Theoretical Calculations for future electron-positron colliders: status and prospects

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Outline

1. Motivation
2. e^+e^- colliders
3. Theory for e^+e^-
4. Conclusions

General motivation

- The Standard Model is the most successful physical model ever
- But there are still many open questions to it
- We believe that it is only an effective theory, but its applicability domain might be limited just by the Planck mass scale
- The primary goal of HEP is to study the physics of our actual microworld
- Discovering physics beyond SM is our hope
- In any case, the research in HEP will not stop by the end of LHC
- Logically, the next step should be a e^+e^- collider

Future e^+e^- collider projects

Linear Colliders

- CLIC, ILC

not to be built in Japan (?)

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$\mathcal{L} \approx 2 \cdot 10^{34}$ cm^{-2}s^{-1}

Stat. uncertainty $\sim 10^{-4}$

Beam polarization:
e^-beam: $P = 80 - 90\%$
e^+beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, CEPC
- Z-factory (ZUNK, LEP3)
- Super Charm-Tau Factory
- $\mu^+\mu^-$ collider

E_{tot}

- 91; 160; 240; 350 GeV

$\mathcal{L} \approx 2 \cdot 10^{36}$ cm^{-2}s^{-1} (4 exp.)

Stat. uncertainty $\sim 10^{-6}$

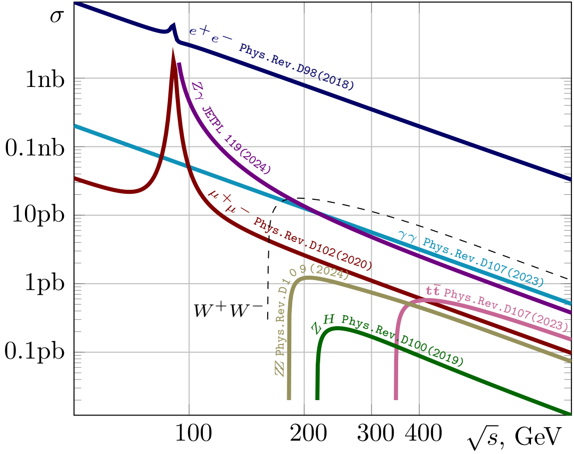
Tera-Z mode!

Beam polarization: desirable

Physics possibilities at future e^+e^- machines

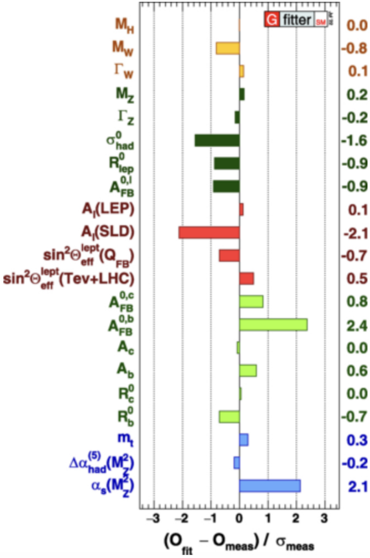
- Indeep verification of the EW sector of the SM
- Dedicated studies of the Higgs boson, EW bosons, and top quark
- Unique possibilities for QCD at the EW scale
- Searches for new physics of SMEFT and other types
- Photon-photon physics
- Properties of tau lepton
- Physics of (exotic) mesons
- N.B. Effective B-factory and Charm-Tau factory

Basic SM processes in e^+e^- interactions

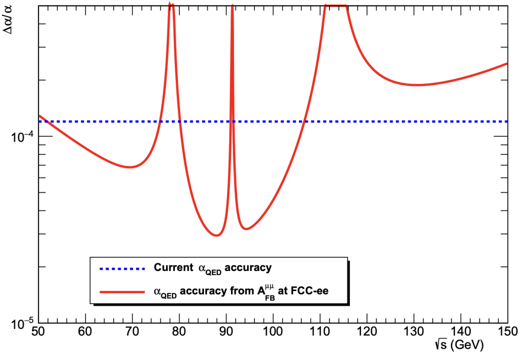


References are given for studies with ReneSANCe Monte Carlo event generator

Where are we now with the SM

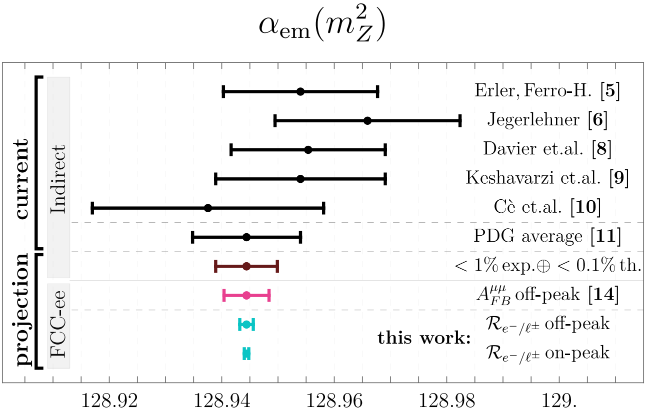


Example: $\alpha_{QED}(m_Z^2)$ (I)



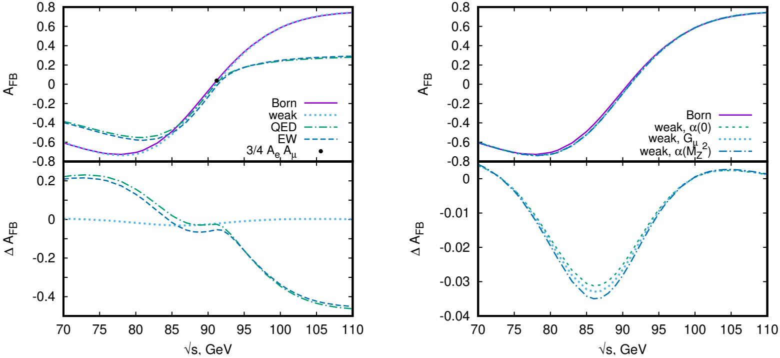
An experimental relative accuracy of 3×10^{-5} on $\alpha_{QED}(m_Z^2)$ can be achieved at FCC-ee, from the measurement of the muon forward-backward asymmetry at energies ~ 3 GeV below and ~ 3 GeV above the Z pole. The corresponding parametric uncertainties on other SM parameters and observables will be reduced. [FCC Coll. EPJC'2019]

Example: $\alpha_{QED}(m_Z^2)$ (II)



A new method of $\alpha_{QED}(m_Z^2)$ extraction is proposed [M. Riembau PRL 2025]

Forward-Backward Asymmetry



$$A_f = \frac{2 g_V g_A}{g_V^2 + g_A^2} \qquad \text{for the given fermion } f$$

[A.A., S.Bondarenko, L.Kalinovskaya, Symmetry'2020]

ISR corrections to $e^+e^- \to Z(\gamma^*)$ ($\sqrt{s} = M_Z$)

LO $\mathcal{O}(\alpha^n L^n)$ and NLO $\mathcal{O}(\alpha^n L^{n-1})$ ISR corrections in % at the Z-peak for $z_{\min} = 0.1$

Type / n	1	2	3	4	5
LO γ	−32.7365	4.8843	−0.3776	0.0034	0.0032
NLO γ	2.0017	−0.5952	0.0710	−0.0019	
LO pair	—	−0.3057	0.0875	0.0016	−0.0001
NLO pair	—	0.1585	−0.0460	0.0038	
Σ	−30.7348	4.1419	−0.2651	0.0069	0.0031

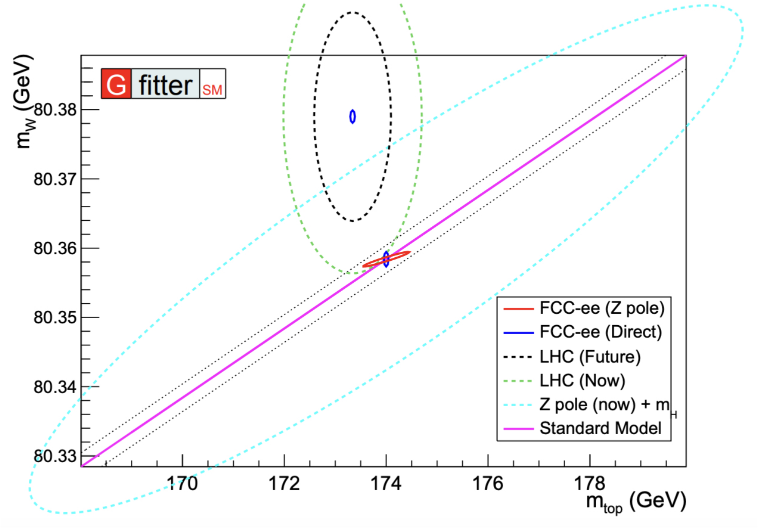
Even higher orders seem to be relevant numerically $\Longrightarrow$ exponentiation

Exponentiation of the leading logs is straightforward and known [Gribov-Lipatov, Kuraev-Fadin, ...]

NLO exponentiation in the MSbar scheme is ambiguous

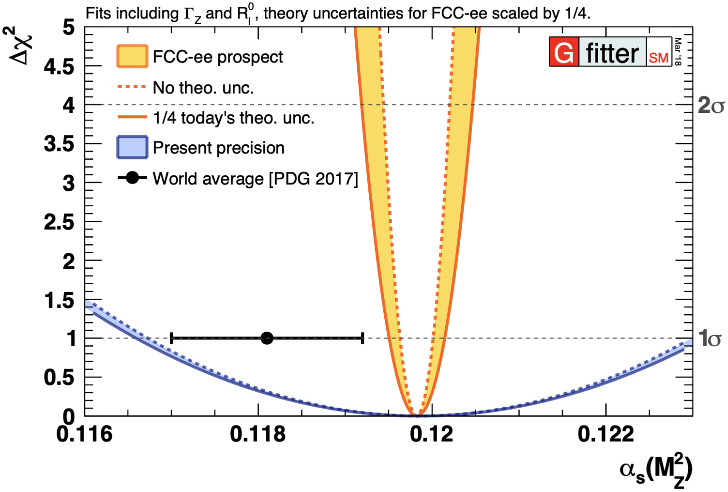
[A.A., U.Voznaya, PRD 2024]

Indirect and direct measurements



[FCC Coll. EPJC'2019]

Alpha QCD



[FCC Coll. EPJC'2019]

EW quasi observables (I)

Observable	Present value	±	error	FCC-ee (statistical)	FCC-ee (systematic)	Source and dominant experimental error
m_Z (keV/c^2)	91 186 700	±	2200	5	100	Z line shape scan Beam energy calibration
Γ_Z (keV)	2 495 200	±	2300	8	100	Z line shape scan Beam energy calibration
R_ℓ^Z ($\times 10^3$)	20 767	±	25	0.06	1	Ratio of hadrons to leptons Acceptance for leptons
$\alpha_s(m_Z)$ ($\times 10^4$)	1196	±	30	0.1	1.6	R_ℓ^Z above
R_b ($\times 10^6$)	216 290	±	660	0.3	<60	Ratio of $b\bar{b}$ to hadrons Stat. extrapol. from SLD [7]
σ^0_{had} ($\times 10^3$) (nb)	41 541	±	37	0.1	4	Peak hadronic cross-section Luminosity measurement
N_ν($\times 10^3$)	2991	±	7	0.005	1	Z peak cross-sections Luminosity measurement
$\sin^2\theta_W^{eff}$($\times 10^6$)	231 480	±	160	3	2–5	$A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z)$($\times 10^3$)	128 952	±	14	4	Small	$A_{FB}^{\mu\mu}$ off peak
$A_{FB}^{b,0}$ ($\times 10^4$)	992	±	16	0.02	<1	b quark asymmetry at Z pole Jet charge

[A.Blondel et al., CERN YR 2019]

EW quasi observables (II)

Observable	Present value	±	error	FCC-ee (statistical)	FCC-ee (systematic)	Source and dominant experimental error
$A_{FB}^{pol,\tau}$ ($\times 10^4$)	1498	±	49	0.15	<2	τ polar. and charge asymm.
						τ decay physics
m_W (keV/c^2)	803 500	±	15 000	600	300	WW threshold scan
						Beam energy calibration
Γ_W (keV)	208 500	±	42 000	1500	300	WW threshold scan
						Beam energy calibration
$\alpha_s(m_W)(\times 10^4)$	1170	±	420	3	Small	R_ℓ^W
$N_\nu(\times 10^3)$	2920	±	50	0.8	Small	Ratio of invis. to leptonic
						in radiative Z returns
m_{top} (MeV/c^2)	172 740	±	500	20	Small	$t\bar{t}$ threshold scan
						QCD errors dominate
Γ_{top} (MeV/c^2)	1410	±	190	40	Small	$t\bar{t}$ threshold scan
						QCD errors dominate
$\lambda_{top}/\lambda_{top}^{SM}$	$m = 1.2$	±	0.3	0.08	Small	$t\bar{t}$ threshold scan
						QCD errors dominate
$t\bar{t}Z$ couplings		±	30%	<2%	Small	$E_{CM} = 365$ GeV run

[A.Blondel et al., CERN YR 2019]

SM for FCC-ee (CEPC): status and needs

Quantity	Current precision	FCC-ee stat. (syst.) precision	Required theory input	Theory status as of today	Needed theory improvement†
m_Z (MeV)	2.0	0.004 (0.1)	non-resonant $e^+e^- \to f\bar{f}$, initial-state radiation (ISR)	NLO, ISR logarithms up to 6th order	NNLO for $e^+e^- \to f\bar{f}$
Γ_Z (MeV)	2.3	0.004 (0.012)			
$\sin^2\theta^\ell_{\rm eff}$	1.6×10^{-4}	$1.2\ (1.2)\times10^{-6}$			
m_W (MeV)	9.9	0.18 (0.16)	lineshape of $e^+e^- \to WW$ near threshold	NLO ($e^+e^- \to 4f$ or EFT framework)	NNLO for $e^+e^- \to WW$, $W \to f\bar{f}'$ in EFT setup
HZZ coupling	–*	0.1%	cross section for $e^+e^- \to ZH$	NLO EW plus partial NNLO QCD/EW	full NNLO EW
$m_{\rm top}$ (MeV)	290	4.2 (4.9)	threshold scan $e^+e^- \to t\bar{t}$	N^{3}LO QCD, NNLO EW, resummations up to NNLL, $\mathcal{O}$(30 MeV) scale uncert.	Matching fixed orders with resummations, merging with MC, α_S (input)

[FCC Coll., 2505.00272 [hep-ex]]

SMEFT

Possible deviations from SM predictions in differential and inclusive observables will be fit within SMEFT extension of the SM by > 4 dim. operators

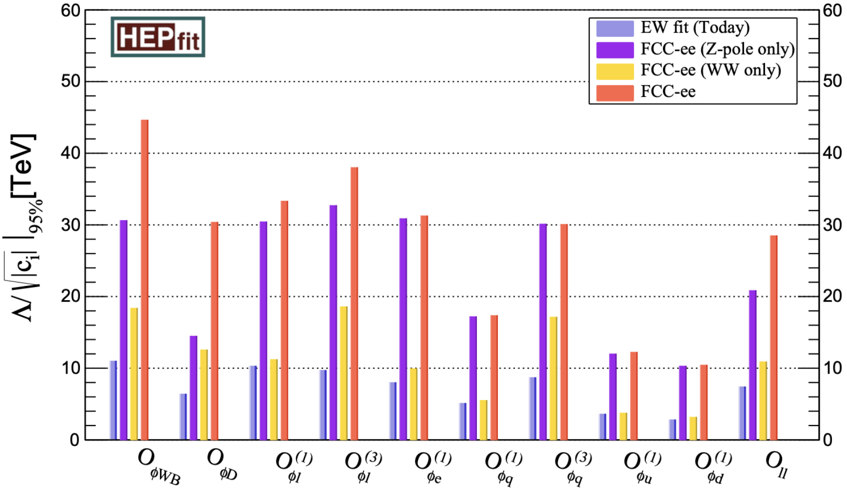
Remind three oblique Peskin–Takeuchi parameters used at LEP.
At a new e^+e^- machine one can (should) do a much more detailed study

Scenarios of specific new physics models can be also verified,
e.g. with long-lived particles

N.B. Having polarized beams would help a lot

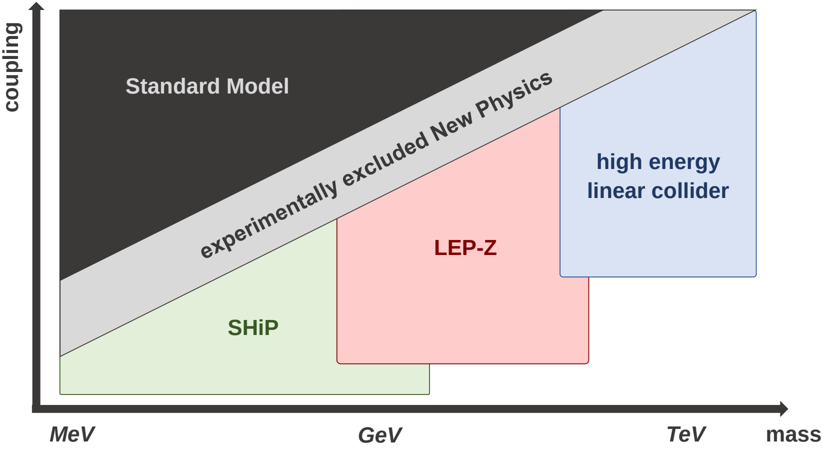
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{C_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \ldots$$

Sensitivity to new physics scale



[FCC Coll. EPJC'2019]

Direct searches for new particles



[M. Drewes, E. Shaposhnikova and M. Shaposhnikov, "A Possible Future Use of the LHC Tunnel," arXiv:2503.17081]

To-do list for theory

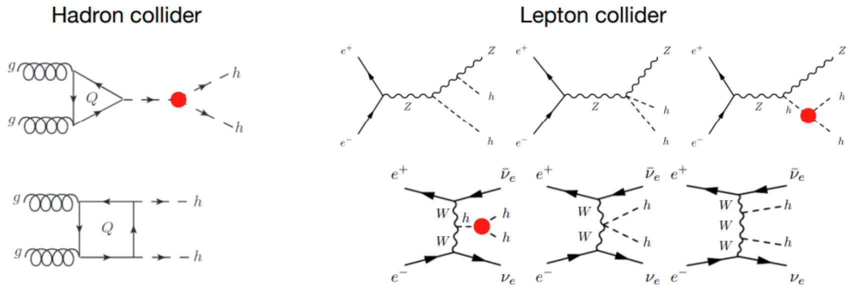
- Develop physical program on SM verification and new physics searches
- Compute 2-loop QED and EW + higher-order QCD radiative corrections to differential distributions of key processes: Bhabha scattering, $e^+e^- \to \mu^+\mu^-;\ \tau^+\tau^-;\ t\bar{t};\ \gamma\gamma;\ ZH$ etc.
- Add higher-order contributions within some approximations
- Account for interplay of QCD and electroweak effects
- Construct reliable Monte Carlo codes

Outlook

- A new high-energy e^+e^- collider is well motivated by the necessity to study SM in more detail and new physics searches
- Such a machine provides unique possibilities for progress in HEP
- Complementarity to hadron-hadron, lepton-hadron machines, fixed target experiments, and low-energy high-precision measurements is essential
- New theoretical calculations of higher-order corrections in SM and BSM are required
- Chains of interfaced Monte Carlo codes to be developed
- The work is started, but there are still many tasks

Backup slides

Higgs^3 self coupling (I)



[J. de Blas et al., JHEP 2020]

Higgs^3 self coupling (II)

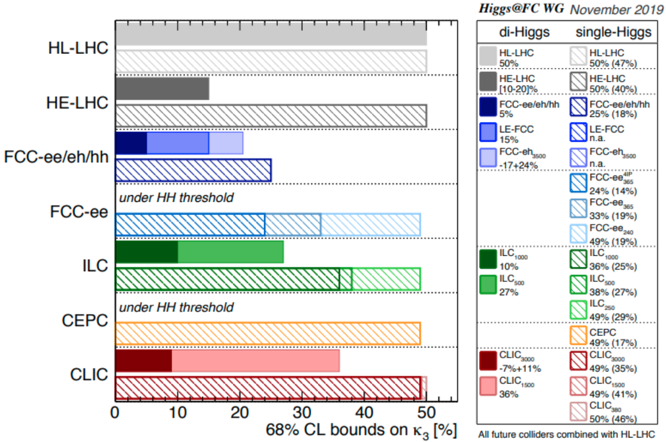


Figure 11. Sensitivity at 68% probability on the Higgs cubic self-coupling at the various FCs.

[J. de Blas et al., JHEP 2020]

Higgs couplings

Coupling	HL-LHC	FCC-ee	FCC-ee + FCC-hh
κ_Z (%)	1.3*	0.10	0.10
κ_W (%)	1.5*	0.29	0.25
κ_b (%)	2.5*	0.38 / 0.49	0.33 / 0.45
κ_g (%)	2*	0.49 / 0.54	0.41 / 0.44
κ_τ (%)	1.6*	0.46	0.40
κ_c (%)	–	0.70 / 0.87	0.68 / 0.85
κ_γ (%)	1.6*	1.1	0.30
$\kappa_{Z\gamma}$ (%)	10*	4.3	0.67
κ_t (%)	3.2*	3.1	0.75
κ_μ (%)	4.4*	3.3	0.42
$\lvert\kappa_s\rvert$ (%)	–	$^{+29}_{-67}$	$^{+29}_{-67}$
Γ_H (%)	–	0.78	0.69
$\mathcal{B}_{inv}$ (<, 95% CL)	1.9×10^{-2} *	5×10^{-4}	2.3×10^{-4}
$\mathcal{B}_{unt}$ (<, 95% CL)	4×10^{-2} *	6.8×10^{-3}	6.7×10^{-3}

[FCC Coll. 2505.00272 [hep-ex]]