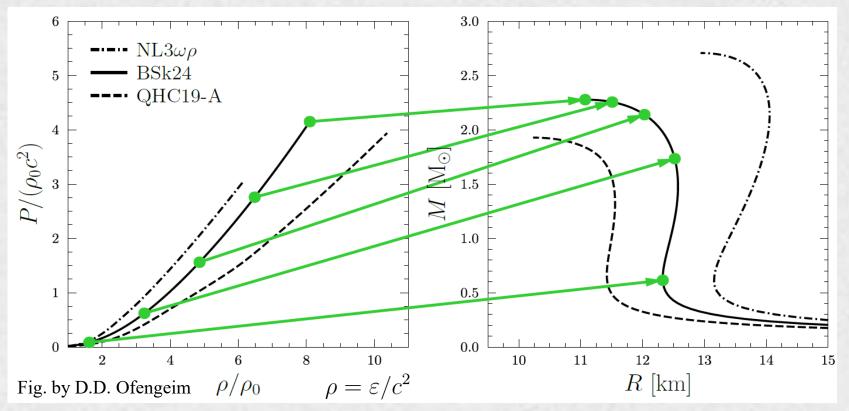
$$\frac{dp}{dr} = -\frac{G \varepsilon M}{r^2} \left(1 + \frac{p}{\varepsilon}\right) \left(1 + \frac{4\pi P r^3}{M}\right) \left(1 - \frac{2GM}{r}\right)^{-1} \qquad \rho = \varepsilon$$

Neutron star configuration



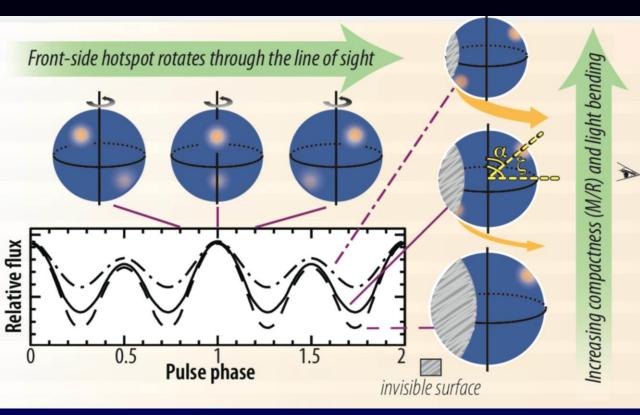
Oppenheimer-Volkoff mapping

[Lindblom ApJ 398 (1992) 569]

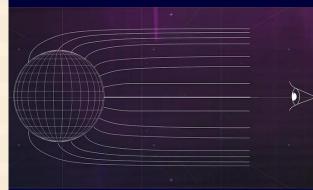


- One can unambiguously restore the $P(\rho)$ function from the M R relation, if the latter is known for every mass
- The maximum NS mass (TOV limit) exists for every EoS
- Stiffer EoSs shift the M R curve to higher masses and radii

The Neutron Star Interior Composition Explorer Mission (NICER)



Lightcurve modeling constrains the compactness (GM/Rc²) and viewing geometry of a non-accreting millisecond pulsar



PSR J0740+6620

Thomas E. Riley *et al* 2021 *ApJL* **918** L27

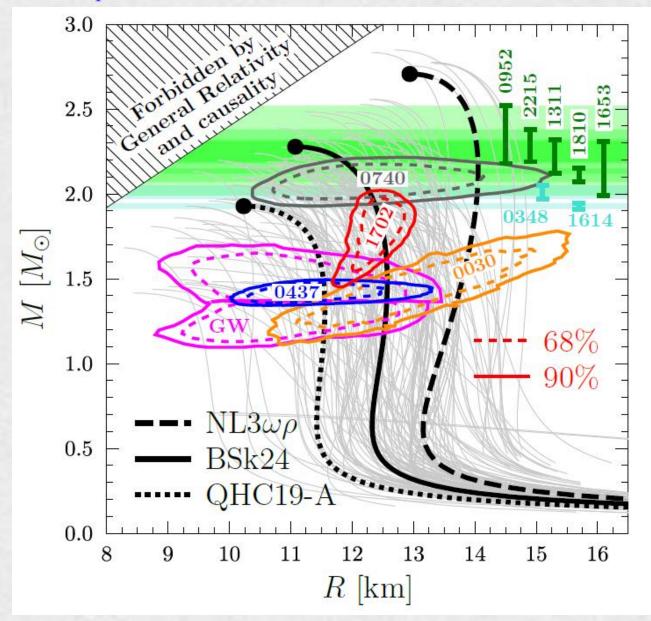
$$R = 12.39^{+1.30}_{-0.98}$$
 km and $M = 2.072^{+0.067}_{-0.066} M_{so}$

M. C. Miller *et al* 2021 *ApJL* **918** L28

 $R = 13.7^{+2.6}_{-1.5} \,\mathrm{km}$ and $M = 2.08 \pm 0.07 \,M_{\mathrm{sol}}$

Selected NS mass and radius measurements

in comparison with M – R relations for various EoSs



Equation of state of nuclear matter

The energy per nucleon of the nuclear matter

$$E(n_p, n_n) = E_0(n) + E_S(n) \frac{(n_p - n_n)^2}{n^2}$$

$$n_p$$
 – proton number density
 n_n – neutron number density

• nuclear matter parameters

$$n = n_p + n_n$$

$$E_0(n) = \frac{E_0}{18} + 0 + \frac{K}{18} \frac{(n-n_0)^2}{n_0^2} + \frac{Q}{162} \frac{(n-n_0)^3}{n_0^3} + O\left(\frac{(n-n_0)^4}{n_0^4}\right)$$
$$n_0 \approx 0.16 \,\mathrm{fm}^{-3}, \ E_0 \approx -16 \,\mathrm{MeV}, \ K \approx 230 \pm 30 \,\mathrm{MeV}, \ Q < 0$$

symmetry energy

$$E_{S}(n) = J + \frac{L}{3} \frac{n - n_{0}}{n_{0}} + \frac{K_{\text{sym}}}{18} \frac{(n - n_{0})^{2}}{n_{0}^{2}} + \frac{Q_{\text{sym}}}{162} \frac{(n - n_{0})^{3}}{n_{0}^{3}} + O\left(\frac{(n - n_{0})^{4}}{n_{0}^{4}}\right)$$

There is a correlation among parameters: J, L, K_{svm}

• Correlations among parameters L-J

$$\varepsilon_S[n] = J + \frac{L}{3} \frac{n - n_0}{n_0} + \frac{K_{\text{sym}}}{18} \frac{(n - n_0)^2}{n_0^2} + \dots$$

Masses: UNEDF0 Skyrme DF+BHF [Kortelainen *et al.*, PRC **82**, 024313 (2010)]

Isobaric analog states+isovector skin: [Danielewicz et al. NPA 958, 147 (2017)]

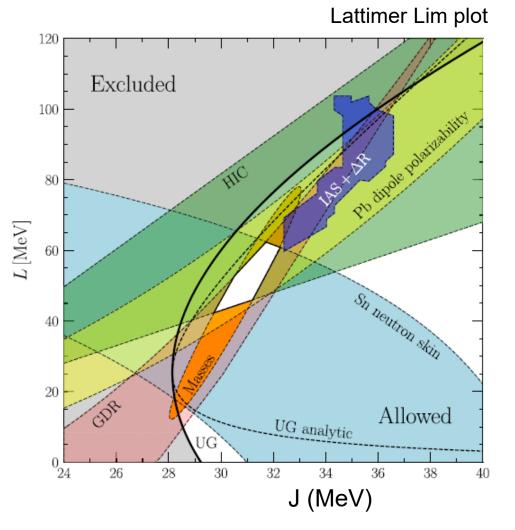
Pb dipole polarizability: [Roca-Maza *et al.*, PRC **88**, 024316 (2013)]

Sn neutron skin: [Chen et al., PRC **82**, 024321 (2010)]

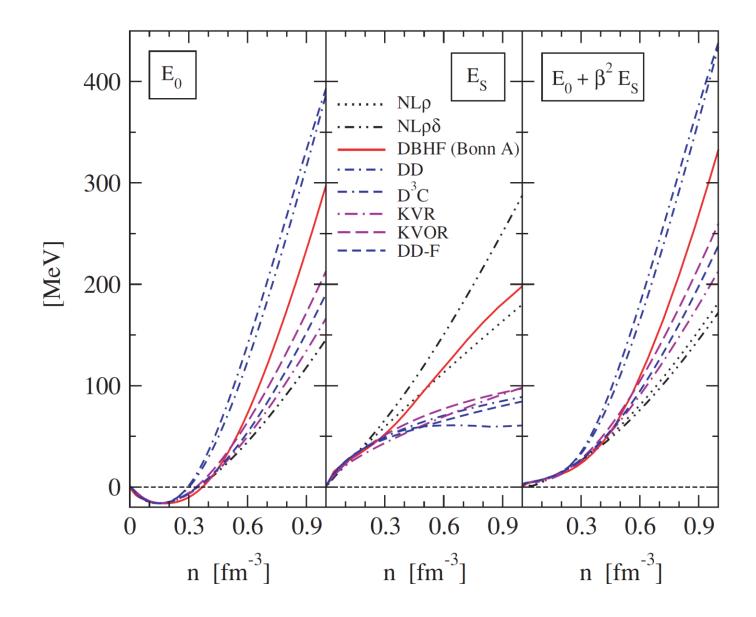
GDR: [Trippa et al., PRC **77**, 061304 (2008)]

Isospin defusion in HIC [Tsang et al., PRL 102, 122701 (2009)]

Behind all calculation are particular models for NN interactions and many-body techniques



Equation of state of nuclear matter

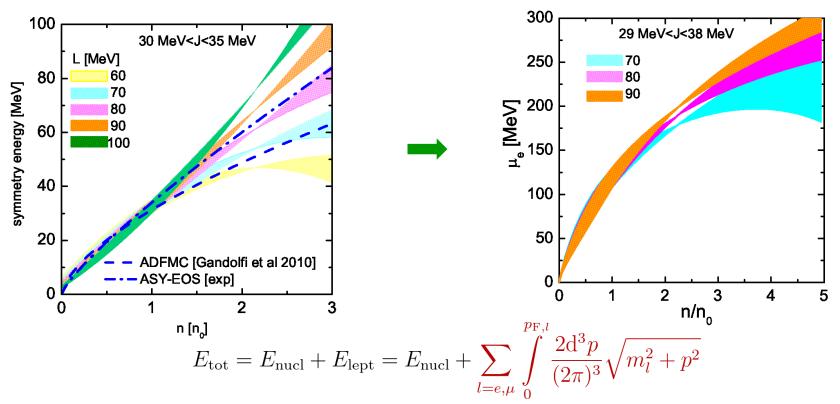


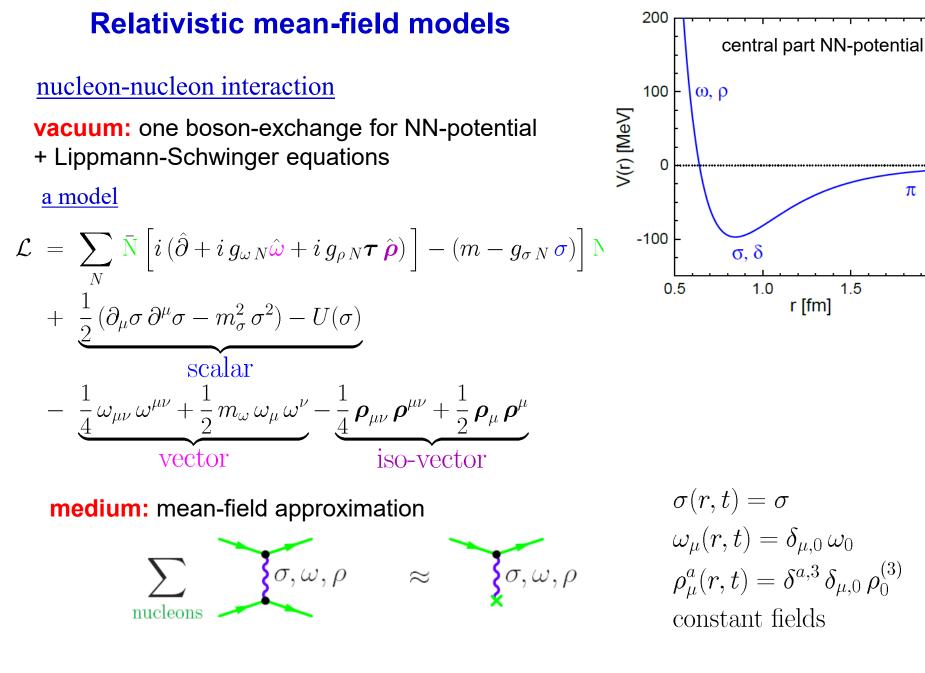
Neutron star composition

chemical potentials: $\mu_{n} = \frac{\partial E(n_{p}, n_{n})}{\partial n_{n}} = \frac{\partial E(n, x)}{\partial n} - \frac{x}{n} \frac{\partial E(n, x)}{\partial x} \quad \mu_{p} = \frac{\partial E(n_{p}, n_{n})}{\partial n_{p}} = \frac{\partial E(n, x)}{\partial n} + \frac{1 - x}{n} \frac{\partial E(n, x)}{\partial x}$ Condition of the beta equilibrium $n \leftrightarrow p + e^{-}$ $n \leftrightarrow p + \mu^{-}$ $\mu^{-} \leftrightarrow e^{-}$ $x = n_{p}/n$ $\mu_{n} = \mu_{p} + \mu_{e} \qquad \mu_{e} = \mu_{n} - \mu_{p} = -\frac{1}{n} \frac{\partial E(n, x)}{\partial x} = 4 E_{\rm S}(n) (1 - 2x)$

equation for the proton concentration

$$n_e(\mu_e) + n_\mu(\mu_e) = \frac{(\mu_e^2 - m_e^2)^{3/2}}{3\pi^2} + \frac{(\mu_e^2 - m_\mu^2)^{3/2}}{3\pi^2} = n_p = x \, n$$





[Serot, Walecka]

pion dynamics falls out completely in this approx.

2.0

nucleon spectrum in MF approximation

$$E_N(p) = \sqrt{m_N^{*2} + p^2} + g_{\omega N} \,\omega_0 + g_{\rho N} \,I_N \,\rho_{03} \quad m_N^* = m_N - g_{\sigma N} \,\sigma$$

Energy-density functional

$$E[n_p, n_n; \sigma] = \frac{m_{\sigma}^2 \sigma^2}{2} + U(\sigma) + C_{\omega}^2 \frac{(n_n + n_p)^2}{2 m_N^2} + C_{\rho}^2 \frac{(n_n - n_p)^2}{8 m_N^2} + \sum_N \int_0^{p_{\mathrm{F},N}} \frac{dp \, p^2}{\pi^2} \sqrt{(m_N - g_{\sigma N} \, \sigma)^2 + p^2}$$

evaluated for ¾ field followed from the equation

$$\frac{\delta E[n_p, n_n, \sigma]}{\delta \sigma} = 0$$

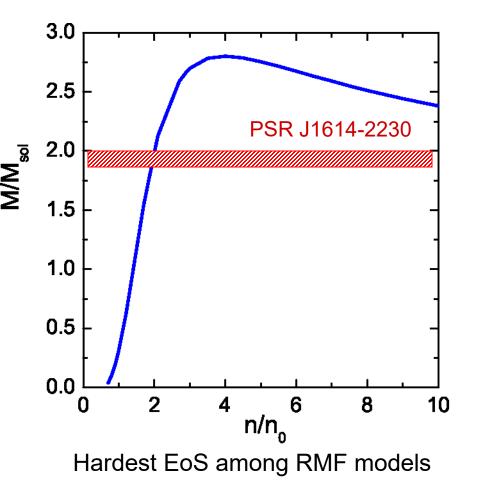
Paremeters $C_i^2 = \frac{g_{iN}^2 m_N^2}{m_i^2}$ are adjusted to properties of nuclear matter at saturation

If we add gradient terms this energy density functional can be used for a description of properties of atomic nuclei.

n_0	\simeq	$0.16 \pm 0.015 \text{ fm}^{-3}$
$E_{ m bind}$	\simeq	$-15.6\pm0.6~{\rm MeV}$
$m^*_N(ho_0)$	\simeq	$(0.75 \pm 0.1) m_N$
K	\simeq	$240 \pm 40 \text{ MeV}$
$a_{ m sym}$	\simeq	$32 \pm 4 \text{ MeV}$

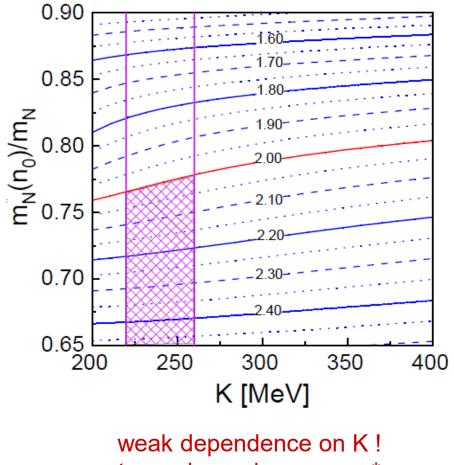
(pure) Walecka model $U(\sigma)=0$ $n_0 = 0.16 \text{fm}^{-3}, \ E_{\text{bind}} = -16 \text{ MeV}$

 $K = 553 \text{ MeV}, \ m_N^*(n_0) = 0.54 m_N$



maximal mass of NS

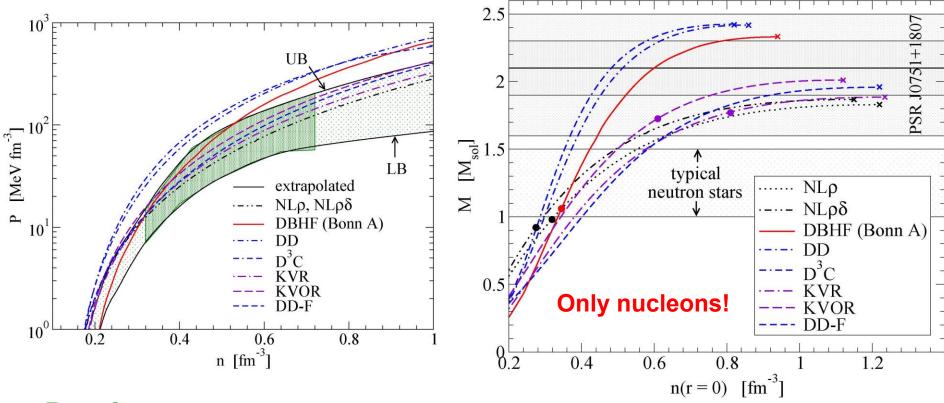
<u>modified Walecka</u> $U(\sigma)=a\sigma^3+b\sigma^4$



strong dependence on m*_N

constraints from heavy-ion collisions

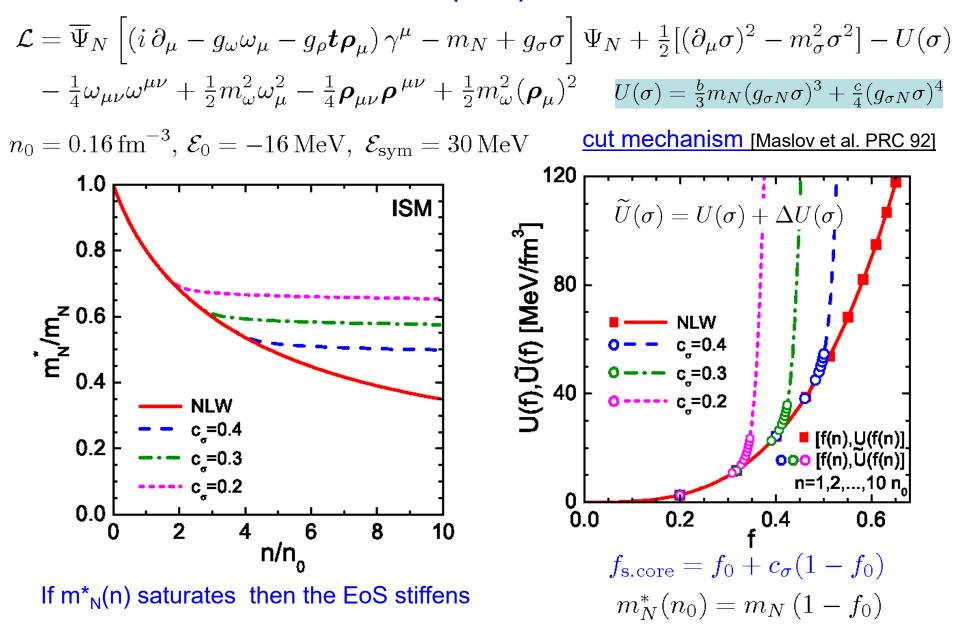
✓ maximum mass constraints



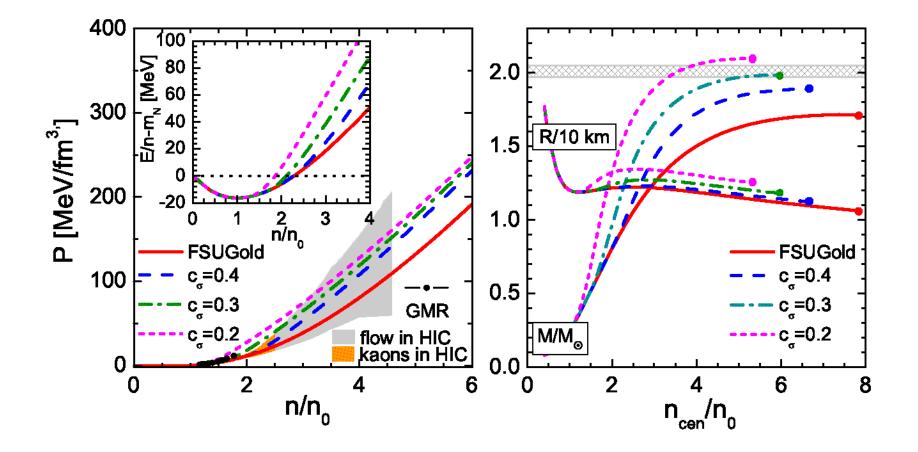
Puzzle

Nucleon part of EoS should be <u>sufficiently stiff</u>. Pressure in isospin symmetric matter should <u>be no to high</u>. (particle flow in HIC) [Klaehn et al., PRC 74 (2006) 035802.]

The standard non-linear Walecka (NLW) model and the cut mechanism



Todd-Rutel, Piekarewicz, Phys. Rev. Lett. 95 (2005) 122501



Alternative FSUGold2 model: W.-Ch. Chen, Piekarewicz, Phys. Rev. C 90 (2014) 044305 $M_{\rm max} = 2.1 \, M_{\odot}$

KVOR model [EEK and D.Voskresensky NPA 759 (2005) 373]

- in standard RMF model m_{σ} , m_{ω} , and m_{ρ} do not change Can the in-medium modification (decrease) of meson masses be included in an RMF model??
- $\bullet~\sigma$ field dependent masses and couplings constant
- decreasing functions of σ : $m^*_{\omega}(\sigma)$, $m^*_{\rho}(\sigma) \leftarrow$ self-consistent σ field results in *increase* of ρ an ω masses
- universal scaling $m_{\sigma}^*/m_{\sigma} \approx m_{\omega}^*/m_{\omega} \approx m_{\rho}^*/m_{\rho} = \Phi(n)$

Lattice QCD (SC-QCD): common drop of meson masses [Ohnishi Miura Kawamoto Mod.Phys.Lett A23, 2459] *Sliding vaccua and double decimation concept* [Brown, Rho PR396(2004)1]

"vector manifestation" [Harada, Yamawaki]

Half-skyrmion model of dense nuclear matter [Vento; Rho, Hyun Kyu Lee 1704.02775]

$$\mathcal{L} = \bar{\Psi}_N \Big(\partial \cdot \gamma - g_\omega \chi_\omega \omega \cdot \gamma - \frac{1}{2} g_\rho \chi_\rho \rho \cdot \gamma \tau \Big) \Psi_N - m_N \Phi_N \bar{\Psi}_N \bar{\Psi}_N$$
$$+ \frac{\partial^\mu \sigma \partial_\mu \sigma}{2} - \Phi_\sigma^2 \frac{m_\sigma^2 \sigma^2}{2} - U(\sigma) - \frac{\omega_{\mu\nu} \omega^{\mu\nu}}{4} + \Phi_\omega^2 \frac{m_\omega^2 \omega_\mu \omega^\mu}{2} - \frac{\rho_{\mu\nu} \rho^{\mu\nu}}{4} + \Phi_\rho^2 \frac{m_\rho^2 \rho_\mu \rho^\mu}{2}$$

Energy-density functional

 $B \in SU(3)$ ground state multiplet scalar field $f = g_{\sigma} \chi_{\sigma} \sigma / m_N$ $E[f, \{n_{\rm B}\}] = \sum E_{\rm kin}(p_{{\rm F},B}, m_B \Phi_B(f)) + \sum E_{\rm kin}(p_{{\rm F},l}, m_l)$ $l=e.\mu$ $+\frac{m_N^4 f^2}{2C^2}\eta_{\sigma}(f)+\frac{1}{2m_{\omega}^2}\left[\frac{C_{\omega}^2 \widetilde{n}_B^2}{n_{\omega}(f)}+\frac{C_{\rho}^2 \widetilde{n}_I^2}{n_{\omega}(f)}+\frac{C_{\phi}^2 \widetilde{n}_S^2}{n_{\omega}(f)}\right],$ $C_i = \frac{g_{iN}m_N}{m_i}, i = \sigma, \omega, \rho$ $C_\phi = m_\omega C_\omega/m_\phi$ effective densities: $\widetilde{n}_B = \sum_{P} x_{\omega B} n_B$ $\widetilde{n}_I = \sum_{P} x_{\rho B} t_{3B} n_B$ $\widetilde{n}_S = \sum_{H} x_{\phi H} n_H$ with coupling constant ratios $x_{\omega(\rho)B} = \frac{g_{\omega(\rho)B}}{g_{\omega(\rho)N}} \quad x_{\phi H} = \frac{g_{\phi H}}{g_{\omega N}}$ scaling functions mass scaling: $\eta_i(f) = rac{\Phi_i^2(f)}{\sqrt{2}(f)}, \quad i = \sigma, \, \omega, \,
ho$ $\Phi_m(f) \approx \Phi_N(f) = 1 - f$ $\Phi_H(f) = 1 - x_{\sigma H} \frac{m_N}{m_H} \xi_{\sigma H} f$

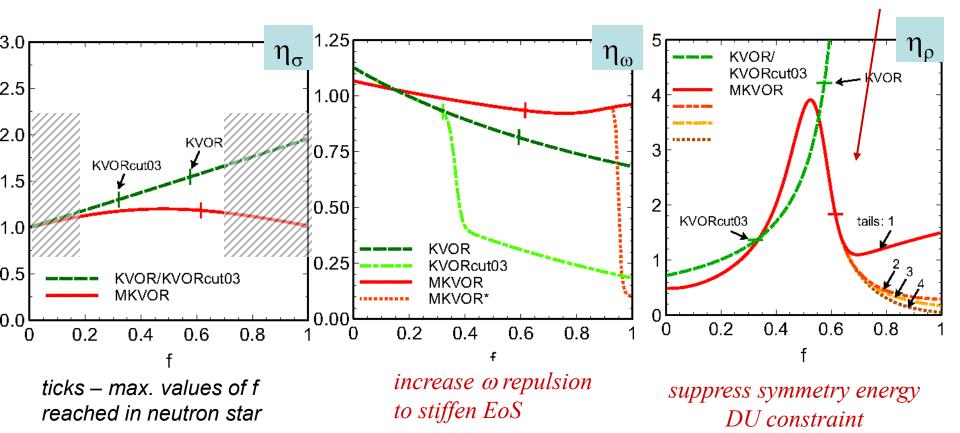
The standard sigma potential can be introduced as $\eta_{\sigma}(f) = 1 + rac{2 C_{\sigma}^2}{m^4 - f^2} U(f)$

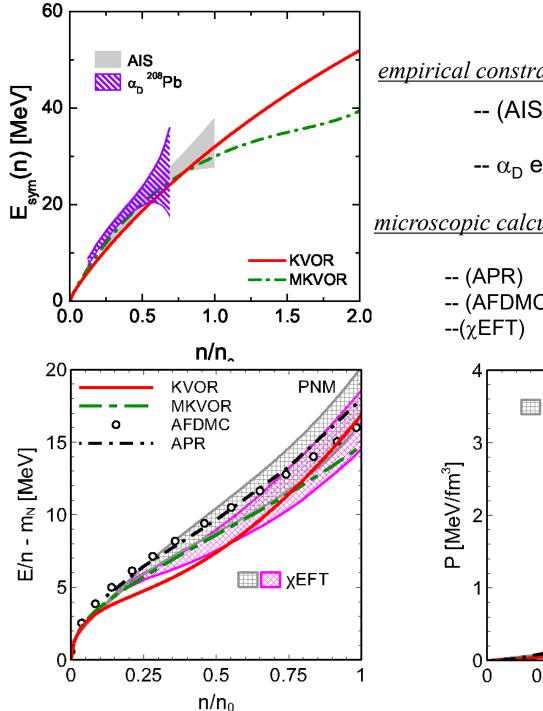
[Maslov, EEK Voskresensky, PLB748,369 (2015); NPA950,64(2016)]

	\mathcal{E}_0	n_0	K	$m_N^*(n_0)$	\widetilde{J}_0	L	K'	$K_{\rm sym}$
EoS	[MeV]	$[fm^{-3}]$	[MeV]	$[m_N]$	[MeV]	[MeV]	[MeV]	[MeV]
MKVOR	-16	0.16	240	0.73	30	41	557	-159

scaling functions for coupling constants vs scalar field:

saturate f growth





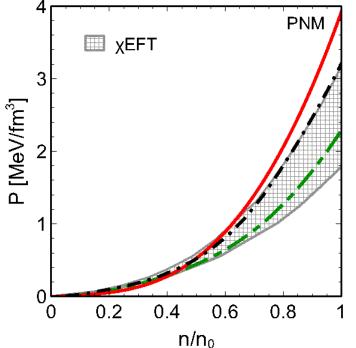
Neutron matter EoS

empirical constraints on symmetry energy

-- (AIS) analog isobar states [Danielewicz, Lee NPA 922 (2014) 1] -- α_{D} electric dipole polarizability ²⁰⁸Pb [Zhang, Chen 1504.01077]

microscopic calculations

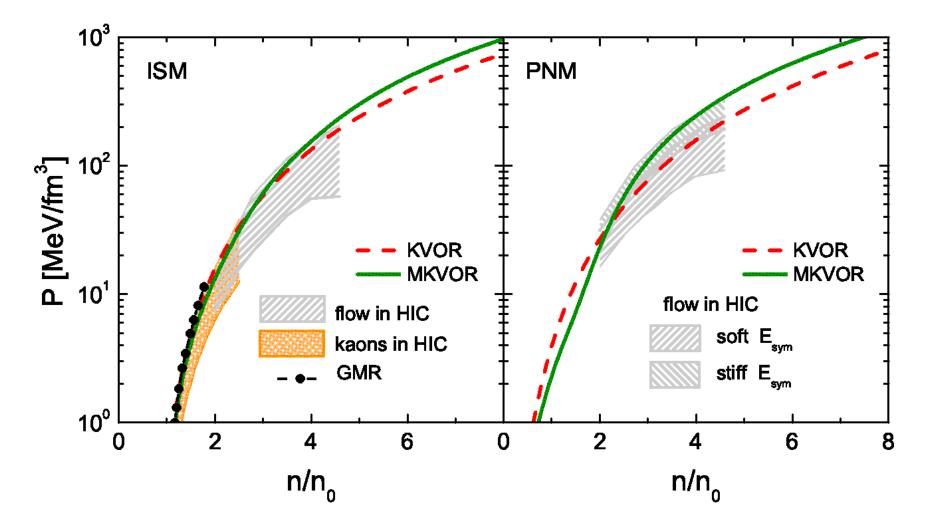
Akmal, Pandharipande, Ravenhall -- (AFDMC) Gandolfi et al.MNRAS 404 (2010) L35 Hebeler, Schwenk EPJA 50 (2014) 11

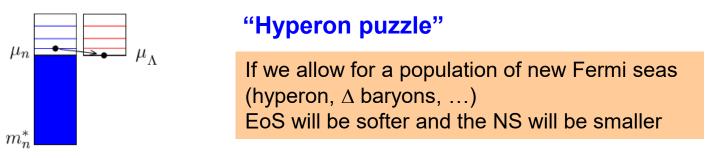


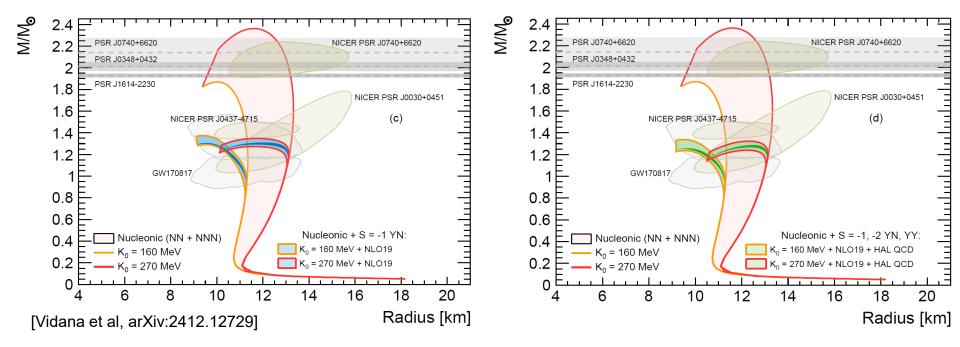
Constraints on EoS from HICs

Particle flow: Danielewicz, Lacey and Lynch, Science 298 (2002) 1592

Kaon production: Fuchs, Prog. Part. Nucl. Phys. 56 (2006) 1







Simple solutions: -- make nuclear EoS as stiff as possible [flow constraint] -- suppress hyperon population (increase repulsion/reduce attraction)

against phenomenology of YN,NN,YY interaction in vacuum +hypernuclear physics

Attempts to solve the hyperon puzzle

play with hyperon coupling constants

$$g_{\omega N}: g_{\omega \Lambda}: g_{\omega \Sigma}: g_{\omega \Xi} = 3:2:2:1$$

 $g_{\rho N}: g_{\rho \Lambda}: g_{\rho \Sigma}: g_{\rho \Xi} = 1:0:2:1$

$$_{H} = \frac{x_{\omega H} n_{0} C_{\omega}^{2} / m_{N}^{2} - U_{H}(n_{0})}{m_{N} - m_{N}^{*}(n_{0})} \qquad \begin{cases} U_{\Sigma}(n_{0}) = -20 \text{ MeV} \\ U_{\Sigma}(n_{0}) = +30 \text{ MeV} \\ U_{\Xi}(n_{0}) = -15 \text{ MeV} \end{cases}$$

extensions

phi meson: HH' repulsion

 x_{σ}

$$g_{\phi N}: g_{\phi \Lambda}: g_{\phi \Sigma}: g_{\phi \Xi} = 0:2:2:1$$
 $g_{\phi \Lambda} = -\frac{\sqrt{2}}{3}g_{\omega N}$

[J. Schaffner et al., PRC71 (1993), Ann.Phys. 235 (94), PRC53(1996)]

SU(3) coupling constants: extra parameters to tune. two effects: $|g_{\omega H}|$ increases; $g_{\phi N}$ non zero

[Weissenborn et al., PRC85 (2012);NPA881 (2012); NPA914(2013)]

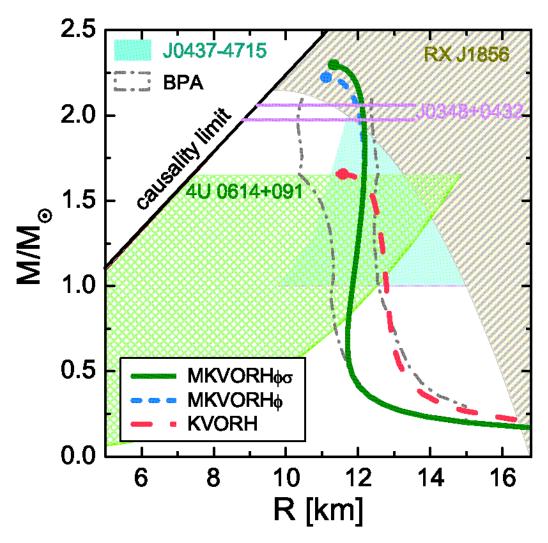
alternative

mass of ϕ meson If we take into account a reduction of the ϕ mass in medium we can increase a HH repulsion

 $x_{mH}=rac{g_{mH}}{g_{mN}}$

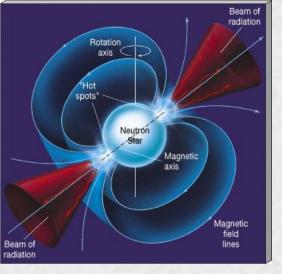
 $(U_{\rm A}(p_{\rm o})) = -28 \,{\rm MeV}$

Mass-radius constraints



BPA: Bayesian probability analysis [Lattimer, Steiner ...]

msp PSRJ0437-4715: 3σ confidence Bogdanov ApJ 762, 96 (2013)



Pulsar age

Pulsar rotation period/frequency changes with time:

$$\dot{\Omega} = -\frac{B^2 R^6}{6 c^3 I} \sin^2 \alpha \, \Omega^3 - G \frac{32\varepsilon^2}{5c^5} I \, \Omega^2$$

e.m. wave emission

angle between rotation a and magnetic axes

ɛ neutron star eccentricity

grav. wave

emission

 $\dot{\Omega} = \varkappa \Omega^n$ $P = 2\pi/\Omega$

 $(n-1)\frac{\dot{P}(t)}{P(t)}t = 1 - \left(\frac{P_0}{P(t)}\right)^{n-1}$

initial period current period

 $P_0/P \ll 1$

spin-down age:

$$\tau_{\rm sd} = \frac{1}{n-1} \frac{P}{\dot{P}}$$

 $n = rac{\ddot{\Omega} \,\Omega}{\dot{\Omega}^2}$

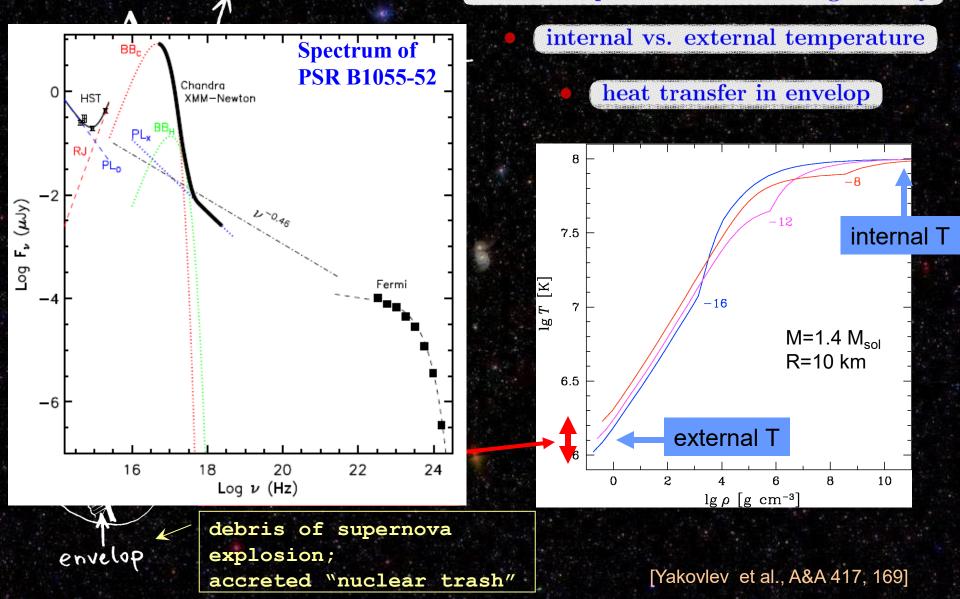
braking index:

kinematic age:

- 1) age of the associated SNR
- 2) pulsar speed and position w.r. to the geometric center of the associated SNR
- 3) historical events Crab : 1054 AD Cassiopeia A: 1680 AD Tycho's SN: 1572 AD

Measuring pulsar temperature

radiation spectrum and source geometry



Neutron star cooling

 $T < T_{\rm opac} \sim 10^{-1} \text{--} 10^0 \text{ MeV}$

neutron star is transparent for neutrino

neutrino emissivity

 $T_9 = T/10^9 \,\mathrm{K} \ll 1$

modified Urca (MU) $\overline{\checkmark}_{\epsilon}$

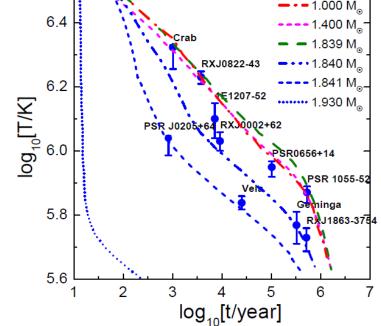
$$\sum_{n}^{n} \sum_{n}^{n} \sum_{n}^{n} \sum_{n}^{e} \frac{e}{\overline{v}} = 10^{22} \times \left(\frac{m_N^*}{m_N}\right)^4 T_9^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\text{erg}}{\text{cm}^3 \, \text{s}}$$

direct Urca (DU)

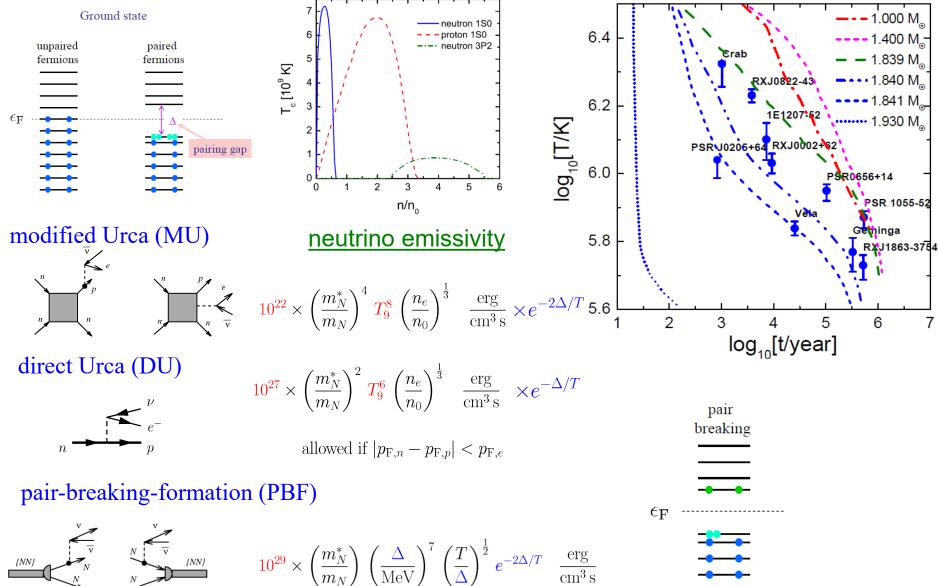
$$n \xrightarrow{\nu} p$$

$$10^{27} \times \left(\frac{m_N^*}{m_N}\right)^2 T_9^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\text{erg}}{\text{cm}^3 \text{ s}}$$
allowed if $|p_{\text{F},n} - p_{\text{F},p}| < p_{\text{F},e}$

$$x = \frac{n_p}{n} \gtrsim 11\%$$



Neutron star cooling



Simple model for NS matter

 $n+p+e+\mu$ matter

Proton concentration $x = n_p/n$ follows from equations

$$\mu_{e} = \mu_{n} - \mu_{p} = \frac{\partial E}{\partial n_{n}} - \frac{\partial E}{\partial n_{n}} = 4 E_{S}(n) (1 - 2x)$$

$$\frac{(\mu_{e}^{2} - m_{e}^{2})^{3/2}}{3\pi^{2}} + \frac{(\mu_{e}^{2} - m_{\mu}^{2})^{3/2}}{3\pi^{2}} = x n$$

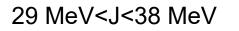
$$\lim_{n \to \infty} \frac{1}{3\pi^{2}} + \frac{1}{3\pi^{2}} + \frac{(\mu_{e}^{2} - m_{\mu}^{2})^{3/2}}{3\pi^{2}} = x n$$

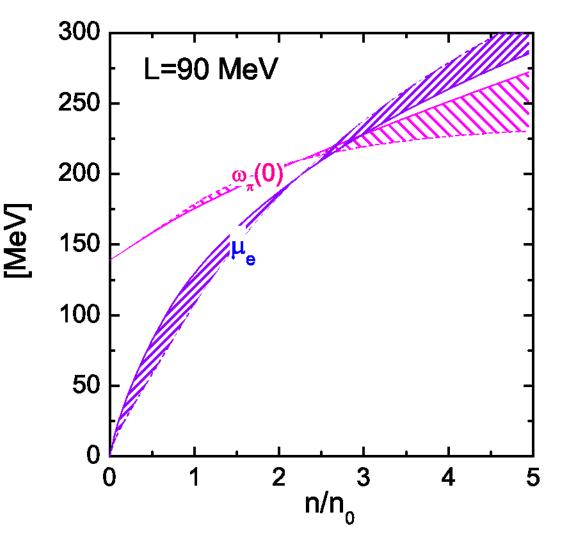
$$\lim_{n \to \infty} \frac{1}{3\pi^{2}} + \frac$$

Chiral symmetry for pion-nucleon interaction

 $T^{(\pm)} = \frac{1}{2} \left[T(\pi^{-}p) \pm T(\pi^{-}n) \right]$ isospin even and odd amplitudes **Chiral symmetry:** for forward scattering amplitude: $T^{(\pm)}(\omega, q^2)$ on mass-shell $q^2 = \omega^2 - \mathbf{k}^2 = m_\pi^2$ $\lim_{\omega \to 0} \omega^{-1} T^{(-)}(\omega, m_{\pi}^2) = \frac{1}{2 f_{\pi}^2} \quad \text{Weinberg Tomazawa theorem}$ polarization operator $T^{(-)}(\omega) \approx \frac{\omega}{2 f_{-}^2}$ $\Pi_{\rm S}(\omega) = -T^{(-)}(\omega) \left(\rho_p - \rho_n\right)$ repulsive in neutron reach matter _____ **spectrum** $D^{-1}(\omega, k = 0) = \omega^2 - m_{\pi}^2 - \prod_S(\omega) = 0$

chiral shield against pionization





Detailed analysis of the possibility of the s-wave pion condensation in Onishi, Jido et al, PRC 80, 038202 (2009)

Conclusion

NSs and HICs are the only sources of the information about properties of the strongly interacting matter under extreme condition.

They provide test for our theories and models in dynamical systems.

Problems in discussion:

Relativistic equation of state? Inclusion of new particles (hyperons, Deltas)? Meson in medium?