Calculation of target dependence of the isotope distributions in heavy-ion reactions at energies from 35 to 140 MeV per nucleon in the modified transport-statistical model

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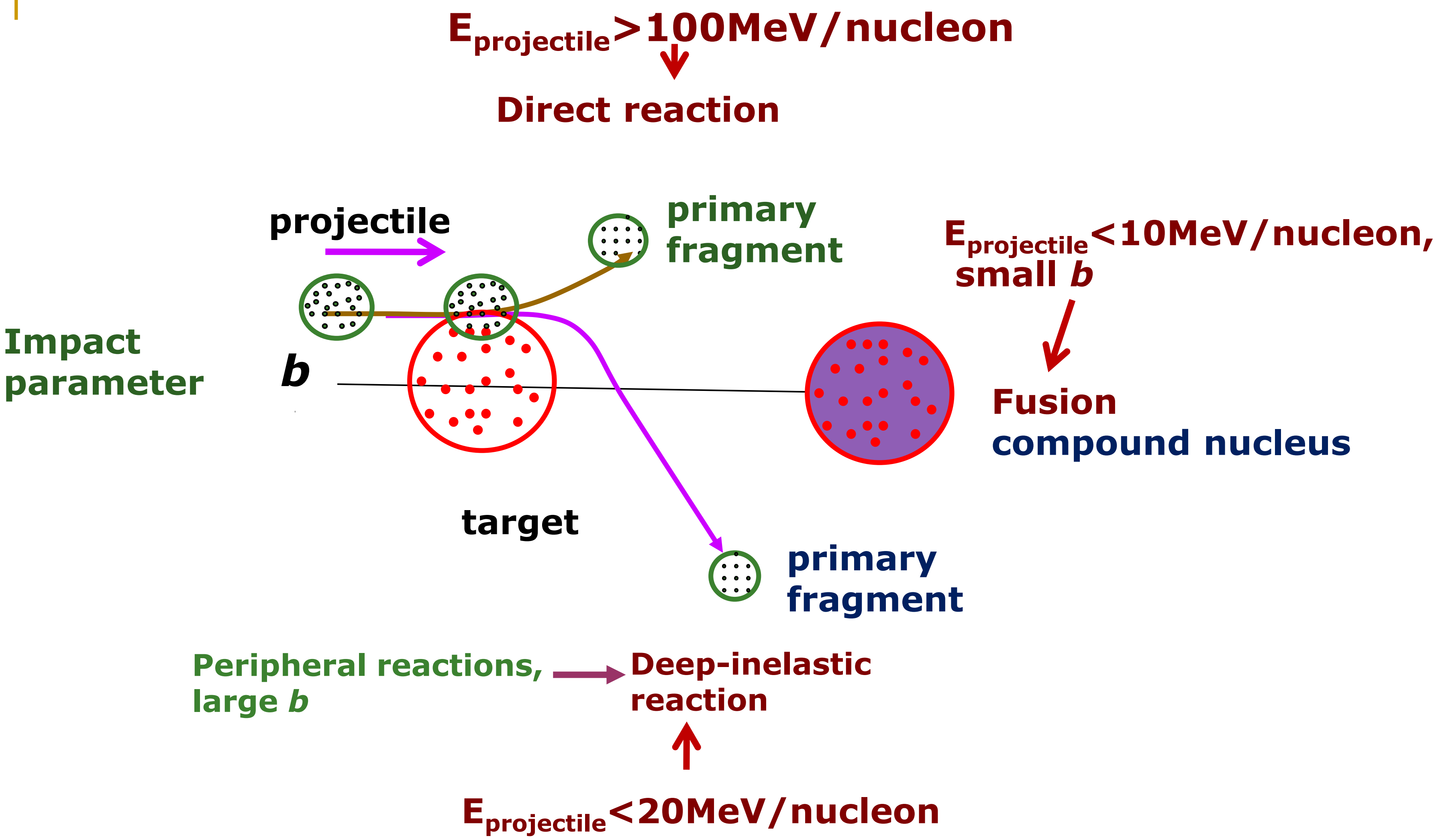
Outline

- Motivation.
- Description of heavy-ion collisions with modified transport (BNV)-statistical (SMM) approach . Comparison with

  experimental data for reactions with two different projectiles ^{18}O (35 MeV per nucleon) and ^{86}Kr (64 MeV per nucleon) on two targets : ^{181}Ta and ^{9}Be
- Comparison with other model calculations (Abrasion-Ablation, EPAX and Fracs)
- Explanation of target dependence from the point of view of BNV-SMM calculations
- Conclusion

Schematic view of the collision of two heavy ions



To model heavy-ion collision microscopically we use kinetic theory:

Transport theory: Boltzmann-Nordheim-Vlasov (BNV) approach

time evolution of the one-body phase space density: f(r,p;t)

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}f-\vec{\nabla}U\,\vec{\nabla}_p f=I_{coll}\left[f,\sigma\right]$$

Physical input:

mean field potential U (equation of state)

and in-medium elastic cross section

F. Bertsch, S. Das Gupta, Phys. Rep. **160** (1988) 189

V. Baran, M. Colonna, M. Di Toro, Phys. Rep., **410** (2005**)** 335

Density functional

U(ρ(r)) = Nuclear Mean Field + Symmetry terms + Coulomb

$$U(\rho)=A\left[\frac{\rho}{\rho_0}\right]+B\left[\frac{\rho}{\rho_0}\right]^d+C(-1)^k\left(\rho_n-\rho_p\right)/(\rho_n+\rho_p)+U_{\mathrm{coul}}$$

$$A=-356MeV, B=303MeV, d=7/6, k=1(p), 2(n), C=36MeV$$

Collision term

$$I_{coll}[f_1,\sigma] = \frac{g}{h}\int d^3p_2\, d^3p_3\, d^3p_4\; \sigma(12,34)\,\delta(\vec{p}_1+\vec{p}_2-\vec{p}_3+\vec{p}_4)\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3+\varepsilon_4)\left[\,\bar{f}_1\bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3\bar{f}_4\,\right]$$

Pauli blocking factors for final state
g degeneracy

$$(1-f(r,v_i;t)) \equiv (1-f_i) := \bar{f}_i$$

Collision term: treatment by stochastic simulation

1. Select in each time step d*t* TPs with distance $d \le \sqrt{\sigma/\pi}$
2. Collide with probability $P = \sigma_{el}/\sigma_{max}$ with random scattering angle
3. Check Pauli blocking of final state in phase space

Computationally most expensive part of calculation

Solution of transport equation

Partial integro-differential equation for *f(r,p;t)* is solved by simulation with the test particle method:
N -finite element test particles (TP) per nucleon.
Each TP carries charge and isospin number.
A – number of nucleons in the system
g – the shape of the TP in space
ρ – the density

$$f(\vec{r},\vec{p},t) = \frac{1}{N}\sum_{i}^{NA} g(\vec{r}-\vec{r}_i(t))\,\bar{g}(\vec{p}-\vec{p}_i(t))$$

$$g = \mathrm{e}^{-(\vec{r}-\vec{r}_i(t))^2/L^2} \ldots;\ \bar{g} = \mathrm{e}^{-(\vec{p}-\vec{p}_i(t))^2/l^2}$$

$$\rho(r;t) = \int d\vec{p}\ f(\vec{r},\vec{p};t)$$

Equations of motion of TP

(Newton equation of motions:):

$$\frac{\partial \vec{p}_i(t)}{\partial t} = -\vec{\nabla}_r U(r_i,t) \qquad \frac{\partial \vec{r}_i(t)}{\partial t} = \frac{\vec{p}_i(t)}{m}$$

Velocity Verlet (or leapfrog) algorithm, accuracy $(dt)^2$:

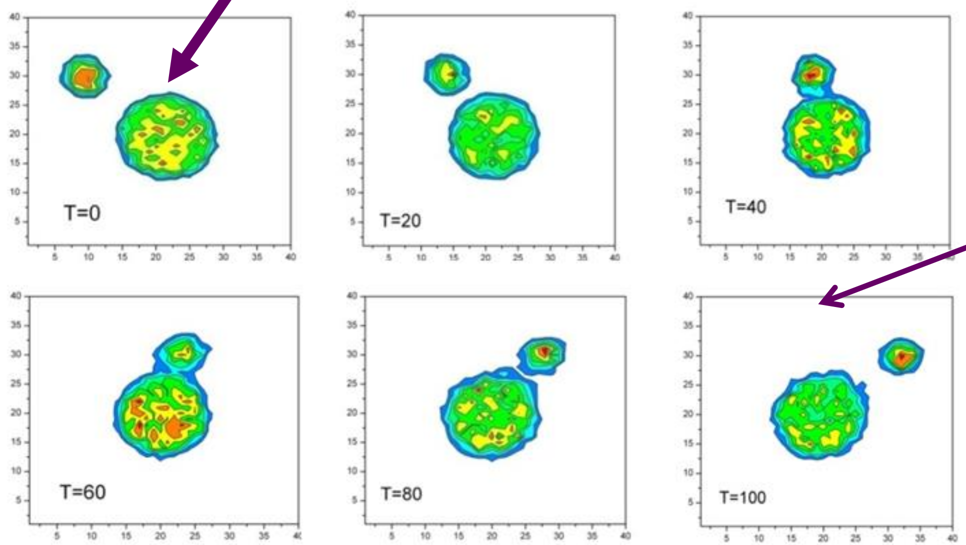
$$\vec{p}_i(t+\frac{1}{2}\Delta t) = \vec{p}_i(t) - \frac{1}{2}\Delta t\vec{\nabla}_r U(r_{i(t)})$$

$$\vec{r}_i(t+\Delta t) = \vec{r}_i(t) + \Delta t\ \vec{p}_i(t+\frac{1}{2}\Delta t)/m$$

$$\vec{p}_i(t+\Delta t) = \vec{p}_i(t+\frac{1}{2}\Delta t) - \frac{1}{2}\Delta t\vec{\nabla}_r U(r_{i(t+\Delta t)})$$

STEP 1: stochastic modeling of the system
of two approaching ions. Minimization of energy in Woods-Saxon potential, taking into account Coulomb and Symmetry energy. The initial values of coordinates ***R*** and momenta ***P*** in the center of mass system are added

STEP 2: Evolution in time
in the mean field
until freeze-out time:
only Coulomb potential,
nuclear forces between
fragments are negligible



Density contour plots in the reaction
^{18}O(35 *A* MeV) + ^{181}Ta at *b* = 9 fm
(t=0, 20, 40, 60, 80, 100 fm / *c* (10 fm/*c*=3.3*10-23 c))

Identify final fragments
by coalescence method
Here:
Cut-off criterion in density

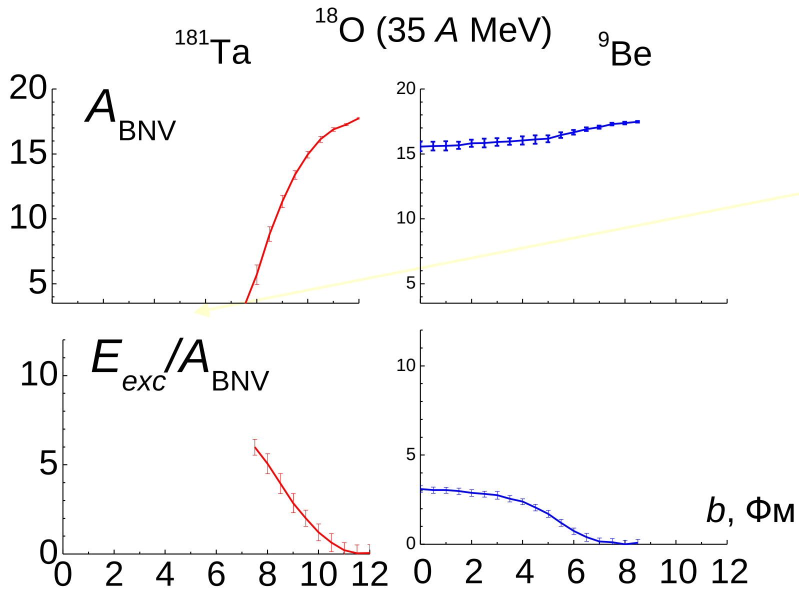
$(\varrho(r;t_{\text{freeze-out}})<0.02)$

Fragments characteristics:
mass *A*, charge *Z*,
intrinsic energy E_{int},
momentum ***P***, coordinates ***R***

Calculations with 200 TP for nucleon

$^{18}O(35\ AMev)+^{181}Ta$

The results smoothly depends on the value of impact parameter *b*



Heavier ion stays practically the unchanged!

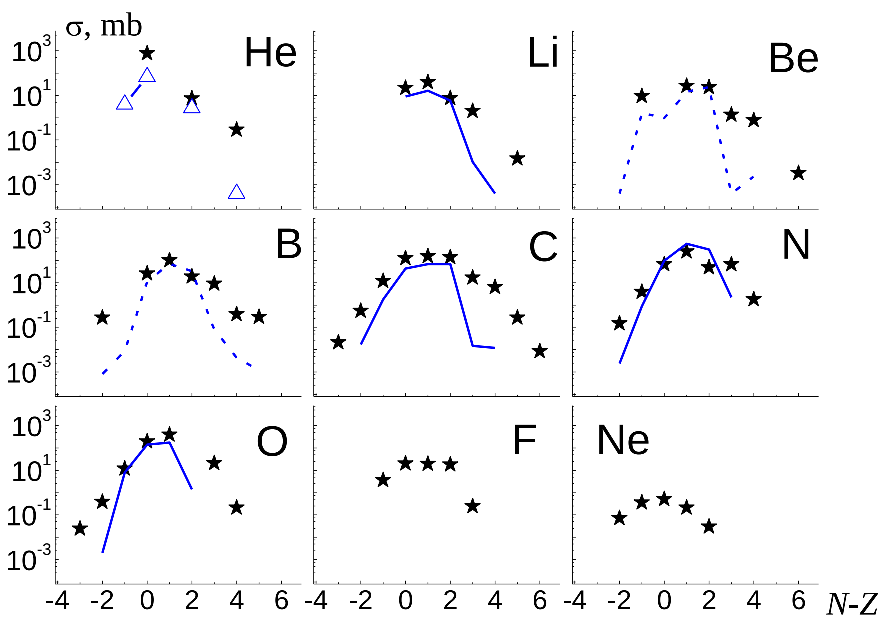
Excitation energy is calculated in self-consisted way with the same potentials as dynamical calculations are fulfilled

Fragments are excited!
Transport approach is semiclassical approach, it can't describe quantum effects.

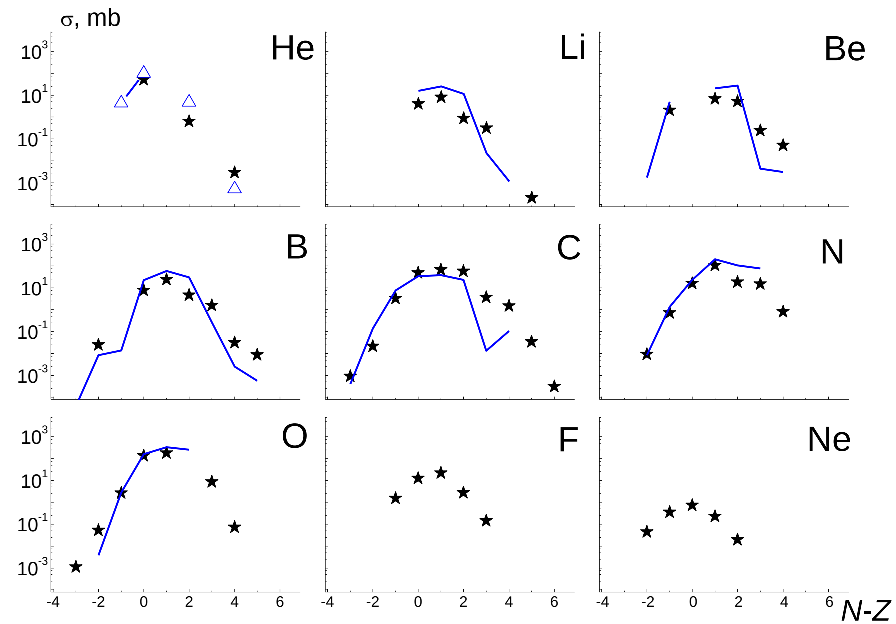
De-excite the fragments with statistical code

STEP 3: Calculating cold evaporation residues:
SMM code, P. Bondorf, et al.,Phys. Rep. 257, 133 (1995)
Input parameters: A_{fr}, Z_{fr}, $\underline{E}_{exc}$ R, P from BNV calculation

^{18}O(35 *A*Mev)+^{181}Ta



^{18}O(35 *A*Mev)+^{9}Be



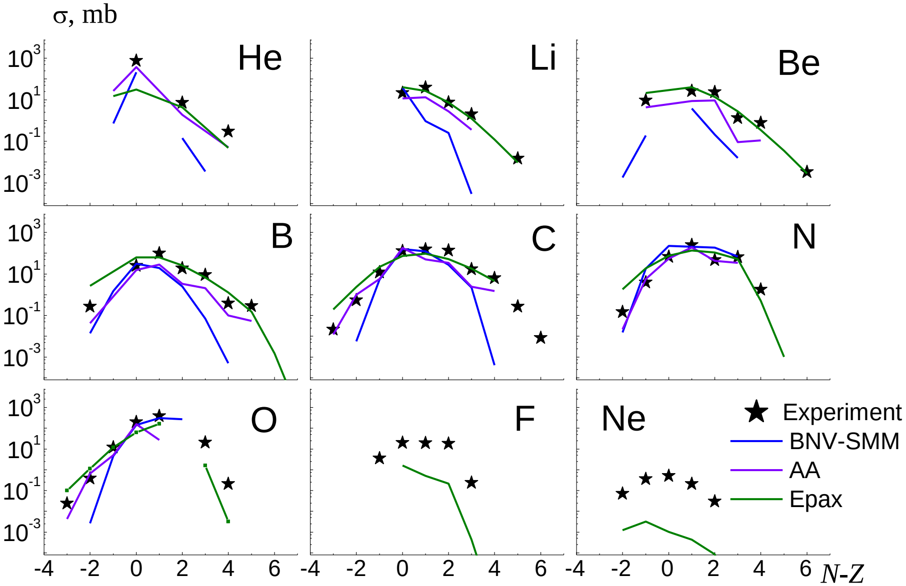
Experiment: measurement of projectile-like fragments emitted at forward angles with velocity close to the beam velocity. Combas set-up, FLNR, JINR
Artukh, A.G, et al. Phys. Part. Nuclei Lett. 18, 19 (2021).

Different approaches can also be used to predict isotope distributions produced in nuclear collisions at Fermi energies

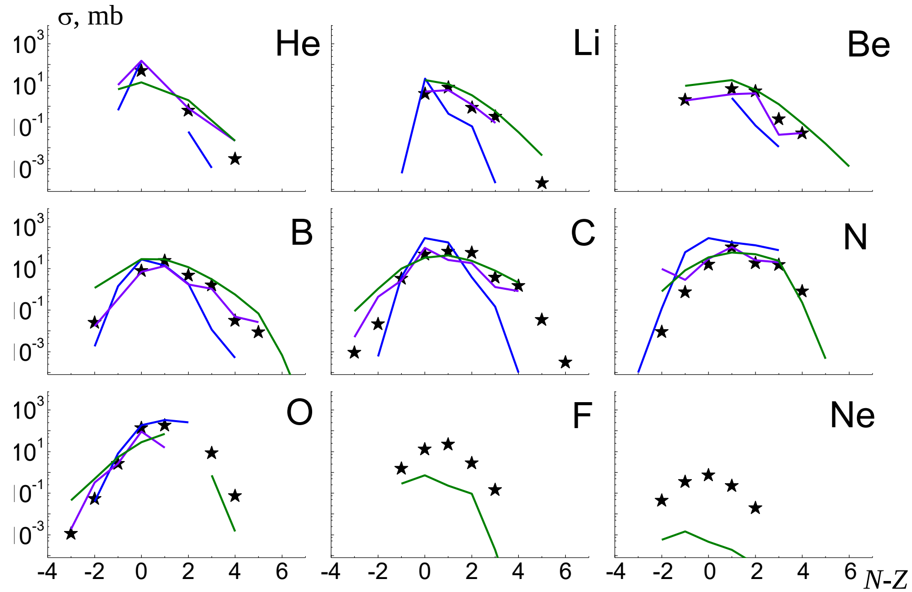
- **Transport approaches: QMD** (quantum molecular dynamics) *J. Aichelin, Phys. Rep.* ***202****, 233 (1991).***,**
  AMD(antisymmetrized/fermionic molecular dynamics)
  A. Ono and H. Horiuchi, Prog. Part. Nucl. Phys. ***53****, 501 (2004);*
- **EPAX (**an Empirical PArametrization of fragmentation CROSS sections)
  K. Summerer and B. Blank, Phys. Rev. C. 61, 034607 (2000).
- **Abration-Ablation model***, Bowman J.D., Swiatecki W.J., Tsang C.F. // LBL Report. 1973. LBL-2908.*
- **FRACS** (Mei B. Improved empirical parameterization for projectile fragmentation cross sections // Phys. Rev. C. 2017. V. 95. P. 034608.
- ***etc*.**

Calculations of isotope distributions with EPAX and Abrasion-Ablation models

$^{18}O(35\ A\mathrm{Mev})+^{181}Ta$



$^{18}O(35\ A\mathrm{Mev})+^{9}Be$

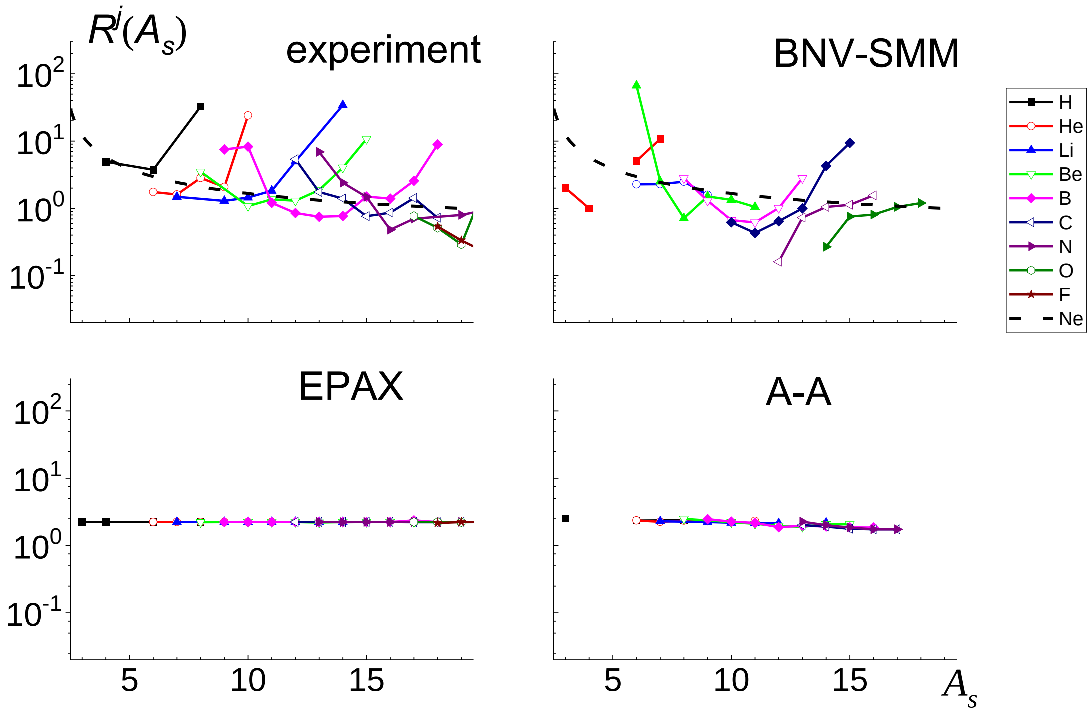


The best coincidence give EPAX calculations, BNV-SMM calculations give reasonable agreement for *N*-*Z* close to zero nuclides

EPAX-[K. Summerer and B. Blank, Phys. Rev. C. 61, 034607 (2000).

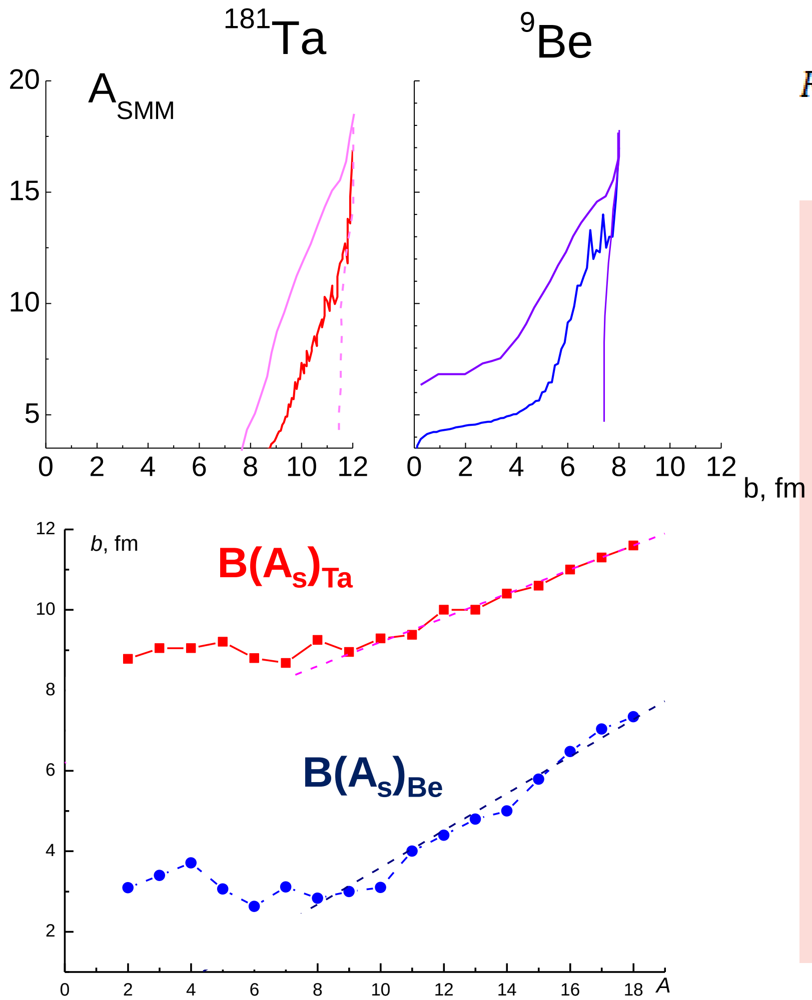
AA - [Bowman J.D.// LBL Report. 1973. LBL-2908.]

Target ratio $R_J(A_s) = \sigma_J(A_s)_{Ta}/\sigma_J(A_s)_{Be}$, for projectile ^{18}O35 (A MeV), experiment and model simulations



$$R(A_s) = b(A_s)_{Ta}\left(\frac{\partial b}{\partial As}\right)_{Ta} \Big/ b(A_s)_{Be}\left(\frac{\partial b}{\partial As}\right)_{Be}$$

Results of SMM calculations for the reaction of ^{18}O beam (35 *A* MeV), 6 degrees acceptance angle for two targets ^{181}Ta and ^{9}Be

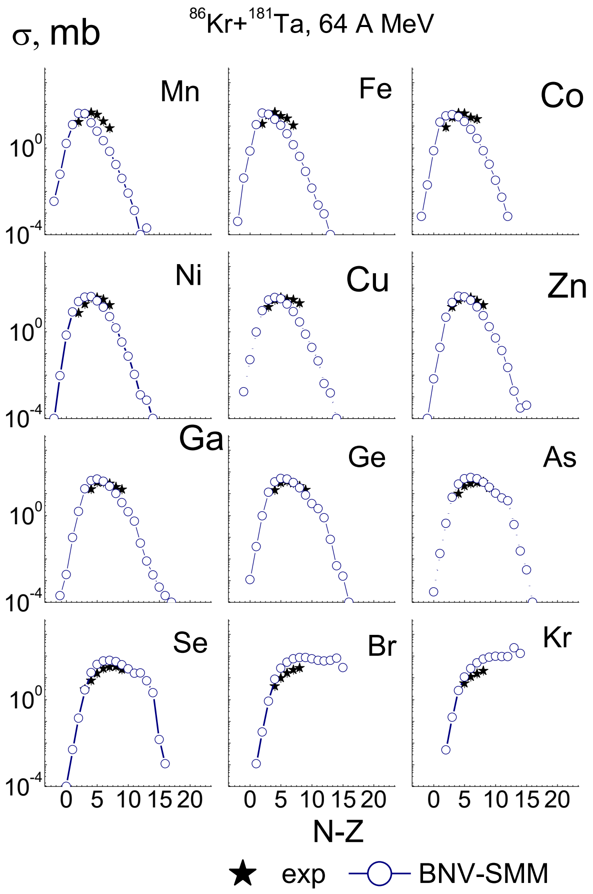


$$R(A_s) = b(A_s)_{Ta}\left(\frac{\partial b}{\partial As}\right)_{Ta} \Big/ b(A_s)_{Be}\left(\frac{\partial b}{\partial As}\right)_{Be}$$

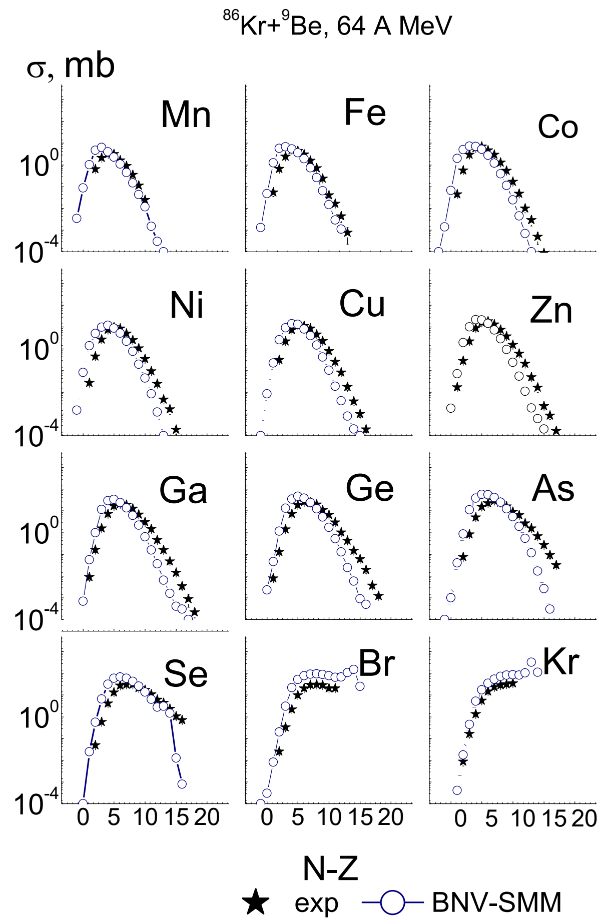
1) The upper panel: dependence of the mass number of the final (cold) fragment on the value of initial impact parameter. One can see, that the range of impact parameters contributing to the final cross-section is much larger for light target ^{9}Be than for heavy ^{181}Ta
2) Lower panel: the dependence of the most probable value of impact parameter on mass numeber of the fragment
Dashed lines - linear fits of the results
3) *R*– the ratio of the inverse functions of the fits from the lower panel

Cold residues for reactions,
Experiment and BNV-SMM calculations:

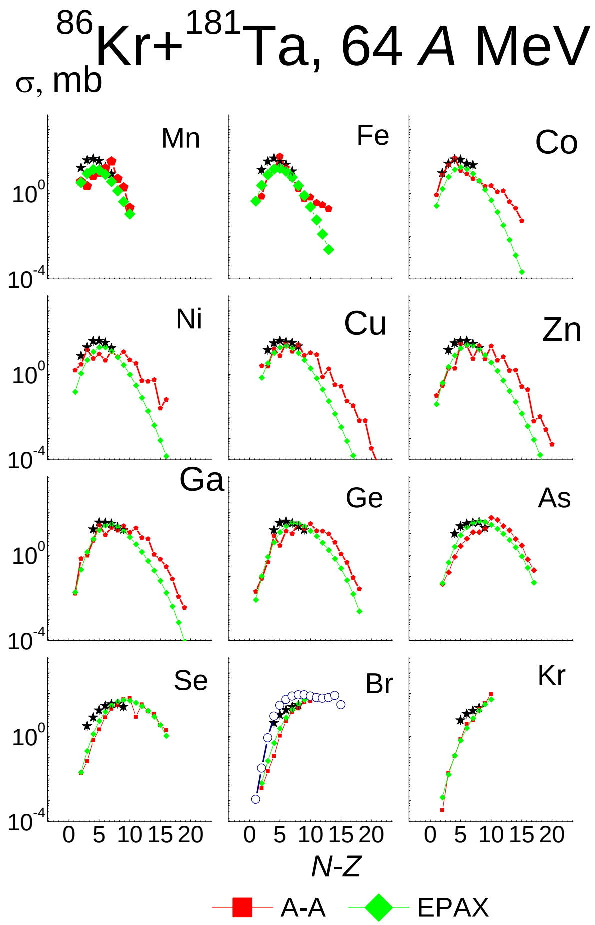
$^{86}Kr(64\ A\text{Mev})+^{181}Ta$

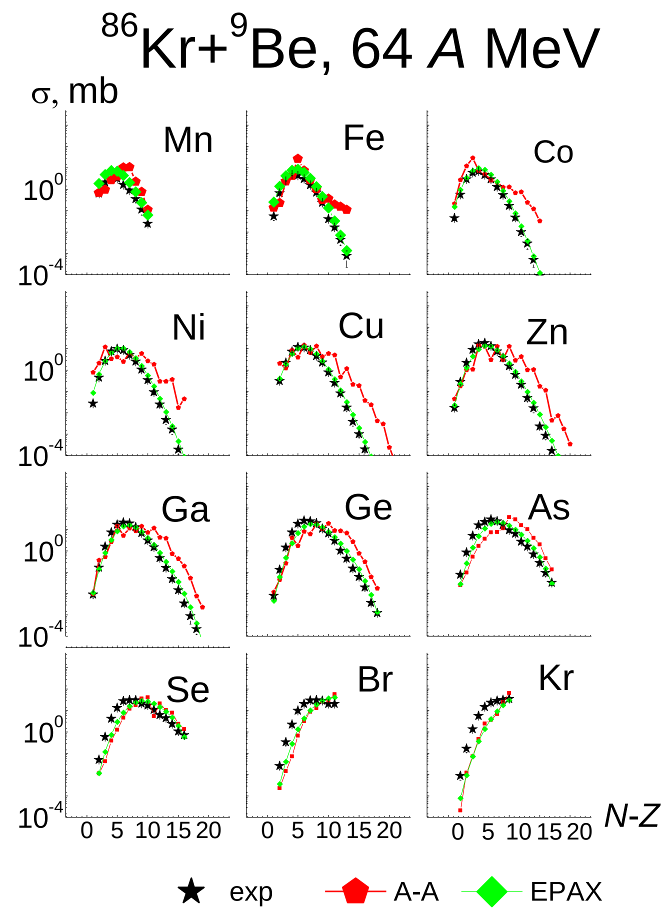


$^{86}Kr(64\ A\text{Mev})+^{9}Be$



Cold residues for reactions, experiment, EPAX and Abrasion-Ablation models





Ratio σ(Kr+Ta) / σ(Kr+Be), 64 *A* MeV
M. Mocko et.al Phys. Rev. C76, 014609(2007)

σ(Kr+Ta) /σ(Kr+Be), 64 *A* MeV,
Experiment and model calculations

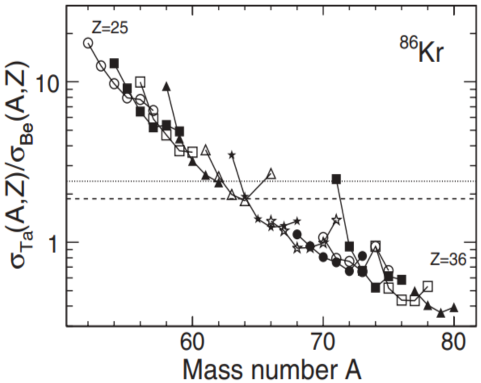
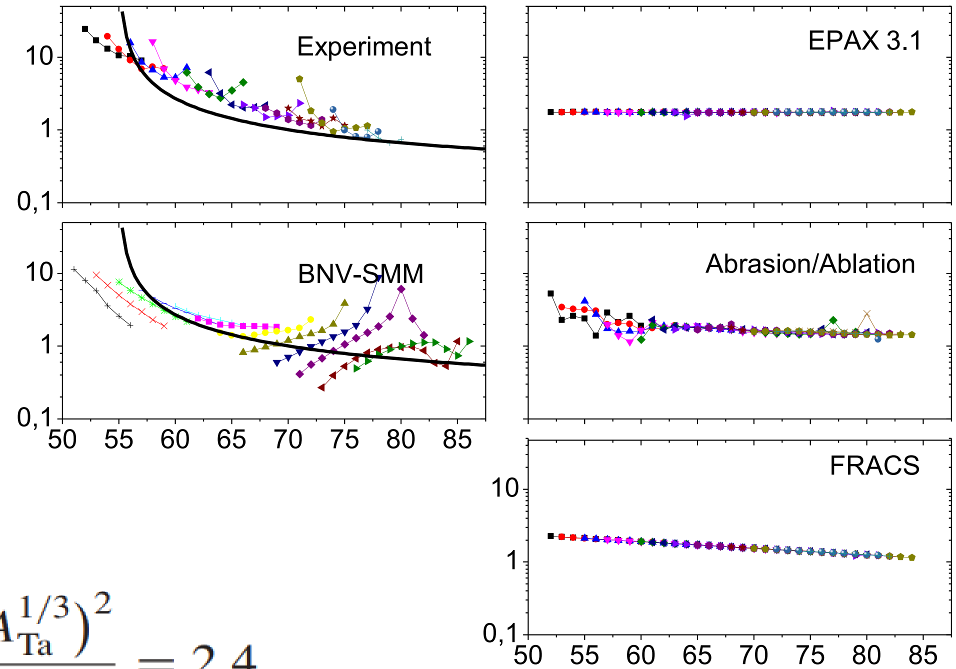


FIG. 10. Ratios of the fragmentation cross sections on Ta and Be targets, $\sigma_{Ta}(A, Z)/\sigma_{Be}(A, Z)$, for fragments with $25 \leqslant Z \leqslant 36$ for the ^{86}Kr beam. Only ratios with relative errors smaller than 25% are shown. Open and solid symbols represent odd and even elements starting with $Z = 25$. The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Eq. (4), respectively.



$$\frac{\sigma_{Ta}(A, Z)}{\sigma_{Be}(A, Z)} = \frac{\left(A_{Kr}^{1/3} + A_{Ta}^{1/3}\right)^2}{\left(A_{Kr}^{1/3} + A_{Be}^{1/3}\right)^2} = 2.4,$$

$$\frac{\sigma_{Ta}(A, Z)}{\sigma_{Be}(A, Z)} = \frac{\left(A_{Kr}^{1/3} + A_{Ta}^{1/3} - 2.38\right)}{\left(A_{Kr}^{1/3} + A_{Be}^{1/3} - 2.38\right)} = 1.9.$$

RARE ISOTOPE PRODUCTION
By

Michal Mocko

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

2006

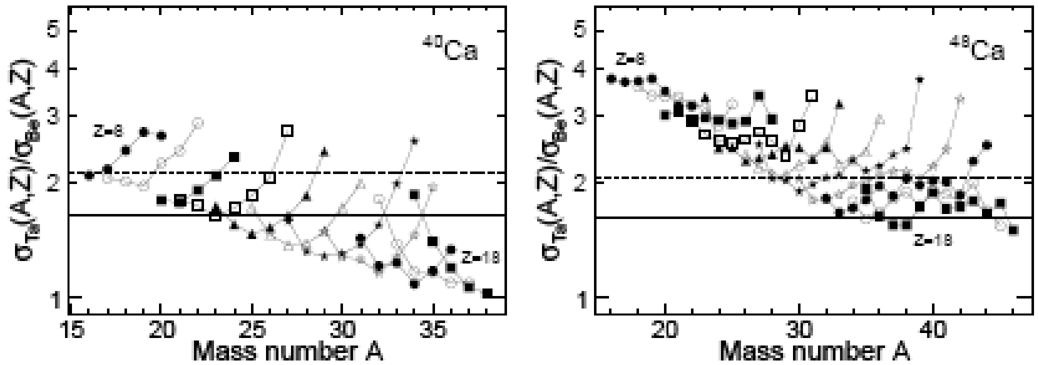


Figure 4.23: Target ratios of the fragmentation cross sections $\sigma_{\mathrm{Ta}}(A, Z)/\sigma_{\mathrm{Be}}(A, Z)$, of fragments $8 \leq Z \leq 18$ for two projectiles ^{40}Ca (left panel) and ^{48}Ca (right panel). The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Equation (4.22), respectively.

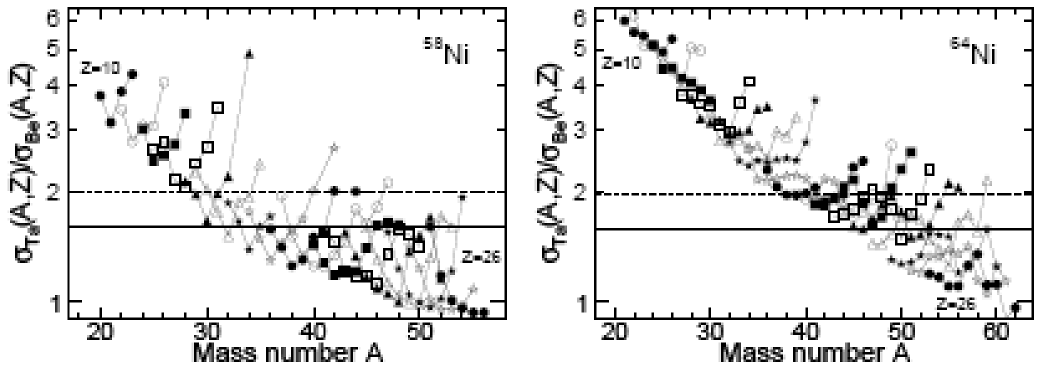
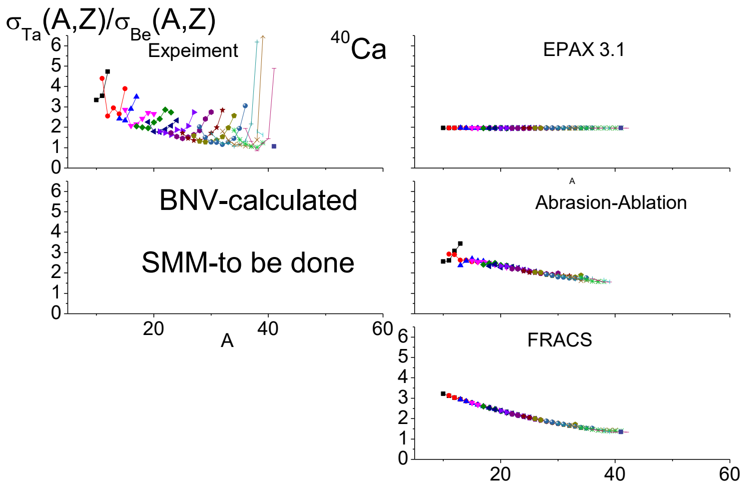
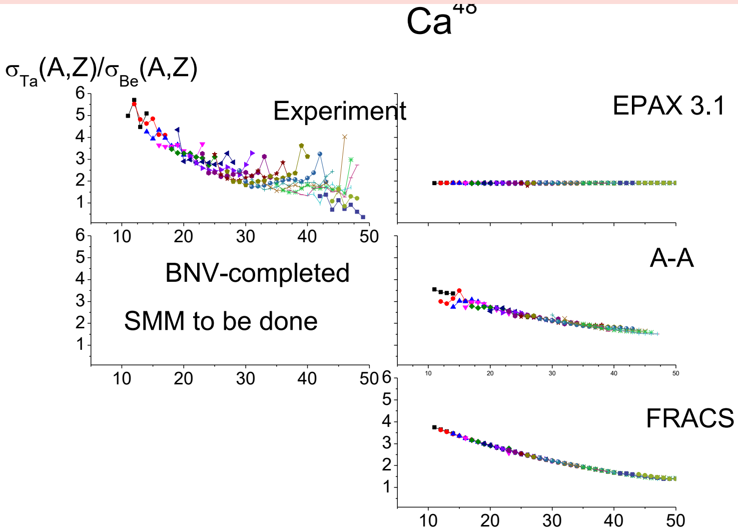


Figure 4.24: Target ratios of the fragmentation cross sections $\sigma_{\mathrm{Ta}}(A, Z)/\sigma_{\mathrm{Be}}(A, Z)$ of fragments $10 \leq Z \leq 26$ for two projectiles ^{58}Ni (left panel) and ^{64}Ni (right panel). The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Equation (4.22), respectively.

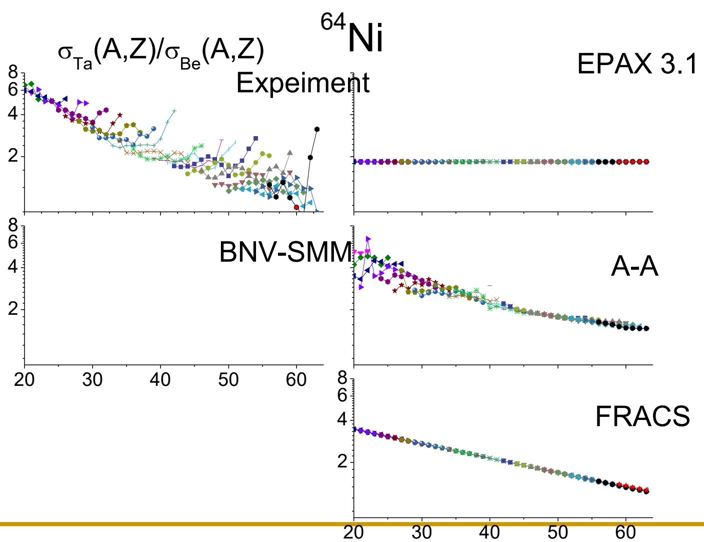
$\sigma(^{40}\mathrm{Ca}+^{181}\mathrm{Ta}) / \sigma(^{40}\mathrm{Ca}+^{9}\mathrm{Be})$, 140 *A* MeV



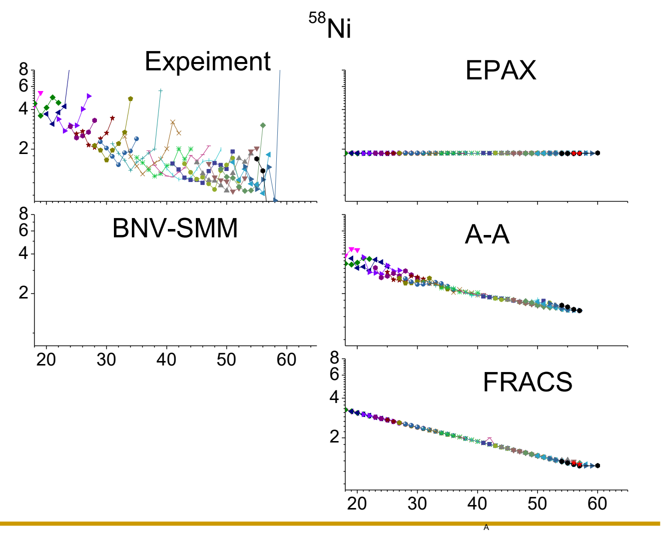
$\sigma(^{48}\mathrm{Ca}+^{181}\mathrm{Ta}) / \sigma(^{48}\mathrm{Ca}+^{9}\mathrm{Be})$, 140 *A* MeV



$\sigma(^{64}\mathrm{Ni}+^{181}\mathrm{Ta}) / \sigma(^{64}\mathrm{Ni}+^{9}\mathrm{Be})$, 140 *A* MeV



$\sigma(^{58}\mathrm{Ni}+^{181}\mathrm{Ta}) / \sigma(^{58}\mathrm{Nir}+^{9}\mathrm{Be})$, 140 *A* MeV



Conclusions

The target ratios point out on the importance of taking into account two crucial characteristics of the reaction:

1) target mass
2) impact parameter value

The excess of the neutron-reach nuclides in the reaction on heavy target can be explained by the fact that in the collision on heavy target even at the first stage of the reaction exotic combinations of *N* and *Z* can be obtained, while in the reactions on light target primary fragments are similar to the projectile

SMM calculations for Ca and Ni are still to be done

Thank you for attention