

# Advances in the study of event-by-event strangeness fluctuations

Rodrigo García Formentí Mendieta

Instituto de Ciencias Nucleares  
UNAM



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  - <https://indico.jinr.ru/event/4578/>
  - <https://indico.jinr.ru/event/5021/>
  - Feedback from previous presentations: need for efficiency, purity, and contamination plots; local efficiency correction; and reducing Monte Carlo dependence.
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# Cumulants and Moments

Let  $\Delta N = N - \bar{N}$  be the net multiplicity of a particle, then the standard deviation is  $\delta N = \Delta N - \langle \Delta N \rangle$ , and the first order cumulants are defined as:

$$\begin{aligned} C_1 &= \langle \Delta N \rangle, \quad C_2 = \langle (\delta N)^2 \rangle, \quad C_3 = \langle (\delta N)^3 \rangle, \\ C_4 &= \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2. \end{aligned} \quad (1)$$

The cumulants are related with the statistical moments as:

$$M = C_1, \quad \sigma^2 = C_2, \quad S = \frac{C_3}{(C_2)^{3/2}}, \quad \kappa = \frac{C_4}{(C_2)^2} \quad (2)$$

# Data Analysis

## Data sample

The following events were generated using UrQMD.

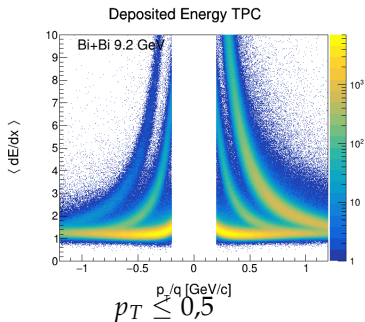
Collision Type	$\sqrt{S_{NN}}$	Events	Analysis
Bi+Bi (Request 25)	9.2 GeV	$\sim 3$ Million	Reconstructed

- The analysis is implemented using the train system.
- A custom wagon was developed that depends on the Centrality and PID wagons.
- This wagon computes the cumulants and only requires an efficiency distribution as input.

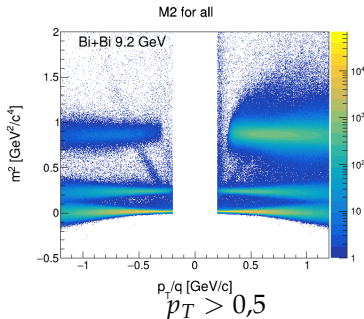
## Particle Identification

TPC and TOF information to identify  $K$  using the PID wagon.

$|\sigma_{DCA}| \leq 2$ , and the following cuts were applied:  $0,2 \leq p_T \leq 1,2$  GeV/c,  $|y| \leq 0,5$ ,  $n\text{Hits} > 15$ .

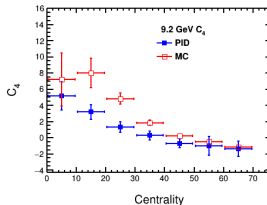
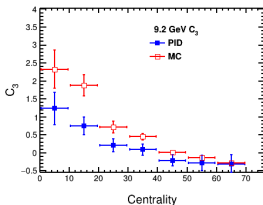
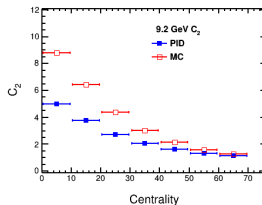
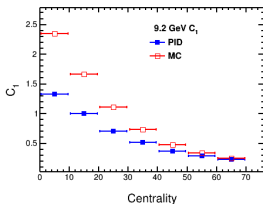


- $|\sigma_{TPC}|, |\sigma_{TOF}| \leq 2$
- Or  $|\sigma_{TPC}| \leq 2$  ( $|\sigma_{\phi,z}| > 2$ )



- $(|\sigma_\phi|, |\sigma_z| \leq 2)$
- $|\sigma_{TPC}| \leq 2$
- and  $|\sigma_{TOF}| \leq 2$

# Calculation of cumulants (uncorrected)

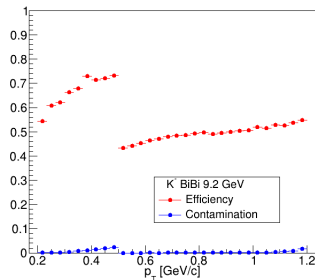
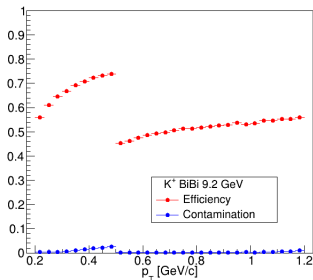


Statistical cumulants without correction.

# Efficiency and purity and contamination

The efficiency, purity, and contamination are defined as

$$\epsilon(p_T) = \frac{N_k(PID)}{N_k(MC)}, \quad C(p_T) = \frac{N_k(FP)}{N_k(PID)}, \quad P(p_T) = 1 - C.$$



Efficiency and contamination of  $K^+$  (left), and  $K^-$  (right). Efficiency shows discontinuity due to PID: TOF is used only within its coverage range.



## Cumulants Efficiency Corrections

To perform the correction, we assume that the difference between the real distribution  $P$  and the measured distribution  $p$  can be modeled as a binomial distribution, so defining the factorial moments of  $p$  and  $P$  as

$$f_{ik} = \left\langle \frac{n_1!}{(n_1 - i)!} \frac{n_2!}{(n_2 - k)!} \right\rangle, \quad F_{ik} = \left\langle \frac{N_1!}{(N_1 - i)!} \frac{N_2!}{(N_2 - k)!} \right\rangle \quad (3)$$

we can get the relation

$$F_{ik} = \frac{1}{p_+^i p_-^k} f_{ik}. \quad (4)$$

$p_+$  and  $p_-$  the efficiency of the identification. With this relation, is possible to obtain the real value of the cumulants.

## Cumulants Corrections

Using the previous relations and by the definition of statistical cumulants, the following equalities are obtained:

$$C_1 = F_{10} - F_{01},$$

$$C_2 = N - C_1^2 + F_{02} - 2F_{11} + F_{20},$$

$$C_3 = C_1 + 2C_1^3 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30} \\ - 3C_1(N + F_{02} - 2F_{11} + F_{20}),$$

$$C_4 = N - 6C_1^4 + F_{04} + 6F_{03} + 7F_{02} - 2F_{11} - 6F_{12} - 4F_{13} \\ + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40} \\ + 12C_1^2(N + F_{02} - 2F_{11} + F_{20}) - 3(N + F_{02} - 2F_{11} + F_{20})^2 \\ - 4C_1(C_1 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}).$$

$C_n$  are the cumulants of the real distribution.

## What if the efficiency is not constant?

To incorporate transverse momentum dependence to the efficiency from the particle  $x$ , and from the negative particle  $\bar{x}$ , we introduce local factorial moments  $A_{ik}$  and  $a_{ik}$ :

$$A_{ik} \quad (x_1, \dots, x_i; \bar{x}_1, \dots, \bar{x}_i) = \langle N(x_1)[N(x_2) - \delta_{x_1, x_2}] \cdots [N(x_i) - \delta_{x_1, x_i} - \cdots - \delta_{x_{i-1}, x_i}] \bar{N}(\bar{x}_2) - \delta_{\bar{x}_1, \bar{x}_2}] \cdots [\bar{N}(\bar{x}_i) - \delta_{x_1, x_i} - \cdots - \delta_{x_{i-1}, x_i}] \rangle \quad (5)$$

$$a_{ik} \quad (x_1, \dots, x_i; \bar{x}_1, \dots, \bar{x}_i) = \langle n(x_1)[n(x_2) - \delta_{x_1, x_2}] \cdots [n(x_i) - \delta_{x_1, x_i} - \cdots - \delta_{x_{i-1}, x_i}] \bar{n}(\bar{x}_2) - \delta_{\bar{x}_1, \bar{x}_2}] \cdots [\bar{n}(\bar{x}_i) - \delta_{x_1, x_i} - \cdots - \delta_{x_{i-1}, x_i}] \rangle \quad (6)$$

## Local efficiency correction

Using

$$F_{ik} = \sum_{x_1, \dots, x_i} \sum_{\bar{x}_1, \dots, \bar{x}_i} A_{ik}(x_1, \dots, x_i; \bar{x}_1, \dots, \bar{x}_i) \quad (7)$$

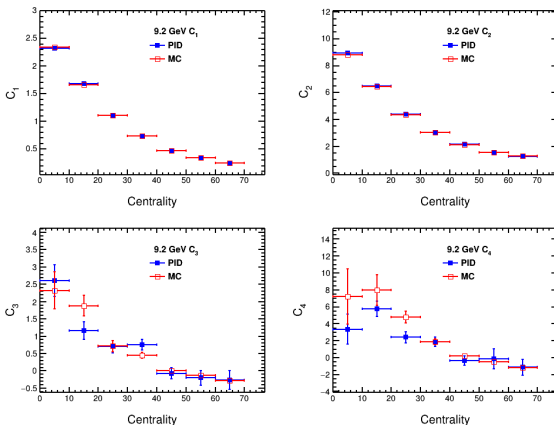
and

$$a_{ik} = \epsilon(x_1) \dots \epsilon(x_i) \bar{\epsilon}(\bar{x}_1) \dots \bar{\epsilon}(\bar{x}_i) A_{ik} \quad (8)$$

where  $\epsilon$  and  $\bar{\epsilon}$  are the local acceptance of  $N$  and  $\bar{N}$ .

$$F_{ik} = \sum_{x_1, \dots, x_i} \sum_{\bar{x}_1, \dots, \bar{x}_i} \frac{a_{ik}(x_1, \dots, x_i; \bar{x}_1, \dots, \bar{x}_i)}{\epsilon(x_1) \dots \epsilon(x_i) \bar{\epsilon}(\bar{x}_1) \dots \bar{\epsilon}(\bar{x}_i)}. \quad (9)$$

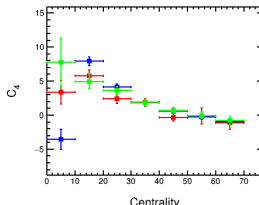
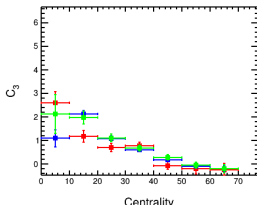
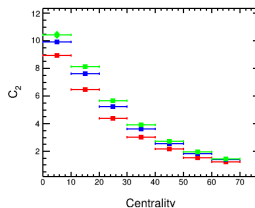
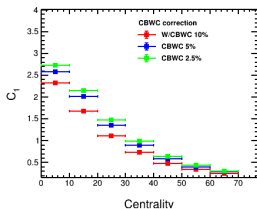
# Cumulants Ratios after efficiency corrections



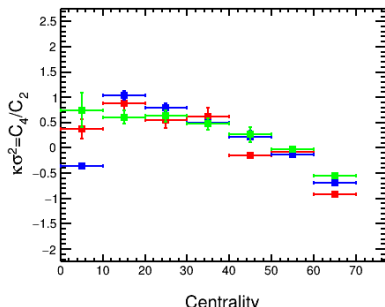
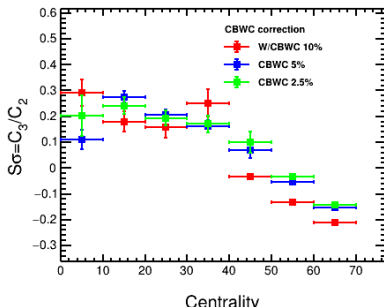
Cumulants ratios with correction. Discrepancy in central collisions.

## Centrality Bin Width Correction (CBWC)

To correct volume fluctuations, we apply the Centrality Bin Width Correction:  $C_i = \frac{\sum_r n_r C_{i,r}}{\sum_r n_r}$ . Where  $n_r$  is the number of events in the  $r$ -th multiplicity.

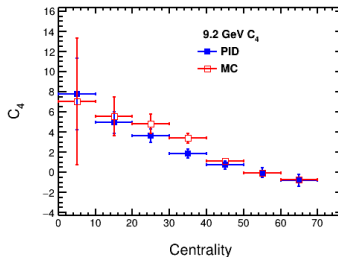
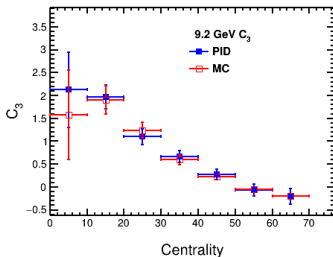
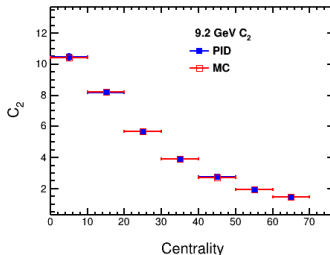
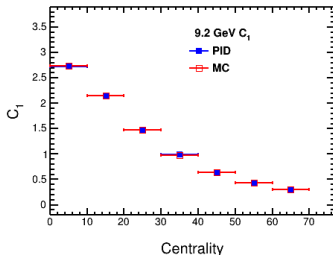


# Centrality Bin Width Correction (CBWC)



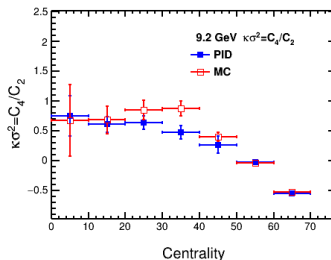
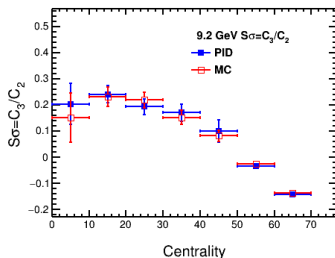
Cumulant ratios. Different CBWC corrections, bin size of 10 %, 5 % and 2.5 %.

# Calculation of cumulants (Corrected by CBWC 2.5 %)





# Cumulant ratios (Corrected by CBWC 2.5) %



Cumulants ratios corrected by CBWC and Local Efficiency

# What's next: New proposal for correcting volume fluctuations

## Proposal-1 for MPD: New Bin Width Correction procedure with the account of Volume fluctuations (CBWC-V)

Ratios of cumulants  $C_2/C_1 = \sigma^2/M$ ,  $C_3/C_2 = S\sigma$ , and  $C_4/C_2 = k\sigma^2$  were used to reduce the volume dependence.

However, the average values of  $\sigma$ ,  $S$  and  $k$  are calculated assuming the fixed value of volume  $V$  in all events!

➤ We propose to use the reduced cumulants, similar to [1], but on the event-by-event basis, following the new CBWC-V procedure with  $V^r$  in each  $r^{\text{th}}$  multiplicity bin:

$$\begin{aligned} M &= \langle N \rangle = C_1, \\ \delta N &= N - \langle N \rangle \\ \sigma^2 &= \langle (\delta N)^2 \rangle = C_2, \\ S &= \langle (\delta N)^3 \rangle / \sigma^3 = \\ &= C_3 / C_2^{3/2}, \end{aligned}$$



$$\begin{aligned} c1 &= M/V^r = \langle N^r/V^r \rangle, \\ \delta N^r &= N^r/V^r - \langle N^r/V^r \rangle \\ c2 &= \sigma_r^2 = \langle (\delta N^r)^2 \rangle, \\ S_r &= \langle (\delta N^r)^3 \rangle / \sigma_r^3, \end{aligned}$$

$$k = \langle (\delta N)^4 \rangle / \sigma^4 - 3 = C_4 / C_2^2$$

$$k_r = \langle (\delta N^r)^4 \rangle / \sigma_r^4 - 3$$

➤ We assume that for any  $r^{\text{th}}$  multiplicity bin the relevant mean volume  $V^r$  is proportional to the mean number of participants  $\langle N^r_{\text{part}} \rangle$ :

$$V^r = \langle N^r_{\text{part}} \rangle V_0$$

Here a volume factor  $V_0 = 2.83 \text{ fm}^3$  (see in [1]).

➤ Thus we obtain the reduced deviation  $\delta N^r = N^r/V^r - \langle N^r/V^r \rangle$  for the relevant distribution of conserved quantity  $N^r$

[1] V. Skokov, B. Friman and K. Redlich, "Volume fluctuations and higher order cumulants of the net baryon number". arXiv:1205.4756v2

This proposal was presented by Gregory at the previous meeting (<https://indico.jinr.ru/event/5294/>).

## Summary and perspectives

The study of strangeness number fluctuations and the calculation of the first 4 cumulants were presented at the reconstruction level in the MPD experiment.

- The cumulants ( $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$ ) for strangeness were calculated and corrected (Centrality Bin Width and Local Efficiency Corrections).
- Optimize correction parameters (Efficiency bins) and analyze corrections with different cuts.
- I've prepared a technical note, Is there any repository to sent it?.
- Analyze the new correction proposal by Grigory.

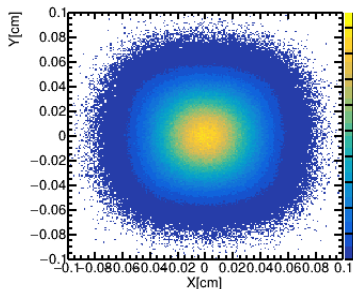
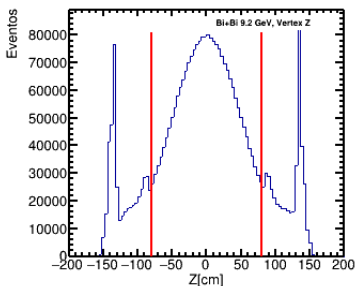
Thank you for your attention <sup>1</sup>.

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<sup>1</sup>Special thanks to Eleazar Cuautle

## Event Selection

Vertex cut  $|z| \leq 80$  cm. Events with at least 1 charged kaon.

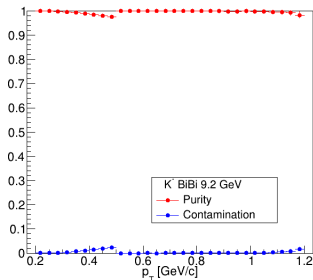
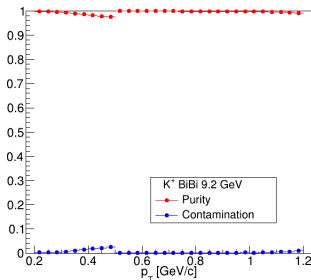


Collision	$\sqrt{s_{NN}}$	Events	Events after cuts
Bi+Bi	9.2 GeV	$\sim 3$ Million	$\sim 1.5$ Million

## Efficiency and contamination and purity

The efficiency, purity, and contamination are defined as

$$\epsilon(p_T) = \frac{N_k(PID)}{N_k(MC)}, \quad C(p_T) = \frac{N_k(FP)}{N_k(PID)}, \quad P(p_T) = 1 - C.$$



Efficiency and contamination of  $K^+$  (left), and  $K^-$  (right).

Efficiency shows discontinuity due to PID: TOF is used only

# Kaon $p_T$

