South Africa - JINR Workshop on Theoretical and Computational Physics

The entrance channel effect in the reactions of heavy-ion collisions

A.K. Nasirov[1,2], E.D. Khusanov[2,3]

[1]Bogoliubov Laboratory of Theoretical Physics, JINR , Dubna, Russia
[2]Institute of Nuclear Physics, Academy of Science of Uzbekistan
[3] National University of Uzbekistan, Tashkent

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Content

- Introduction
- Nature of hindrance in complete fusion of the massive nuclei in heavy ion collisions.
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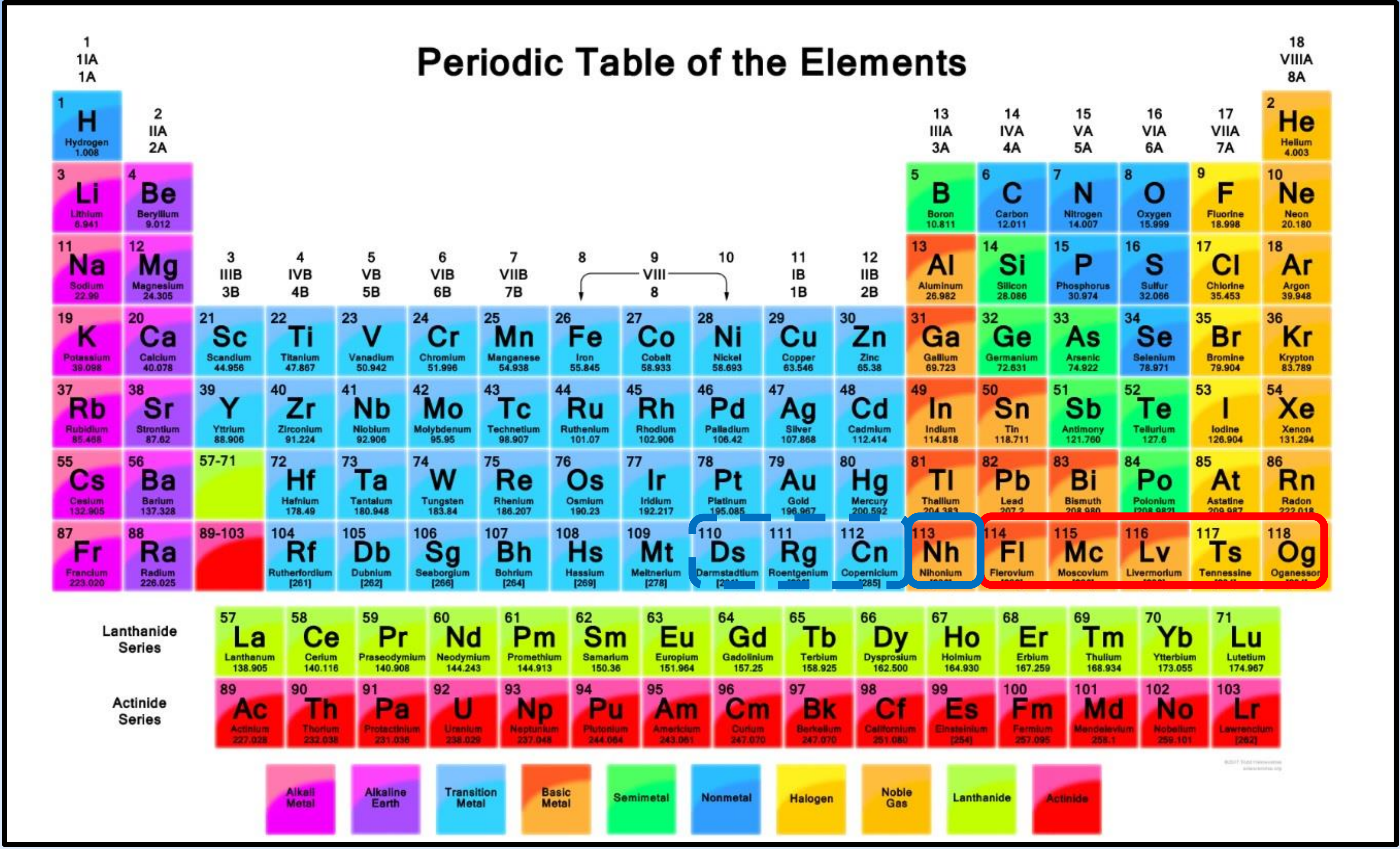
Conclusions.



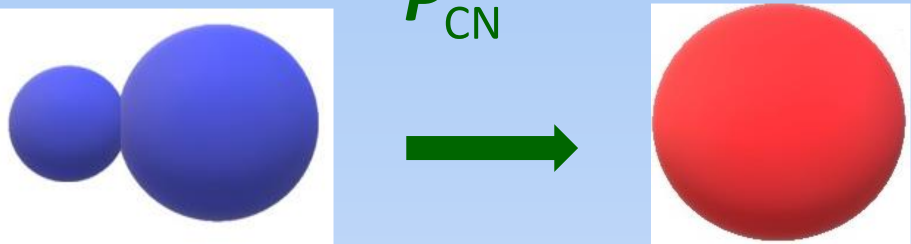
Periodic Table of the Elements
1 1IA 1A
2 IIA 2A
13 IIIA 3A
14 IVA 4A
15 VA 5A
16 VIA 6A
17 VIIA 7A
18 VIIIA 8A
3 IIIB 3B
4 IVB 4B
5 VB 5B
6 VIB 6B
7 VIIB 7B
8 9 10 VIII 8
11 IB 1B
12 IIB 2B
1 H Hydrogen 1.008
2 He Helium 4.003
3 Li Lithium 6.941
4 Be Beryllium 9.012
5 B Boron 10.811
6 C Carbon 12.011
7 N Nitrogen 14.007
8 O Oxygen 15.999
9 F Fluorine 18.998
10 Ne Neon 20.180
11 Na Sodium 22.99
12 Mg Magnesium 24.305
13 Al Aluminum 26.982
14 Si Silicon 28.086
15 P Phosphorus 30.974
16 S Sulfur 32.066
17 Cl Chlorine 35.453
18 Ar Argon 39.948
19 K Potassium 39.098
20 Ca Calcium 40.078
21 Sc Scandium 44.956
22 Ti Titanium 47.867
23 V Vanadium 50.942
24 Cr Chromium 51.996
25 Mn Manganese 54.938
26 Fe Iron 55.845
27 Co Cobalt 58.933
28 Ni Nickel 58.693
29 Cu Copper 63.546
30 Zn Zinc 65.38
31 Ga Gallium 69.723
32 Ge Germanium 72.631
33 As Arsenic 74.922
34 Se Selenium 78.971
35 Br Bromine 79.904
36 Kr Krypton 83.789
37 Rb Rubidium 85.468
38 Sr Strontium 87.62
39 Y Yttrium 88.906
40 Zr Zirconium 91.224
41 Nb Niobium 92.906
42 Mo Molybdenum 95.95
43 Tc Technetium 98.907
44 Ru Ruthenium 101.07
45 Rh Rhodium 102.906
46 Pd Palladium 106.42
47 Ag Silver 107.868
48 Cd Cadmium 112.414
49 In Indium 114.818
50 Sn Tin 118.711
51 Sb Antimony 121.760
52 Te Tellurium 127.6
53 I Iodine 126.904
54 Xe Xenon 131.294
55 Cs Cesium 132.905
56 Ba Barium 137.328
57-71
72 Hf Hafnium 178.49
73 Ta Tantalum 180.948
74 W Tungsten 183.84
75 Re Rhenium 186.207
76 Os Osmium 190.23
77 Ir Iridium 192.217
78 Pt Platinum 195.085
79 Au Gold 196.967
80 Hg Mercury 200.592
81 Tl Thallium
82 Pb Lead
83 Bi Bismuth
84 Po Polonium
85 At Astatine
86 Rn Radon
87 Fr Francium 223.020
88 Ra Radium 226.025
89-103
104 Rf Rutherfordium [261]
105 Db Dubnium [262]
106 Sg Seaborgium [266]
107 Bh Bohrium [264]
108 Hs Hassium [269]
109 Mt Meitnerium [278]
110 Ds Darmstadtium
111 Rg Roentgenium
112 Cn Copernicium [285]
113 Nh Nihonium
114 Fl Flerovium
115 Mc Moscovium
116 Lv Livermorium
117 Ts Tennessine
118 Og Oganesson
Lanthanide Series
57 La Lanthanum 138.905
58 Ce Cerium 140.116
59 Pr Praseodymium 140.908
60 Nd Neodymium 144.243
61 Pm Promethium 144.913
62 Sm Samarium 150.36
63 Eu Europium 151.964
64 Gd Gadolinium 157.25
65 Tb Terbium 158.925
66 Dy Dysprosium 162.500
67 Ho Holmium 164.930
68 Er Erbium 167.259
69 Tm Thulium 168.934
70 Yb Ytterbium 173.055
71 Lu Lutetium 174.967
Actinide Series
89 Ac Actinium 227.028
90 Th Thorium 232.038
91 Pa Protactinium 231.036
92 U Uranium 238.029
93 Np Neptunium 237.048
94 Pu Plutonium 244.064
95 Am Americium 243.061
96 Cm Curium 247.070
97 Bk Berkelium 247.070
98 Cf Californium 251.080
99 Es Einsteinium [254]
100 Fm Fermium 257.095
101 Md Mendelevium 258.1
102 No Nobelium 259.101
103 Lr Lawrencium [262]
Alkali Metal
Alkaline Earth
Transition Metal
Basic Metal
Semimetal
Nonmetal
Halogen
Noble Gas
Lanthanide
Actinide

5 new superheavy elements obtained in FLNR, in JINR (Dubna).

48 new isotopes

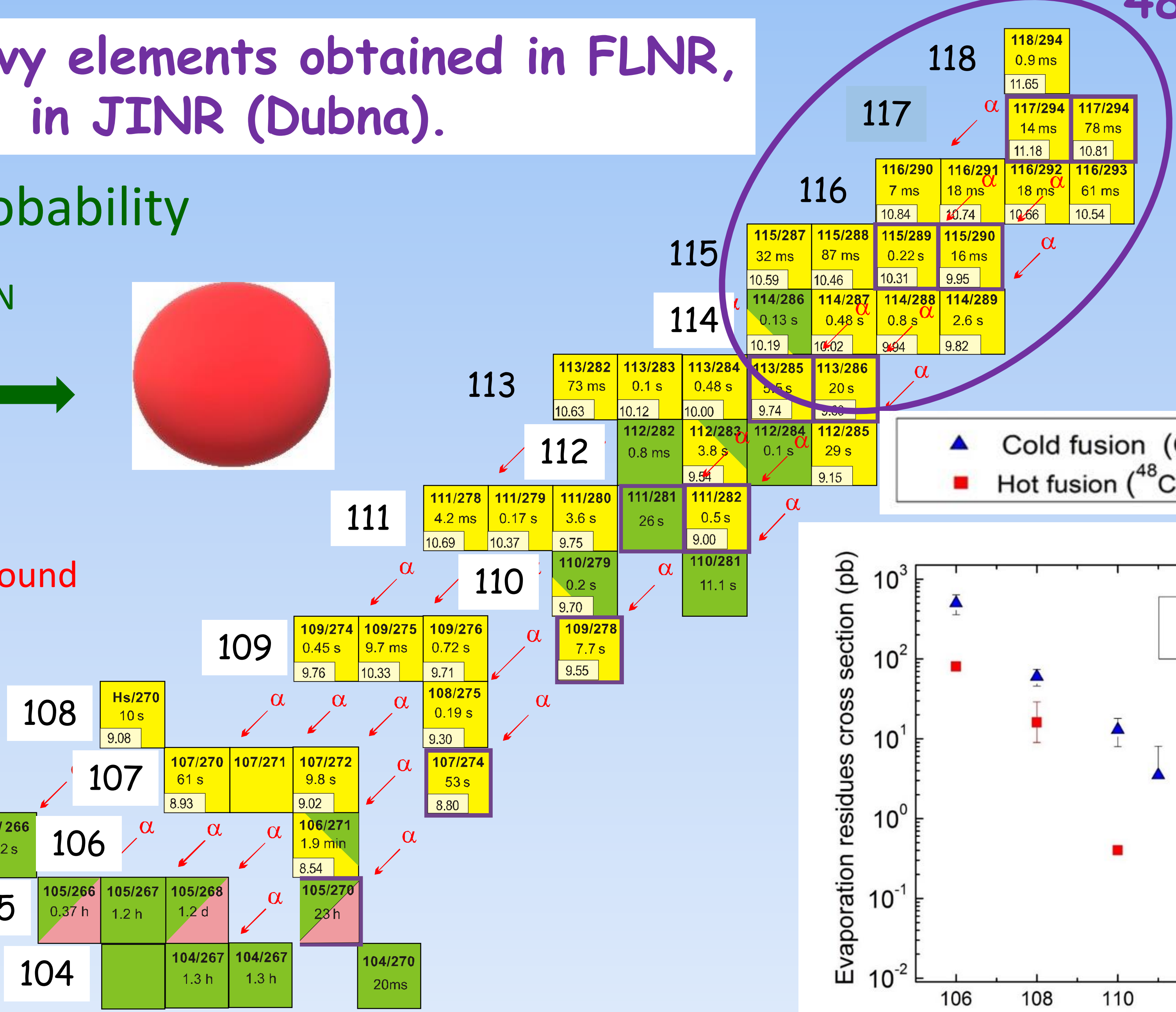
Fusion probability

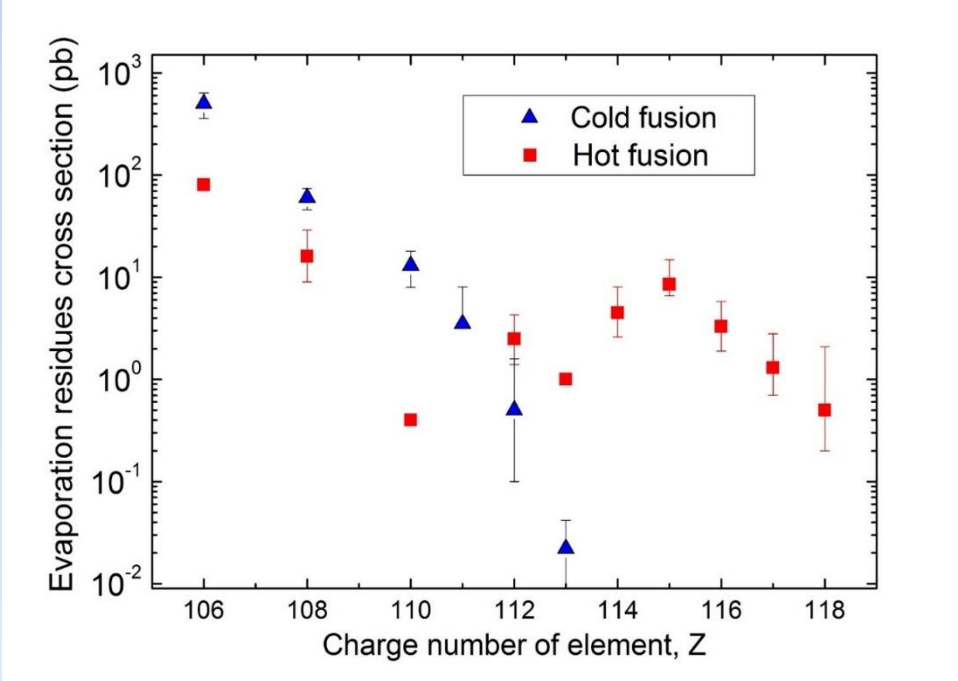
P_{CN}



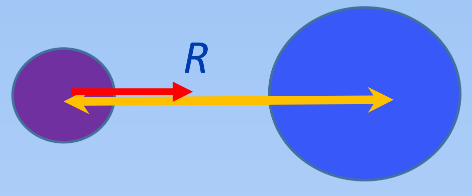
$Z_1+Z_2=Z_{Compound}$

$A_1+A_2=A_{Compound}$

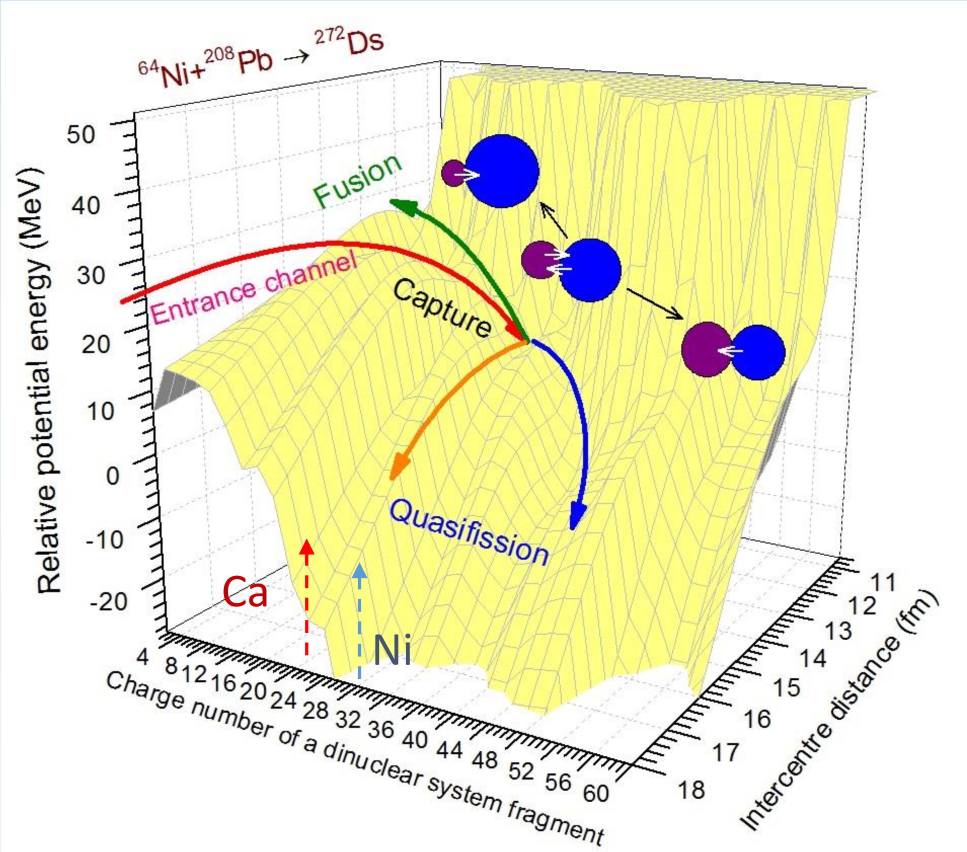


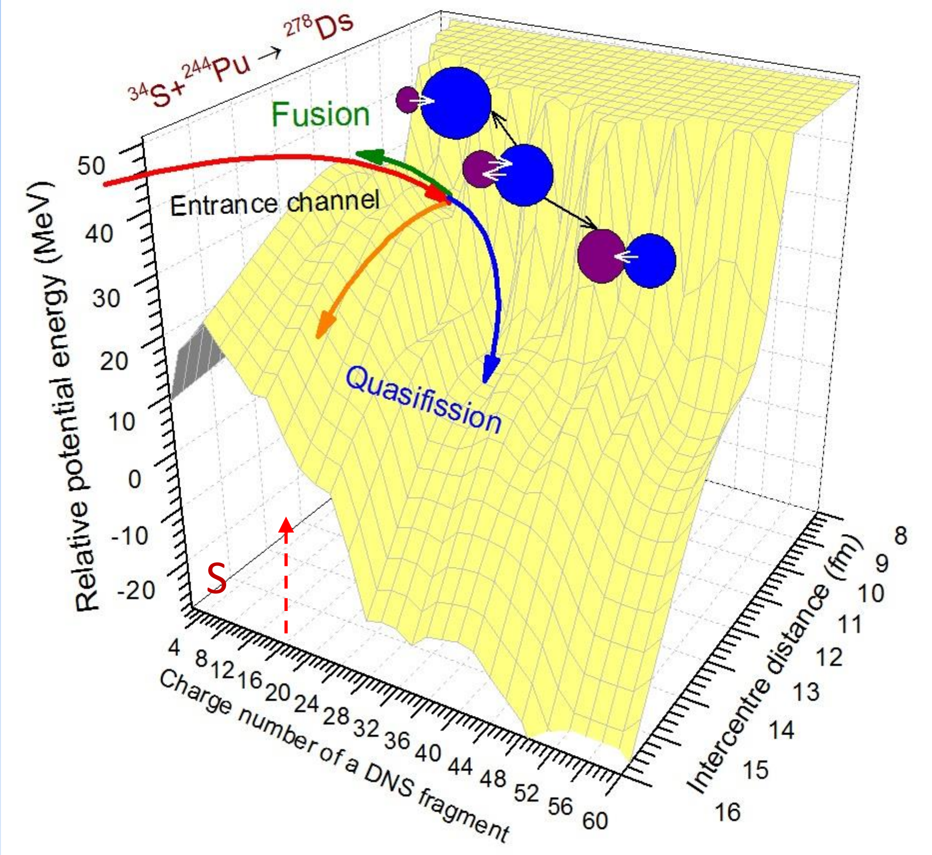


Differences in the mechanisms of cold and hot fusion according to dinuclear system model



A.K. Nasirov , A.I.Muminov, G. Giardina, and G. Mandaglio,
Physics of Atomic Nuclei, 2014, Vol. 77, No. 7, pp. 881–889

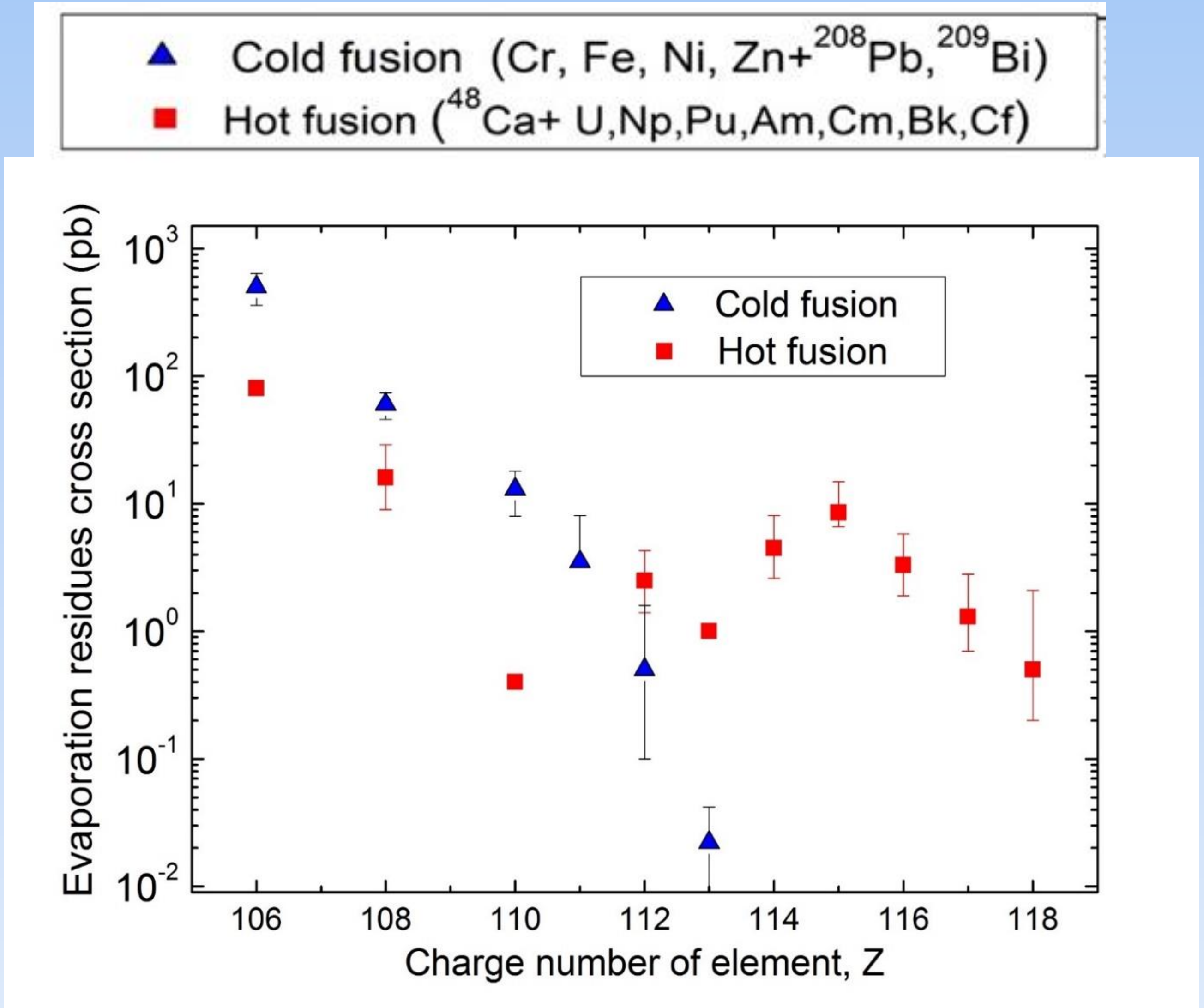


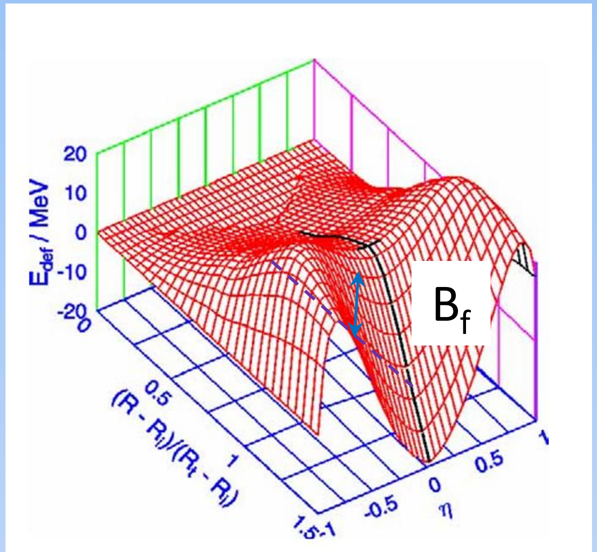


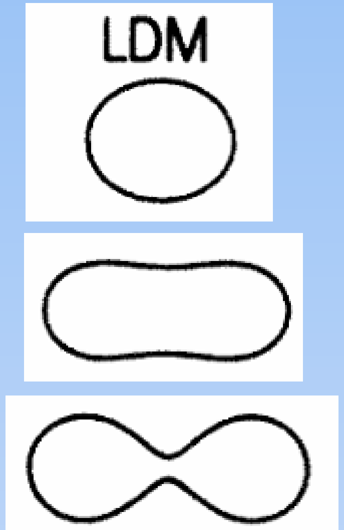
Potential energy surface for the dinuclear system, Z and A charge-mass numbers of the light fragment.

$U(Z,A,R)=V_{int}(R,Z,A)+B_1(Z,A)+B_2(Z_{CN}-Z,A_{CN}-A)-B_{CN}$; B_i $(i=1,2,\mathrm{CN})$ binding energies

The cross sections of the synthesis of the same superheavy elements, obtained for the cold and hot fusion reactions, shows the dependence of the fission barrier on neutron numbers.







M. Kowal et al. Phys. Rev. C82,014303 (2010)

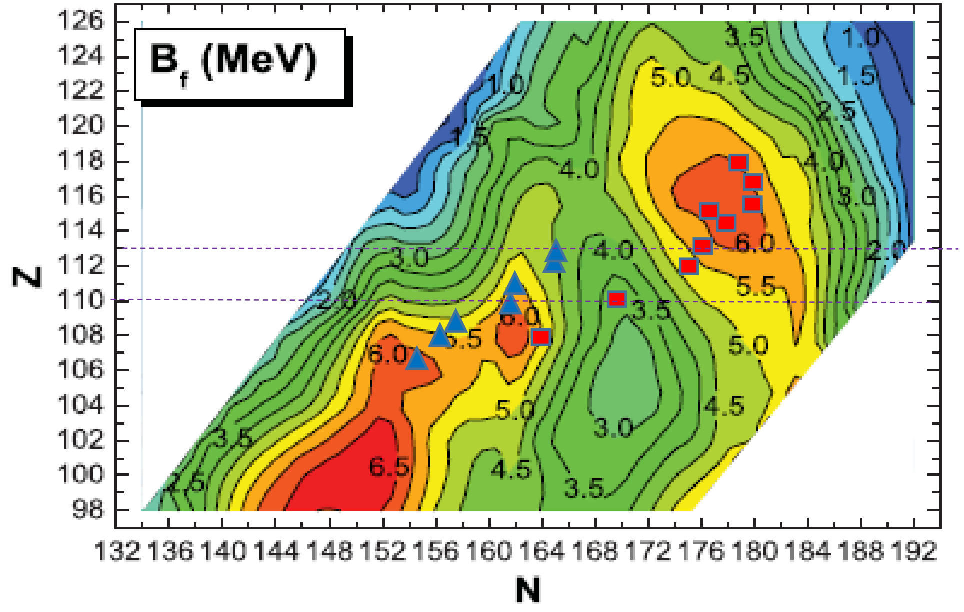


FIG. 6. (Color online) Contour map of calculated fission barrier heights B_f for even-even superheavy nuclei.

The experimental and theorerical methods studying a mechanism of the complete fusion reactions leading to synthesis the new super heavy elements have open problems.

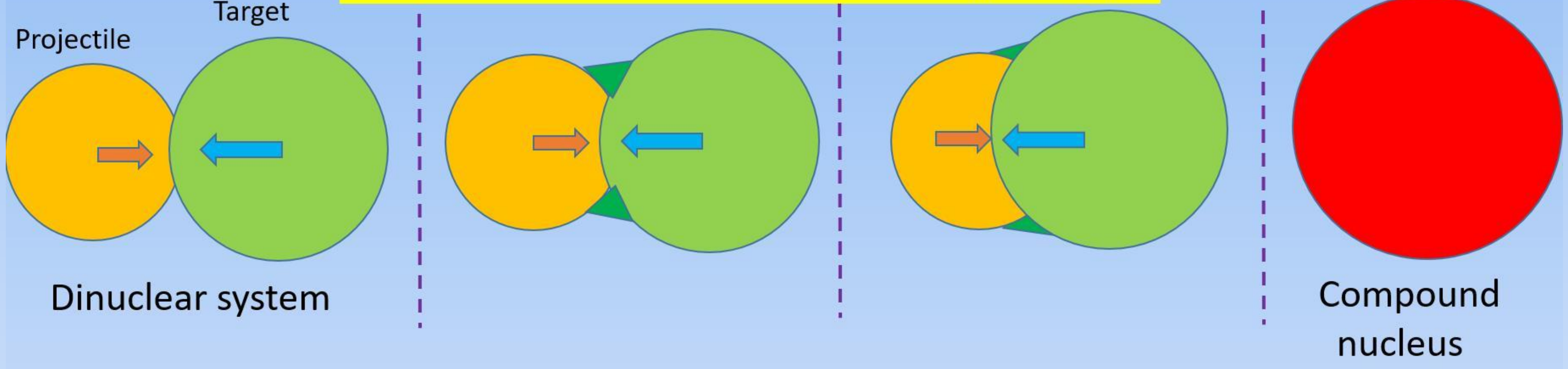
1. Theoretical methods are developed on the base of the knowledge of the reaction mechanism obtained from the experimental studies.

2. At the same time the analysis of the experimental data requires the relevant methods allowing to identify surely the mechanism producing the observed nuclei by detectors.

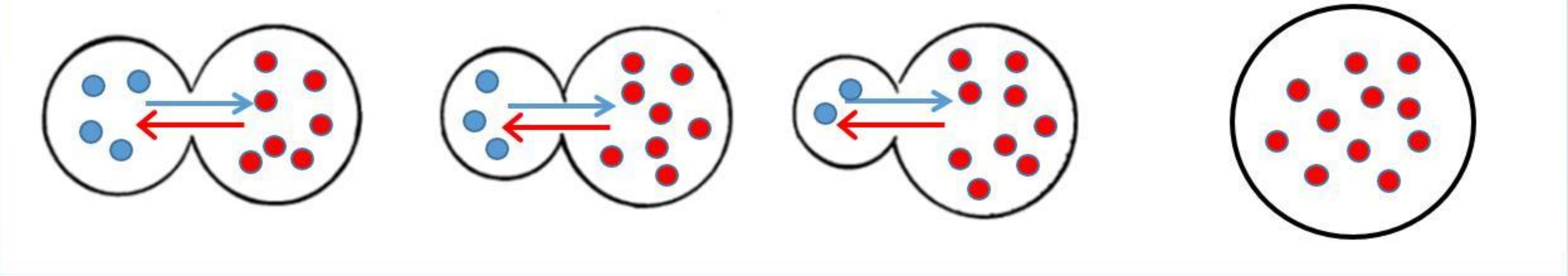
3. The successful choice of the degrees of freedom of the interacting system depends on the assumption about the way of complete fusion.
   There are two ways of complete fusion.

Two different mechanisms of the complete fusion.

Fusion mechanism as confluence of liquid drops



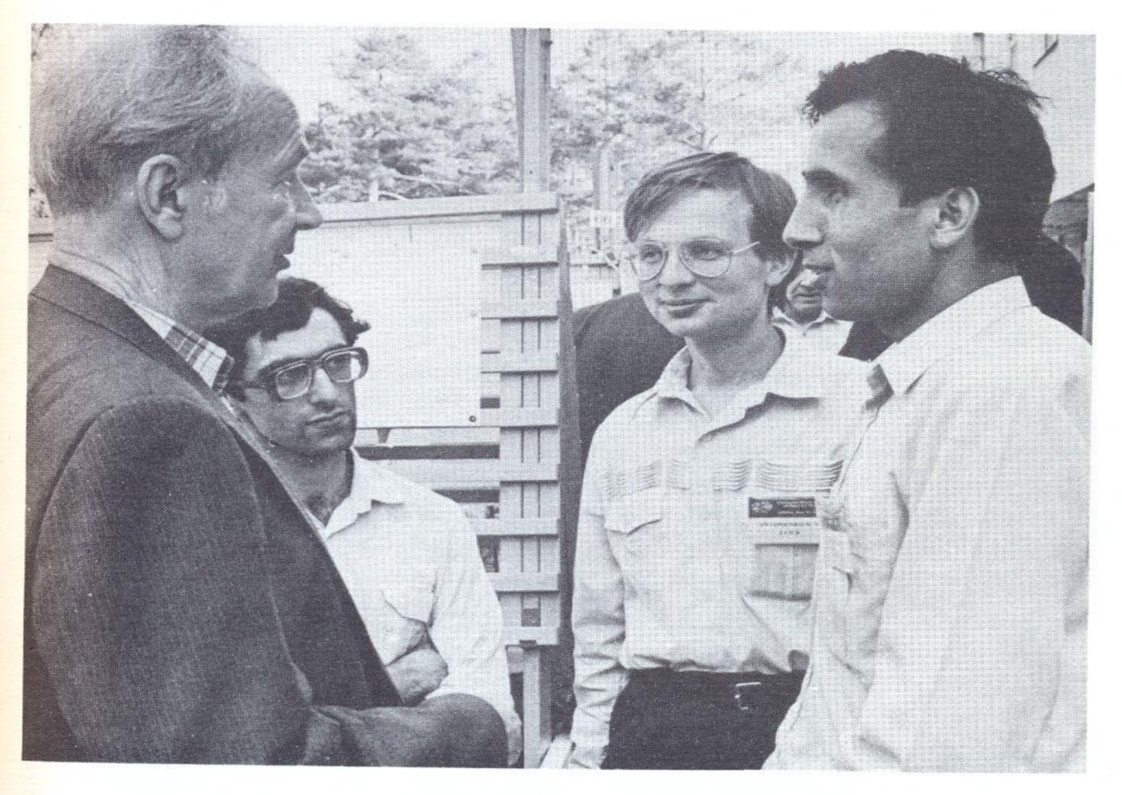
Multinucleon transfer mechanism of the complete fusion suggested by Prof. Vadim Volkov



Heavy Ion Conference in Dubna, Bogoliubov Laboratory of Theoretical Physics, JINR, September 1966

International School-Seminar on Heavy Ion Physics in Dubna, Bogoliubov Laboratory of Theoretical Physics, JINR, September 1993.

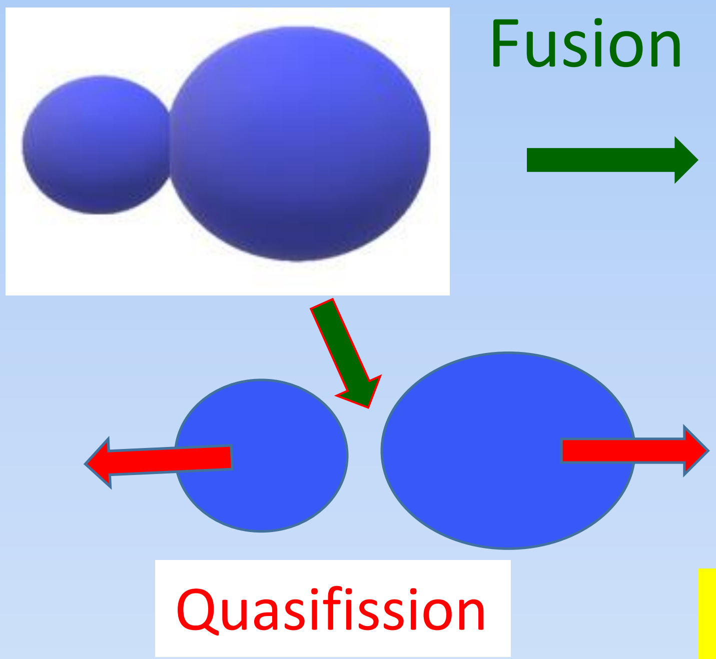
Rudolf Bock, Vadim Volkov, and Wolter Greiner

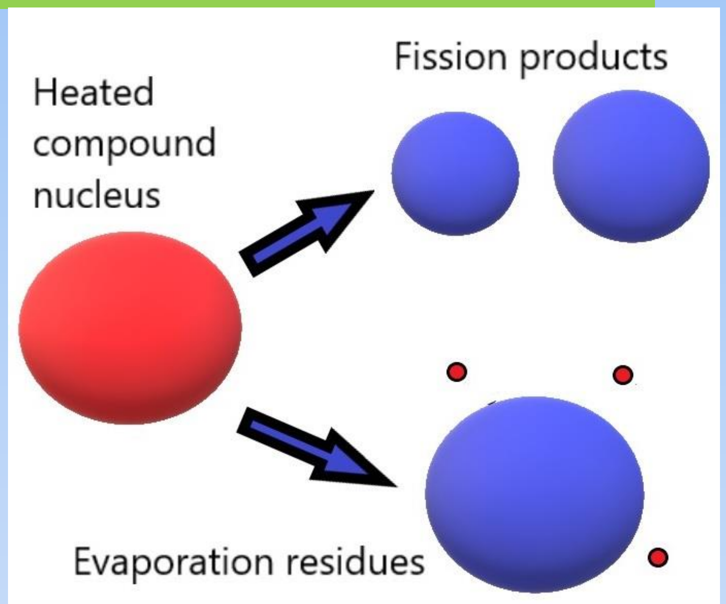


Vadim Volkov, Gurgen Adamian, Nikolai Antonenko, and Avazbek Nasirov

Reasons causing a hindrance to formation of the evaporation residues in synthesis of the superheavy elements complete fusion.

Competition between quasifission and formation of the compound nucleus is the other reason causing decreasing of the probability of synthesis of superheavy elements. The quasifission is dominant in cold fusion processes.





$$\sum_{l=0}^{l=l_f} \sigma_{cap}(E_{c.m.}, l)\, 1\, W_{surv}$$

$$\sigma_{ER}(E^*) = \sum_{\ell=0}^{\ell=\ell_f} \sigma_{cap}(E_{c.m.}, \ell)\, P_{CN}(E^*_{DNS}, \ell)\, W_{surv}(E_{CN}^*, \ell)$$

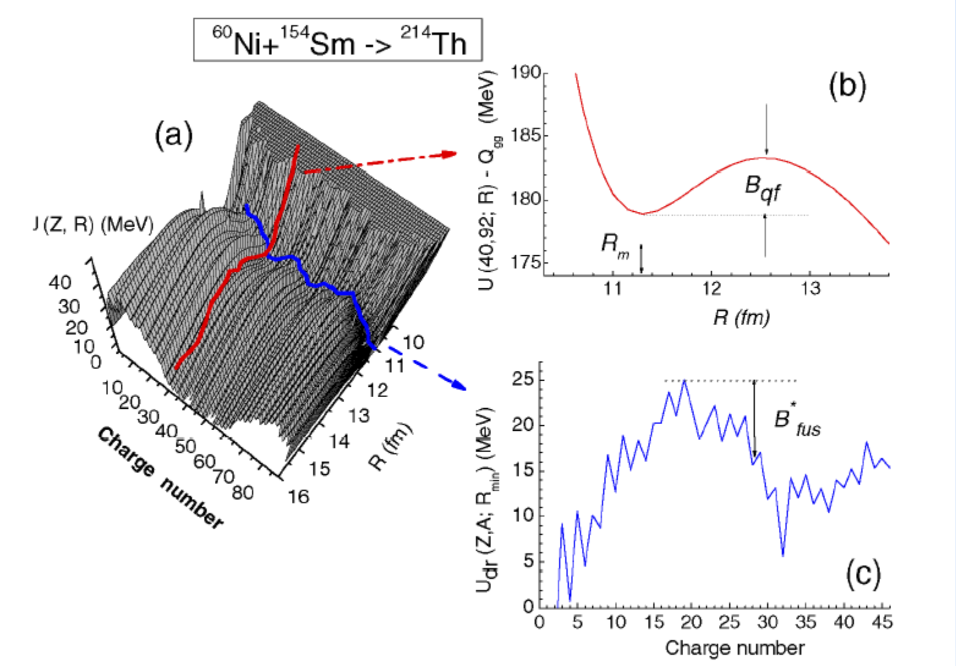
N.V. Antonenko, V.V. Volkov, E.A. Cherepanov, A.K. Nasirov, V.P. Permyakov Phys.Lett. B 319 (1993) p.425; Phys.Rev.C 51, (1995) p.2635. For first time

Calculation of the competition between complete fusion and quasifission: $P_{cn}(E_{DNS},L)$.
Influence of the nuclear shell effects are in intrinsic barrier B^*_{fus} and in Y_Z charge (mass) distributions.

$$P_{CN}(E^*_{DNS},\ell) = \sum_{Z_{sym}}^{Z_{max}} Y_Z(E^*_{DNS},\ell) P^{(Z)}_{CN}(E^*_{DNS},\ell)$$

where

$$P^{(Z)}_{CN}(E^*_{DNS},\ell) = \frac{\rho(E^*_{DNS}(Z) - B^*_{fus}(Z),\ell)}{\rho(E^*_{DNS}(Z) - B^*_{fus}(Z),\ell) + \rho(E^*_{DNS}(Z) - B^*_{qf}(Z),\ell) + \rho(E^*_{DNS}(Z) - B^*_{sym}(Z),\ell)}$$

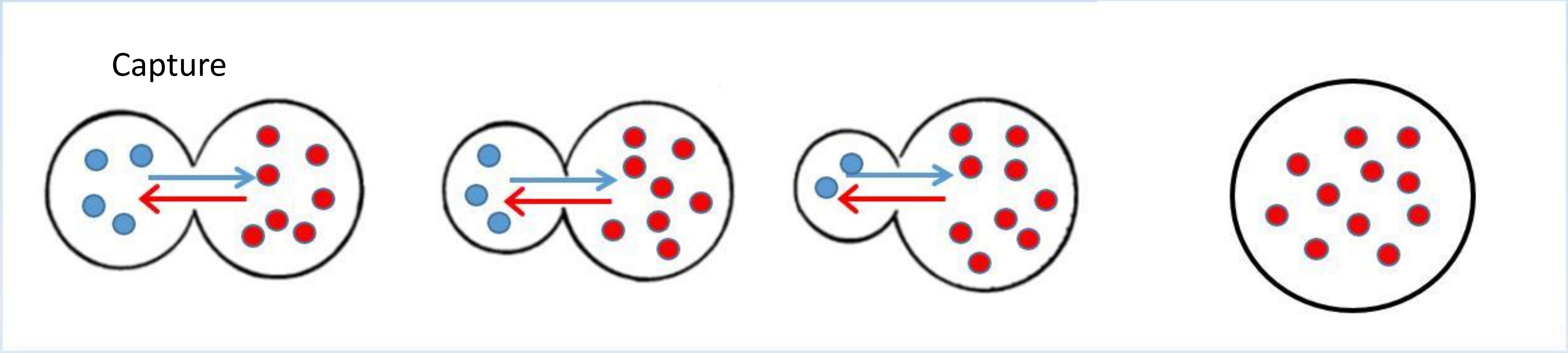


$E^*_{DNS}(Z)=E_{c.m.}-V_{min}+(B_P+B_T)-(B_Z+B_{ztot-z})$

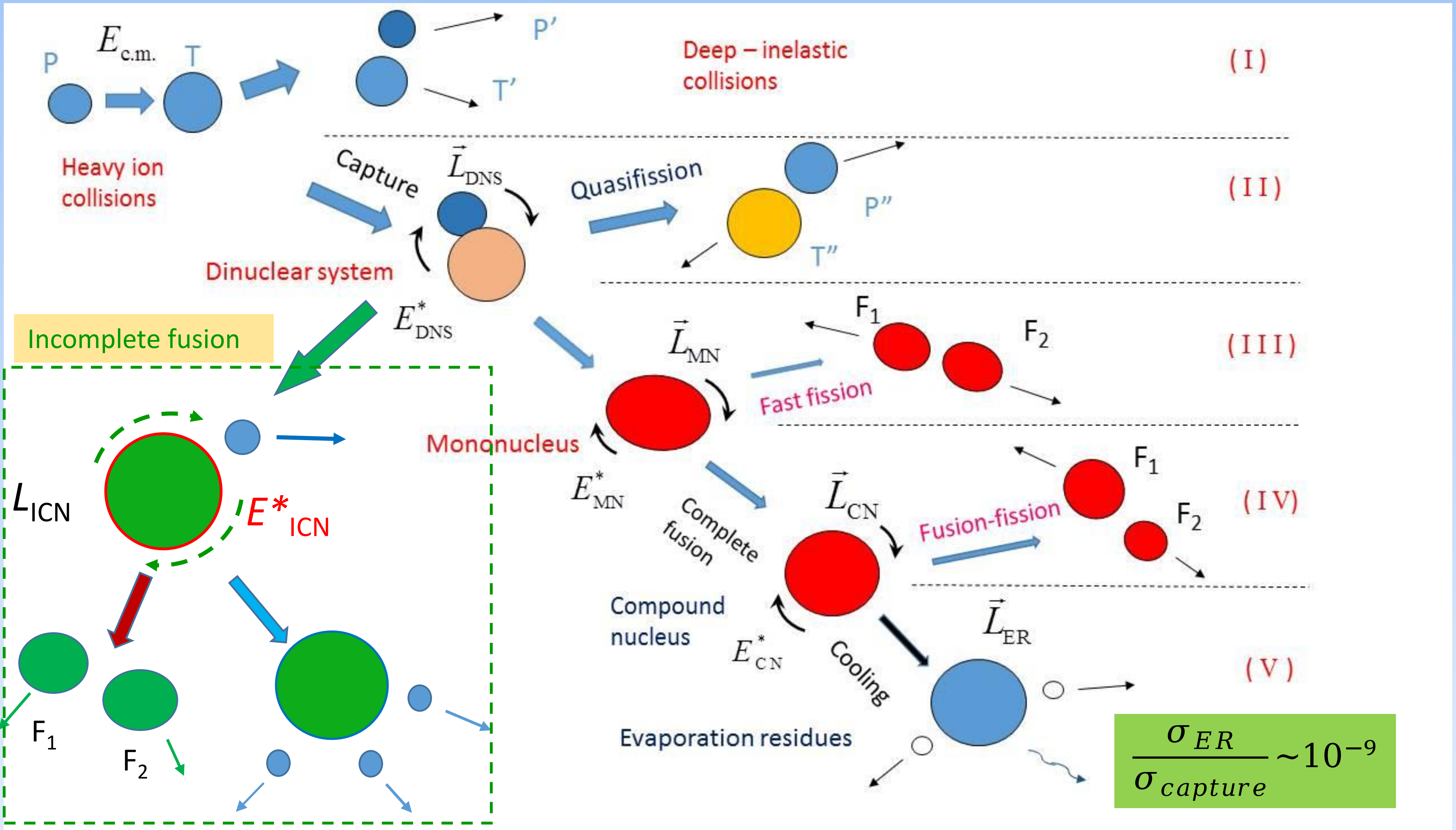
Nasirov A.K. et al. Nuclear Physics A 759 (2005) 342.

Fazio G. et al, Modern Phys. Lett. A 20 (2005) p.391

How we can prove that complete fusion occurs by multinucleon transfer through the small window (or neck) between two fragments of the dinuclear system?



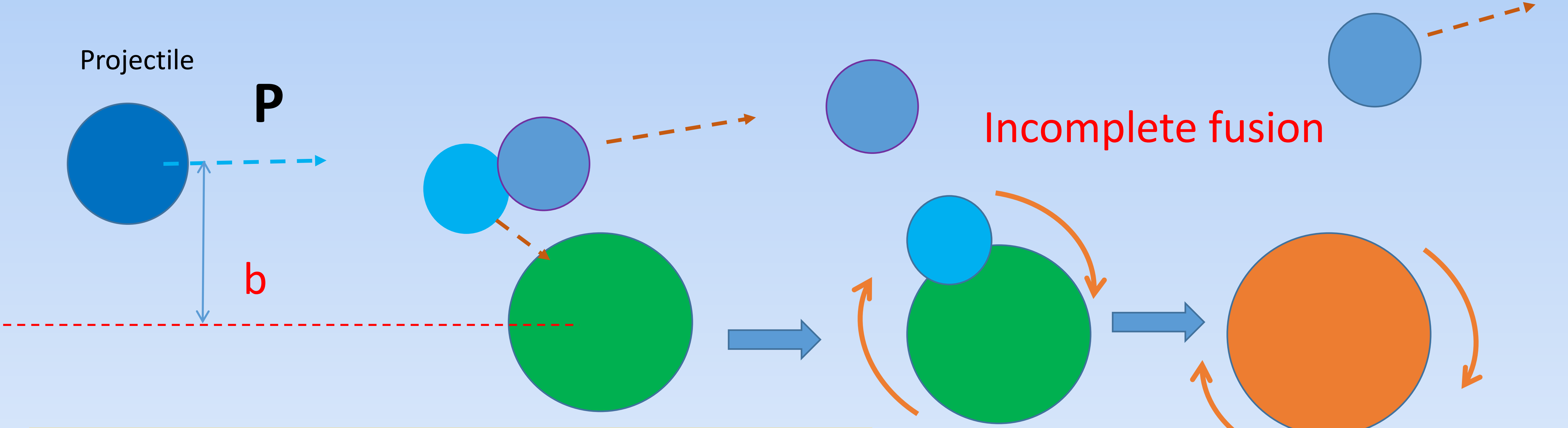
Main reaction channels of the heavy ion collisions



1. About incomplete fusion mechanism---as a type of the quasifission.

- The authors of the recent experiments concluded that “none of the theoretical models is able to explain satisfactorily the incomplete fusion reaction dynamics at lower energies below 10 MeV/nucleon” .

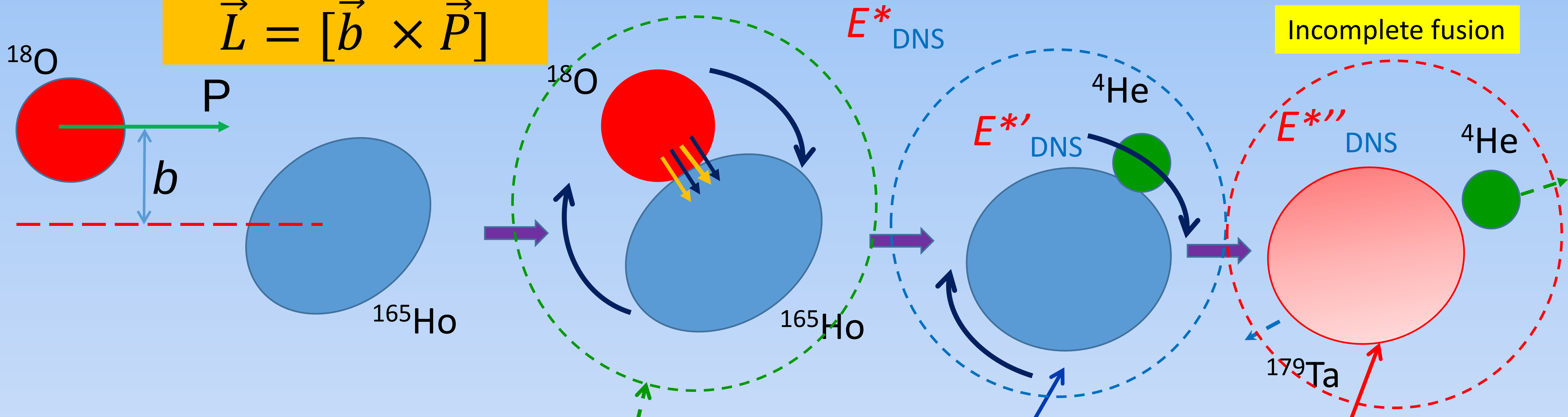
[A. Agarwal *et al.* PHYSICAL REVIEW C 103, 034602 (2021)]



Mechanism of incomplete fusion which is popular during 60 years since the first experiment was performed.

A.K.Nasirov, et. Al. Phys. Lett. B **942** (2023) 137976

$$\vec{L} = [\vec{b} \times \vec{P}]$$



Calculation of the incomplete fusion cross section

$$\sigma_{icf}(E_{\text{Lab}}, Z) = \sum_{l=0}^{l_d} \sigma_{cap}(E_{\text{Lab}}, l)\; Dz\,(E_{\text{Lab}}, l)\; \Lambda_Z(E_{\text{Lab}}, l, Z)$$

Classical equations of the radial and tangential motions with the kinetic coefficients which are calculated microscopically.

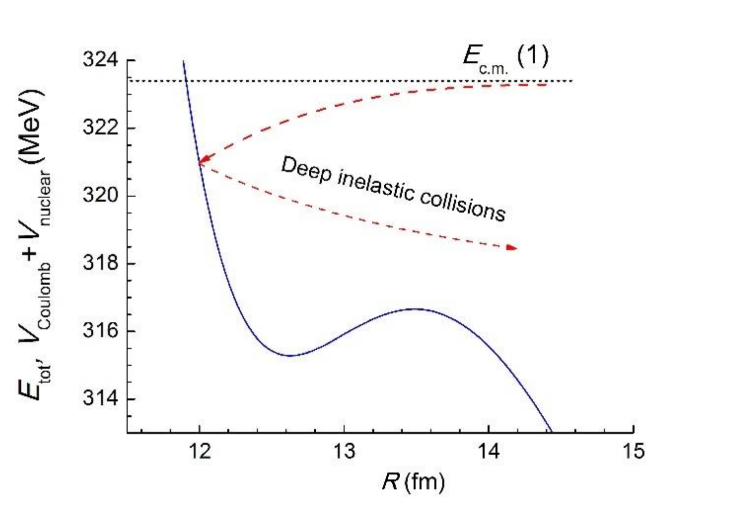
$$\frac{d\left(\mu(R)\dot{R}\right)}{dt} + \gamma_R(R)\dot{R}(t) = -\frac{\partial V(R)}{\partial R}$$

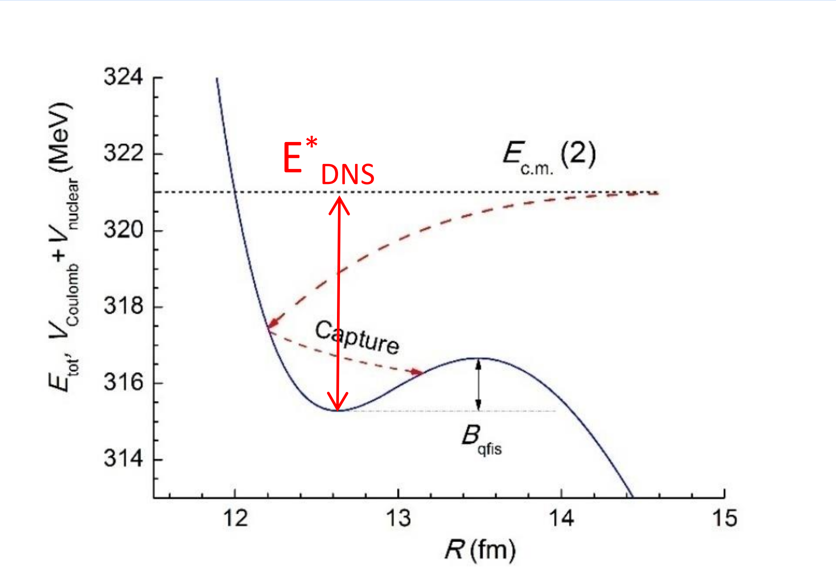
$$\frac{dL}{dt} = \gamma_\theta(R)R(t)\left[\dot{\theta}R(t) - \dot{\theta}_1 R_{1eff} - \dot{\theta}_2 R_{2eff}\right]$$

$$L_0 = J_R\dot{\theta}_R + J_1\dot{\theta}_1 + J_2\dot{\theta}_2$$

$$E_{rot} = \frac{J_R\dot{\theta}^2}{2} + \frac{J_1\dot{\theta}_1^2}{2} + \frac{J_2\dot{\theta}_2^2}{2}$$

$$\Delta E(\Delta t) = \int_0^{\Delta t} \gamma\left(R(t)\right)\dot{R}^2(t)dt$$





Nucleus-nucleus interaction potential

$$V_C(R,\alpha_1,\alpha_2) = \frac{Z_1 Z_2}{R} e^2$$

$$+ \frac{Z_1 Z_2}{R^3} e^2 \left\{ \left(\frac{9}{20\pi} \right)^{1/2} \sum_{i=1}^{2} R_{0i}^2 \beta_2^{(i)} P_2(\cos\alpha_i) + \frac{3}{7\pi} \sum_{i=1}^{2} R_{0i}^2 \left[\beta_2^{(i)} P_2(\cos\alpha_i) \right]^2 \right\}$$

$$V_{nucl}(R,\alpha_1,\alpha_2) = \int \rho_1^{(0)}(\vec{r} - \vec{R}) f_{eff}\left[\rho_1^{(0)} + \rho_2^{(0)}\right] \rho_2^{(0)}(\vec{r}) d^3\vec{r}$$

$$\rho_i^{(0)}(\vec{r},\vec{R}_i,\alpha_i,\theta_i,\beta_{2(i)}) = \left\{ 1 + \exp\left[\frac{\left|\vec{r} - \vec{R}_i(t)\right| - R_{oi}(1 + \beta_2^{(i)} Y_{20}(\theta_i,\alpha_i))}{a} \right] \right\}^{-1}.$$

$$V_{rot} = \hbar^2 \frac{l(l+1)}{2\mu[R(\alpha_1,\alpha_2)]^2}$$

Density dependent effective nucleon-nucleon forces

$$f_{eff}(r) = C_0\left(f + f'\vec{\tau}_1\vec{\tau}_2 + (g + g'\vec{\tau}_1\vec{\tau}_2)\vec{\sigma}_1\vec{\sigma}_2 \right)$$

$$f(r) = f^{ex} + (f^{in} - f^{ex})\frac{\rho(r)}{\rho(0)}$$

Constants	Versions	
	I	II
f_{in}	- 0.09	+0.09
f_{ex}	- 2.23	- 2.59
f'_{in}	0.89	0.42
f'_{ex}	0.06	0.54
g	0.7	0.7
g'	0.83	0.83
C_0=300 MeV fm^{-3}		

The values of the constants of the effective nucleon-nucleon forces from the textbook A.B. Migdal, "*Theory of the Finite Fermi-Systems and properties of Atomic Nuclei*", Moscow, Nauka, 1983. The constants of version II were used in our calculations.

Expressions for the friction coefficients

$$\gamma_R(R(t)) = \sum_{i,i'} \left| \frac{\partial V_{ii'}(R(t))}{\partial R} \right|^2 B_{ii'}^{(1)}(t), \tag{B.1}$$

$$\gamma_\theta(R(t)) = \frac{1}{R^2} \sum_{i,i'} \left| \frac{\partial V_{ii'}(R(t))}{\partial \theta} \right|^2 B_{ii'}^{(1)}(t), \tag{B.2}$$

and the dynamic contribution to the nucleus-nucleus potential

$$\delta V(R(t)) = \sum_{i,i'} \left| \frac{\partial V_{ii'}(R(t))}{\partial R} \right|^2 B_{ii'}^{(0)}(t), \tag{B.3}$$

$$B_{ik}^{(n)}(t) = \frac{2}{\hbar} \int_0^t \mathrm{d}t' (t-t')^n \exp\left(\frac{t'-t}{\tau_{ik}}\right) \times \sin\left[\omega_{ik}\left(\mathbf{R}(t')\right)(t-t')\right]\left[\tilde{n}_k(t') - \tilde{n}_i(t')\right], \tag{B.4}$$

$$\hbar\omega_{ik} = \epsilon_i + \Lambda_{ii} - \epsilon_k - \Lambda_{kk}\,. \tag{B.5}$$

$$H(\xi, R, L) = H_{in}(\xi) + H_{coll}(R, L) + \delta V\,(\xi, R, L)$$

$$H_{in}(\xi) = \sum_i^{A1} a_i^+ a_i + \sum_j^{A2} a_j^+ a_j$$

$$H_{coll}(R, L) = \frac{P^2}{2\mu} + V(R) + \frac{l(1+l)\hbar^2}{2J_{DNS}}$$

$$\delta V\,(\xi, R) = \sum_{i,j} g_{i,j}(R)(a_i^+ a_j + a_j^+ a_i) + \sum_{i,j} V_{i,j} a_i^+ a_i$$

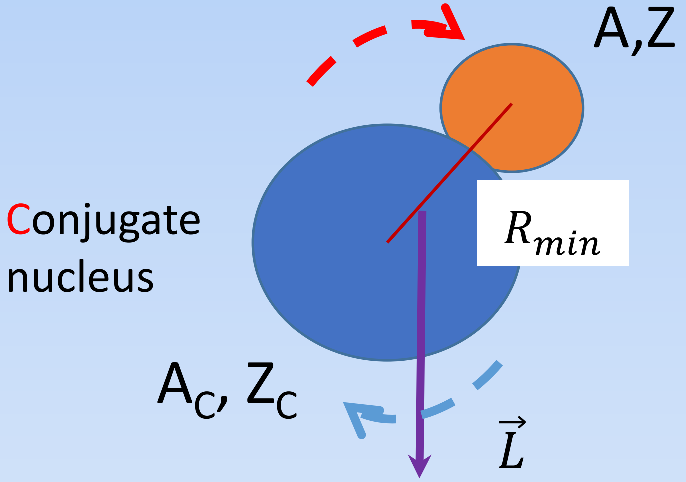
$i \in A_1$, $j \in A_2$

The role of the orbital angular momentum

$U(Z,A,L,R)=V_{coul}(Z,A,R)+V_{nucl}(Z,A,R,L)+V_{rot}(Z,A,R,L)+Q_{gg}(Z,A)-V_{rot}^{(CN)}(L)$

$$V_{rot}(Z,A,R_{min},L)=\frac{l(l+1)\hbar^2}{2J_{DNS}(Z,A,R_{min})}$$

$$J_{DNS}(Z,A,R_{min})=\mu_Z R_{min}^2+\left(J_1(Z,A)+J_2(Z_C,A_C)\right)/2$$



Calculation of the charge distributions.

$$\frac{\partial}{\partial t} Y_Z(E_Z^*, \ell, t) = \Delta_{Z+1}^{(-)} Y_{Z+1}(E_Z^*, \ell, t) + \Delta_{Z-1}^{(+)} Y_{Z-1}(E_Z^*, \ell, t) - (\Delta_Z^{(-)} + \Delta_Z^{(+)} + \Lambda_Z^{qf}) Y_Z(E_Z^*, \ell, t)$$

for $Z = 2, 3, \ldots, Z_{tot} - 2$

$E^*_{DNS}(Z) = E_{c.m.} - V_{min} + (B_P + B_T) - (B_Z + B_{ztot-z})$

$$\Delta_Z^{(\pm)} = \frac{1}{\Delta t} \sum_{P_z, T_Z} \left| g_{P_z T_Z} \right|^2 n_{T_Z(P_Z)}(t)(1 - n_{T_Z(P_Z)}(t)) W_{P_z T_Z}(\Delta t) \Big/ (\varepsilon_{Pz} - \varepsilon_{Tz})^2$$

$$W_{P_z T_Z}(\Delta t) = (1 + e^{-2(\Gamma_{Pz} + \Gamma_{Tz})\Delta t/\hbar} - 2.\, e^{-2(\Gamma_{Pz} + \Gamma_{Tz})\Delta t/\hbar} \cos((\varepsilon_{Pz} - \varepsilon_{Tz}) \Delta t/\hbar))$$

Details of calculation of the matrix elements are presented in papers:

G.G. Adamian, et al. Phys. Rev. C**53**, (1996) p.871-879
R.V. Jolos et al., Eur. Phys. J. A **8, 115–124 (2000)**

$$\frac{1}{\tau_i^{(\alpha)}} = \frac{\sqrt{2}\pi}{32\hbar\varepsilon_{F_K}^{(\alpha)}} \left[(f_K - g)^2 + \frac{1}{2}(f_K + g)^2 \right] \times \left[\left(\pi T_K\right)^2 + \left(\tilde{\varepsilon}_i - \lambda_K^{(\alpha)}\right)^2 \right] \left[1 + \exp\left(\frac{\lambda_K^{(\alpha)} - \tilde{\varepsilon}_i}{T_K} \right) \right]^{-1}, \quad (A.1)$$

where

$$T_K(t) = 3.46 \sqrt{\frac{E_K^*(t)}{\langle A_K(t) \rangle}} \quad (A.2)$$

Application of the dinuclear system model for the interpretation of the experimental data measured at the Inter-University Accelerator Centre (IUAC), New Delhi by the group of Prof. Indranil Mazumdar from Tata Institute of Fundamental Research, Mumbai, India.

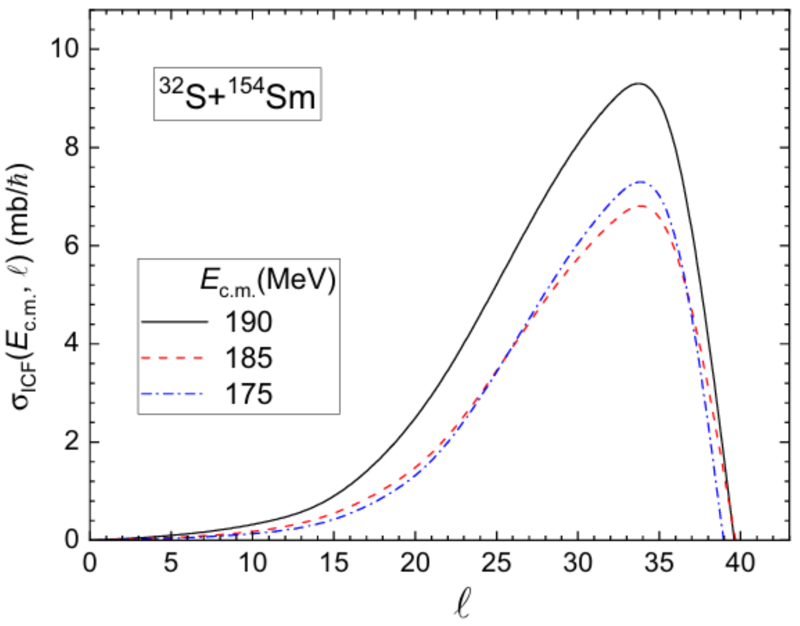


FIG. 16. The partial cross section of the incomplete fusion [$\sigma_{ICF}(E_{c.m.}, \ell)$] calculated in this work for the $^{32}S + ^{154}Sm$ reaction as a function of the collision energy $E_{c.m.}$ and orbital angular momentum ℓ.

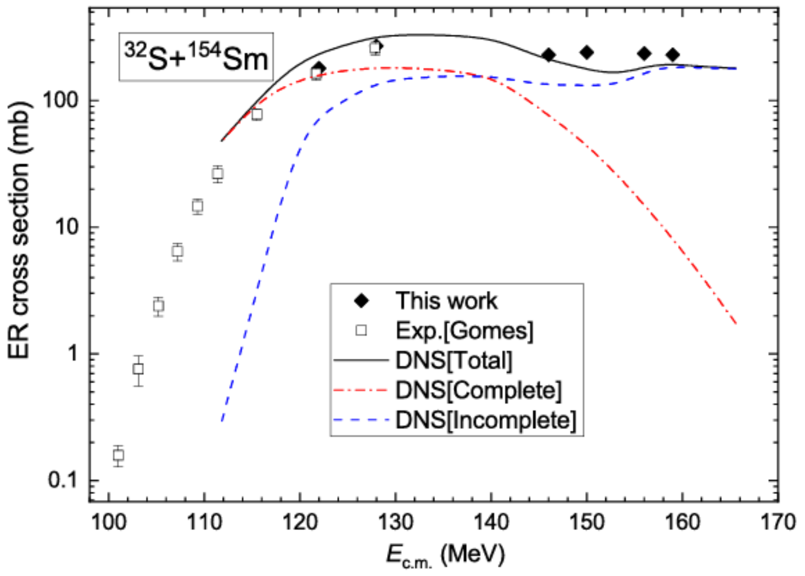
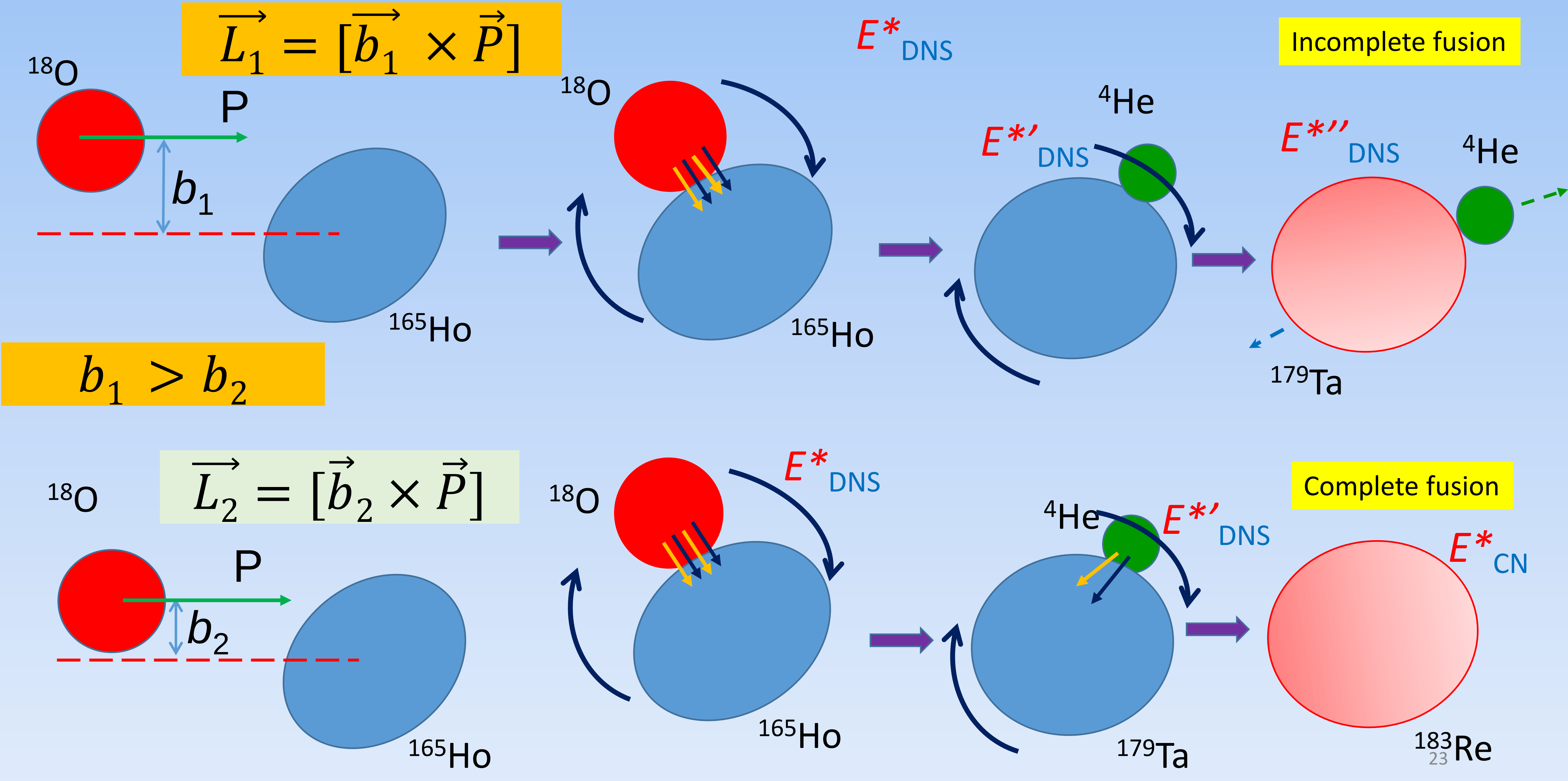
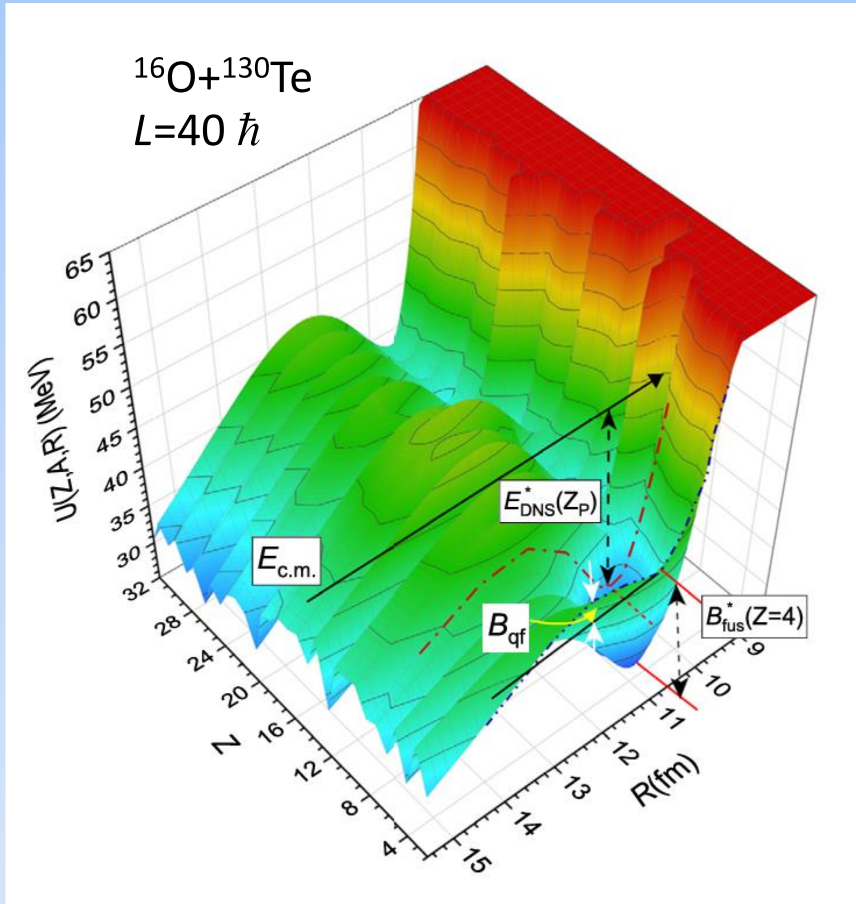


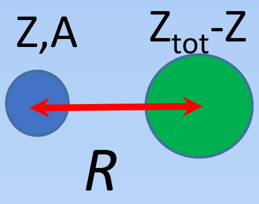
FIG. 17. Comparison of the theoretical (solid curve) and measured total cross sections of the evaporation residues formed in the $^{32}S + ^{154}Sm$ reactions. The filled diamonds and open squares present the experimental data of this work and the ones obtained from Ref. [14], respectively. The dashed and dot-dashed curves are contributions of the complete and incomplete fusion mechanisms estimated by the DNS model, respectively.

Role of the orbital angular momentum in the reaction mechanism.

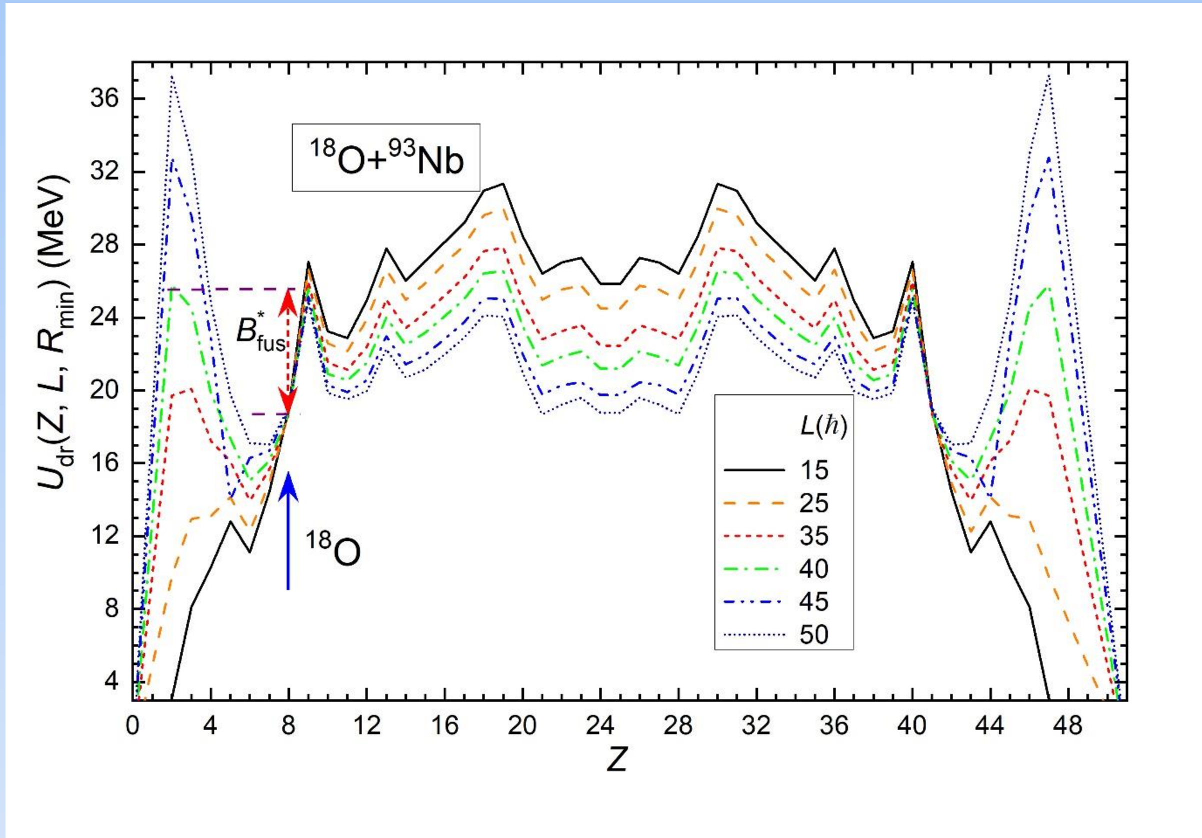


The appearance of the hindrance to complete fusion in reactions with the light nuclei due to centrifugal forces in collisions with the large impact parameters.





R=$R_m(Z)$



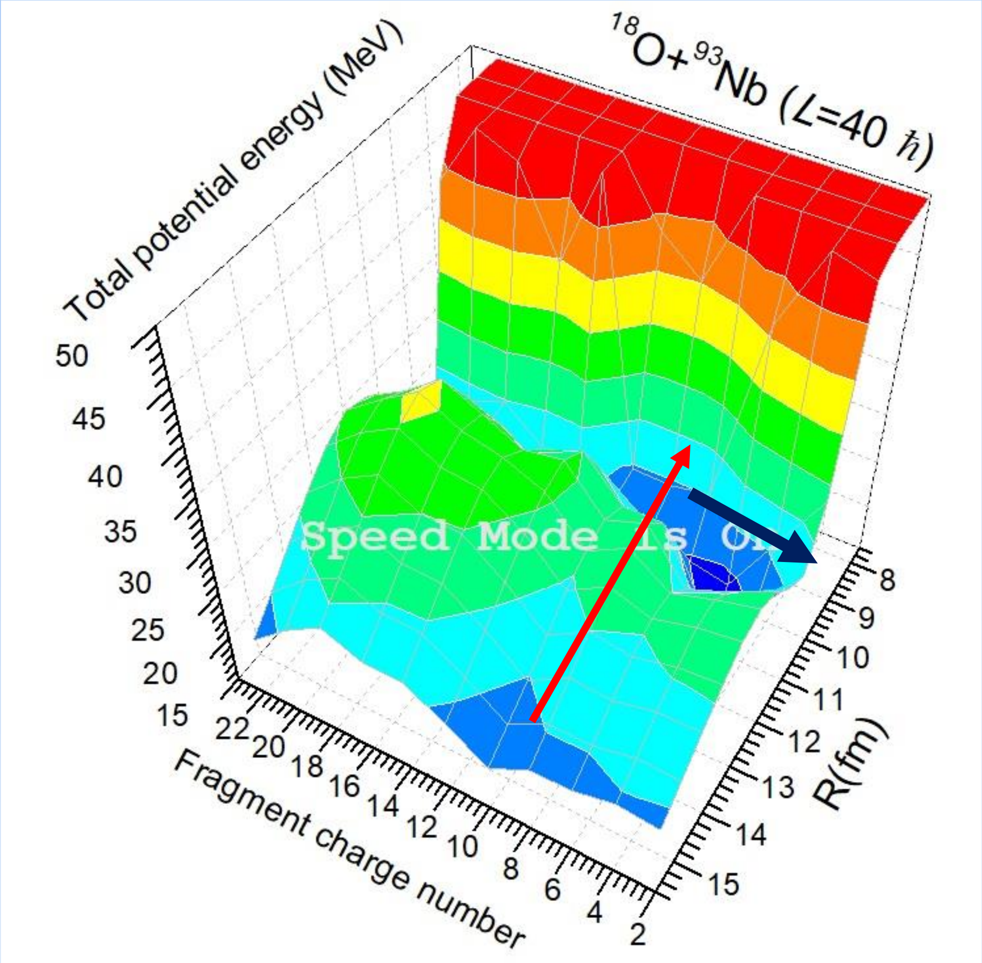
(A.K. Nasirov et al. Physics Letters B 842: 137976 (2023))

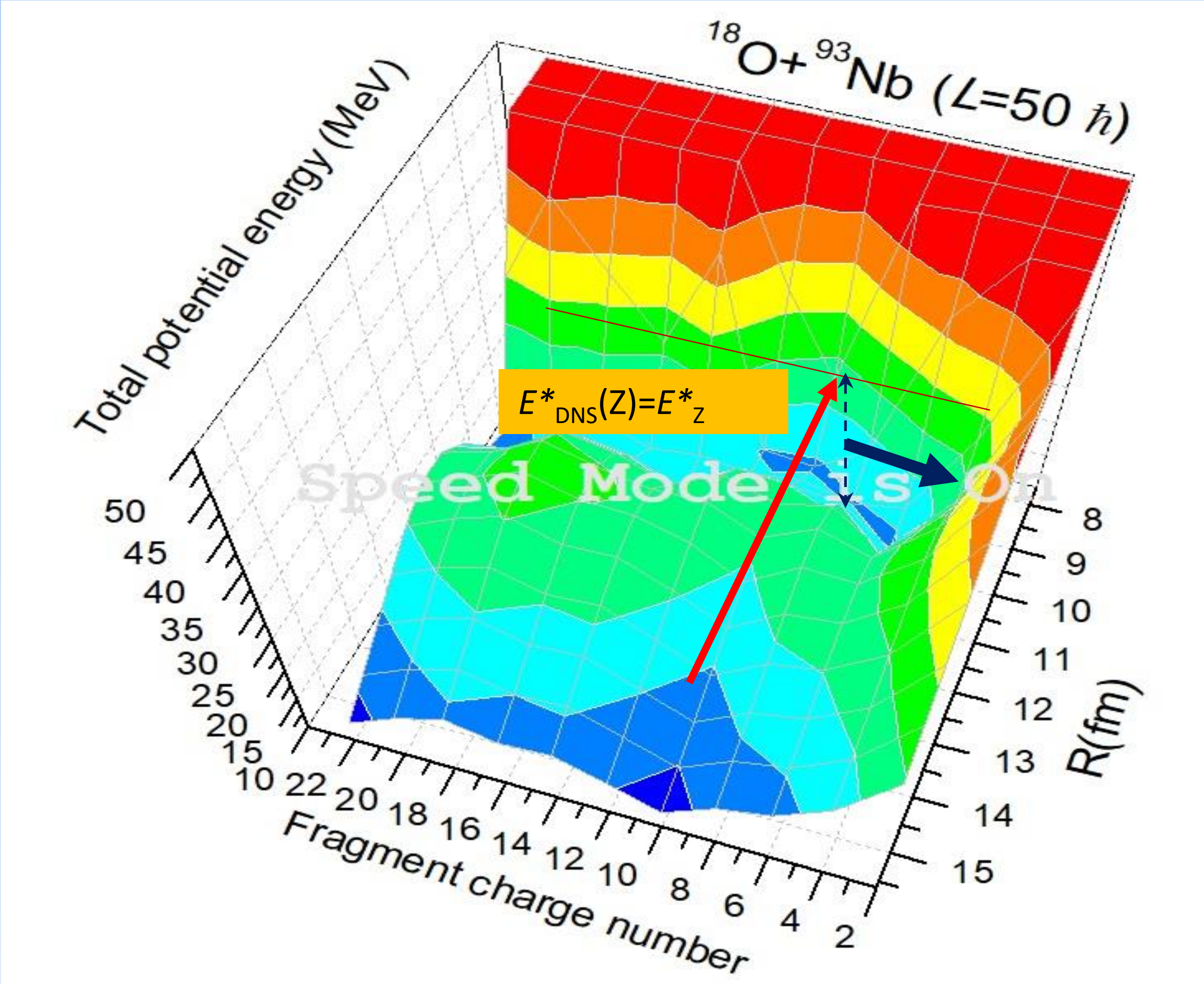
Potential energy surface for the dinuclear system, Z and A charge-mass numbers of the light fragment.

$U(Z,A,R)$=$V_{int}(R,Z,A)+B_1(Z,A)+B_2(Z_{CN}-Z,A_{CN}-A)-B_{CN}$; B_i (i=1,2, CN) binding energies

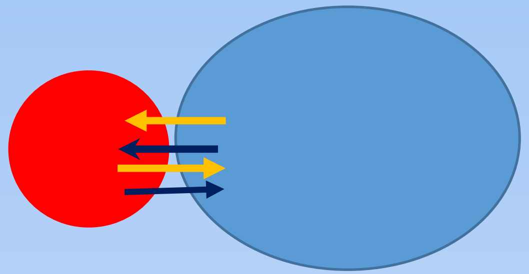
Appearance of the hindrance to complete fusion

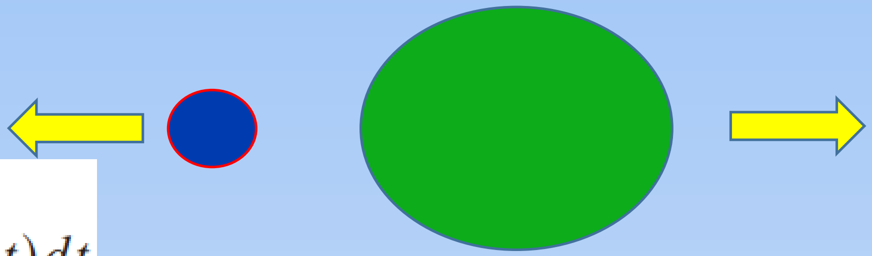
$E^*_{DNS}(Z)=E_{c.m.}-V_{min}+(B_P+B_T)-(B_Z+B_{ztot-z})$



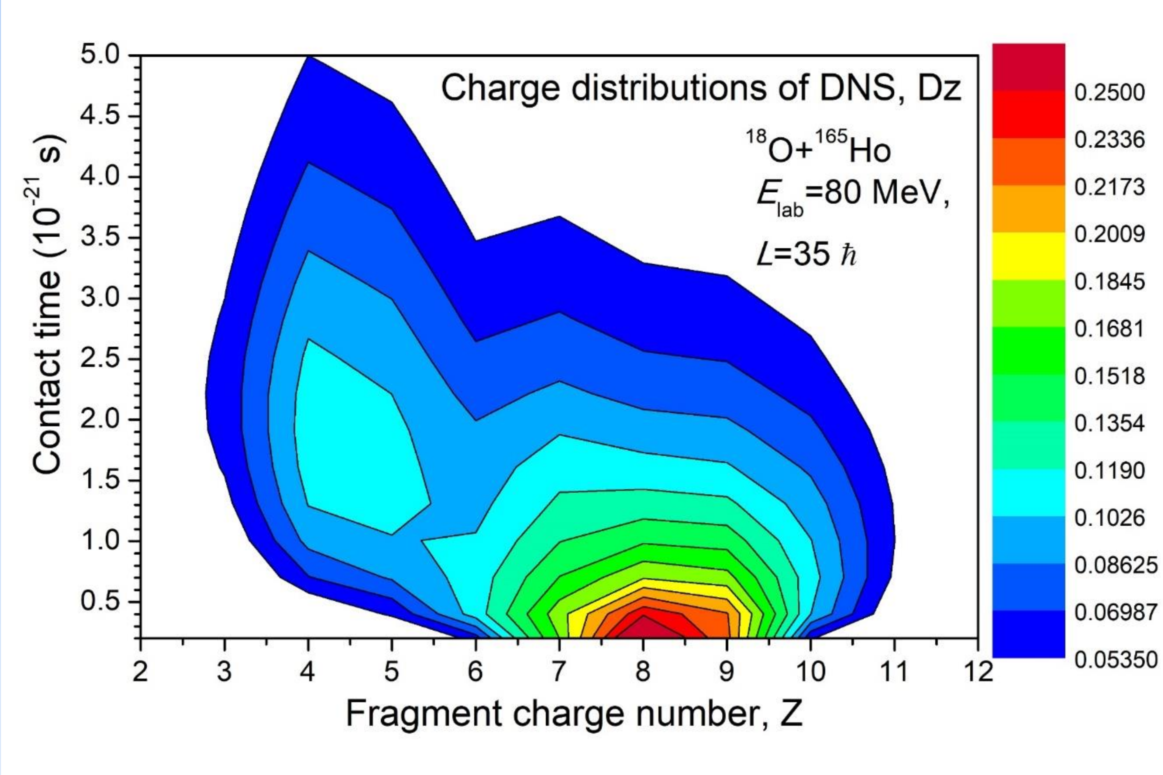


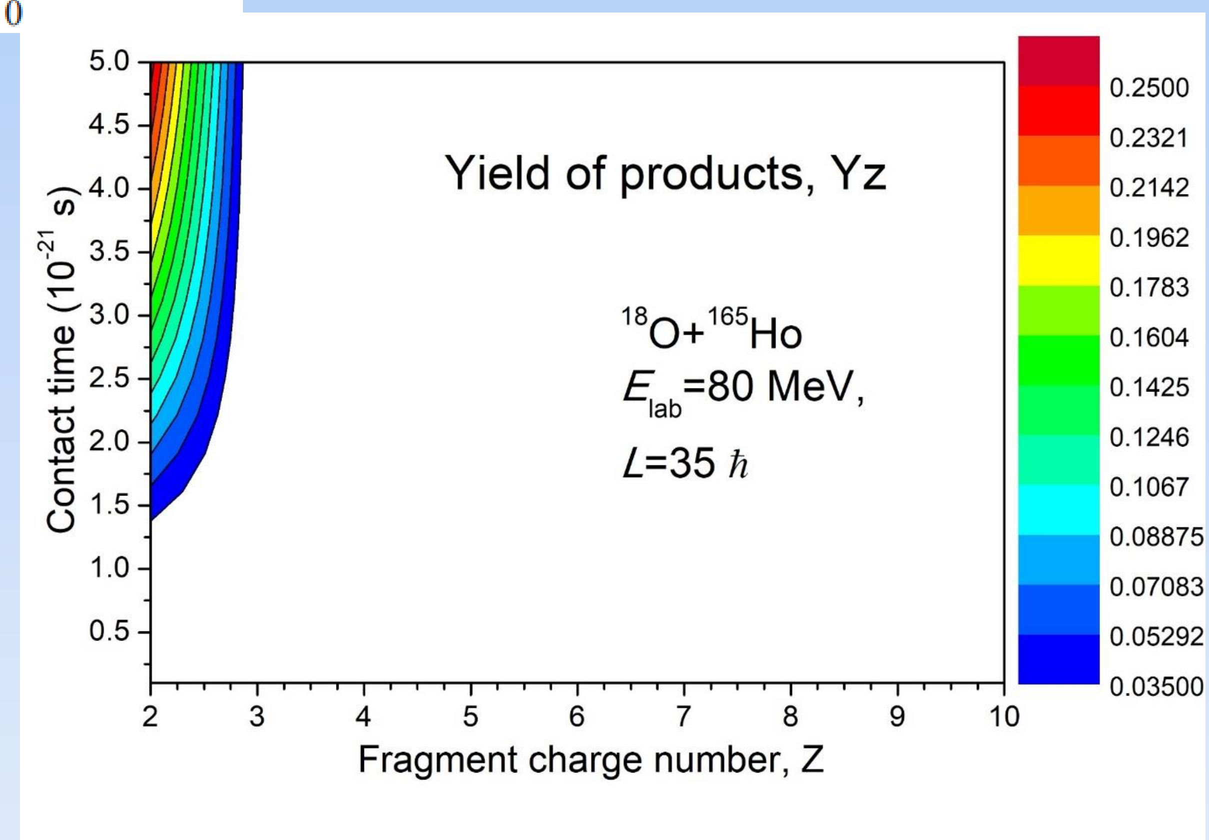
Solutions of the transport master equations for the evolution and decay dinuclear system formed in reaction $^{18}O+^{165}Ho$



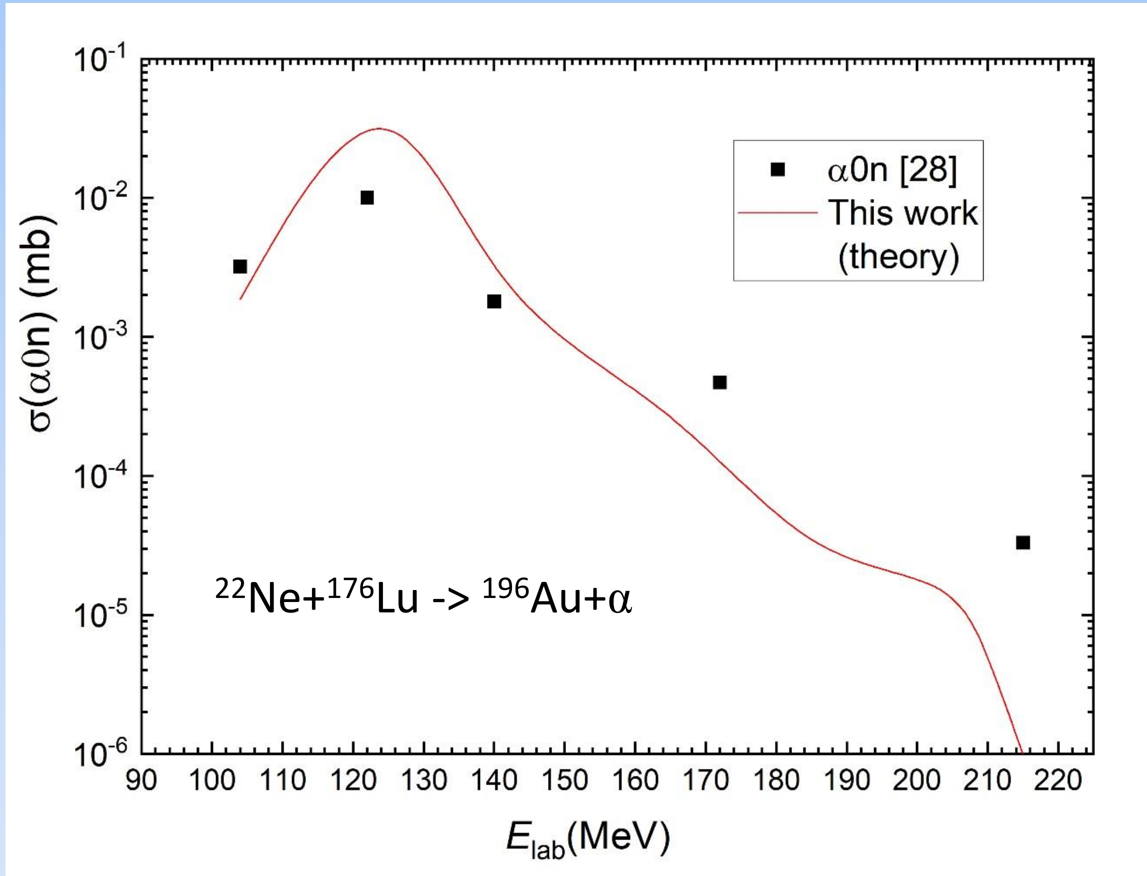


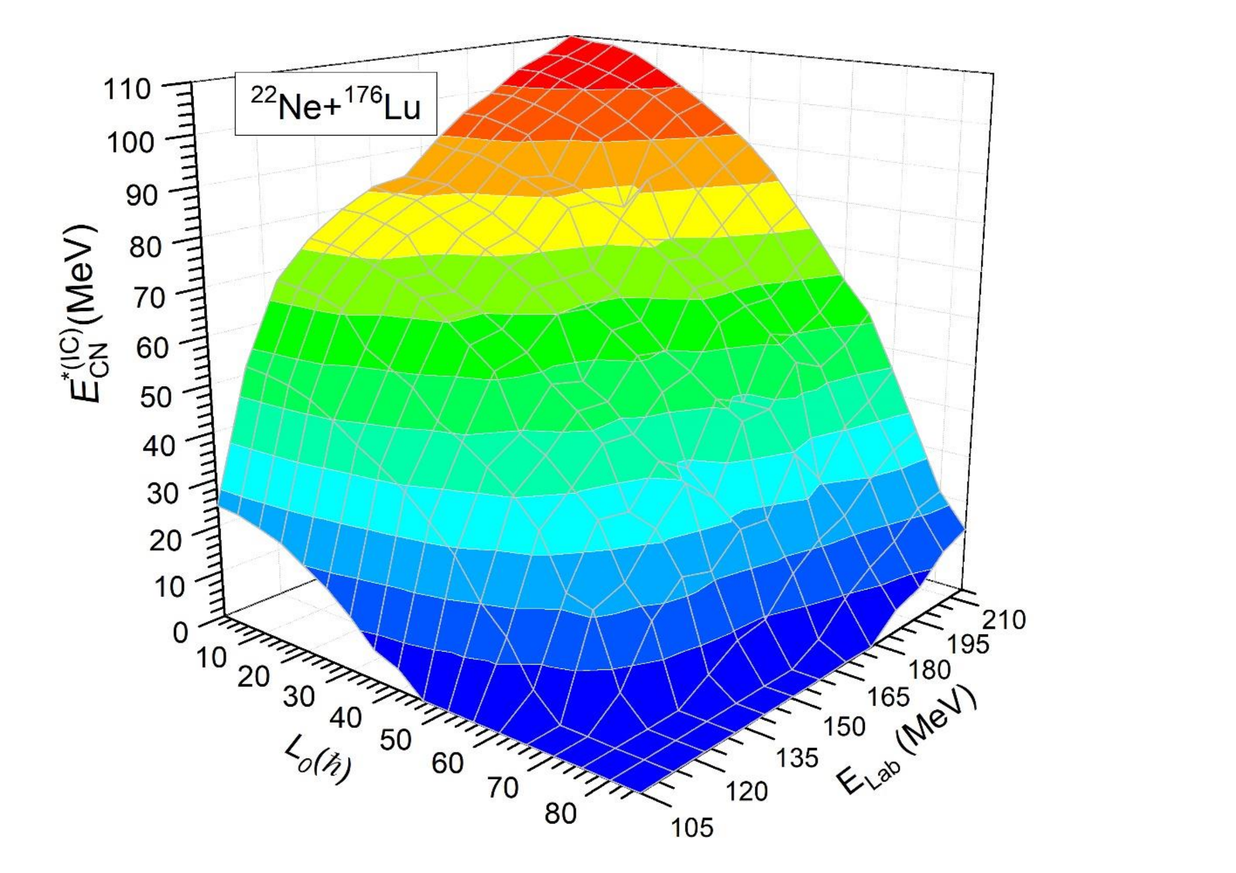
$$Y_Z(t) = \Lambda_Z^{qf} \int_0^t P_Z(t)dt$$





Explanation of the formation of the cold conjugate nucleus in the incomplete fusion. (A.K. Nasirov et al. Physics Letters B 842: 137976 (2023))



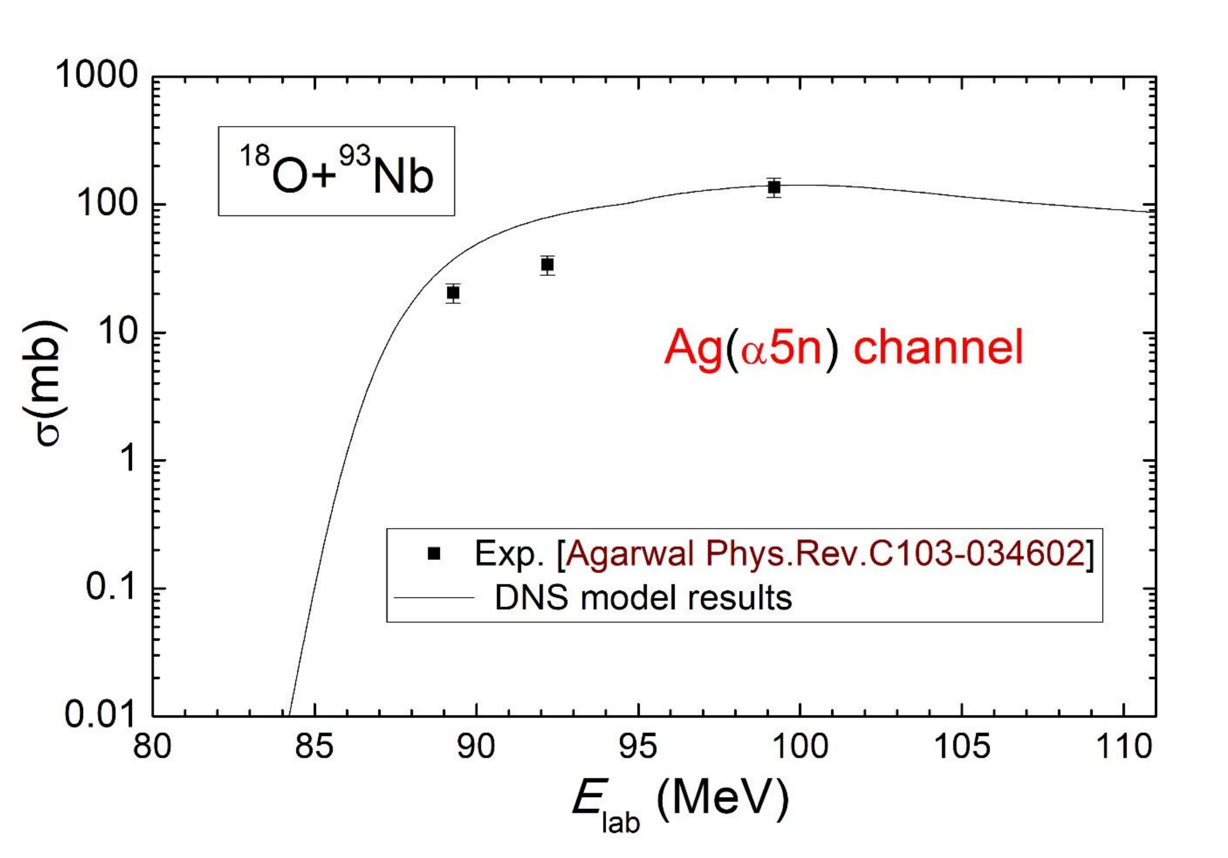


28. H. Bruchertseifert et al., Soviet Journal of Nuclear Physics; v33(6),p778 (1981)).

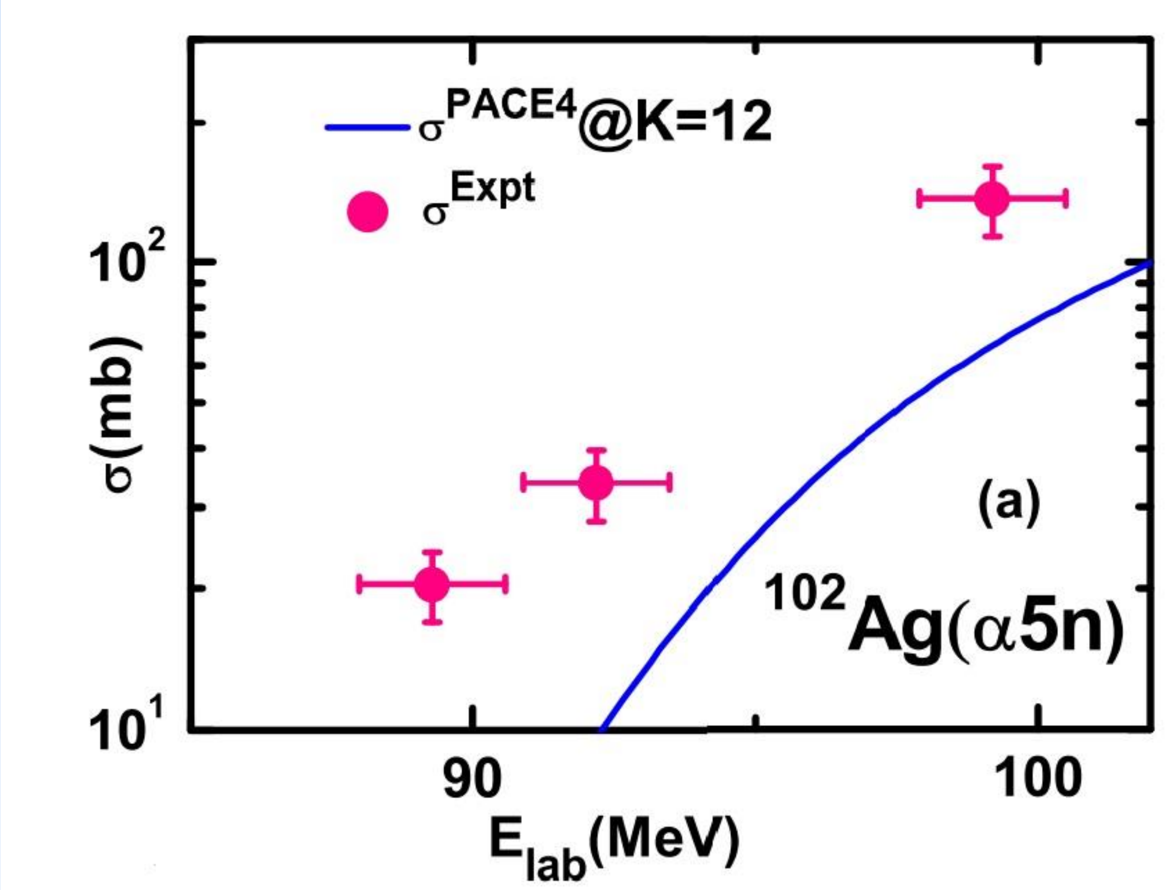
$$E^{*(IC)}_{CN}(Z,L)=E_{c.m.}-V_{min}(Z,L)+(B_P+B_T)-B_{CN}$$

Comparison of the results by dinuclear system model and PACE4 code.

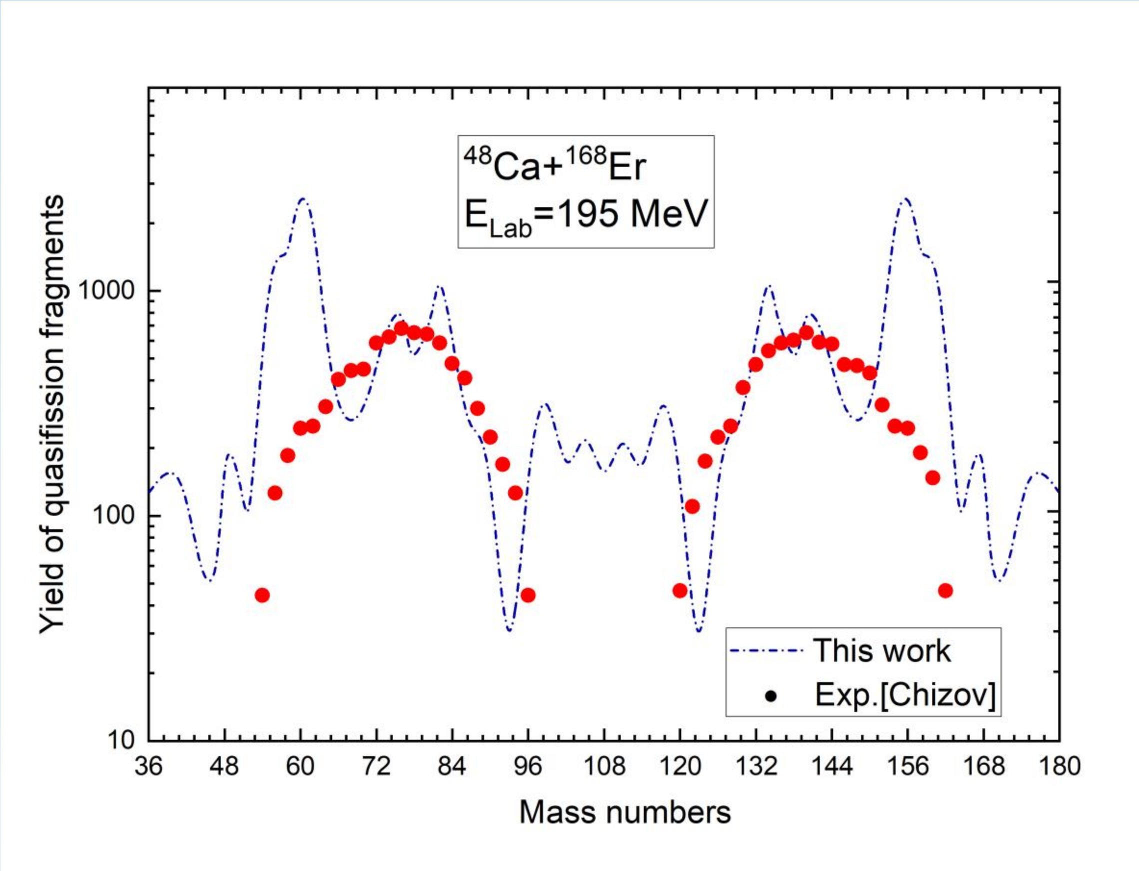
Theoretical result of this work

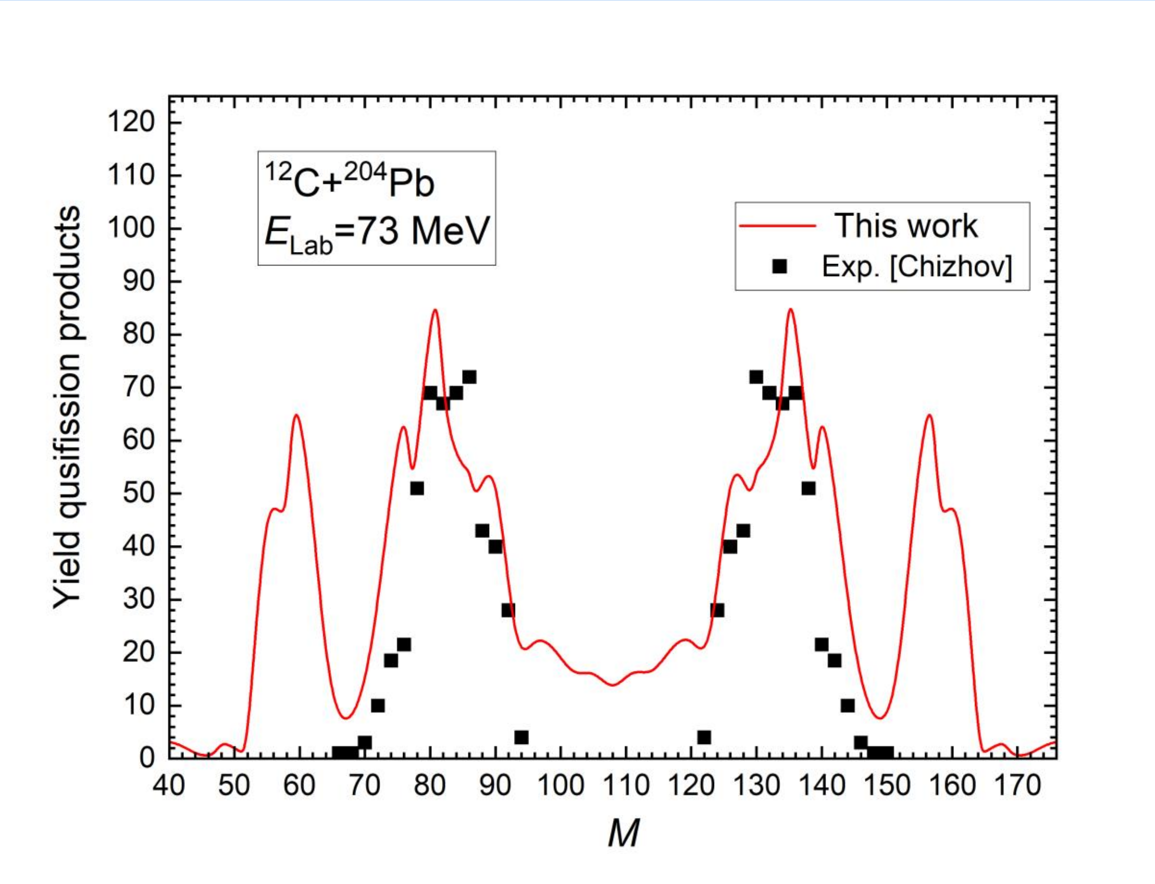


Avinash Agarwal, Phys.Rev.C103-034602 (2021)]



“Unexpected entrance-channel effect in the fission of 216Ra”
A. Yu. Chizhov, M. G. Itkis, G. N. Kniajeva, et al, Phys. Rev. C 67, 011603, (2003). Experiment.





A.K. Nasirov, E.D. Khusanov, M.M. Nishonov, Phys. Rev.C (accepted 2024)

Conclusion

1. Experience of the synthesis of superheavy elements shows that there is a huge hindrance for complete fusion of the colliding nuclei as a function of the mass asymmetry of the entrance channel.
2. The hindrance to fusion increases by the increase of the angular momentum of collision.
3. Complete fusion occurs by multinucleon transfer through the neck connecting two fragments of dinuclear system.

Thank you for your attention !