



# *“Gluon dominance model and multiparticle production”*

E. Kokoulina, A. Kutov, V. Nikitin, V. Popov  
JINR & GSTU



26 International Baldin Seminar on High Energy Physics Problems  
"Relativistic Nuclear Physics and Quantum Chromodynamics"

# Multiparticle processes in HEP

Electron-positron annihilation ( $e^+e^-$ ).

Proton-antiproton annihilation ( $p\bar{p}$ ).

Three-gluon decay of quarkonium ( $\Upsilon$ ).

High multiplicity in  $pp$  interactions.

Heavy ion collisions ( $AA$ )

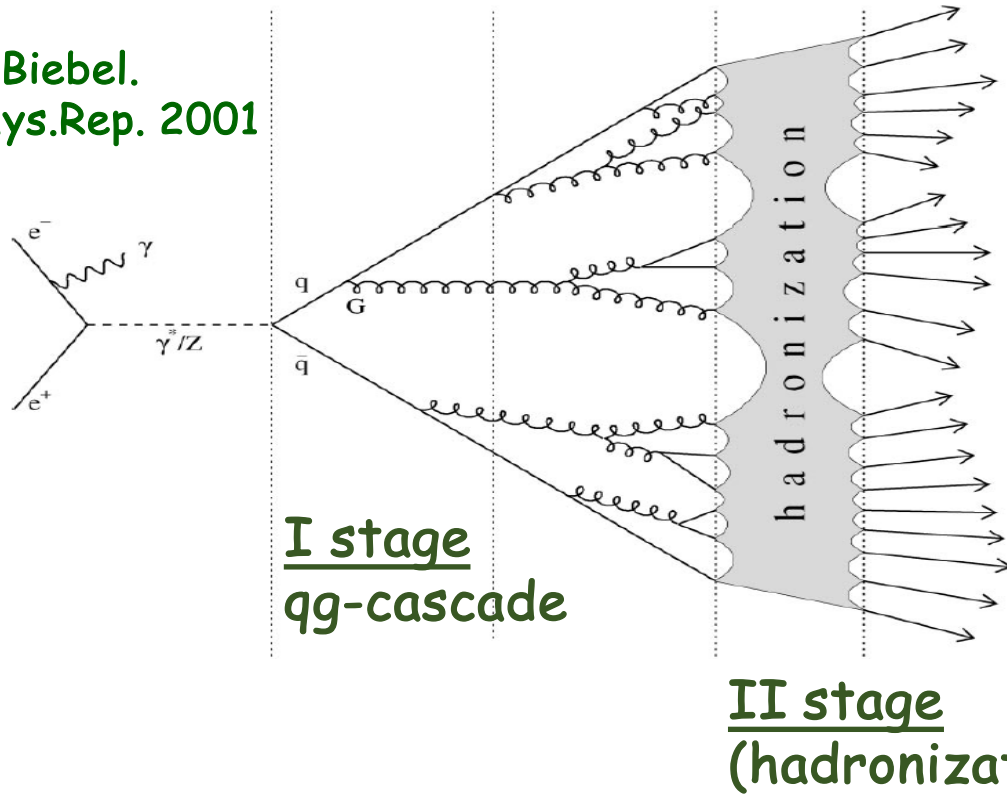
"Gluons are carriers of the strong force, bind quarks together inside nucleons and nuclei and generate nearly all of the visible mass in the Universe.

Despite their importance, fundamental questions remain about the role of gluons in nucleon and nuclei." **Xiangdong Jin**

# $e^+e^-$ - annihilation

$$e^+e^- \rightarrow \gamma(Z^0) \rightarrow q\bar{q} \rightarrow (q, g) \rightarrow ? \rightarrow \text{hadrons}$$

O.Biebel.  
Phys.Rep. 2001



**Multiplicity  
Distribution (MD)**

$$P_n(s) = \frac{\sigma_n}{\sum_m \sigma_m}$$

**Generation  
function (GF):**

$$Q(s, z) = \sum_n P_n(s) z^n$$

**GF  $\leftrightarrow$  MD**

$$P_n(s) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} Q(s, z) \Big|_{z=0}$$

**Correlative moments,  $F_k$ :**

$$F_k(s) = \overline{n(n-1)\dots(n-k+1)} = \frac{\partial^k}{\partial z^k} Q(s, z) \Big|_{z=1}$$

# $e^+e^-$ - annihilation

Konishi et al. & Giovannini [NP, 1979] described the  $qg$ -cascade in pQCD as Markovian branching processes of elementary events:

- 1) quark emission of gluon -  $q \rightarrow q + g, (\tilde{A})$
- 2) gluon fission -  $g \rightarrow g + g, (A)$
- 3) quark-antiquark pair creation from gluon - (B)  $g \rightarrow q + \bar{q}$  .

$$\left\{ \begin{array}{l} \frac{\partial G}{\partial Y} = -AG + AG^2, \\ \frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG. \end{array} \right. \quad \text{System of diff. eq. describing branching processes, leads to Pólya (NBD) for } q\text{-jet and Yule-Furry MD for } g\text{-jet:}$$

$$P_m^g = \frac{1}{\bar{m}} \left( 1 - \frac{1}{\bar{m}} \right)^{m-1},$$

$$P_m^q = \frac{k_p(k_p+1)\dots(k_p+m-1)}{m!} \left( \frac{\bar{m}}{\bar{m}+k_p} \right)^m \left( \frac{k_p}{\bar{m}+k_p} \right)^{k_p}.$$

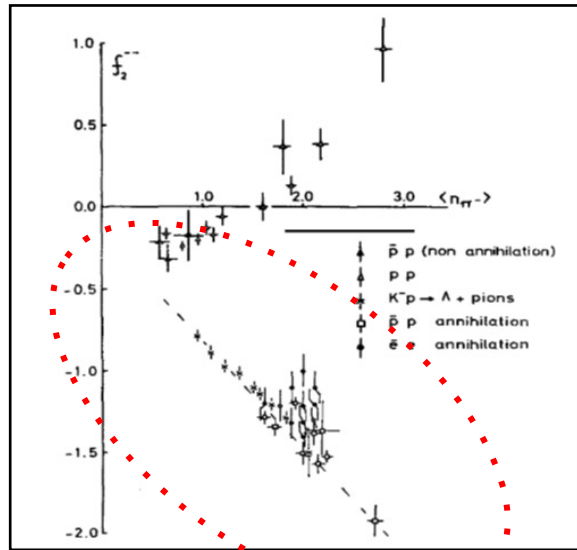
Evolutinary parameter:

$$Y = \frac{1}{2\pi b} \ln[1 + ab \ln(Q^2 / \mu^2)], \quad \tilde{A} \text{ и } A - \text{probabilities of 1) и 2) events, } k_p = \tilde{A}/A.$$

## $e^+e^-$ annihilation - II stage

pQCD is unable to describe hadronization. The choice of MD at this stage is based on experimental behavior of the second correlative moment  $f_2$ . It is always **positive** for **Pólya (NBD)** (and **Furry** also):

$$f_2 = \overline{n(n-1)} - \bar{n}^2 \rightarrow \frac{\overline{m}^2}{k_p} > 0$$



J.G. Rushbrooke,  
B.R. Webber. Phys. Rep.  
44 (1978) 1

We chose **binomial MD (Bernoulli)** for II-stage:

$$P_P^H(n) = C_{N_p}^n \left( \frac{\bar{n}_p^h}{N_p} \right)^n \left( 1 - \frac{\bar{n}_p^h}{N_p} \right)^{N_p - n}, P = q, g.$$

$$f_2 = - \frac{(\bar{n}_p^h)^2}{N_p} < 0.$$

## Convolution of two stages. Two-stage model

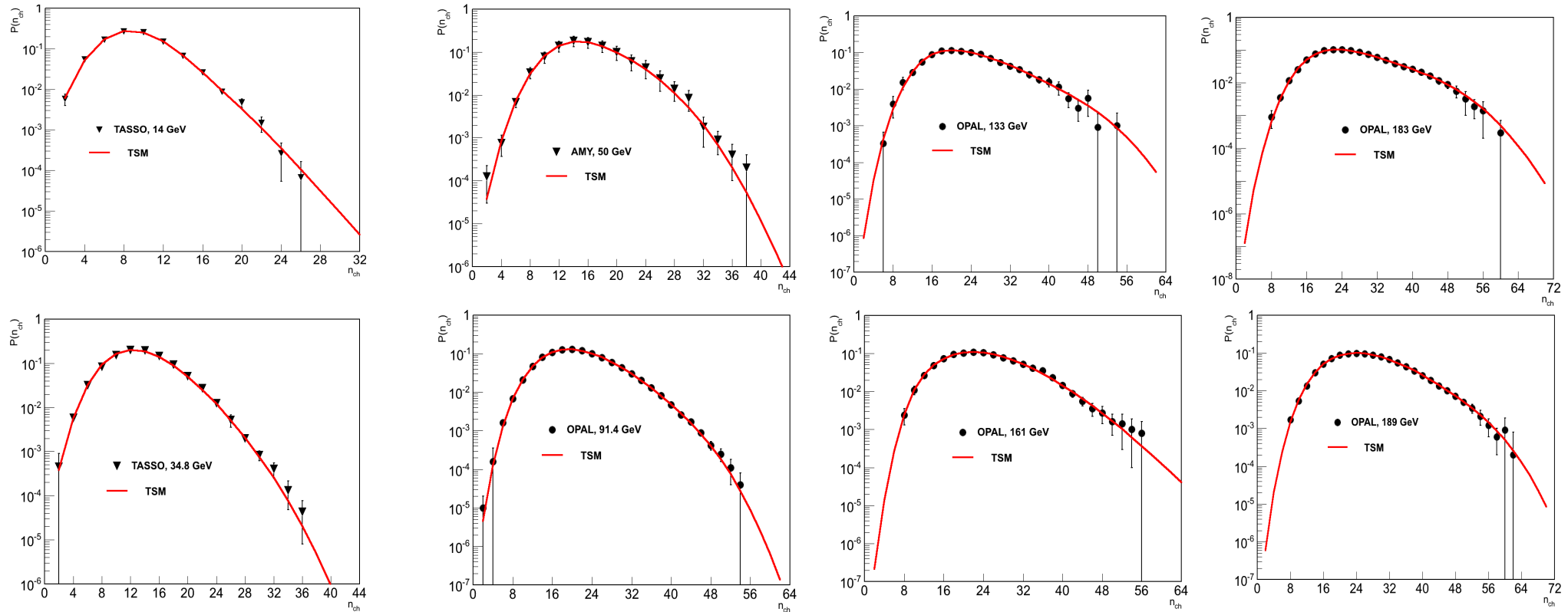
$$Q(s, z) = \sum_m P_m^P Q^H(m, s, z) \quad (\text{soft discoloration}).$$

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C_{(2+\alpha m)N}^n \left( \frac{\bar{n}^h}{N} \right)^n \left( 1 - \frac{\bar{n}^h}{N} \right)^{(2+\alpha m)N-n}$$

$$Q_p^H = \left[ 1 + \frac{\bar{n}_p^h}{N_p} (z-1) \right]^{N_p}, \quad \mathbf{p} = \mathbf{q}, \mathbf{g},$$

**Model parameters:**  $k_p$ ,  $\bar{m}$ ,  $N_q = N$ ,  $\bar{n}_q^h$ ,  $N_g = \alpha N$ ,  $\bar{n}_g^h = \alpha \bar{n}_q^h$ .

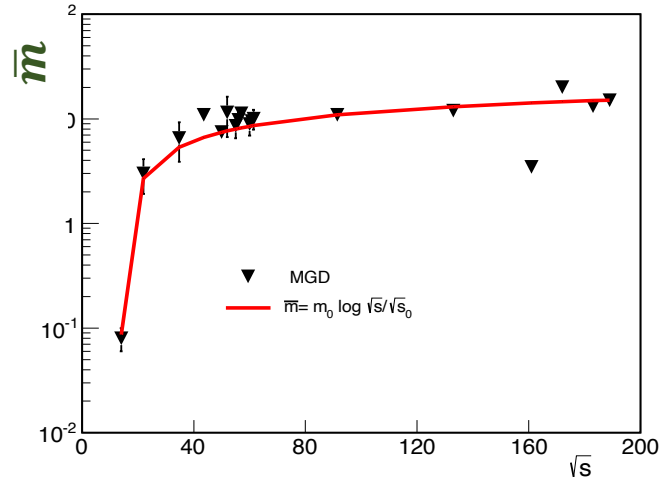
# MD in $e^+e^-$ annihilation (14 -189 GeV)



**Prediction:** mean multiplicity in  $e^+e^-$  annihilation at 500 GeV:  $\langle n \rangle \sim 50$



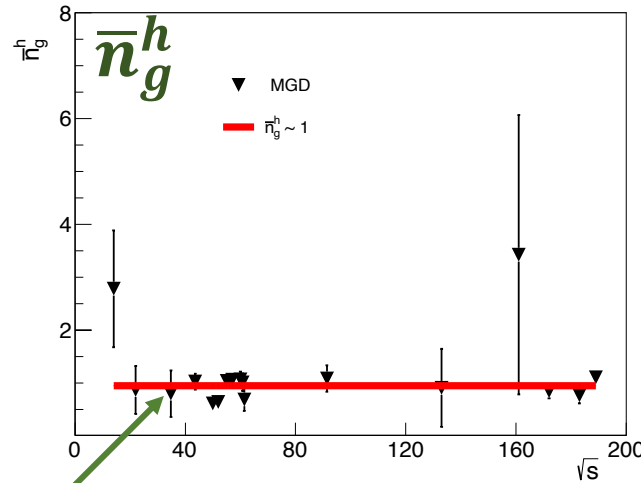
# Parameters of model



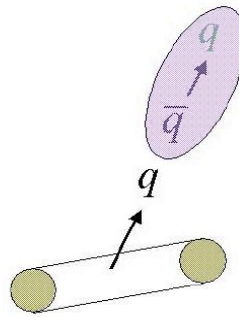
$$\bar{m} \sim \log s.$$

Hypothesis of Local  
Parton-Hadronic  
Duality (LoPHD)

$$\langle m \rangle = \rho \langle n \rangle, \rho \sim 1.$$

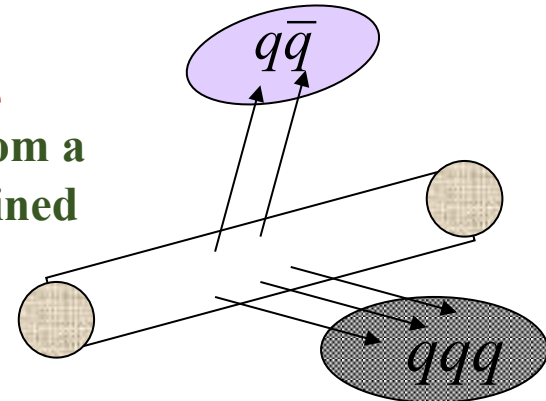


Average number of  
hadrons,  $\bar{n}_g^h$ , formed  
from gluon, is close to 1,  
which testifies the  
**fragmentation** mechanism  
of hadronization.



**Fragmentation  
mechanism** (in vacuum)

**Recombination  
mechanism** (from a  
thermal, deconfined  
medium)



[ B. Muller. 2004]

## Summary from $e^+e^-$ -annihilation study:

1. The process of  $e^+e^-$ -annihilation is the **simple one** for the QCD description for the first (parton) stage because the creation of gluons and following its fission occurs in a vacuum.
2. pQCD is not applied at the 2nd stage (hadronization). Our choice of MD at the hadronization is based on the data:  $f_2$  for quark and gluon jets is always positive. Experimental  $f_2$  is negative at low energies and becomes positive at higher energies.
3. At low energy, hadronization dominates over the  $qg$ -cascade. We use **Binomial distribution** for hadronization. **Convolution** of those two stages **describes well** experimental MD up to 200 GeV including low energies.

## Summary from $e^+e^-$ -annihilation study:

4. Average number of hadrons formed from single gluon (  $\bar{n}_g^h$  ) at the second stage is close to 1 at the wide energy region ( $\sim 10$ - $200$  GeV):  $1 \text{ gluon} \rightarrow 1 \text{ hadron}$  (confirmation of the **fragmentation mechanism** of hadronization in vacuum).
5. Average gluon multiplicity (  $\bar{m}$  ) rises logarithmically.
6. Gluon fission ( $g \rightarrow g + g$ ) commences prevailing over bremsstrahlung ( $q \rightarrow q + g$ ) with increasing of energy and  $f_2$  changes sign.
7. Hadronization of gluons is softer than quark one at the hadronization.

# pp interactions

The “Thermalization” project was aimed to the searching for collective phenomena in pp interactions at 50 GeV/c in the region of high multiplicity ( $n_{ch} \gg \bar{n}_{ch}$  ).

We waited for the manifestation of pionic jets creation, Cherenkov radiation of gluons, Bose-Einstein condensation of pions, excess of soft photon yield and others.

The important part of our SVD-2 setup was high multiplicity trigger (suppressed registration low multiplicity events).

## pp interactions

The simulated MD underestimated data obtained at the Mirabelle setup by 3 orders of magnitude at max registered  $n_{ch}=18$ .

We modified our TSM, which still consisted of two stages:  $qg$ -branching and hadronization. In the beginning, we included in the scheme all valence quarks and few active gluons. In that case parameter  $\bar{n}^h$  was  $\ll 1$  ( $\bar{n}^h \sim 1$  in  $e^+e^-$ ).

A natural step would be to assume: not all 3 pairs of valence quarks are involved in the interaction (central collisions are rare). The gradual exclusion of one, then 2 pairs from scheme, still left the value of  $\bar{n}^h \ll 1$ .

## pp interactions

And only the complete exclusion of all valence quarks from this scheme immediately improved the description of data, but at the same time, parameter  $\bar{n}^h$  slightly exceeded 1. That small excess over 1 is the evidence of the **recombination** mechanism of hadronization (in **denser qq-medium**).

In that way, all valence quarks are staying in the **leading** particles. Sources of all newly born secondaries turn out to be **active gluons**. And we renamed TSM the Gluon Dominance Model (**GDM**).

GDM describes well MD from 50 GeV/c (lab system) up to 60 GeV (ISR), and observe growth of  $\bar{m}$  and hadronization parameter  $\bar{n}^h$ .

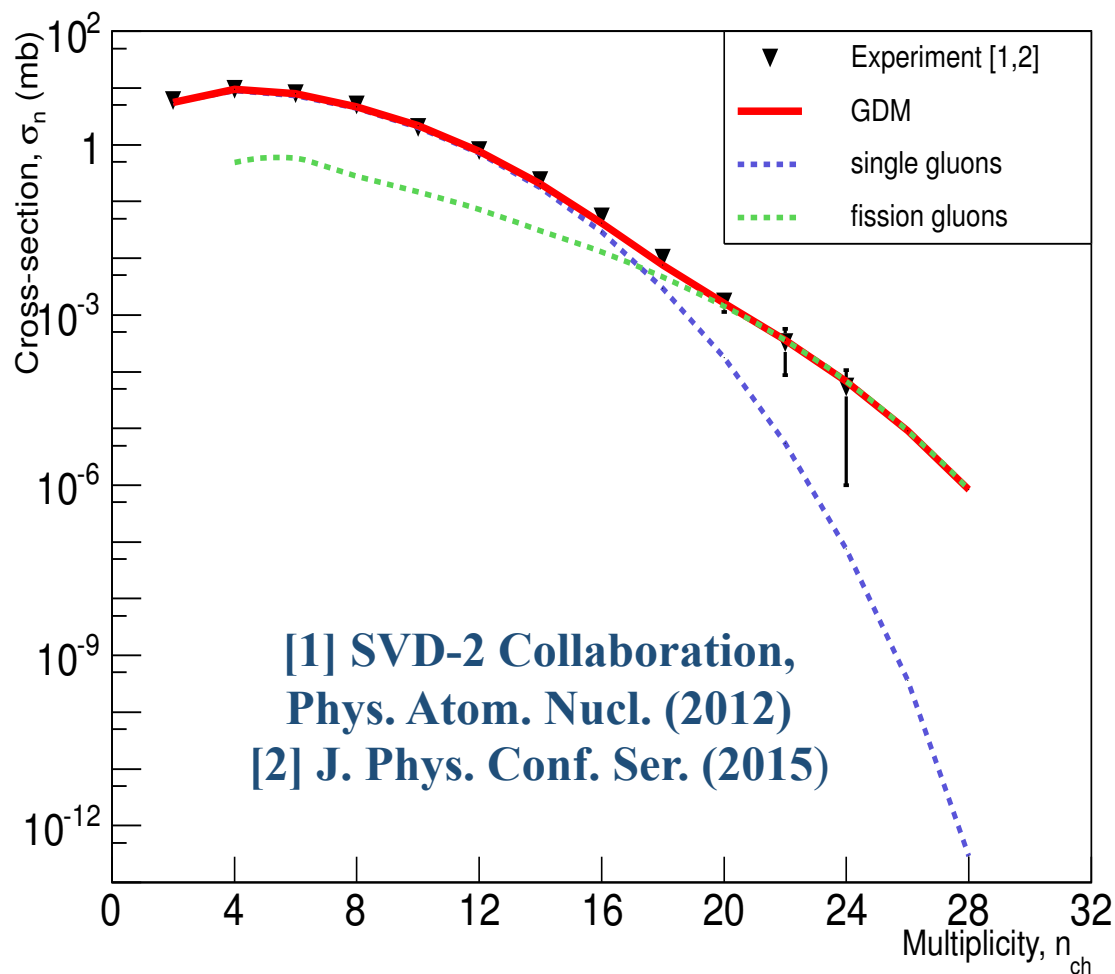
## Scheme with branching:

$$P_n(s) = \sum_{k=1}^{MK} \frac{\bar{k}^k e^{-\bar{k}}}{k!} \sum_{m=k}^{MG} \frac{1}{\bar{m}^k} \frac{k(k+1)(k+2)\dots(m-1)}{(m-k)!} \left(1 - \frac{1}{\bar{m}}\right)^{m-k} \times \\ \times C_{\alpha m N}^{n-2} \left(\frac{\bar{n}^h}{N}\right)^{n-2} \left(1 - \frac{\bar{n}^h}{N}\right)^{\alpha m N - (n-2)} ;$$

## Scheme without branching:

$$P_n(s) = \sum_{m=1}^{ME} \frac{\bar{m}^m e^{-\bar{m}}}{m!} C_{mN}^{n-2} \left(\frac{\bar{n}^h}{N}\right)^{n-2} \left(1 - \frac{\bar{n}^h}{N}\right)^{mN - (n-2)} \quad (n > 2)$$

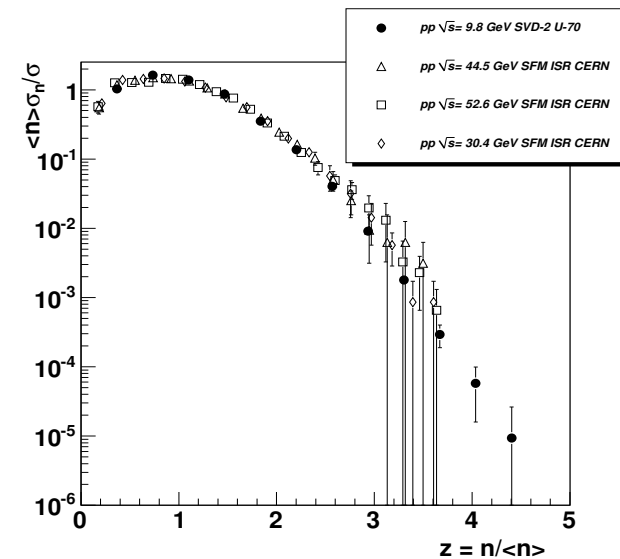
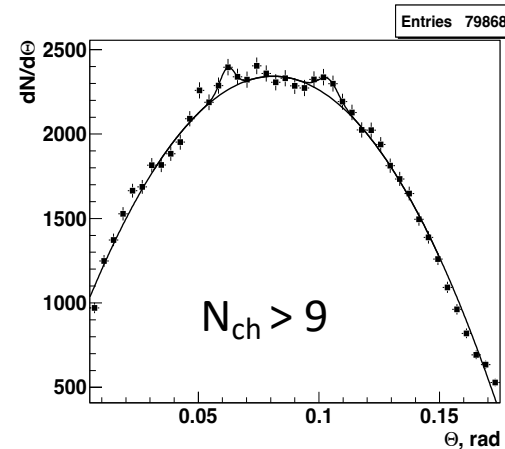
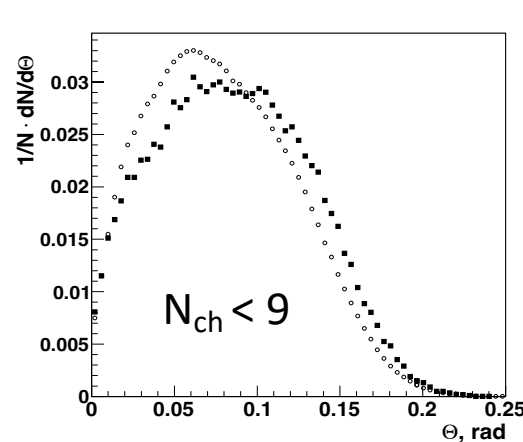
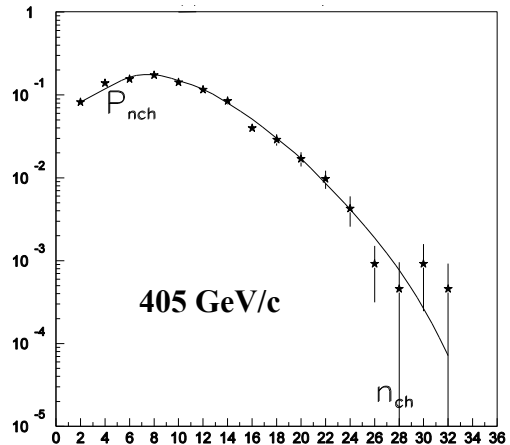
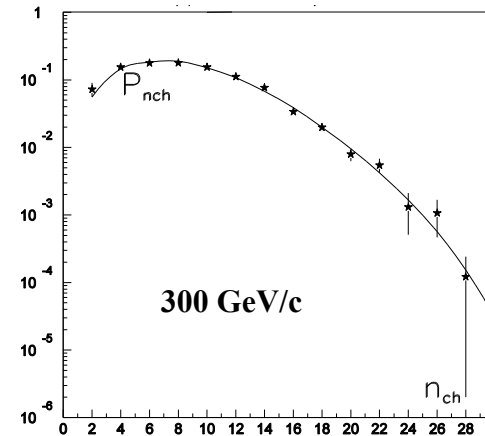
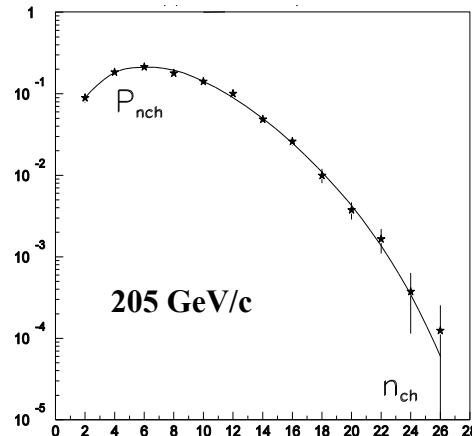
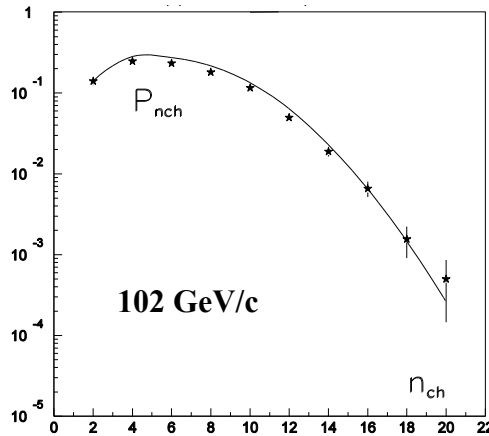
## GDM with gluon fission



SVD-2 and Mirabelle (50 GeV) have been stitched along  $\sigma_n$ . GDM in HM region takes into account 2 types of contributions: without g-fission (blue line) and with (green line). Their superposition is shown by red line. HM stipulates namely by gluon fission. Ratio of bremsstrahlung to gluon fission is equal to  $\sim 1/9$ . More than 64% of  $E$  is converted into a mass of newborn pions.



# pp interactions at higher energies:



The world KNO distribution with addition of the SVD-2 points (9.8 GeV).

Cherenkov radiation at high multiplicity  
the two-humped structure  
in polar angle distribution

## Summary from study of pp interactions:

1. We described pp interactions by two schemes: without gluon branching and with it.
2. The first scheme can't describe the tail of high multiplicity. The second one describes well.
3. The fraction of active gluons that do not fragment into hadrons (they are captured by secondary particles) turned out to be about 47%, which is close to the estimate of A.H. Muller (50%).
4. The estimation of the charge exchange is about 50% ( $p p \rightarrow p n \pi^+$ ).
5. The two-humped structure in the distribution over the polar angle at HM is interpreted as Cherenkov radiation of gluons by quarks and allows us to determine the refractive index of the nuclear medium. It is close to 1 (rarefied medium). At RHIC it is about 3.

# $p\bar{p}$ annihilation

Pure  $p\bar{p}$  annihilation is obtained by subtracting of pp contribution (diffraction interactions) from  $\sigma_n(p\bar{p})$ :

$$\Delta\sigma_n(p\bar{p} - pp) = \sigma_n(p\bar{p}) - \sigma_n(pp)$$

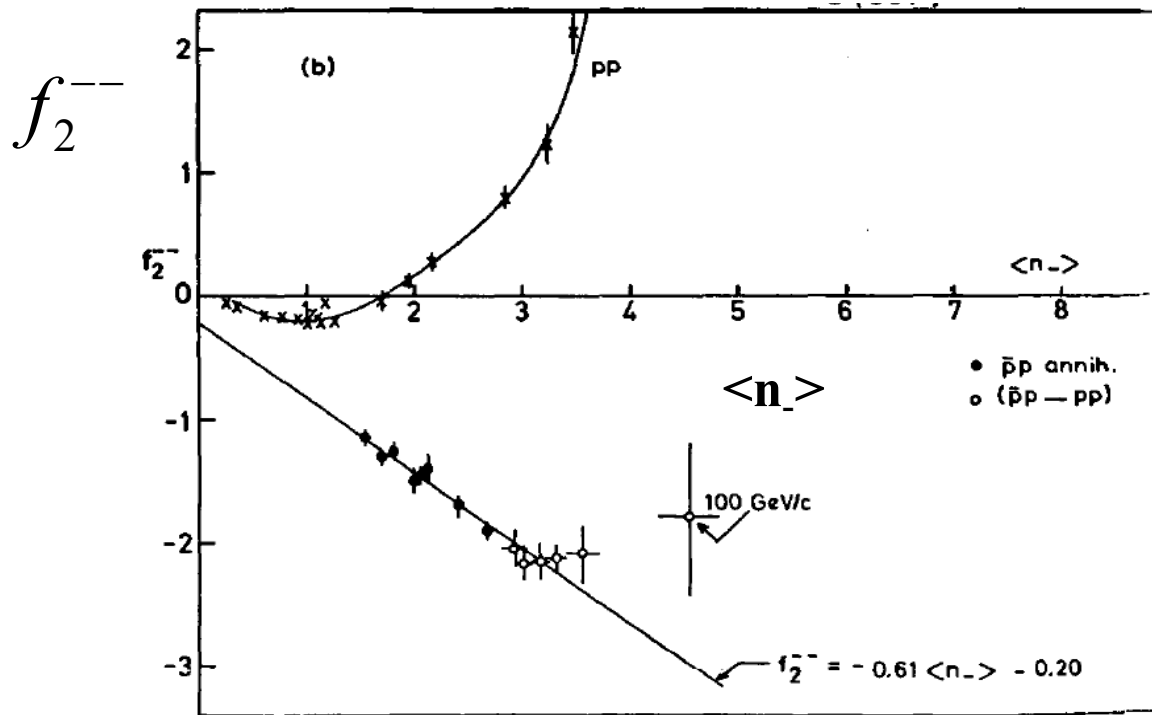
What we had before the description of it?

Experimental data:

- 1) absence of two leading baryons (p and  $\bar{p}$ );
- 2) leading pions separate across a large rapidity gap from  $3\pi$  clusters (3 pion jets);
- 3)  $f_2$  is staying negative in the wide energy region.

From J.G. Rushbrooke and B.R. Webber. Phys. Rep. 44 (1978) 1.

## Variation of $f_2$ with $\langle n_- \rangle$ for annihilation & non-annihilation data

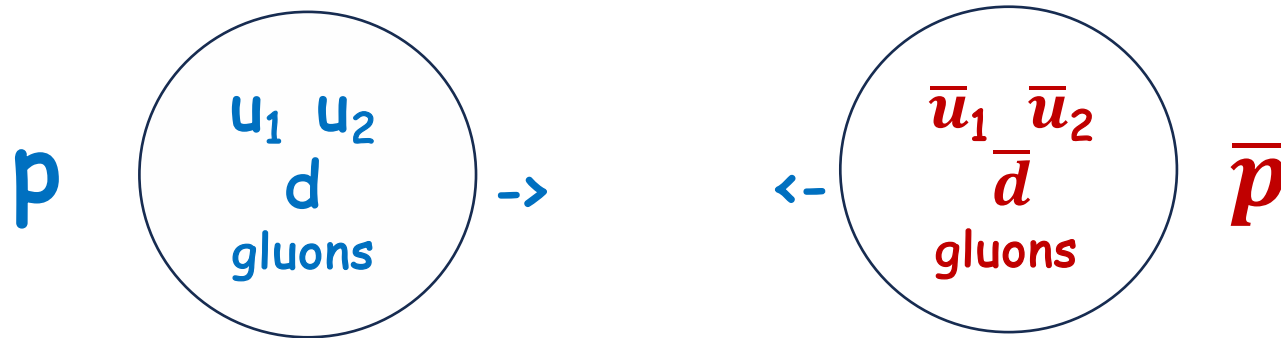


$$f_2 = \langle n(n-1) \rangle - \langle n \rangle^2 = D_2 - \langle n \rangle < 0;$$

J.G. Rushbrooke,  
B.R. Webber. Phys. Rep.  
44 (1978) 1

# $p\bar{p}$ annihilation

GDM offers description of  $p\bar{p}$  annihilation by the formation of 3 and more intermediate charged quark topologies with corresponding contributions  $c_0$ ,  $c_2$  and  $c_4$ , which are stipulated by all kinds of permutations of valence quarks with antiquarks and the formation of three leading pions.



# Variants of q-topologies

2 variants of "0"- topology ( $3 \pi^0$ ):

$$u_1 \bar{u}_1 + u_2 \bar{u}_2 + d \bar{d} \quad \text{and} \quad u_1 \bar{u}_2 + u_2 \bar{u}_1 + d \bar{d};$$

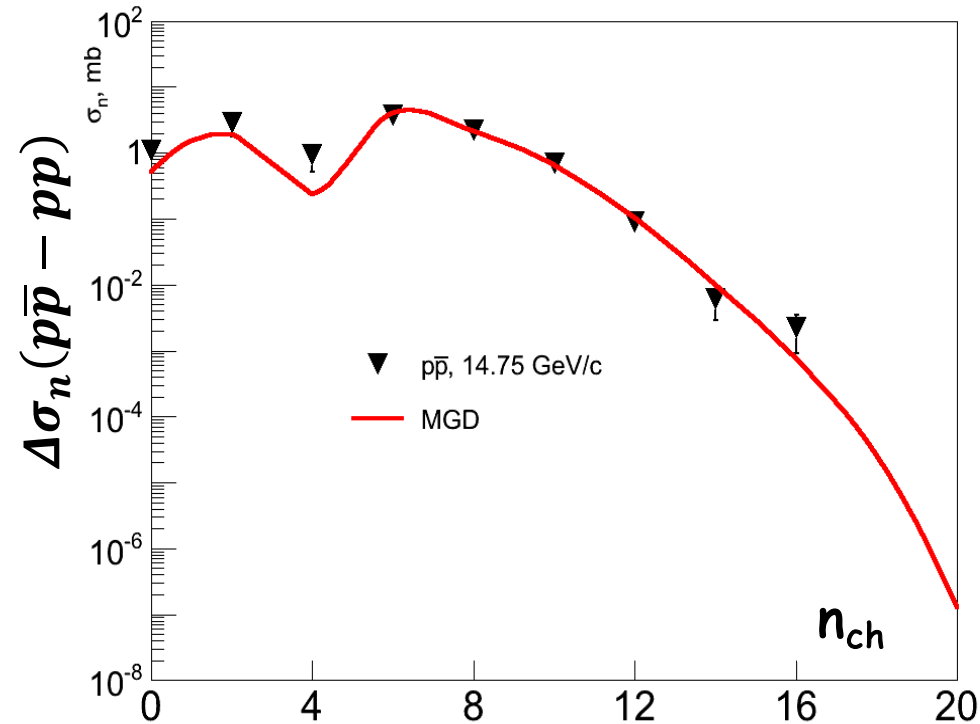
4 variants of "2"- topology ( $\pi^0, \pi^+, \pi^-$ ):

$$u_1 \bar{d} + u_2 \bar{u}_1 + d \bar{u}_2, \quad u_1 \bar{u}_2 + u_2 \bar{d} + d \bar{u}_1 \\ u_1 \bar{u}_1 + u_2 \bar{d} + d \bar{u}_2, \quad \text{and} \quad u_1 \bar{d} + u_2 \bar{u}_2 + d \bar{u}_1;$$

Topology "4" (and higher) is formed by adding to a valence quark (an antiquark) the corresponding antiquark (quark), which are born from active gluons ( $g \rightarrow q + \bar{q}$ ):

$$u_1 \bar{d} + \bar{u}_1 d + u_2 \bar{d} + d \bar{u}_2 + \dots (\pi^+, \pi^-, \pi^-, \pi^+):$$

# $p\bar{p}$ annihilation



Generation function in GDM:

$$\begin{aligned} Q(z) = & \\ = & c_0 \sum_m P_m^G \left[ 1 + \frac{\bar{n}^h}{N} (z - 1) \right]^{mN} + \\ & + c_2 z^2 \sum_m P_m^G \left[ 1 + \frac{\bar{n}^h}{N} (z - 1) \right]^{mN} + \\ & + c_4 z^4 \sum_m P_m^G \left[ 1 + \frac{\bar{n}^h}{N} (z - 1) \right]^{mN}. \end{aligned}$$

The "4" topology is responsible for the tail of HM.

## $p\bar{p}$ annihilation

Comparing GDM with the data at 14.25 GeV/c gives the following values of its parameters:  $\bar{m} = 3.36 \pm 0.18$ ,  $\bar{n}^h = 1.74 \pm 0.26$ ,  $N = 4.01 \pm 0.61$ ,  $c_0 : c_2 : c_4 = 15 : 40 : 0.05$ . Maximum number of active gluons at this energy is equal 4 at  $\chi^2/\text{n.d.f.} = 5.77/4$ .

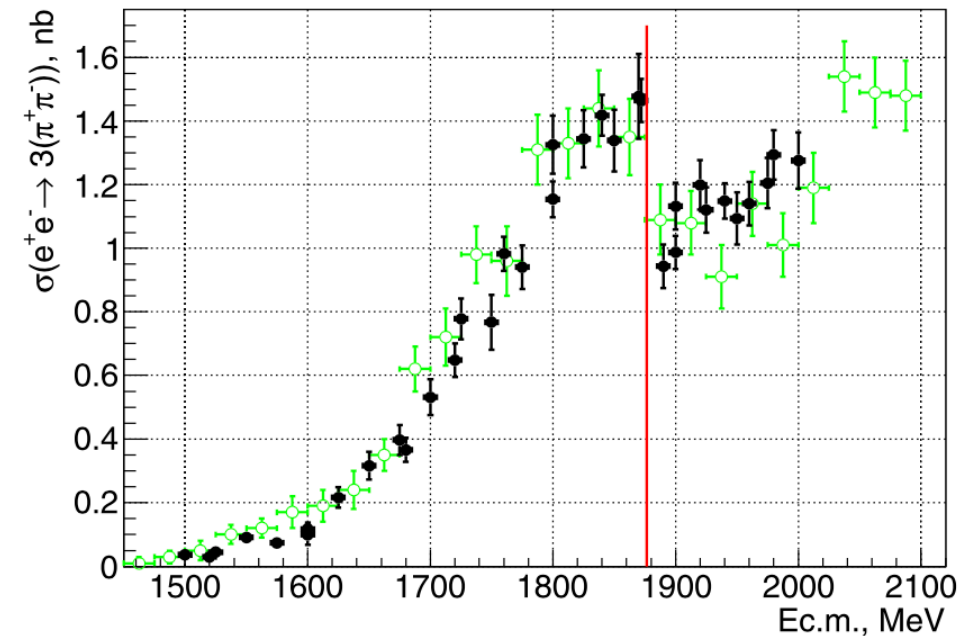
Hadronization parameters coincide in values with those obtained in pp interactions in close energy. The ratio of possible permutations for "0"-topology to "2"-topology (1/2) is close to the ratio of  $c_0:c_2$ .



## Study of the process $e^+e^- \rightarrow 3(\pi^+\pi^-)$ in the c.m. energy range 1.5-2.0 GeV with the CMD-3 detector

" $\sigma(e^+e^- \rightarrow 3(\pi^+\pi^-))$  is measured at the CMD-3 (VEPP-2000) in the c.m. energy range 1.5-2.0 GeV. The measured  $\sigma$  exhibits a sharp drop near the  $p\bar{p}$  threshold ( $\sim 1.88$  GeV). A first study of dynamics of 6-pion production has been performed".

R.R. Akhmetshin et al. Phys. Lett. B 723 (2013) 82–89.



$\sigma(e^+e^- \rightarrow 3(\pi^+\pi^-))$  measured with the CMD-3 at VEPP-2000 (black). BaBar data are shown by (green). The red line shows the  $p\bar{p}$  threshold.

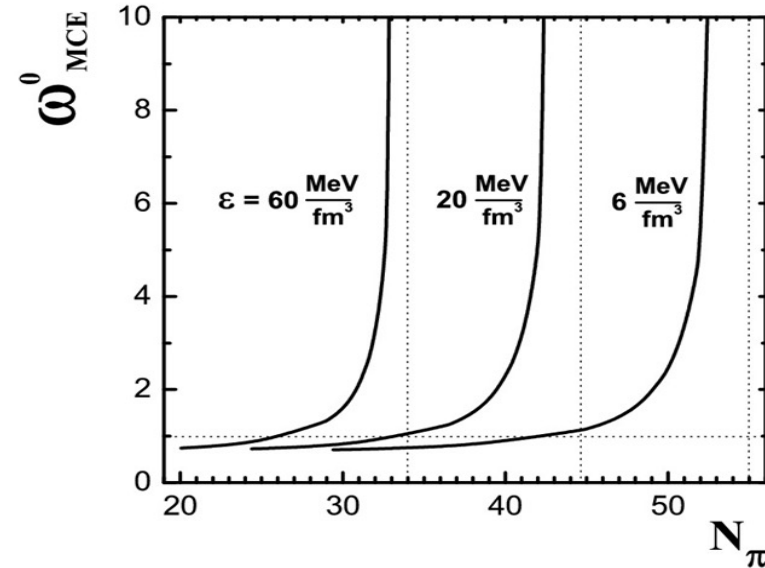
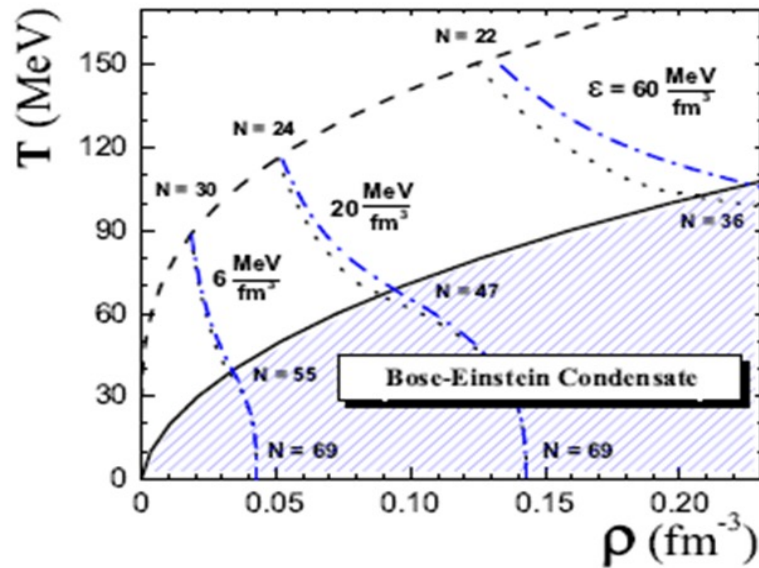
## Neutral pion fluctuations at high total multiplicity

V. Begun & M. Gorenstein put us the task on searching for pionic (Bose-Einstein, BEC) condensate [Phys.Lett., 2007, Phys.Rev. 2008] in pp interactions at U-70 for HM. For this purpose, we only had to measure the scaled variance

$$\omega^0 = D / \langle N_0(N_{\text{tot}}) \rangle, \quad D = \langle N_0^2 \rangle - \langle N_0 \rangle^2,$$

of  $\pi^0$ -meson number with growth of total multiplicity ( $n_{\text{tot}} = n_{\text{ch}} + n_0$ ). Abrupt growth of  $\omega^0$  would be signal of BEC formation.

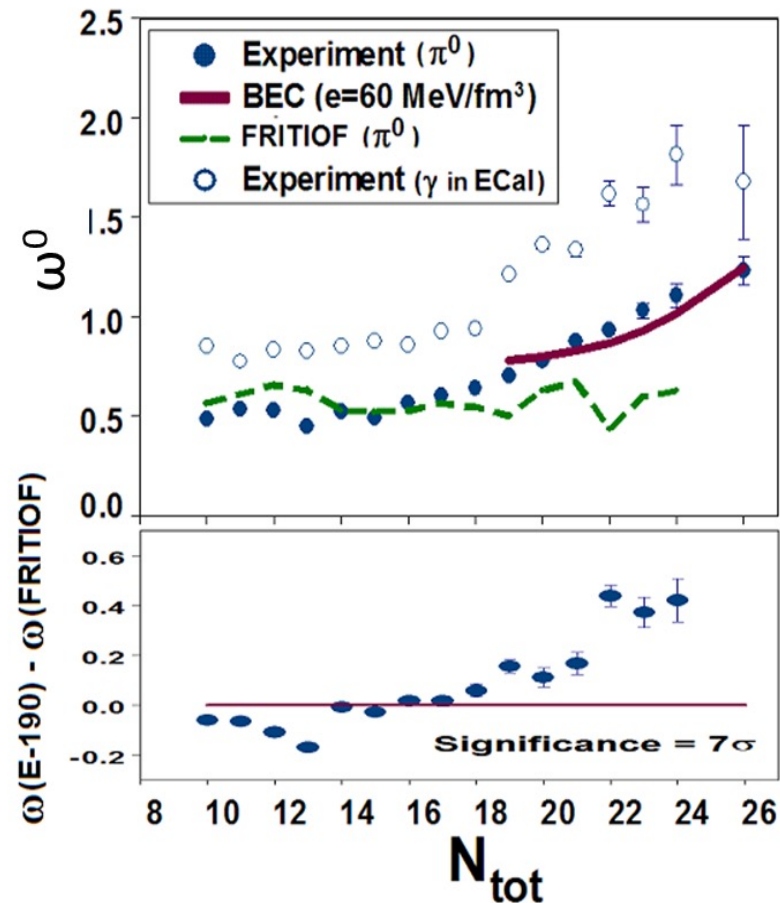
# Fluctuations of $\pi^0$ -mesons at high multiplicity



Phase diagram of pionic gas at  $\mu_Q = 0$ . Dash line corresponds to  $\rho_\pi(T, \mu_\pi = 0)$ , solid - BEC. Energy densities 6, 20 и 60 MeV/fm<sup>3</sup>.

$$\frac{T_C(\pi)}{T_C(A)} \approx \frac{m_A}{m_\pi} \left( \frac{r_A}{r_\pi} \right)^2 \cong \frac{m_A}{m_\pi} \cdot 10^{10} \rightarrow T_C(\pi) \gg T_C(A).$$

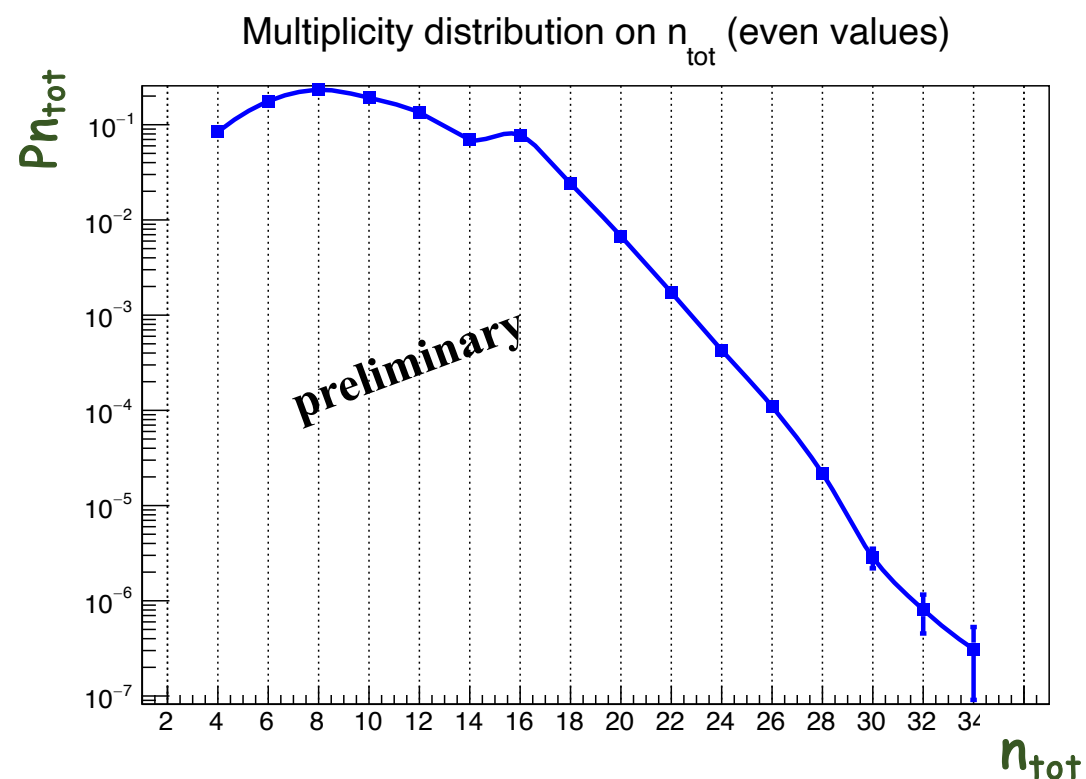
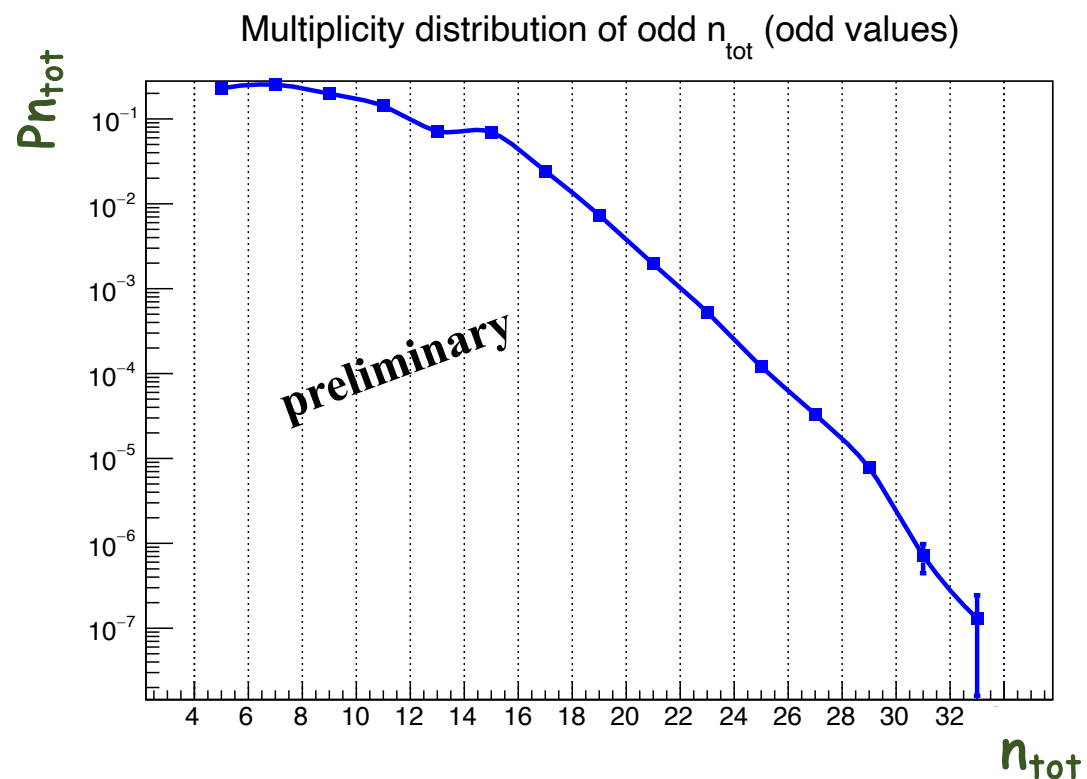
# Fluctuations of $\pi^0$ -mesons at high multiplicity



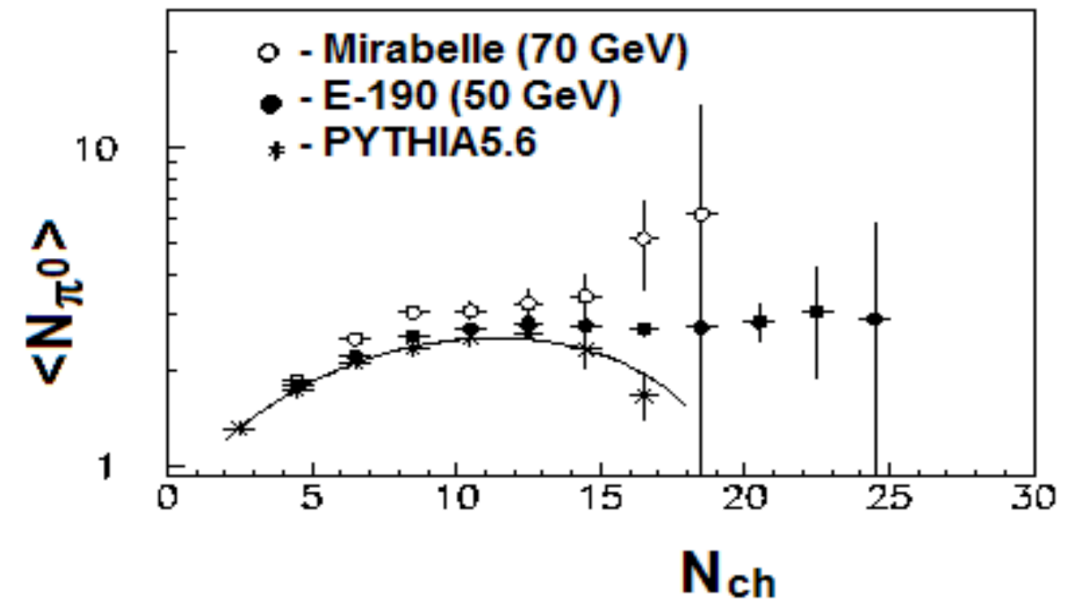
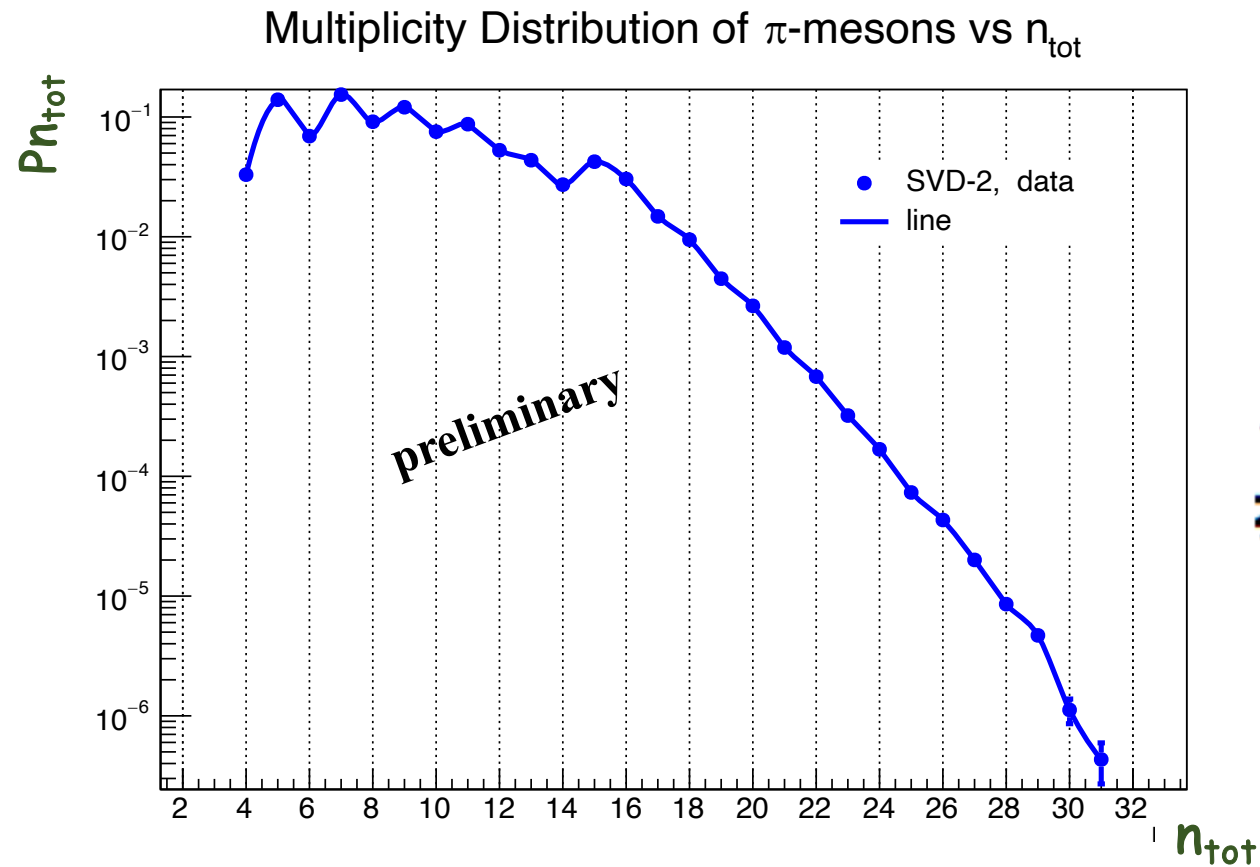
The deviation of the scaled variance,  $\omega^0$  measured at the SVD-2 from the Monte Carlo predictions in the HM region is  $7\sigma$  at  $N_{\text{tot}} \sim 25$  [EPJ, 2012, ICHEP 2012].

Our study testify, BEC formation starts forming from  $n_{\text{ch}} \sim 16$ . At  $n_{\text{tot}} > 18$ ,  $\omega^0$  rises, leading particles disappear, the hadron system becomes isotropic in all directions.

# Multiplicity distributions of $\pi$ -mesons at U70



# Multiplicity distributions of $\pi$ -mesons at U70



# Multiplicity distributions of $\pi^0$ – mesons at U70

(GDM's scheme without fission of gluons)

$$P_n = \alpha \sum_m^{Mg} \frac{e^{-\bar{m}} \bar{m}^m}{m!} C_{mN}^n \left( \frac{\bar{n}^h}{N} \right)^n \left( 1 - \frac{\bar{n}^h}{N} \right)^{mN-n}$$

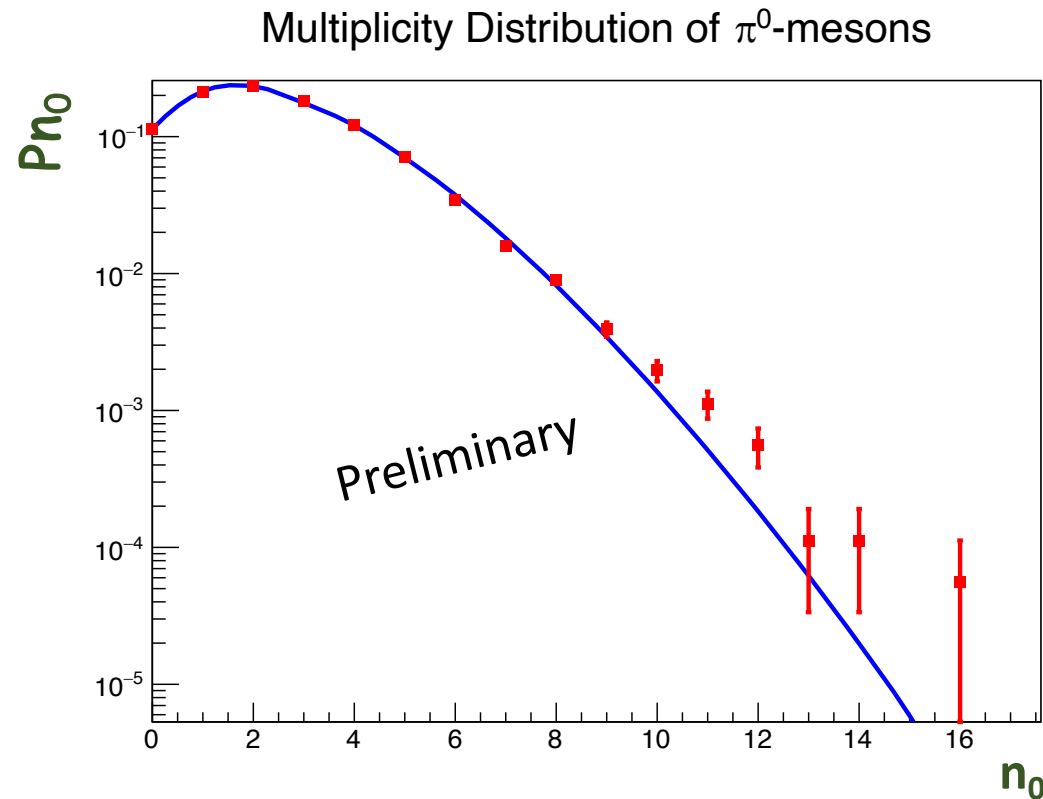
$P_n$  - Multiplicity distribution,  $m$  - number of gluons,

$\bar{m}$  - average number of gluons,

$\bar{n}^h$  ( $N$ ) - average (max) number of hadrons formed  
from single gluon

$C_{mN}^n$  - binomial coefficient

# Multiplicity distributions of $\pi^0$ – mesons at U-70



**Chi2 = 31.4703**

**NDf = 12**

**without fission of gluons**

**p0  $\bar{m}$  = 2.38727 +/- 0.177867**

**p1  $N$  = 2.0001 +/- 2.54369e-05**

**p2  $\bar{n}^h$  = 0.990043 +/- 0.062349**

**p3  $\alpha$  = 1.06469 +/- 0.0151777**

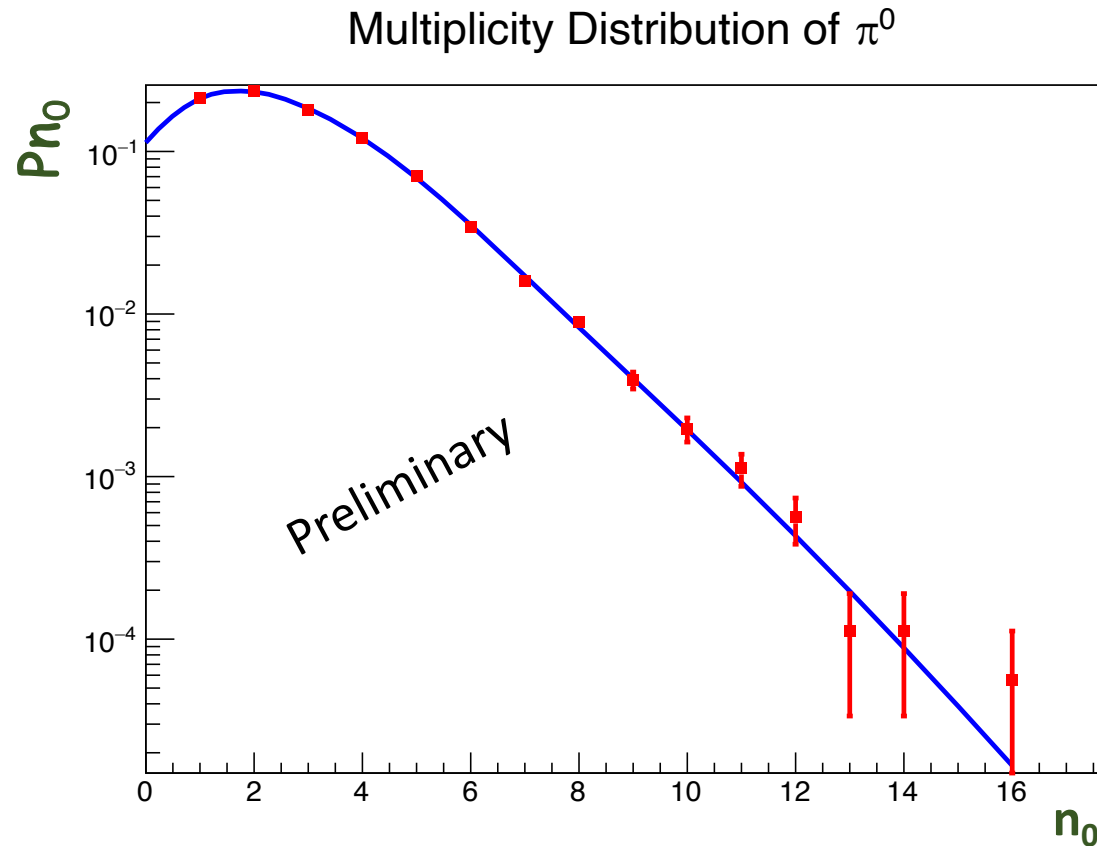


# Multiplicity distributions of $\pi^0$ – mesons at U-70

## Scheme with fission of gluons

$$Pn = \alpha_1 \sum_m^M g \frac{e^{-\bar{m}_1} \bar{m}_1^m}{m!} C_{mN}^n \left( \frac{\bar{n}^h}{N} \right)^n \left( 1 - \frac{\bar{n}^h}{N} \right)^{mN-n} +$$
$$+ \alpha_2 \sum_m^M g \frac{e^{-\bar{m}_2} \bar{m}_2^m}{m!} C_{2mN}^n \left( \frac{\bar{n}^h}{N} \right)^n \left( 1 - \frac{\bar{n}^h}{N} \right)^{2mN-n}$$

# Multiplicity distributions of $\pi^0$ – mesons at U-70



$$\text{Chi2} = 8.46469$$

$$\text{Ndf} = 10$$

with fission of gluons

$$p0 \quad \overline{m1} = 1.24634 \quad \pm 0.12041$$

$$p1 \quad N = 7.55451 \quad \pm 2.57284$$

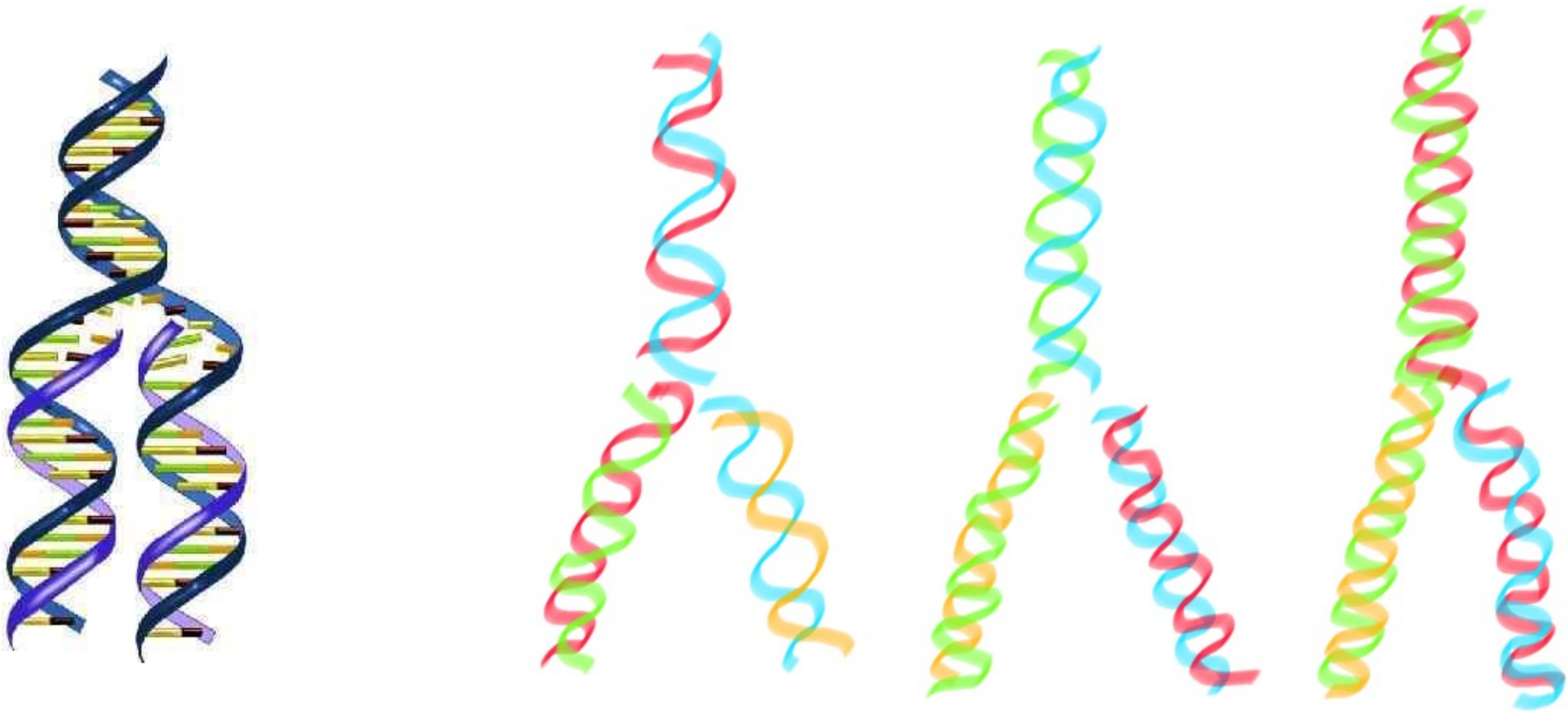
$$p2 \quad \overline{n}^h = 1.41561 \quad \pm 0.0550313$$

$$p3 \quad \alpha_1 = 1.02585 \quad \pm 0.0974778$$

$$p4 \quad \overline{m2} = 0.28488 \quad \pm 0.138065$$

$$p5 \quad \alpha_2 = 1.18518 \quad \pm 0.370877$$

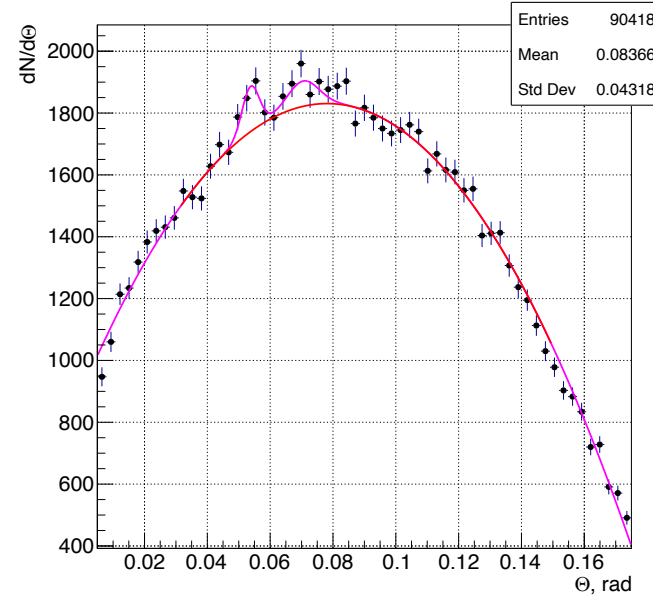
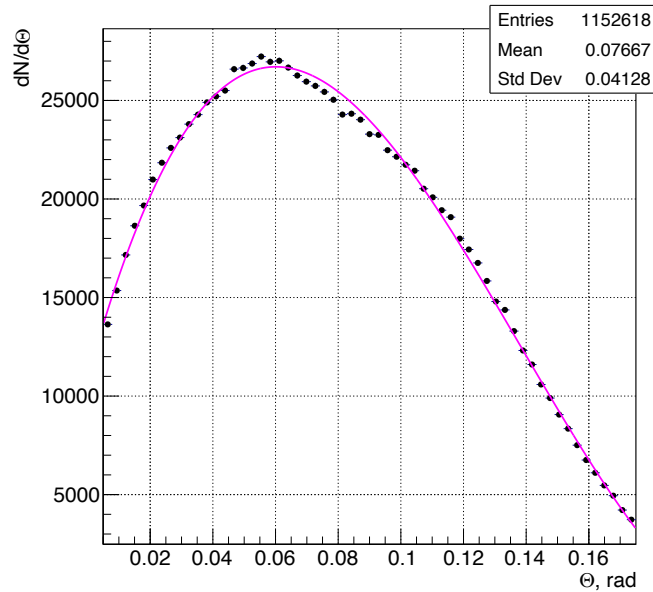
And as afterword, I would like to compare the formation of secondary hadrons from gluons with **DNA replication**. Nature unexpectedly repeats itself ~ **gluons -> hadrons**



Thank you for attention

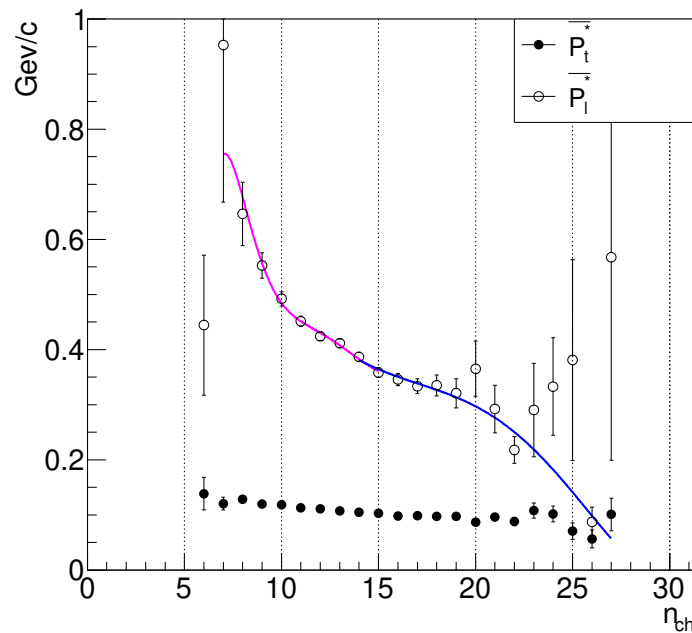
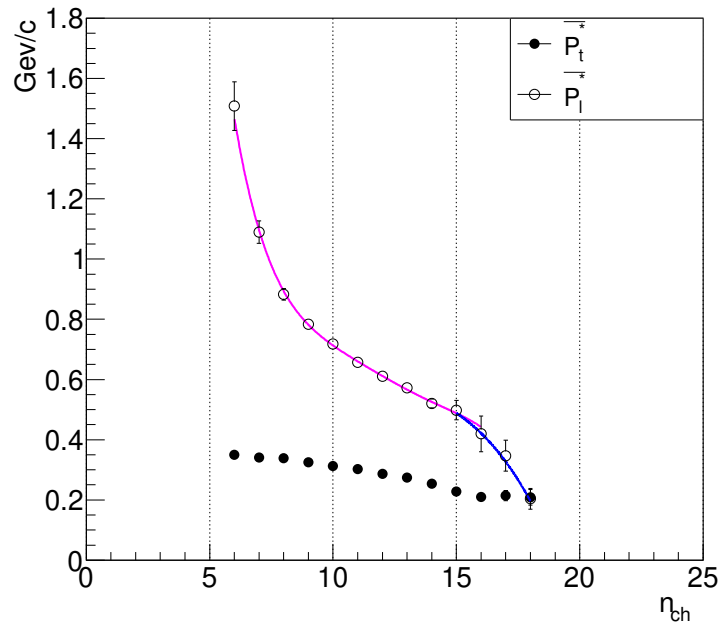
# Spare slides

# Polar angle ( $\theta$ ) distributions: small mult. and HM



Angle distributions on the polar angle  $\Theta$ . In HM region we observe two-humped structure, which is interpreted as Cherenkov radiation gluon by quark.  $\Theta_{\text{Cher}} = 0.05377 \pm 0.00273$  rad with  $CL_{3.1} \sigma$ . For gluon rings  $\cos \Theta = 1/\beta n_r$ , where  $n_r$  refraction coefficient  $n_r = 1.0016 \pm 0.0001(4)$ , close to 1. It testifies about rarity of  $qg$ -medium.

# Longitudinal & transversal components of p at HM



$\langle p_T \rangle$  and  $\langle p_{||} \rangle$   
components of charged  
particles.  
Left: M.C.-simulation,  
right: experimental  
data.

BEC formation starts to form from  $n_{ch} \sim 16$  (inflection point).  
At  $n_{tot} > 18$   $\omega^0$  rises, leading particles disappear, hadron system  
becomes isotropic in all directions.