EMPIRICAL SYSTEMATICS OF SPONTANEOUS FISSION HALF-LIVES OF HEAVY AND SUPERHEAVY NUCLEI

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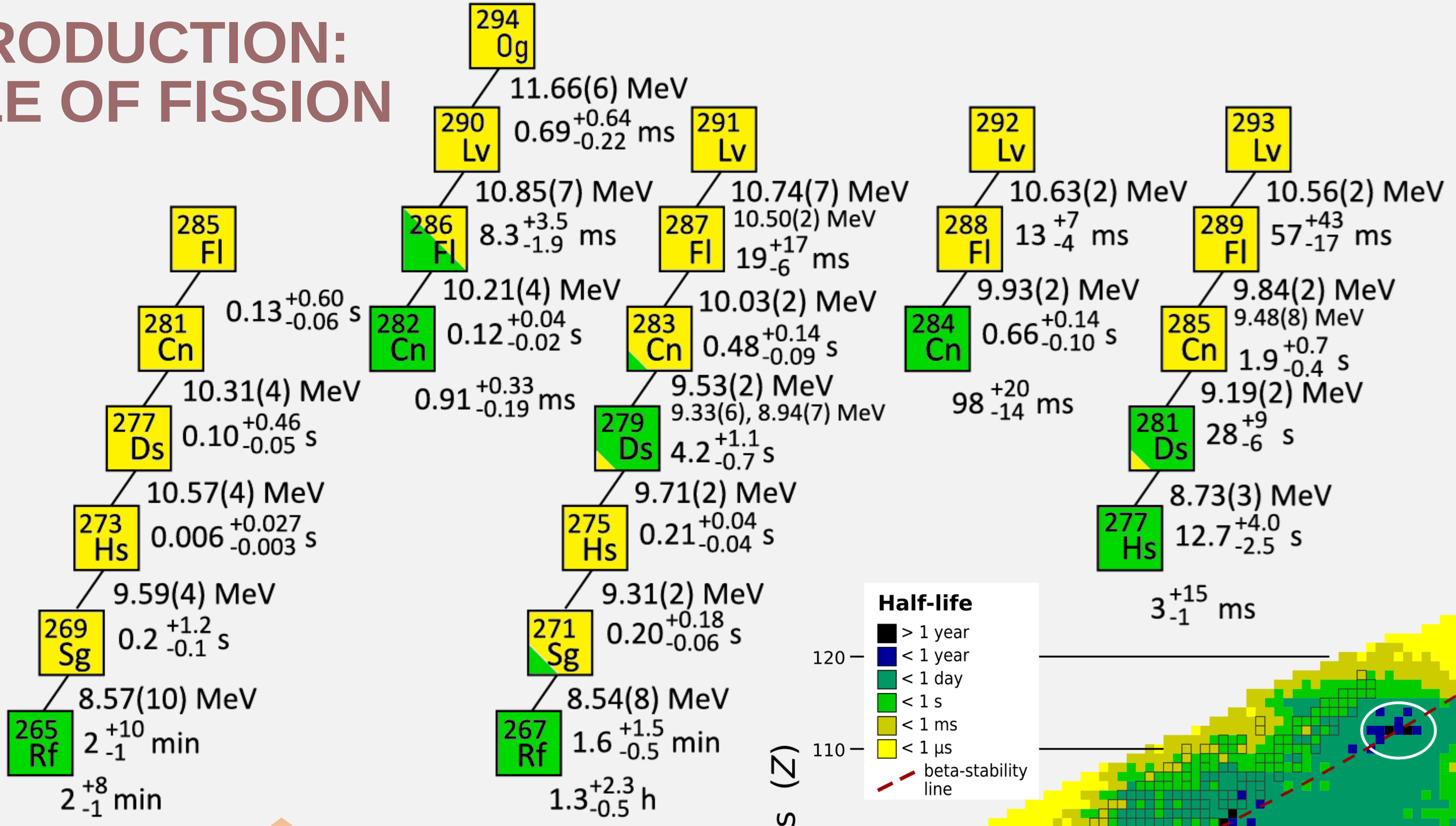
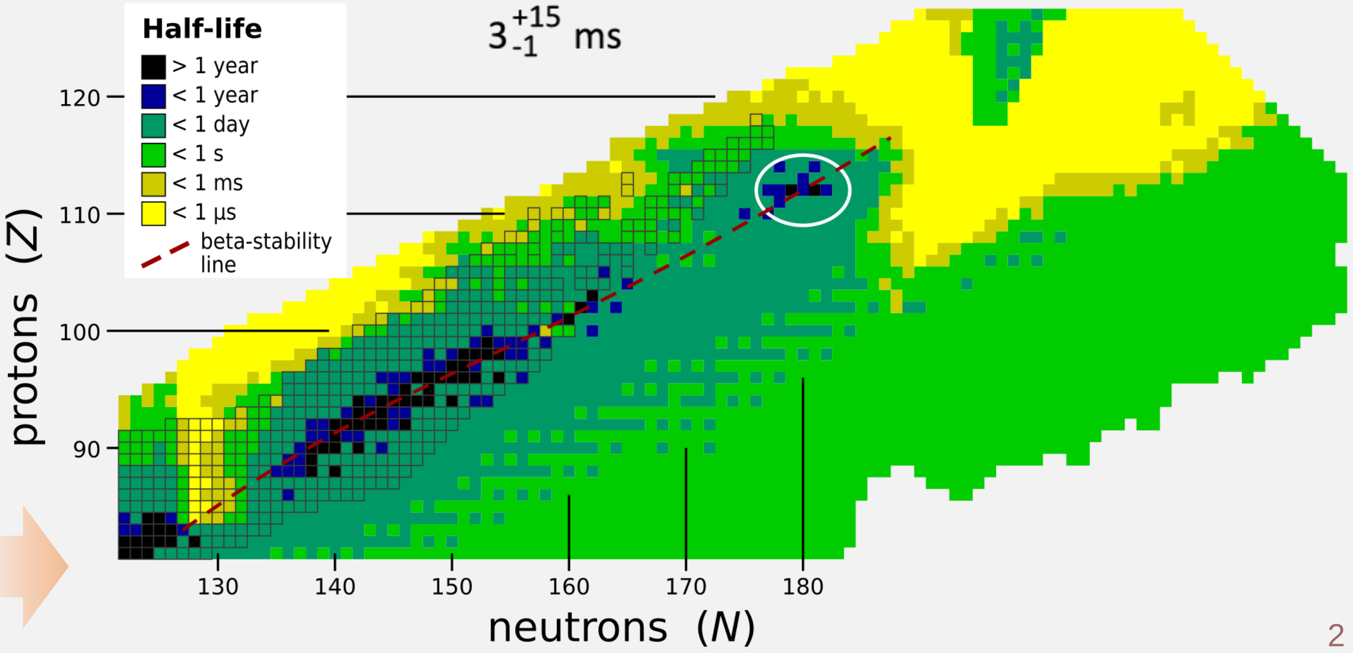
INTRODUCTION: ROLE OF FISSION



Summary decay properties of the isotopes of even-Z elements synthesized in the reactions of ^{48}Ca with ^{238}U, ^{242,244}Pu, ^{245,248}Cm, and ^{249}Cf target nuclei.



Predicted half-lives of superheavy nuclides. The expected location of the island of stability around $Z = 112$ is circled.

THEORETICAL MODELS

MICROSCOPIC APPROACHES

- Fundamental approaches
- Mean Field (MF) method, Hartree-Fock method
- Relativistic Mean Field (RMF) method

MICROSCOPIC-MACROSCOPIC APPROACHES

- FRDM (Finite Range Droplet Model)
- FRLDM (Finite Range Liquide Droplet Model)
- WS (Weizsäcker-Skyrme)
- WS+RBF (WS + Radial Basis Functions)

PHENOMENOLOGICAL APPROACHES

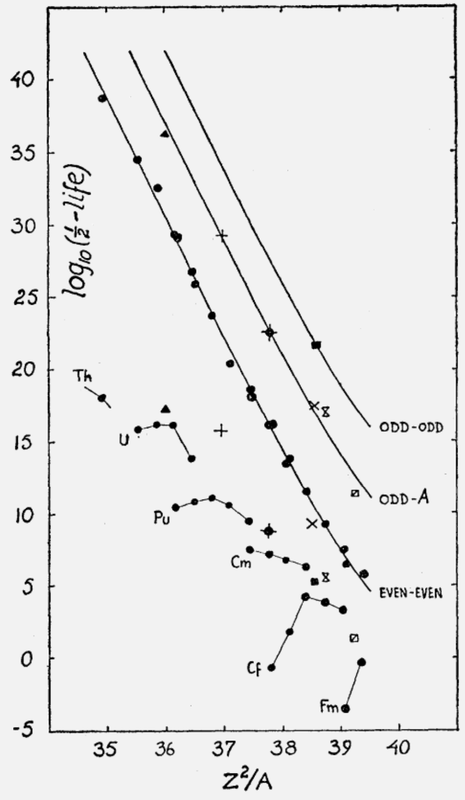
- Local Mass Relations (LMR) method

PROBLEMS

- complexity of the fission process
- uncertainty in the height and shape of the fission barrier
- analysis of inter-nucleon interaction potentials
- distribution of forces in a multi-nucleon system
- difficulty of calculations
- models do not always have sufficient accuracy

Review of formulas for $T_{1/2}^{\mathrm{SF}}$

Świątecki, 1955
Dorn, 1961
Viola & Seaborg, 1965
Ren & Xu, 2005
Xu & Ren, 2005
Xu *at al.*, 2008
Karpov *at al.*, 2008
Santhosh *at al.*, 2010
Zdeb *et al.*, 2014
Bao *et al.*, 2015
Santhosh & Nithya, 2016
Yuan *et al.*, 2024
Babu *et al.*, 2024



$$\log_{10} T_{1/2}^{\mathrm{SF}}\ (\text{years}) = -7.8\theta + 0.35\theta^2 + 0.073\theta^3 - (5-\theta)\delta M + \\ + \begin{cases} 18.2, & \text{even-even}, \\ 24.8, & \text{odd-}A, \\ 29.7, & \text{odd-odd}, \end{cases}$$

$$\theta = (Z^2/A) - 37.5,$$

$$\delta M = M_{\mathrm{exp}} - M_{\mathrm{ref}},$$

$$M_{\mathrm{ref}} = 1000A - 8.3557A + 19.120A^{2/3} + 076278\frac{Z^2}{A^{1/3}} + \\ + 25.444\frac{(N-Z)^2}{A} + 0.420(N-Z).$$

Władysław J. Świątecki
(1926 – 2009)

AIM:

TO EXPLORE A **NOVEL APPROACH TO THE SYSTEMATICS OF SPONTANEOUS FISSION HALF-LIVES**, AND TO INVESTIGATE THE ROLE OF CLUSTER DEGREES OF FREEDOM IN BOTH THE STRUCTURE AND FISSION PROCESS OF ATOMIC NUCLEI

OBJECTIVES:

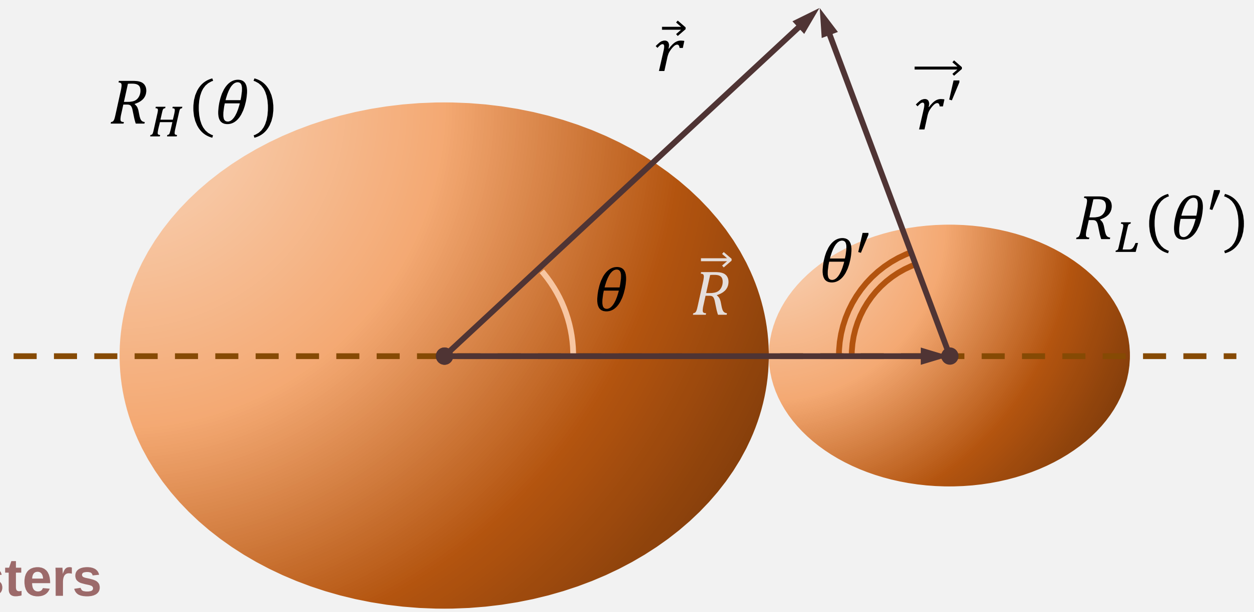
REVIEW OF FORMULAS FOR THE SPONTANEOUS FISSION HALF-LIFE OF HEAVY AND SUPERHEAVY NUCLEI

EXAMINATION OF THE SYSTEMATICS OF SPONTANEOUS FISSION HALF-LIVES

DEVELOPMENT OF AN **EMPIRICAL FORMULA** FOR THE SPONTANEOUS FISSION HALF-LIFE OF HEAVY AND SUPERHEAVY NUCLEI

OBTAINING ESTIMATES OF SPONTANEOUS FISSION HALF-LIVES FOR EXPERIMENTALLY UNKNOWN ISOTOPES

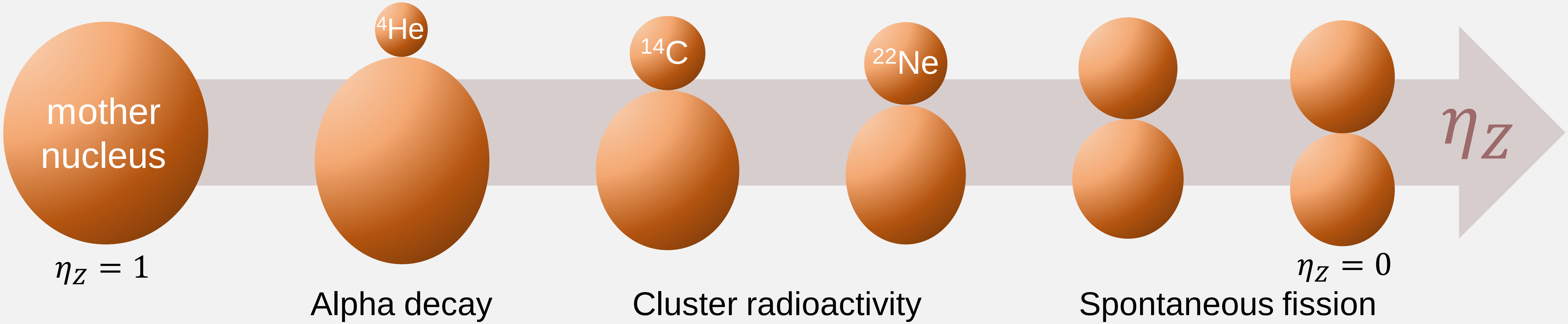
DNS



Schematic representation of the DNS
in the case of axially symmetric mutual arrangement of clusters

Characteristics of DNS:

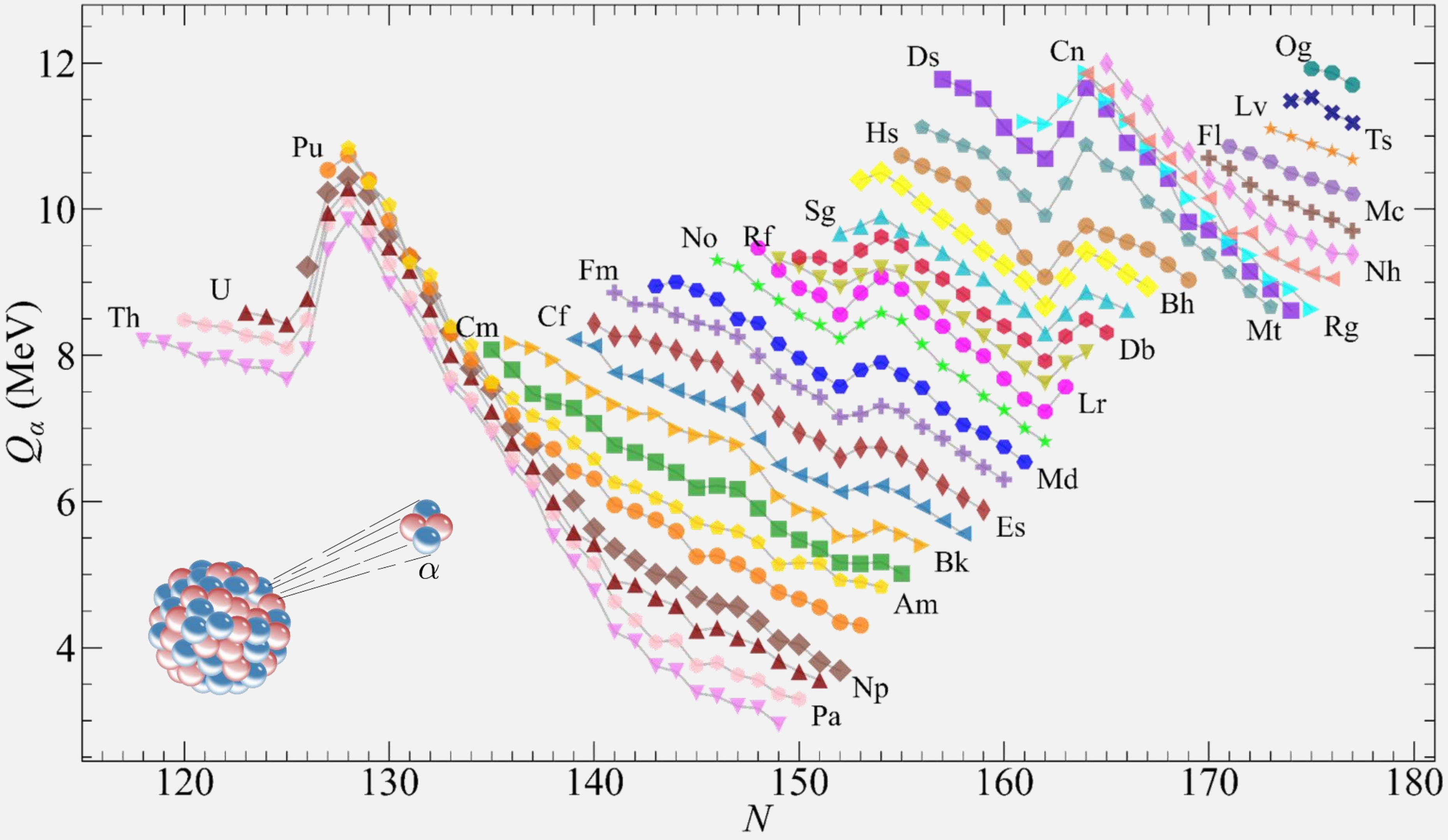
- relative **distance $\vec{R}$ between clusters**
- **Euler angles**
- **charge asymmetry** $\eta_Z = \frac{Z_H - Z_L}{Z_H + Z_L}$, Z_H and Z_L are charge numbers of the heavy and light clusters

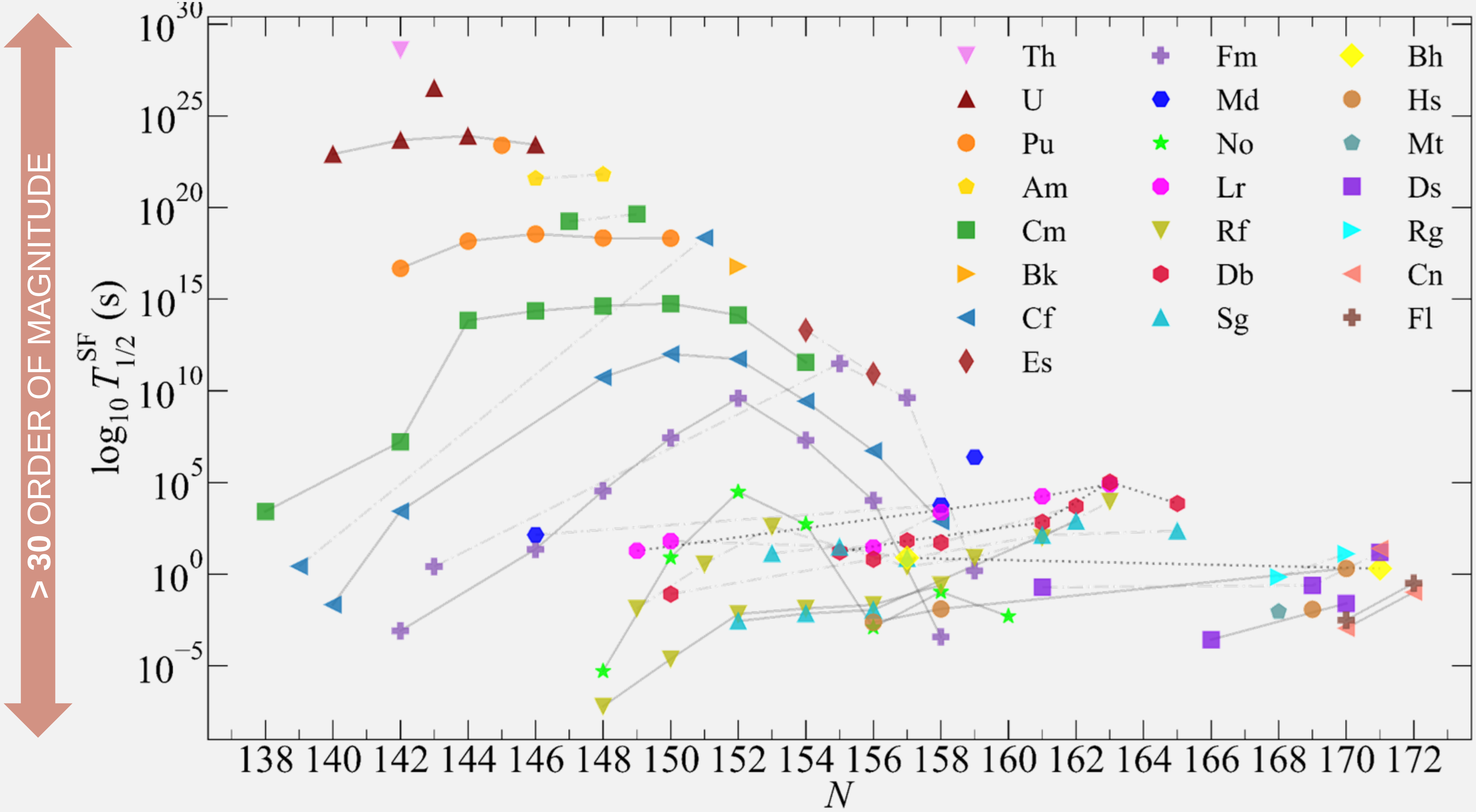


[Adamian, G., Antonenko, N., & Scheid, W. (2012). Clustering effects within the dinuclear model. *Clusters in Nuclei, Vol. 2*, 165-227.]

$$Q_\alpha(Z,A) = M(Z,A) - M(Z-2,A-4) - M(^4\mathrm{He})$$

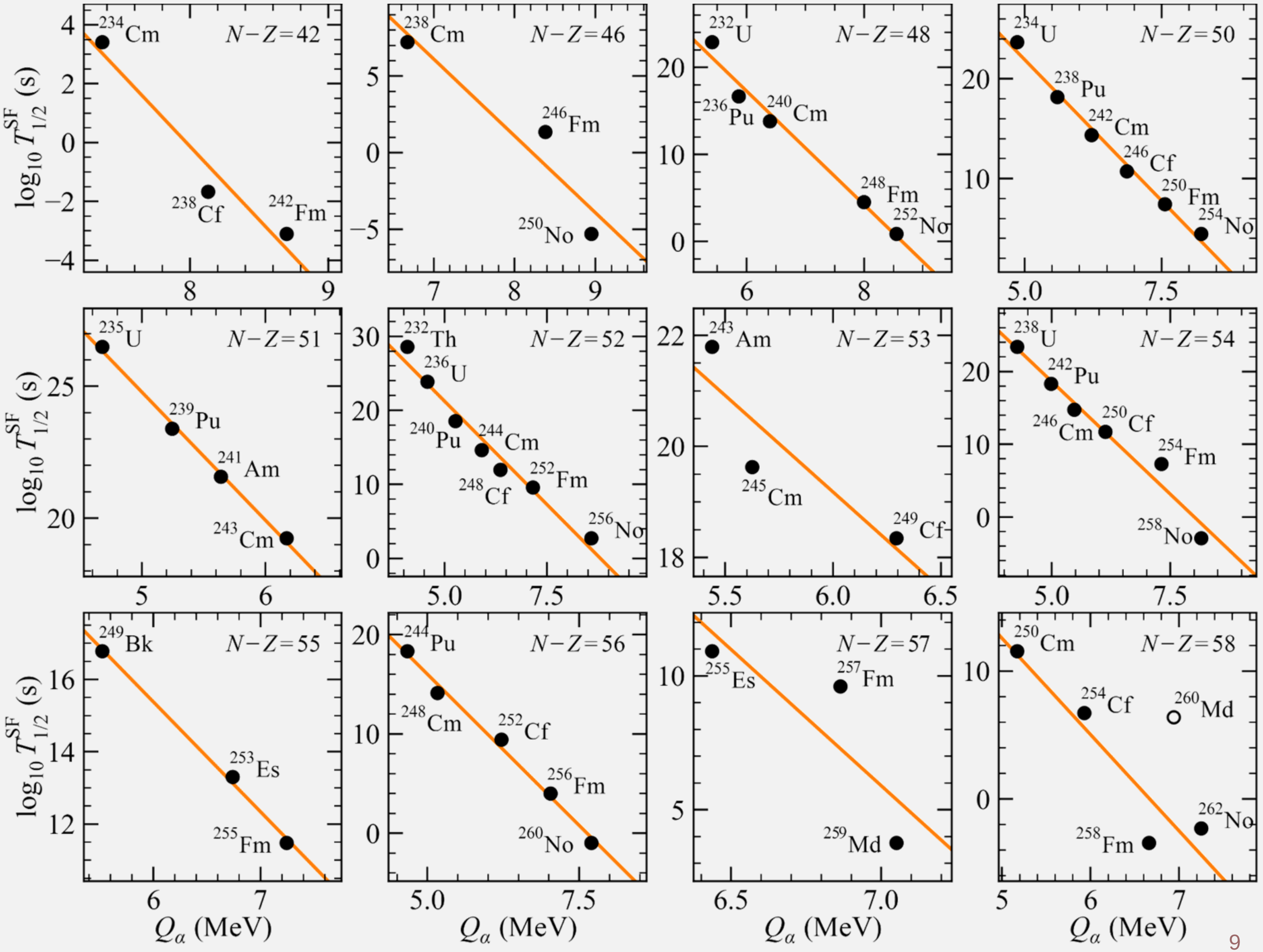
$A > 200$
$Q_\alpha = (4-11)$ MeV





Spontaneous fission half-lives $T_{1/2}^{SF}$ for nuclei with $90 \leq Z \leq 114$ [Wang, 2021]. Solid lines are even-even nuclei, dashed-dotted lines are even-odd and odd-even nuclei, and dotted lines are odd-odd nuclei.

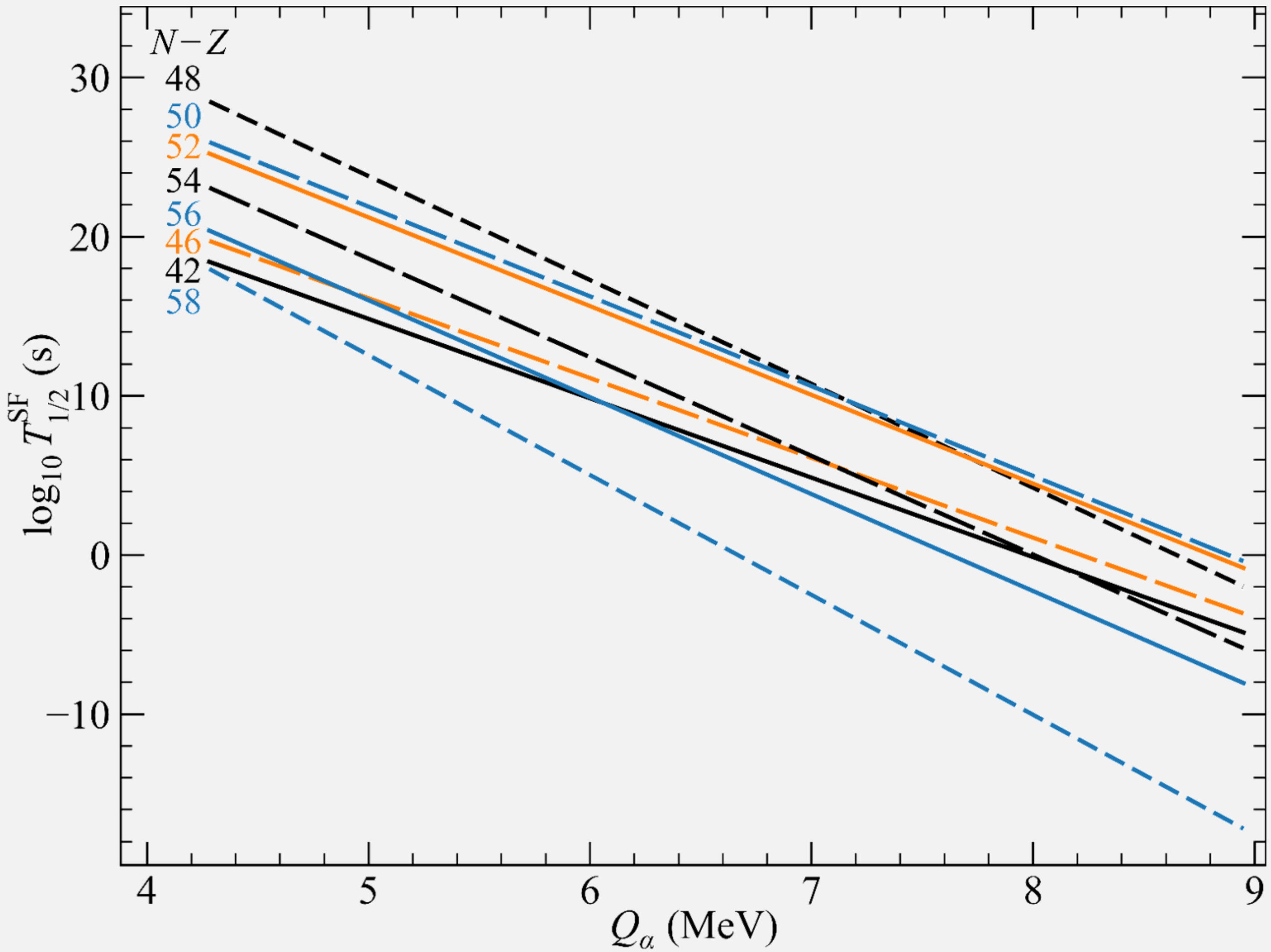
Peculiarities of dependencies of $T_{1/2}^{SF}$ on Q_α



Decimal logarithm of the experimental $T_{1/2}^{SF}$ as a function of Q_α for fixed values of the neutron excess $N-Z$ for nuclei with $90 \le Z \le 102$ (**black round symbols**) and their approximation (orange lines) by

$$\log_{10} T_{1/2}^{SF}(Q_\alpha) = c_Q Q_\alpha + f$$

Comparison of approximating lines



Values of slope and intercept obtained by linear approximation of experimental points

$N-Z$	c_Q	f
42	−4.989	39.78
46	−5.014	41.21
48	−6.528	56.45
50	−5.640	50.09
51	−4.859	49.07
52	−5.577	49.10
53	−3.475	40.03
54	−6.198	49.62
55	−3.039	33.61
56	−6.090	46.46
57	−10.1791	77.15
58	−7.533	50.23

$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha) = c_Q Q_\alpha + f$$

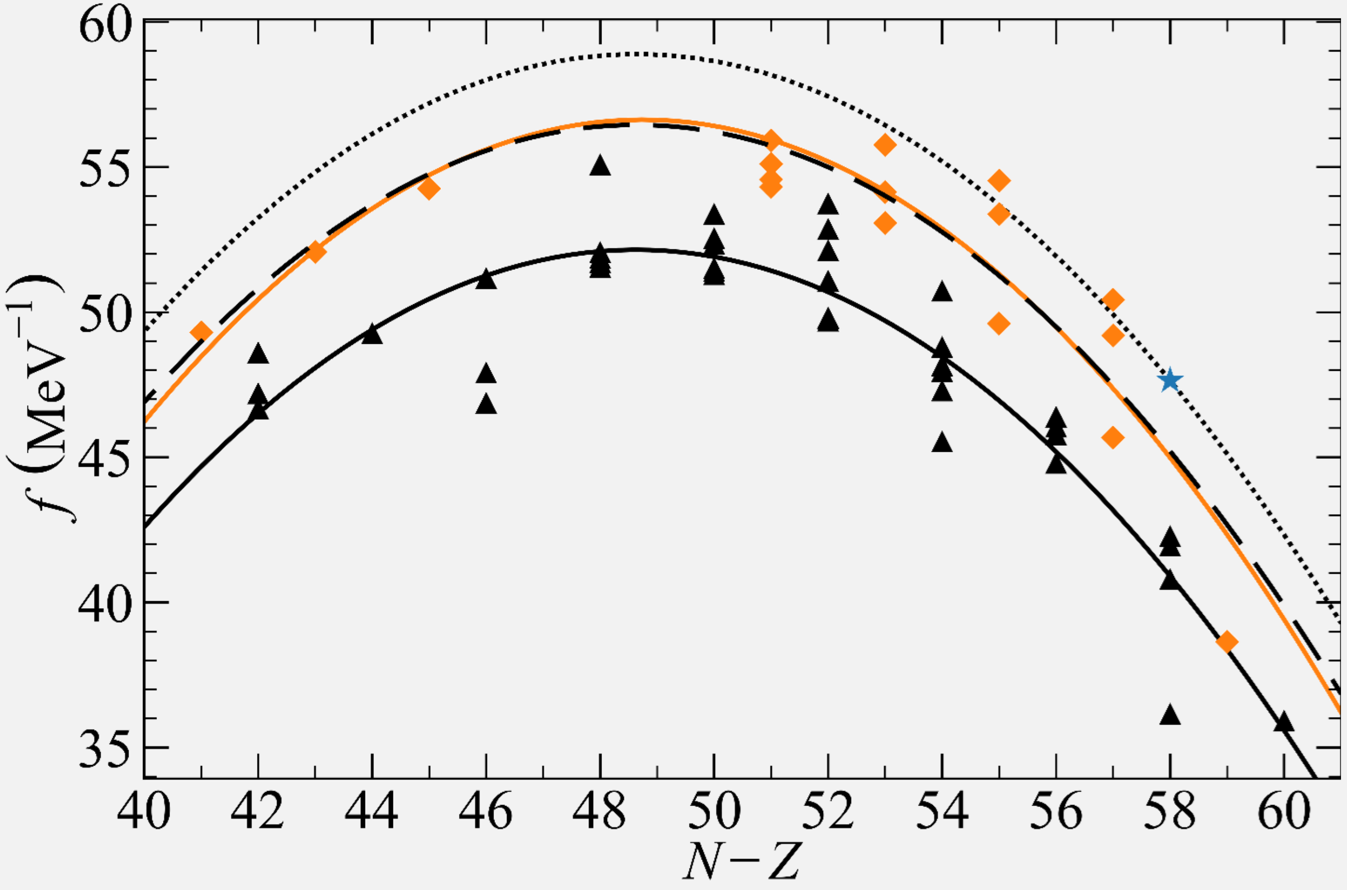
$$c_Q = c_Q^{\mathrm{mean}} = -5.946$$

$$f = f(A,Z) \Rightarrow f = f(N-Z)$$

$$f(N-Z) = \log_{10} T_{1/2}^{\mathrm{SF}} - c_Q^{\mathrm{mean}} Q_\alpha$$

$$c_Q^{\mathrm{mean}} = -5.946$$

$$f(N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_{\mathrm{shift}}^{\text{o-}A/\text{o-o}}$$



even-even: ▲

$n_{\mathrm{nucl}} = 41$

$c_0 = -250.93,\ c_1 = 12.464,\ c_2 = -0.1281$

$\mathrm{rms} = 1.65$

odd-A: ◆

$n_{\mathrm{nucl}} = 17$

$c_0 = -266.44,\ c_1 = 13.255,\ c_2 = -0.1360$

$\mathrm{rms}_{\mathrm{fit}} = 1.82$

$c_{\mathrm{shift}} = 4.30$

$\mathrm{rms}_{\mathrm{shift}} = 1.84$

odd-odd: ★

$n_{\mathrm{nucl}} = 1$

$c_{\mathrm{shift}} = 6.74$

SETTING OF THE APPROXIMATING FUNCTION

$$\log_{10} T_{1/2}^{\mathrm{SF}} = \log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha, N-Z)$$

Form of the approximating function for even-even (**e-e**) nuclei:

$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha, N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_Q Q_\alpha$$

Form of the approximating function for odd-even and even-odd (**o-**A) nuclei / odd-odd (**o-o**) nuclei:

$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha, N-Z) = c_{\mathrm{shift}}^{\text{o-}A/\text{o-o}} + c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_Q Q_\alpha$$

NEW EMPIRICAL FORMULA FOR SPONTANEOUS FISSION HALF-LIVE

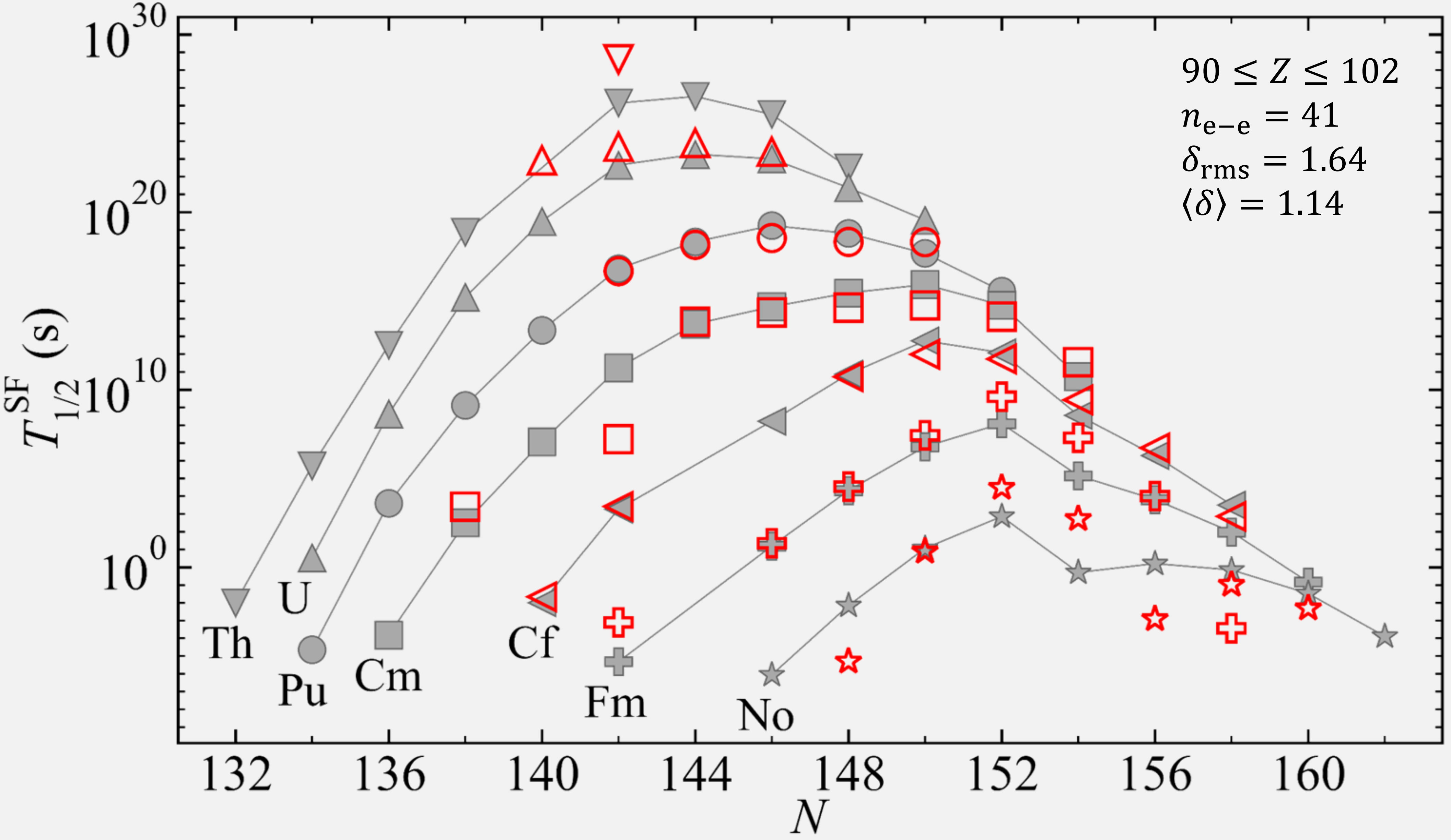
$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha, N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_Q Q_\alpha + \begin{cases} 0, & \text{for even-even}, \\ h, & \text{for odd-}A, \\ 1.575h, & \text{for odd-odd}. \end{cases}$$

$90 \leqslant Z \leqslant 102$:

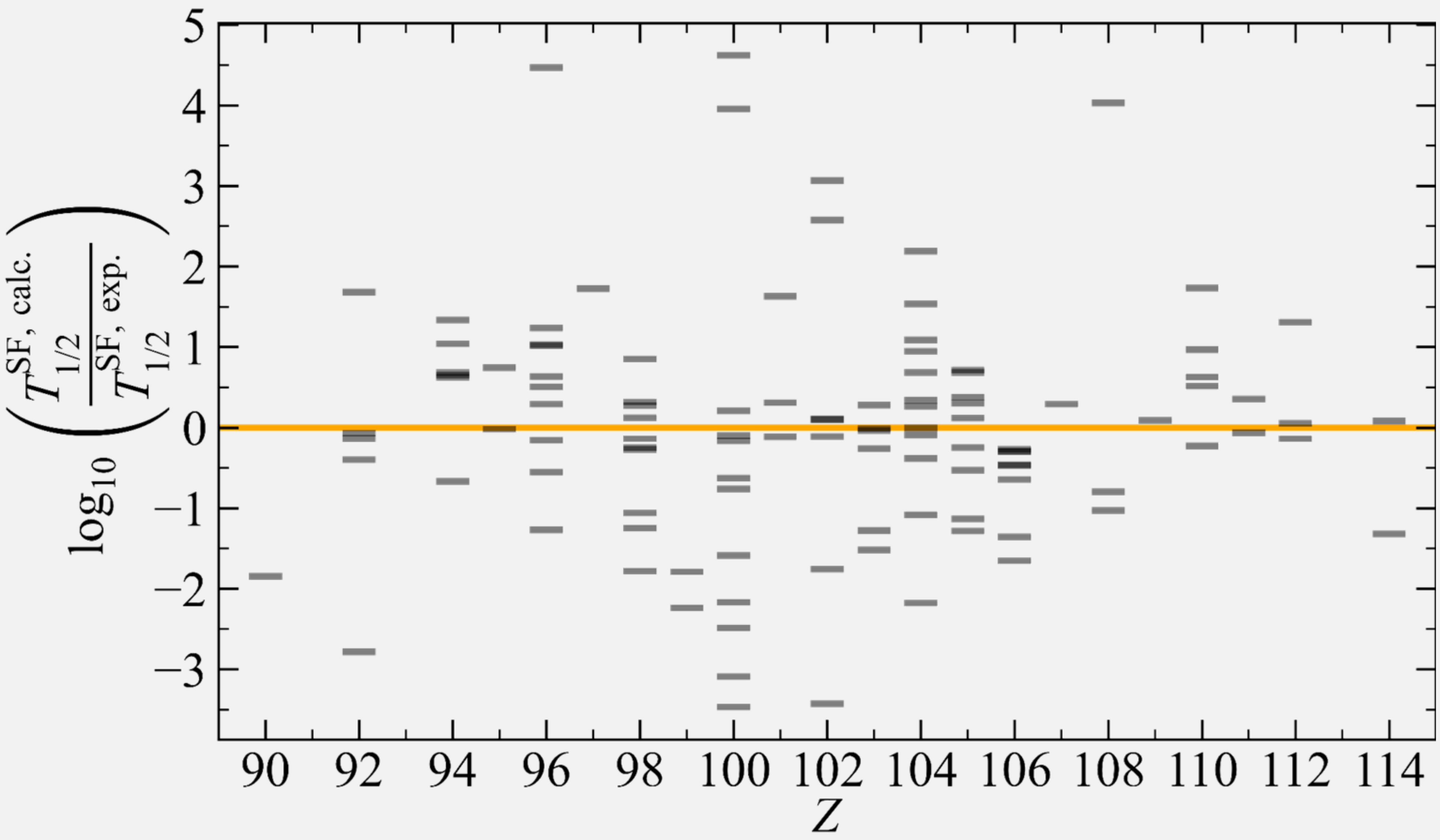
$$c_0 = -244.845, \quad c_1 = 12.2843, \quad c_2 = -0.12658,$$
$$c_Q = -6.1023, \quad h = 4.309,$$

$Z \geqslant 103$:

$$c_0 = -0.414, \quad c_1 = 0.1379, \quad c_2 = 0,$$
$$c_Q = -0.9097, \quad h = 2.327.$$



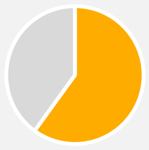
Calculated spontaneous fission half-lives of even-even nuclei with charge numbers $90 \leq Z \leq 102$ (solid gray symbols connected by lines) in comparison to the experimental values [Kondev, 2021] (open red symbols).



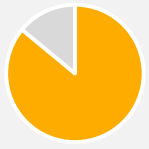
Deviations of predictions from the experimental half-life values [Kondev, 2021] against charge number of parent nuclei.

$n_{\mathrm{nucl}} = 111$
$\delta_{\mathrm{rms}} = 1.43$
$\langle\delta\rangle = 1.00$
$\delta_{\max} = 4.62$

$\delta < 1$:
60% *nuclei*

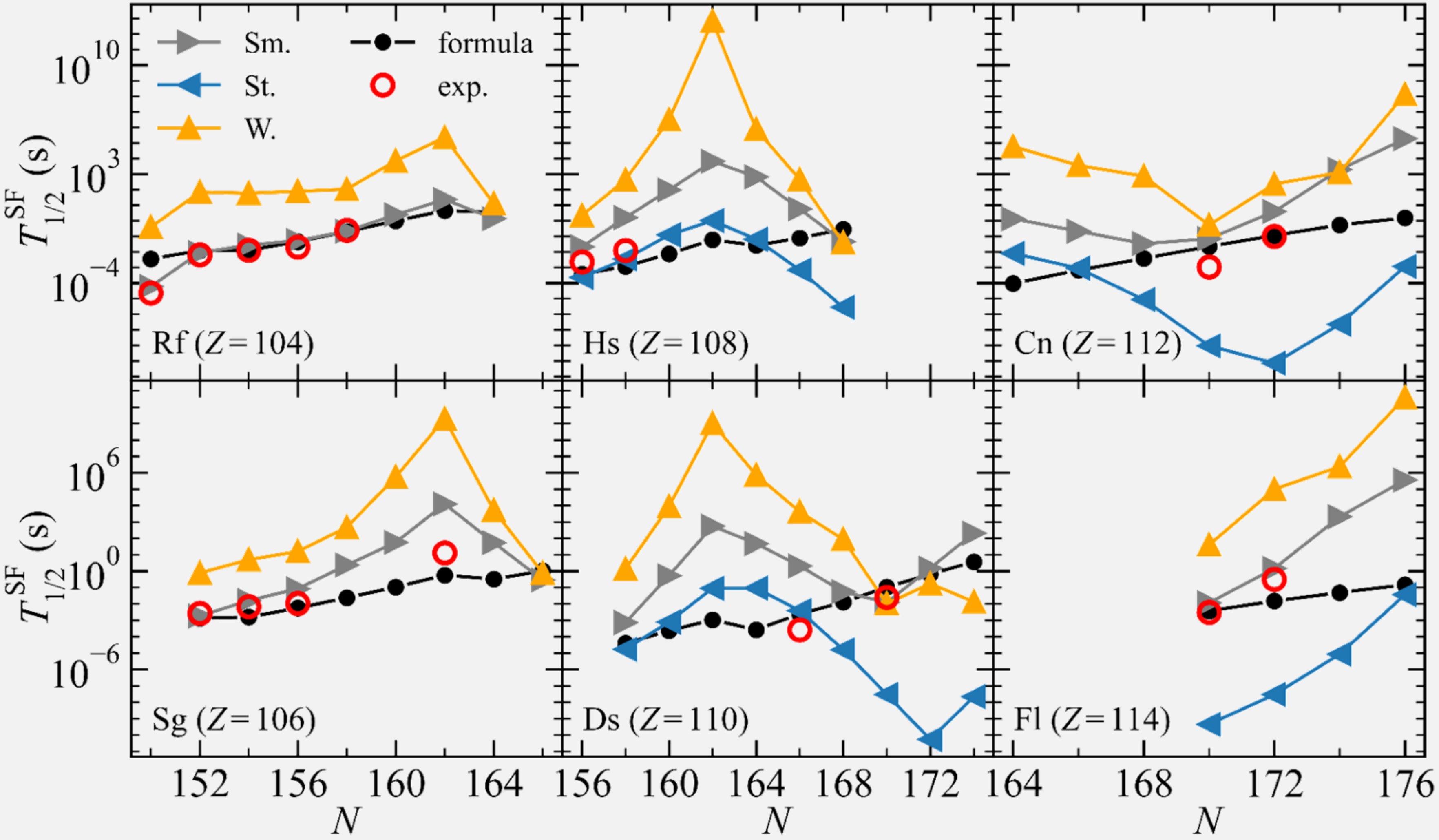


$\delta < 2$:
86% *nuclei*



$\delta > 4$:
3 *nuclei:*
^{238}Cm, ^{258}Fm, ^{277}Hs

COMPARISON WITH THEORETICAL MODEL PREDICTIONS



Half-lives of even-even superheavy nuclei, calculated using experimental Q_α values and systematics from [Wang, 2021] and the empirical formula (solid black circles connected by lines), compared with theoretical predictions from Smolanczuk, Staszczak и Warda (solid gray, blue and orange triangles, connected by lines), and experimental values (open circles).

COMPETITION BETWEEN SF AND α-DECAY

Viola-Seaborg semi-empirical relationship

$$\log_{10} T^{\alpha}_{1/2} = \frac{(aZ+b)}{\sqrt{Q_\alpha}} + (cZ+d) + h_i$$

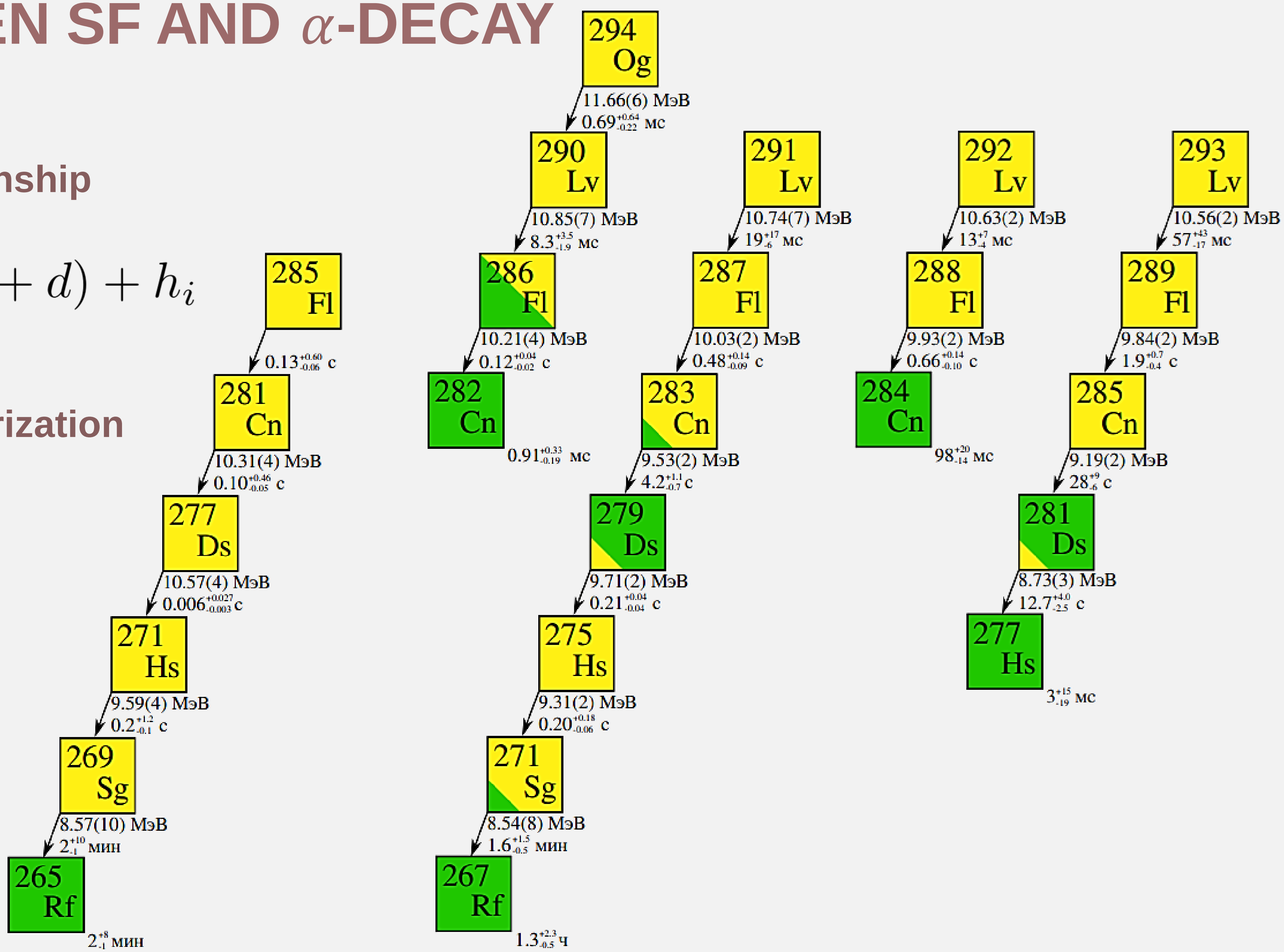
Parkhomenko-Sobichevsky parametrization

[Parkhomenko, 2005]

$$a = 1.3892, b = 13.862,$$

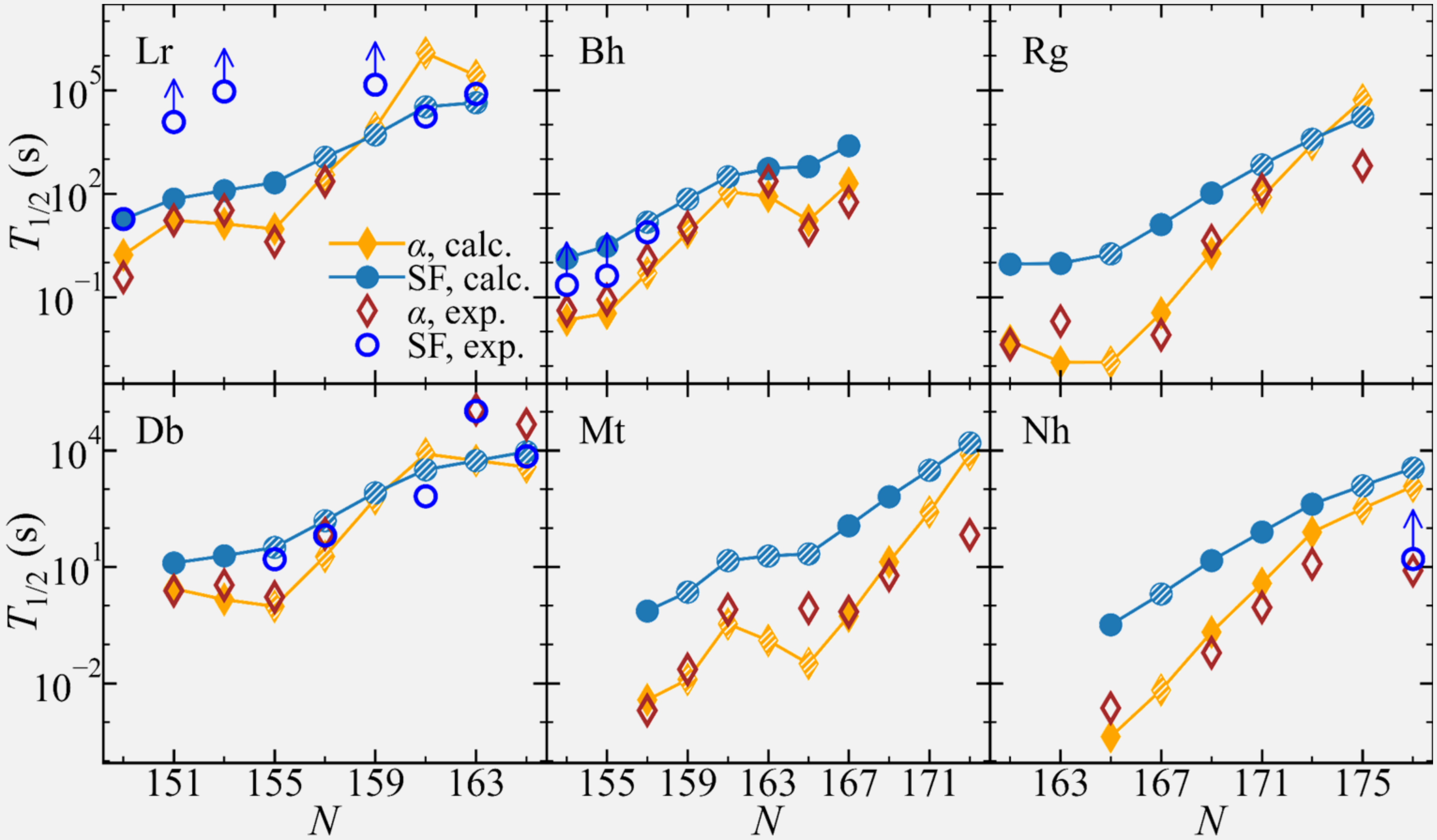
$$c = -0.1086, d = -41.458,$$

$$h_i = \begin{cases} 0, & \text{even-even}, \\ 0.437, & \text{odd-even}, \\ 0.641, & \text{even-odd}, \\ 1.024, & \text{odd-odd} \end{cases}$$



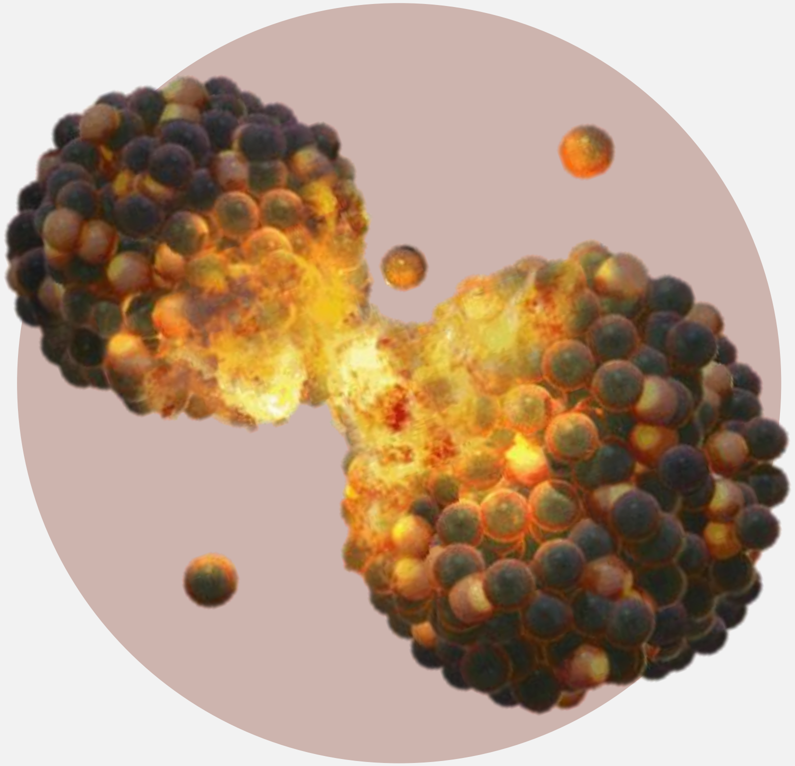
α-decay chains for even-Z elements synthesized in complete fusion reactions ^{48}Ca + ^{238}U, $^{242, 244}$Pu, $^{245, 248}$Cm, ^{249}Cf.

COMPETITION BETWEEN SF AND α-DECAY



Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **odd-odd** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [Wang, 2021]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [Kondev, 2021]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.

CONCLUSIONS



influence of neutron excess on systematics of **SF half-lives** of actinides and SHN was revealed;

linear dependence of the decimal **logarithm of SF half-life** on **energy release of α decay** at fixed values of neutron excess was found;

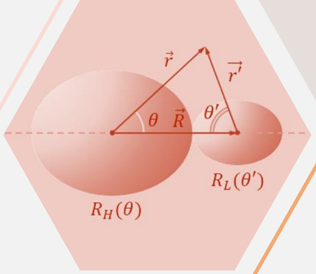
parameter sets were determined to best reproduce experimental SF half-lives **in two regions** with $90 \leq Z \leq 102$ and $Z \geq 103$;

semi-empirical formula was proposed **for the SF half-lives**, depending on Q_α and $N - Z$;

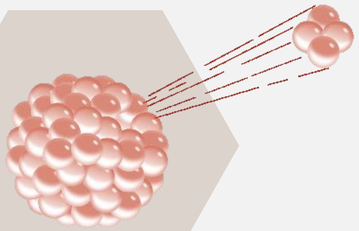
comparison with **theoretical models** demonstrated that the formula **accounts for shell effects** and predicts a **slower increase in SF half-lives** with increasing neutron number **in Cn** and **Fl**;

neutron excess preserved in α-decay chains was shown to **play a significant role** in the **competition between α decay and SF**;

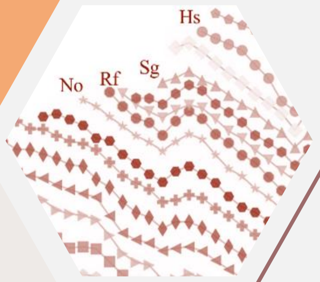
discovered systematics underscores the importance of considering the nuclear structure from the perspective of α-cluster formation and confirms the fundamental **connection between SF and α decay**, suggesting that these processes are **different manifestations of nuclear fission**.



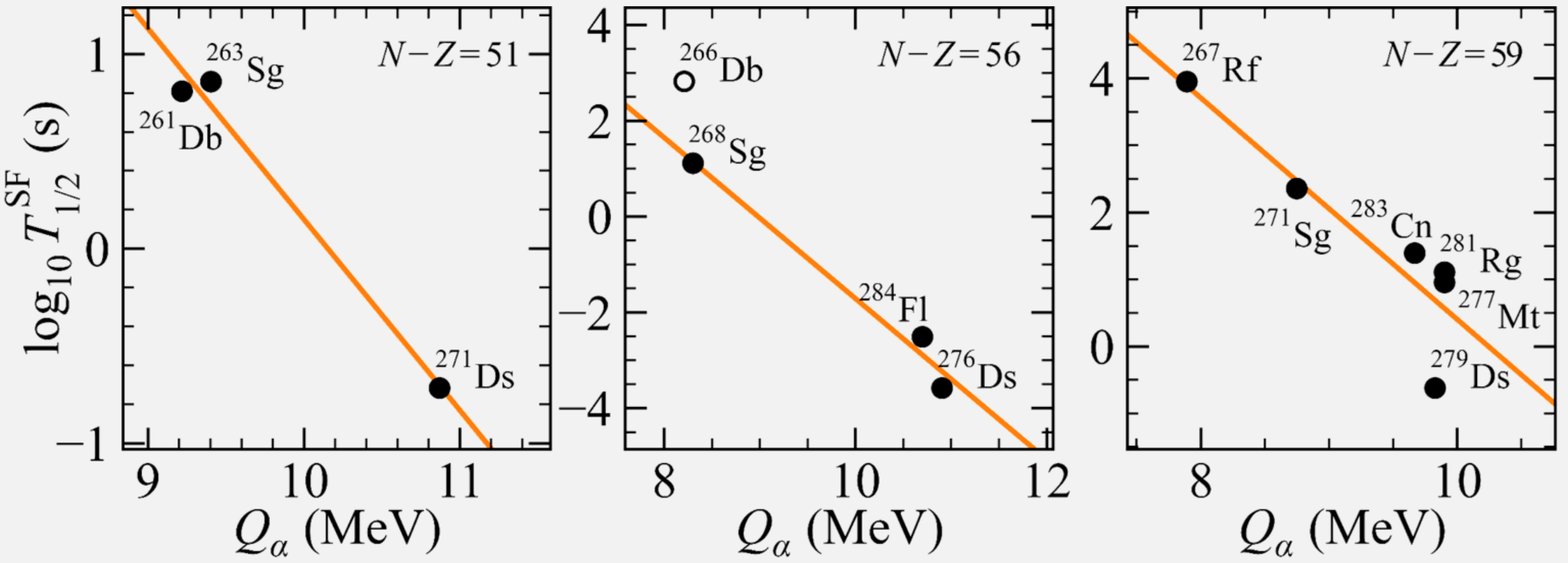
$T_{1/2}^{SF}$



THANK YOU
FOR ATTENTION



BACKUP SLIDES



Decimal logarithm of the experimental $T_{1/2}^{\mathrm{SF}}$ as a function of Q_α for fixed values of the neutron excess $N-Z$ for nuclei with $103 \le Z \le 118$ (**black round symbols**) and their approximation (orange lines) by

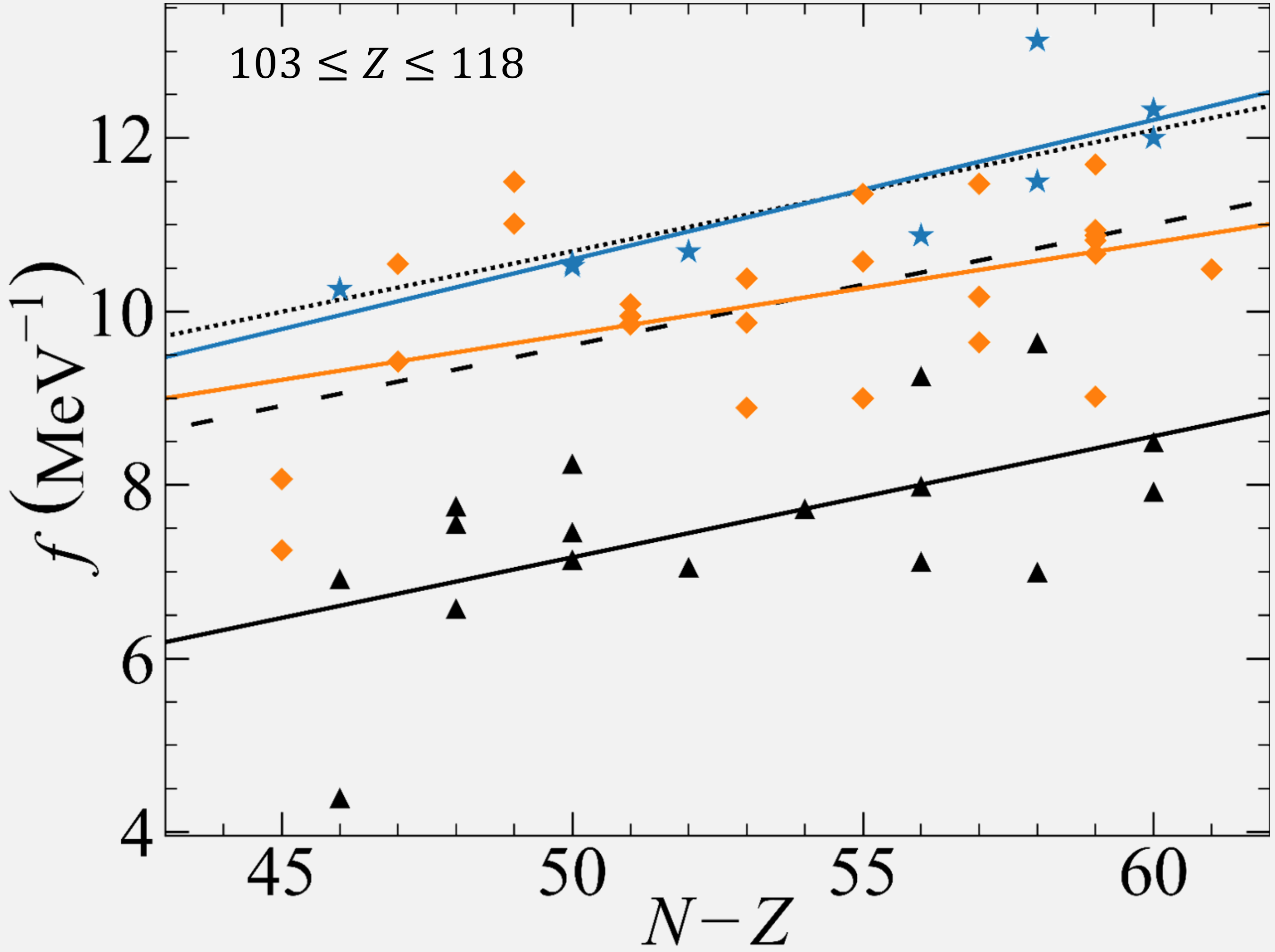
$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha) = c_Q Q_\alpha + f$$

Values of slope and intercept obtained by linear approximation of experimental points

$N-Z$	c_Q	f
51	-0.98	9.96
56	-1.68	15.09
59	-1.65	16.93

$$f(N-Z) = \log_{10} T_{1/2}^{\mathrm{SF}} - c_Q^{\mathrm{mean}} Q_\alpha$$

$$f(N-Z) = c_0 + c_1(N-Z)$$



$c_Q^{\mathrm{mean}} = -0.981$

even-even: ▲

$n_{\mathrm{nucl}} = 17$

$c_0 = 0.19,\ c_1 = 0.140$

$\mathrm{rms} = 0.91$

odd-A: ◆

$n_{\mathrm{nucl}} = 25$

$c_0 = 4.46,\ c_1 = 0.106$

$\mathrm{rms}_{\mathrm{fit}} = 0.94$

$c_{\mathrm{shift}} = 2.44$

$\mathrm{rms}_{\mathrm{shift}} = 0.95$

odd-odd: ★

$n_{\mathrm{nucl}} = 9$

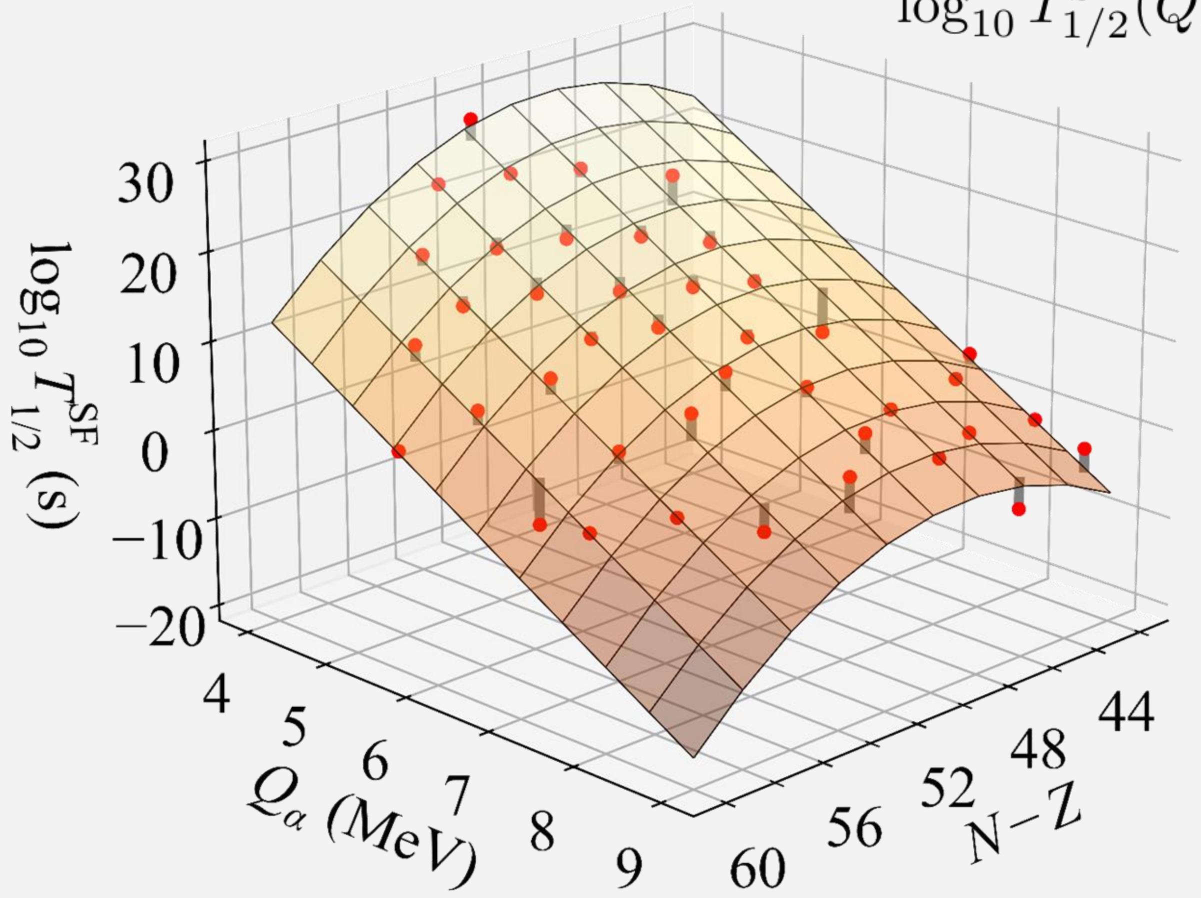
$c_0 = 2.56,\ c_1 = 0.161$

$\mathrm{rms}_{\mathrm{fit}} = 0.51$

$c_{\mathrm{shift}} = 3.53$

$\mathrm{rms}_{\mathrm{shift}} = 0.5$

NEW EMPIRICAL FORMULA FOR SPONTANEOUS FISSION HALF-LIVE



$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha, N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_Q Q_\alpha + \begin{cases} 0, & \text{for even-even,} \\ h, & \text{for odd-}A, \\ 1.575h, & \text{for odd-odd.} \end{cases}$$

$90 \leqslant Z \leqslant 102$:

$c_0 = -244.845, \quad c_1 = 12.2843, \quad c_2 = -0.12658,$
$c_Q = -6.1023, \quad h = 4.309,$

$Z \geqslant 103$:

$c_0 = -0.414, \quad c_1 = 0.1379, \quad c_2 = 0,$
$c_Q = -0.9097, \quad h = 2.327.$

Decimal logarithm of the spontaneous fission half-life of even-even isotopes with atomic numbers $90 \leq Z \leq 102$, plotted as a function of Q_α and $N - Z$. The surface is based on calculations using proposed formula. Experimental values [Kondev, 2021] are shown as red circles. Gray segments visualize the deviations of the experimental data from the calculated surface.

AVERAGE VALUES

Root mean square deviation (rms) of calculated values of decimal logarithms of spontaneous fission half-lives from experimental values:

$$\delta_{\text{rms}} = \left\{ \frac{1}{n} \sum_{i=1}^{n} \left[\log_{10} T_{1/2}^{SF} \left((Q_\alpha)_i \,, N_i - Z_i \right) - \log_{10} \left(T_{1/2}^{SF\ exp} \right)_i \right]^2 \right\}^{1/2}$$

n is the number of cores used to estimate the standard deviation.

Average absolute deviation:

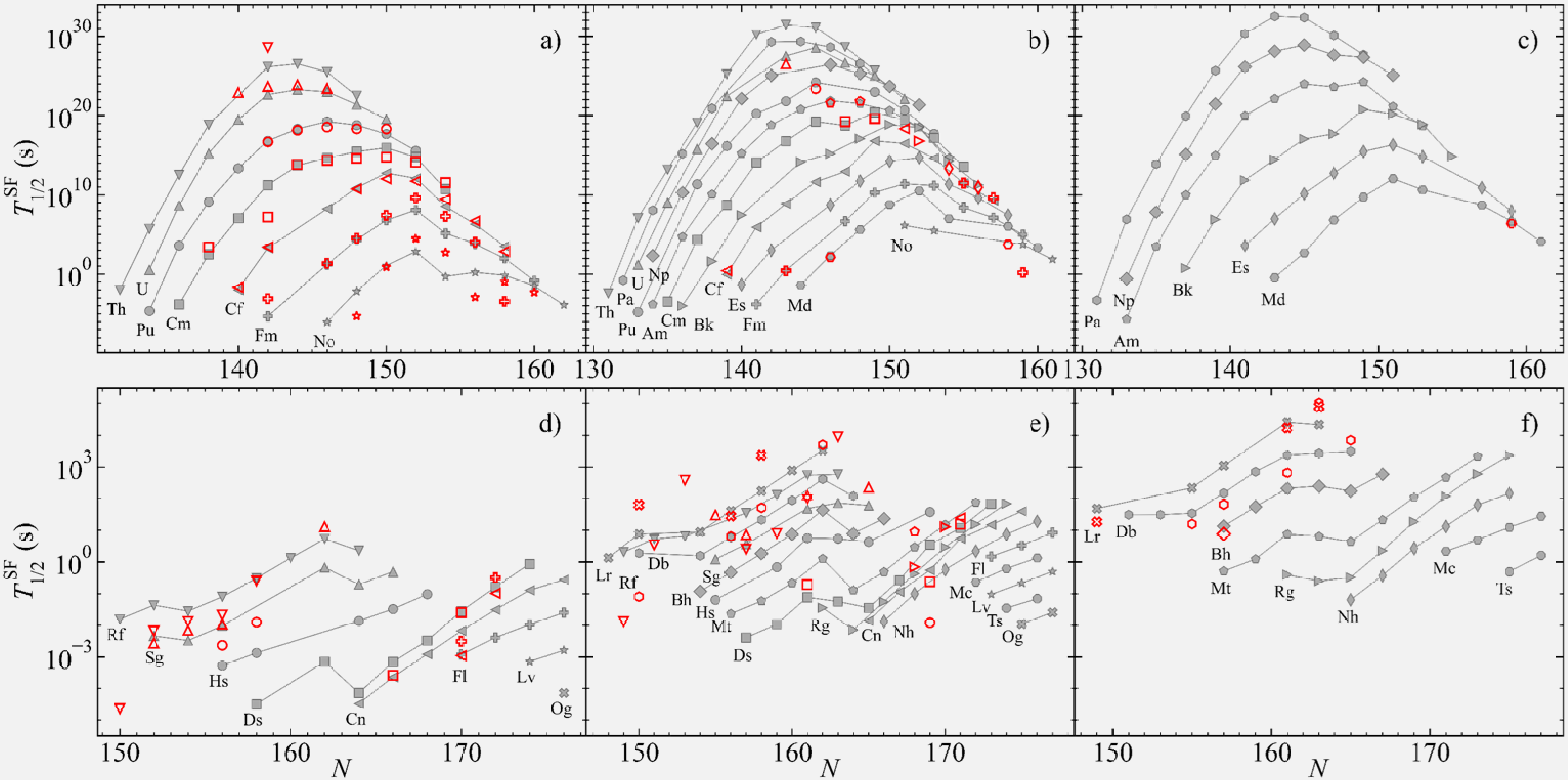
$$\langle \delta \rangle = \frac{1}{n} \sum_{i=1}^{n} \left| \log_{10} T_{1/2}^{SF} \left((Q_\alpha)_i \,, N_i - Z_i \right) - \log_{10} \left(T_{1/2}^{SF\ exp} \right)_i \right| = \frac{1}{n} \sum_{i=1}^{n} \left| \log_{10} \frac{\left(T_{1/2}^{SF\ \text{calc.}} \right)_i}{\left(T_{1/2}^{SF\ \text{exp.}} \right)_i} \right|$$

Maximum absolute deviation:

$$\delta_{\max} = \max_i \left| \log_{10} T_{1/2}^{SF} \left((Q_\alpha)_i \,, N_i - Z_i \right) - \log_{10} \left(T_{1/2}^{SF\ exp} \right)_i \right|$$

Mean absolute deviation $\langle\delta\rangle$, rms deviation δ_{rms}, and maximum absolute deviation $\delta_{\max}$ of logarithmic spontaneouse fission half-lives calculated with proposed formula and the experimental data from Refs. [Kondev, 2021] for different nuclide regions; n is the number of experimentally studied nuclei in each region.

Nuclei	Z	n	$\langle\delta\rangle$	δ_{rms}	$\delta_{\max}$
e-e		41	1.14	1.64	4.62
o-A	$Z \leq 102$	17	1.49	1.89	3.95
o-o		1	0.11	0.11	0.11
e-e		17	0.69	0.90	2.19
o-A	$Z \geq 103$	26	0.90	1.24	4.03
o-o		9	0.40	0.54	1.28
e-e		58	1.01	1.46	4.62
o-A	$\forall\ Z$	43	1.13	1.53	4.03
o-o		10	0.37	0.51	1.28
all	$\forall\ Z$	111	1.00	1.43	4.62



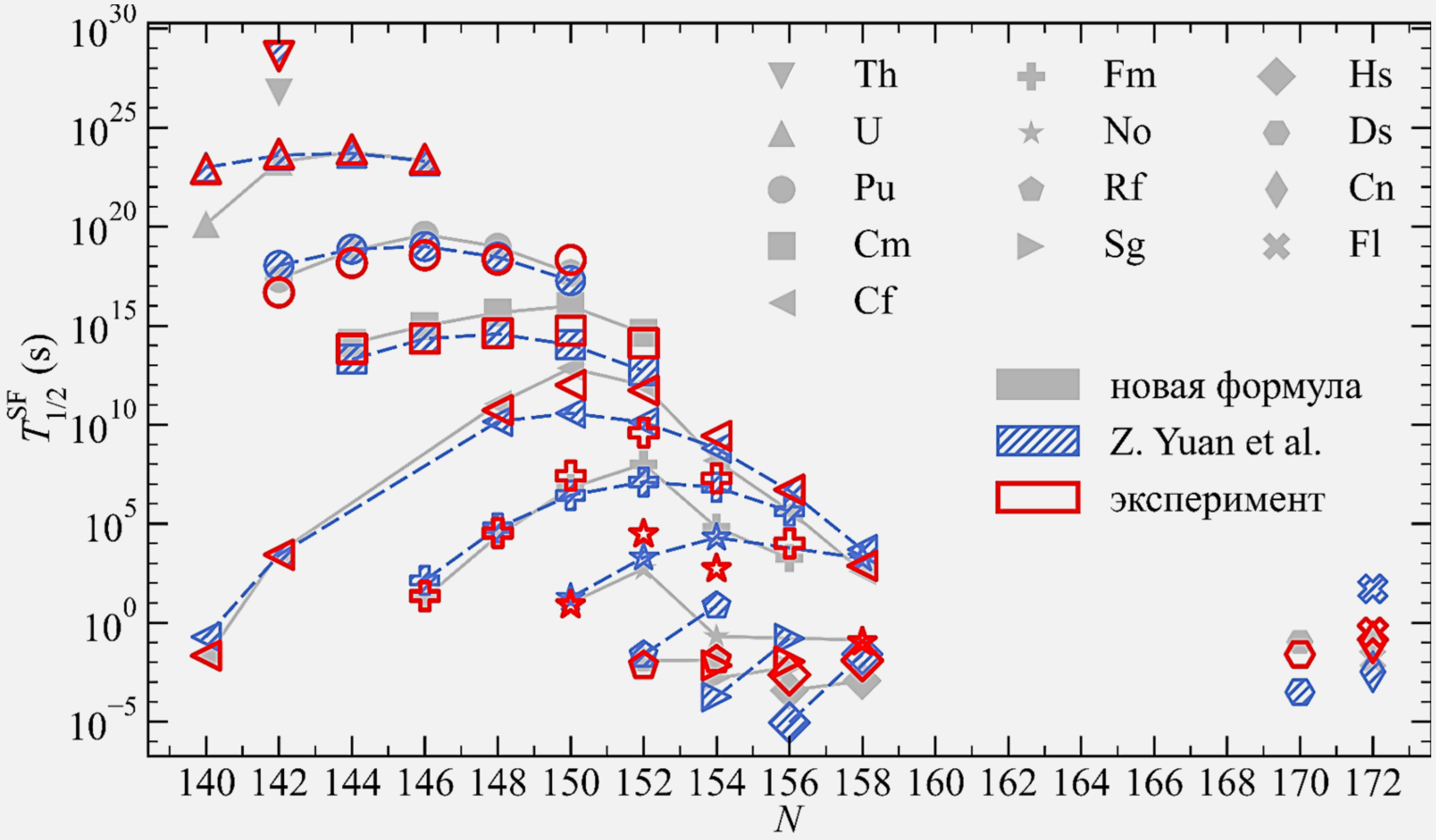
Half-lives of spontaneous fission (filled gray symbols connected by lines) of **even-even** (a,d), **odd-**A (b,e) and **odd-odd** (c,f) nuclei with $90 \leq Z \geq 102$ (top line) and $Z \geq 103$ (bottom line) calculated using the proposed semi-empirical formula in comparison with the experimental data (open red symbols).

COMPARISON WITH ANOTHER EMPIRICAL FORMULA

Z. Yuan, D. Bai, Zh. Wang, and Zh. Ren, Improved formulas of spontaneous fission half-lives for heavy and superheavy nuclei, 2024

$$\log_{10} T^{\mathrm{SF}}_{1/2}\ (\mathrm{s}) = 28.60 + c_1 \frac{Z-90}{A} + c_2 \frac{(Z-90)^2}{A} + c_3 \frac{|Z-90|^{1/2}(N-Z-52)^2}{A} + c_4 \frac{(Z-90)(N-152)^2}{A} + c_5 \frac{(Z-90)^2(N-162)}{A} + c_6 \frac{|Z-90|^{5/2}|N-162|^{3/2}}{A} + h$$

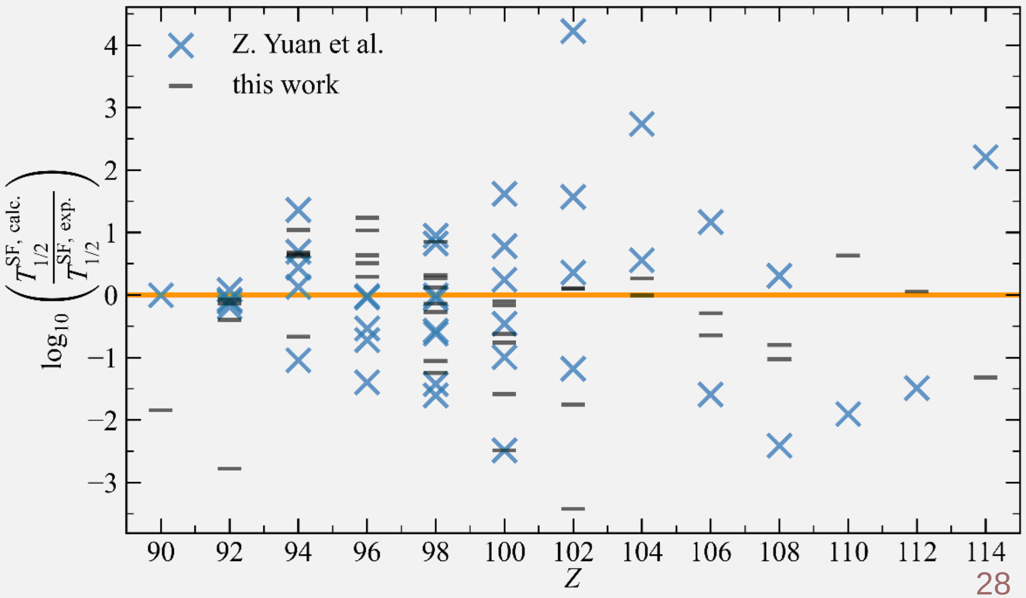
$$c_1 = -646.834955,\ c_2 = 10.058292,\ c_3 = -11.472012,\ c_4 = 0.699861,$$
$$c_5 = -0.209631,\ c_6 = -0.016230,\ h = 0\ (\text{e-e}),\ h = 3.128822\ (\text{o-}A,\ \text{o-o})$$

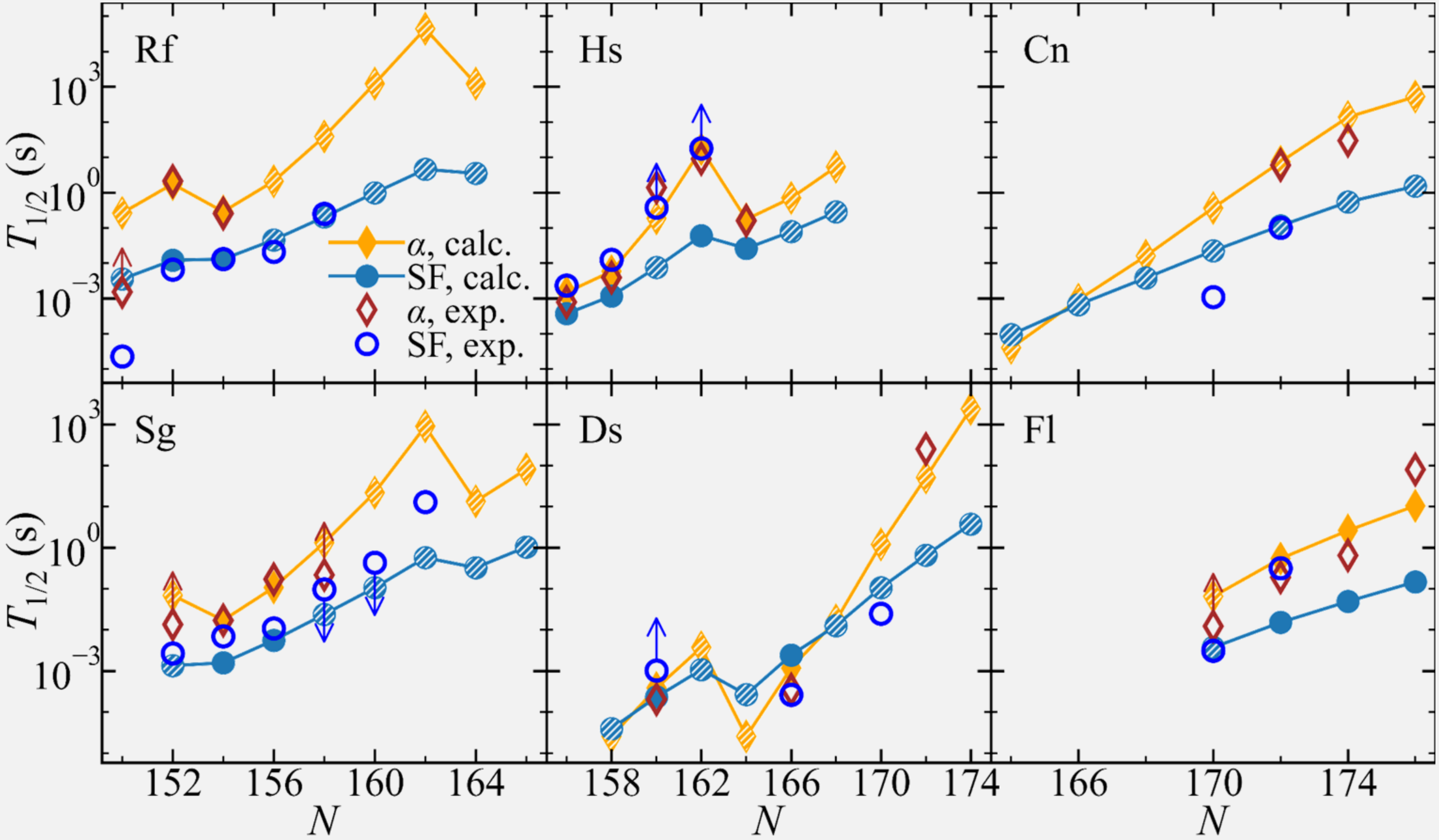


$n_{\mathrm{nucl}} = 42$

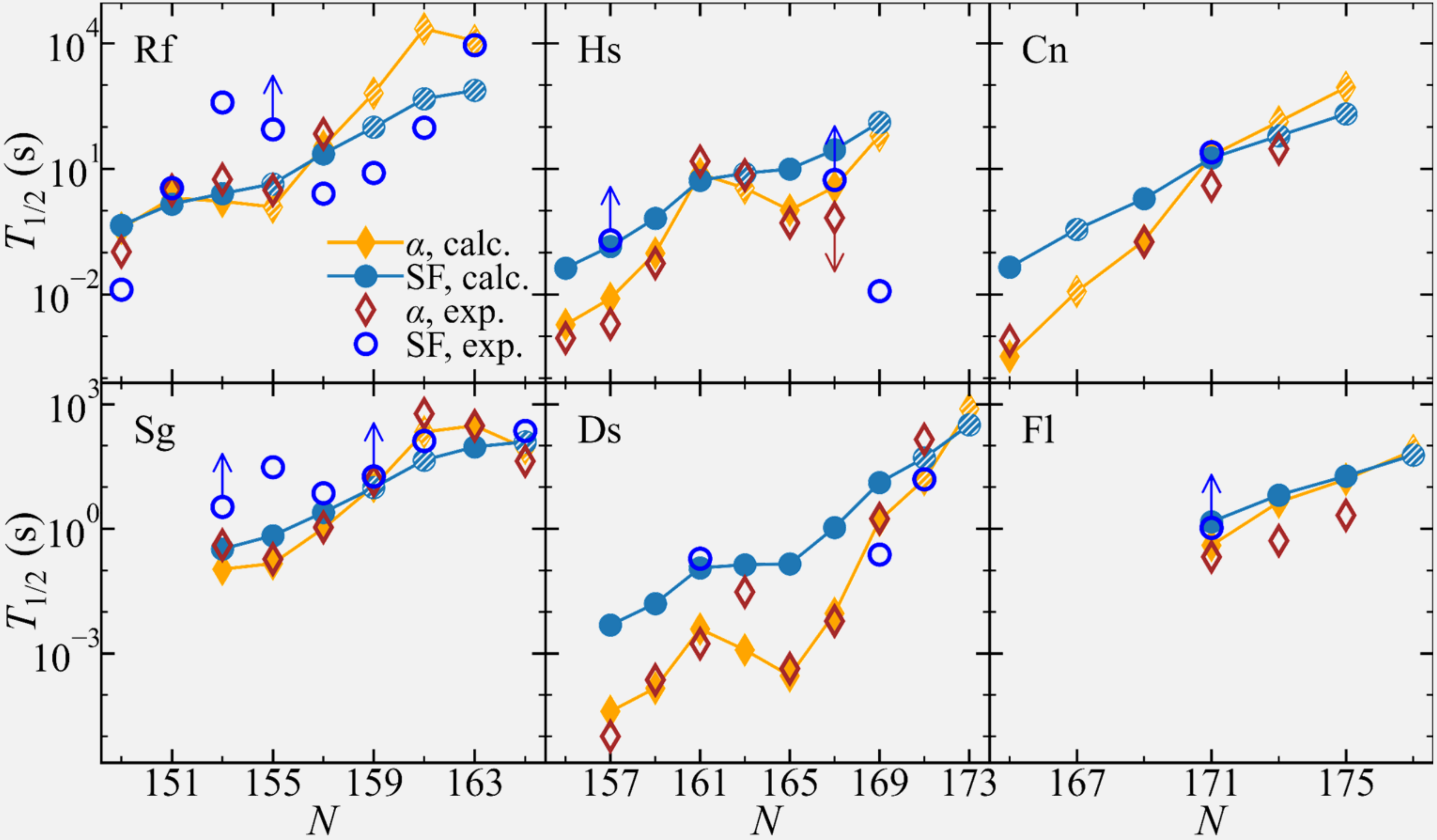
[Yuan *et al.*, 2024]
$\delta_{\mathrm{rms}} = 1.33$
$\langle\delta\rangle = 0.98$

this work
$\delta_{\mathrm{rms}} = 1.09$
$\langle\delta\rangle = 0.79$

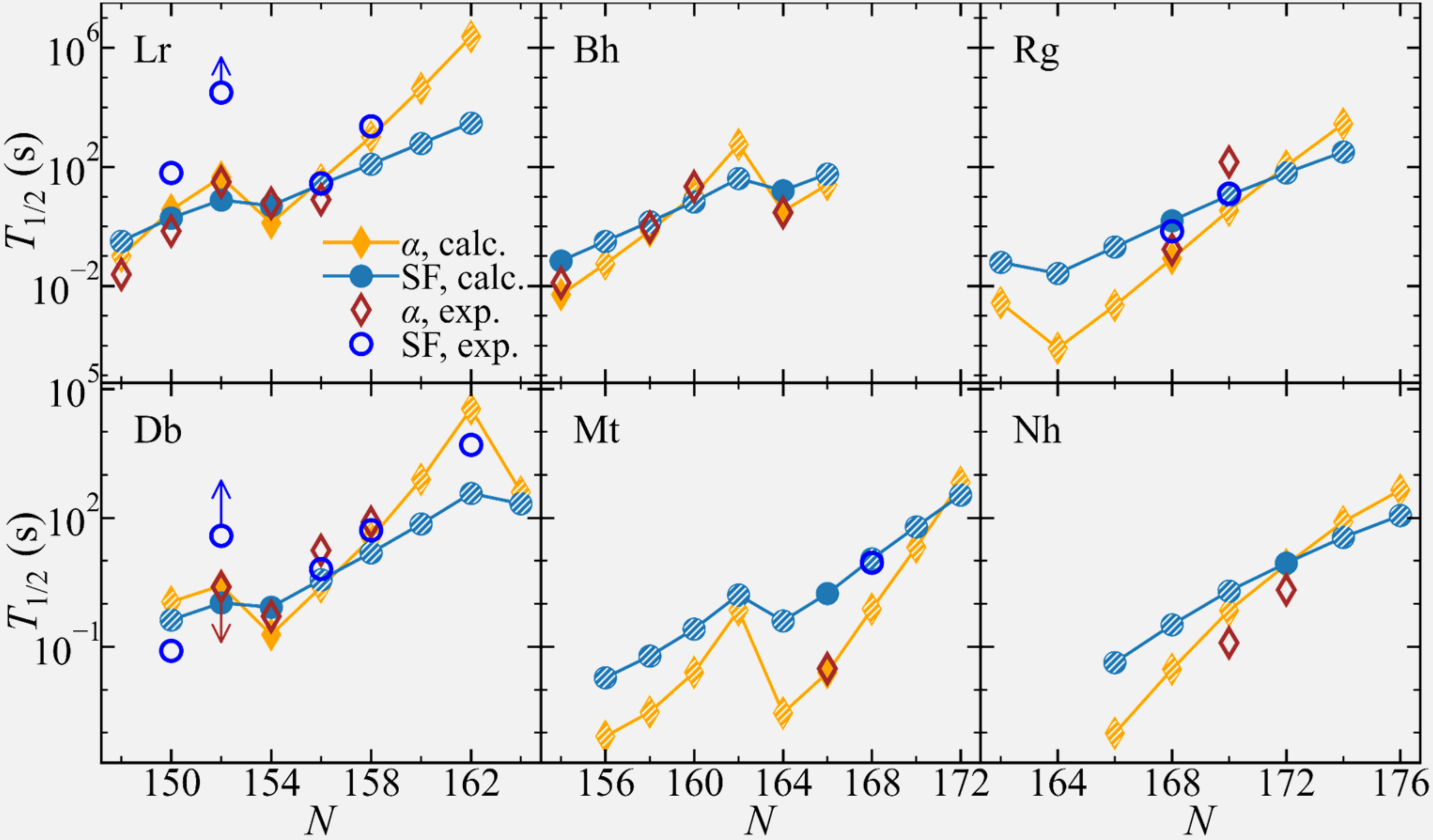




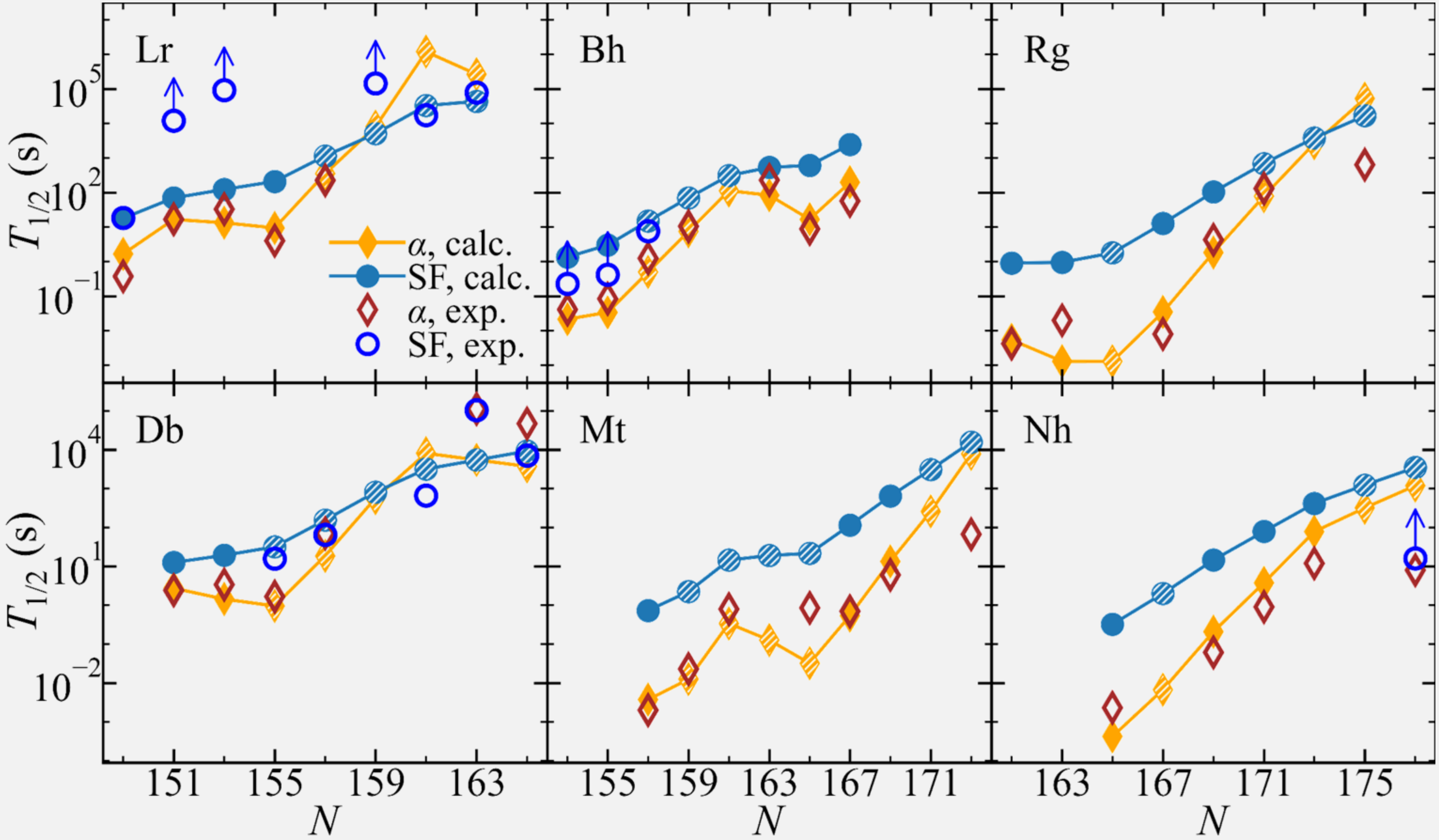
Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **even-even** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [*Wang, 2021*]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [*Kondev, 2021*]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



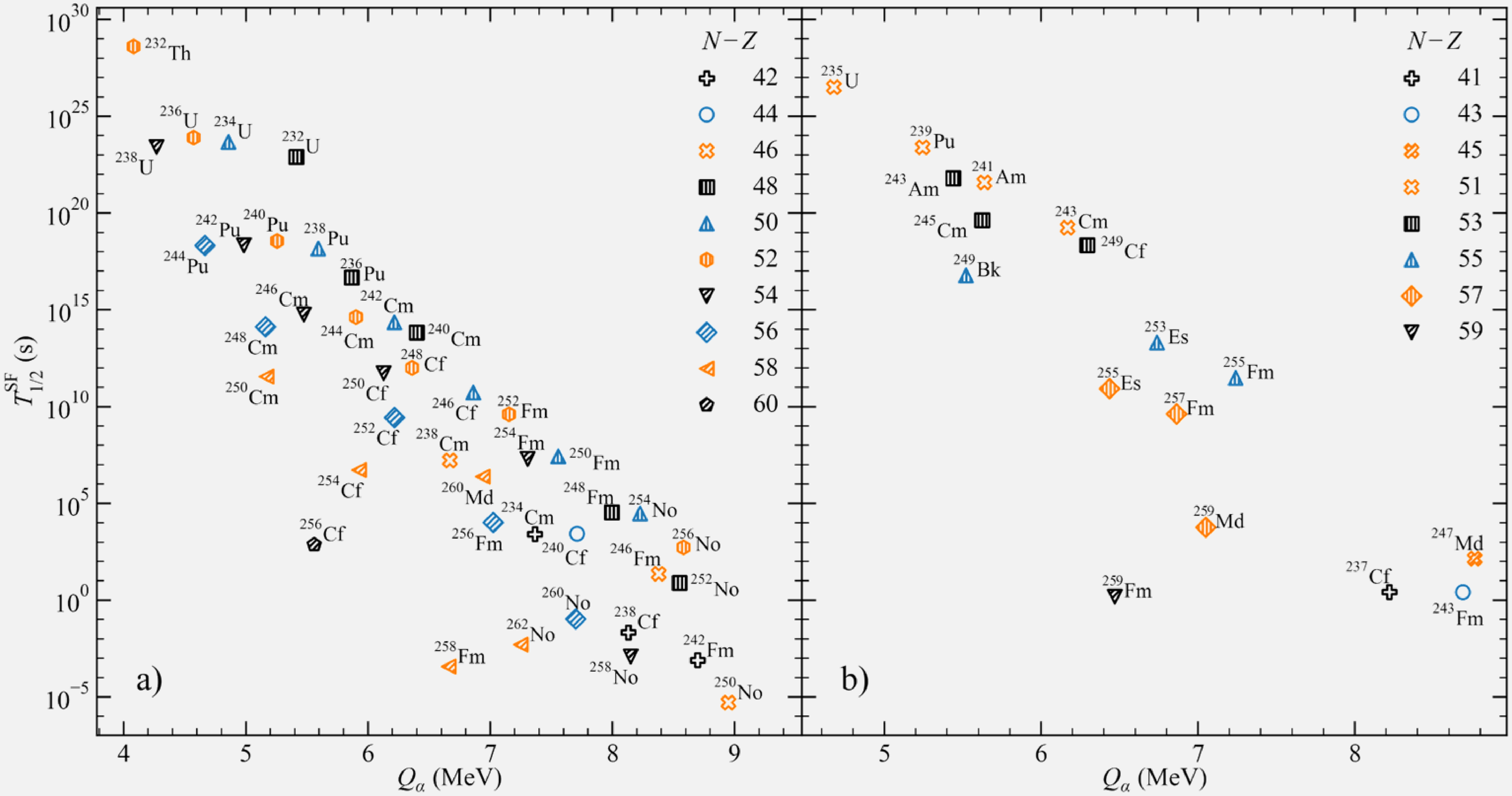
Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **even-odd** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [Wang, 2021]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [Kondev, 2021]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **odd-even** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [*Wang, 2021*]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [*Kondev, 2021*]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **odd-odd** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [Wang, 2021]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [Kondev, 2021]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



Spontaneous fission half-life versus alpha separation energy for nuclei with $90 \le Z \le 102$, shown for even (left) and odd (right) neutron excess $N - Z$. Experimental data from [Wang, 2021; Kondev, 2021].