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Essence of Charge and Spin with General Covariant Dirac Equations

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Motivation for Our Study

First of all we very shortly formulate the central tenets of the field-theoretical approach to the physical reality.
1. A function of multiple variables is considered as an elementary object and a physical field as the system of functions of multiple variables with known low of

transformation under the transition from the initial system of coordinates to any other one. Fields of such kind are called general covariant.

2.Concept of field has principal meaning and the concept of point particle of the classical mechanics is of secondary importance.

3. A definite system of fundamental general covariant fields form one whole and are characterized by a very restricted number of parameters known as coupling constants. To any state of such system of fields we put into correspondence its integral characteristics, for example energy. This is a field-theoretical model of a particle and, in particulary, nuclei. Emphasize, that fields form one whole and only particles interact.

One basic aspect of field-theoretical approach was represented at the previous Baldin conference

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ISHEPP-23 (PEPAN, 2024,Vol.55, No.4). It is connected with the new representations about space and time. Here, I will speak about charge and spin in the framework of general covariant Dirac equations, which represent else one fundamental aspect of the field or quantum theory. A need for these equations is provided by the fact that in the framework of the field-theoretical approach we can not operate with the well-known Pauli's and Dirac's matrixes which are used to describe Spin and that is why we need to create a field-theoretical representation of Spin.

Space and Time

Concepts of space and time are very important to bring the concept of Spin into correspondence with field-theoretical approach. To this end, let us remind the

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definition and some main properties of natural Time existing independently of an observers and their devices. An important element of the theory of functions of multiple variables and hence, general covariant theory, is the reference space R^4 in which a point is defined as 4-tuple of real numbers

$$x = (x^1, x^2, x^3, x^4), \quad -\infty < x^i < \infty.$$

The independent variables x^1, x^2, x^3, x^4 are considered on the absolutely equal footing.

Definition: a moment of natural Time is a number that we put into correspondence to any point of the reference space R^4. Hence, a moment of Time can be found as follows:

$$t = \chi(x^1, x^2, x^3, x^4) = \chi(x),$$

where the function $\chi(x)$ is the scalar field.
All points of the reference space that correspond to the same moment of Time t constitute physical space $S(t)$ or isochrone. An isochrone is a three–dimensional surface in R^4 which is defined by the equation

$$\chi(x^1, x^2, x^3, x^4) = \chi(x) = t = constant.$$

The gradient of natural Time is the vector field $\mathbf{t}$ with the components

$$t^i = (\nabla\chi)^i = g^{ij}\partial_j\chi = g^{ij}t_j,$$

where g^{ij} are the contravariant components of the symmetric tensor field $g_{ij}(x)$ that defines a positive definite general covariant scalar product in the linear space of the vector fields

$$(u|v) = g_{ij}u^i v^j, \quad (u|u) = g_{ij}u^i u^j \geq 0, \quad (u|u) = 0$$

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if and only if $u^i = 0$. From the physical point of view g^{ij} is the potential of the gravitational field. The gradient of Time defines fundamental general covariant bilateral symmetry(discrete internal symmetry of left and right). The connection between the right–hand sided and left–hand sided vector fields is represented as a linear transformation (reflection)

$$\overline{v}^i = R^i_j v^j, \quad R^i_j = 2t^i t_j - \delta^i_j,$$

$$R^i_k R^k_j = \delta^i_j, \quad \mathrm{Det}(R^i_j) = -1.$$

The t^i and g_{ij} are invariant under reflection since

$$R^i_j t^j = t^i, \quad R^i_k R^j_l g_{ij} = g_{kl}.$$

We consider bilateral symmetry as fundamental discrete internal symmetry of Nature.

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The potential $\chi(x)$) of natural Time is a solution to the equation

$$g^{ij}\frac{\partial\chi}{\partial x^i}\frac{\partial\chi}{\partial x^j} = 1, \tag{1}$$

which can be considered as condition that singles out the temporal scalar field $\chi(x)$ from manifold of other one. The fundamental observation (from a physical point of view) reads that equation (1) has not only general solution but also a special solution known as the function of the geodesic distance. This means that there are two different Times in Nature and, hence, two different kinds of natural dynamical processes. An important example: under the condition $g_{ij} = \delta_{ij}$ equation (1) has the following two solutions:

$$\chi(x) = a_i x^i, \quad a_i a^i = 1,$$

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$$\chi(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}.$$

We consider the first solution as a natural time tightly connected with the translation motion and second one as dual time that is tightly connected with rotational motion. We consider known nuclei as different states of the system of fields which form one whole and live in the time connected with rotational motion. One of these key fields will be subject of our consideration.

Spin as emergent property

We put forward an idea that spin is emergent property of the system of fields and our task here is to built this system and get this surprise.
An emergent property is a characteristic of a complex system that is not present in its individual components but arises from their interactions and relationships.

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These properties are "more than the sum of their parts," meaning that they cannot be deduced by simply examining the qualities of the individual elements. Let us consider an ordered tuple of real fields:

$$\mathbf{A(x)} = \Big(a(x),\ a_i(x),\ a_{ij}(x),\ a_{ijk}(x),\ a_{ijkl}(x)\Big), \qquad (2)$$

$$i, j, k, l = 1, 2, 3, 4,$$

where a is the scalar field, a_i is the covector field and further antisymmetric tensor fields. Now we can question about the relations of these fields. These relations exist in a form of transformations which are defined by the antisymmetrical tensor fields of the type (p,q) $T^{i_1 \cdots i_p}_{j_1 \cdots j_q}$, $\quad (p, q = 0, 1, 2, 3, 4)$. as follows

$\overline{\mathbf{A}} = T\mathbf{A}(\mathbf{x})$, where $\overline{\mathbf{A}}$ is a new tuple

$$\overline{\mathbf{A}} = \left(\overline{a}, \overline{a}_i, \overline{a}_{ij}, \overline{a}_{ijk}, \overline{a}_{ijkl}\right), \quad (i, j, k, l = 1, 2, 3, 4)$$

and the element of this tuple are defined by the system of equations of the form

$$\overline{a} = Ta + T^m a_m + \frac{1}{2!} T^{mn} a_{mn} + \frac{1}{3!} T^{mnp} a_{mnp} + \frac{1}{4!} T^{mnpq} a_{mnp\ldots}$$

$$\overline{a}_i = T_i a + T_i^m a_m + \frac{1}{2!} T_i^{mn} a_{mn} + \frac{1}{3!} T_i^{mnp} a_{mnp} + \frac{1}{4!} T_i^{mnpq} a_{mn\ldots}$$

$$\overline{a}_{ij} = T_{ij} a + T_{ij}^m a_m + \frac{1}{2!} T_{ij}^{mn} a_{mn} + \frac{1}{3!} T_{ij}^{mnp} a_{mnp} + \frac{1}{4!} T_{ij}^{mnpq} a_{m\ldots}$$

$$\overline{a}_{ijk} = T_{ijk} a + T_{ijk}^m a_m + \frac{1}{2!} T_{ijk}^{mn} a_{mn} + \frac{1}{3!} T_{ijk}^{mnp} a_{mnp} + \frac{1}{4!} T_{ijk}^{mnpq} \ldots$$

$$\overline{a}_{ijkl} = T_{ijkl} a + T_{ijkl}^m a_m + \frac{1}{2!} T_{ijkl}^{mn} a_{mn} + \frac{1}{3!} T_{ijkl}^{mnp} a_{mnp} + \frac{1}{4!} T_{ijkl}^{mnp\ldots}$$

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The identical transformation E is defined by the following natural conditions

$$E^{i_1\cdots i_p}_{j_1\cdots j_q} = 0, \quad \text{if} \quad p \neq q, \quad E^{i_1\cdots i_p}_{j_1\cdots j_p} = \delta^{i_1\cdots i_p}_{j_1\cdots j_p},$$

where $\delta^{i_1\cdots i_p}_{j_1\cdots j_p}$ is the generalized Kronecker delta. These finite transformations general covariant internal symmetry form an algebraic structure which is called ring. But our interest to the infinitesimal transformations of this internal symmetry since it is well-known that such transformations have a fundamental meaning in the quantum theory where these operators are defined by the vector fields. To this end, let us consider the natural algebraic operators $\overline{\mathbf{A}} = \mathrm{E}_{\mathbf{v}}\mathbf{A}$ and $\overline{\mathbf{A}} = \mathrm{I}_{\mathbf{v}}\mathbf{A}$ defined by the vector field v^i as follows:

$$\mathrm{E}_{\mathbf{v}}\mathbf{A} = \Big(0,\ v_i a,\ v_{[i}a_{j]},\ v_{[i}a_{jk]},\ v_{[i}a_{jkl]}\Big),$$

$$\mathrm{I}_{\mathbf{v}}\mathbf{A} = \Big(v^m a_m,\ v^m a_{mi},\ v^m a_{mij},\ v^m a_{mijk},\ 0\Big),$$

where the square brackets $[\cdots]$ denote the process of alternation and $v_i = g_{ij}v^j$. To complete, let us introduce the numerical operator Z that is defined as follows

$$\mathrm{Z}\mathbf{A} = \Big(a, -a_i, a_{ij}, -a_{ijk}, a_{ijkl}\Big).$$

Now we introduce the fundamental operators:

$$Q_{\mathbf{v}} = \mathrm{E}_{\mathbf{v}} - \mathrm{I}_{\mathbf{v}}, \quad \widetilde{Q}_{\mathbf{v}} = (\mathrm{E}_{\mathbf{v}} + \mathrm{I}_{\mathbf{v}})\mathrm{Z}$$

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for which the very important relations

$$Q_{\mathbf{v}} Q_{\mathbf{w}} + Q_{\mathbf{w}} Q_{\mathbf{v}} = -2(\mathbf{v}|\mathbf{w}), \quad \widetilde{Q}_{\mathbf{v}} \widetilde{Q}_{\mathbf{w}} + \widetilde{Q}_{\mathbf{w}} \widetilde{Q}_{\mathbf{v}} = -2(\mathbf{v}|\mathbf{w}),$$
$$\widetilde{Q}_{\mathbf{v}} Q_{\mathbf{w}} = Q_{\mathbf{w}} \widetilde{Q}_{\mathbf{v}}$$

are fulfilled at any vector fields $\mathbf{v}$ and $\mathbf{w}$.
At last, let us consider three orthogonal and unit vector fields $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$, and put

$$S_1 = \frac{i}{2} Q_{\mathbf{v}\wedge\mathbf{w}}, \quad S_2 = \frac{i}{2} Q_{\mathbf{w}\wedge\mathbf{u}}, \quad S_3 = \frac{i}{2} Q_{\mathbf{u}\wedge\mathbf{v}},$$

$$\widetilde{S}_1 = \frac{i}{2} \widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}}, \quad \widetilde{S}_2 = \frac{i}{2} \widetilde{Q}_{\mathbf{w}\wedge\mathbf{u}}, \quad \widetilde{S}_3 = \frac{i}{2} \widetilde{Q}_{\mathbf{u}\wedge\mathbf{v}},$$

where $2Q_{\mathbf{v}\wedge\mathbf{w}} = Q_{\mathbf{v}} Q_{\mathbf{w}} - -Q_{\mathbf{w}} Q_{\mathbf{v}}$ and the same for the $\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}}$. One can verify that for these operators the following relations are fulfilled

$$[S_j, S_k] = i\varepsilon_{jkl} S_l, \quad S_1^2 + S_2^2 + S_3^2 = s(s+1) = \frac{3}{4}, \quad (3)$$

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$$[\widetilde{S}_j,\, \widetilde{S}_k] = i\varepsilon_{jkl}\widetilde{S}_l, \quad \widetilde{S}_1^2 + \widetilde{S}_2^2 + \widetilde{S}_3^2 = \frac{3}{4}, \quad [S_j,\, \widetilde{S}_k] = 0. \tag{4}$$

Thus, tuple

$$\mathbf{A}(\mathbf{x}) = \left(a, a_i, a_{ij}, a_{ijk}, a_{ijkl}\right) \tag{5}$$

shows that Spin is the field-theoretical realization of the concept of Top of the classical mechanics. That is why tuple can be called a general covariant spinor field or shortly spin field. The positive definite bilinear quadratic form $(\mathbf{A}|\mathbf{B})$ for the spin fields is defined as follows

$$(\mathbf{A}|\mathbf{B}) = \sum_{p=0}^{4} \frac{1}{p!} a_{i_1 \cdots i_p} b^{i_1 \cdots i_p}.$$

Operators $Q_{\mathbf{v}}$ and $\widetilde{Q}_{\mathbf{v}}$ are anti self-conjugated with respect to this scalar product since

$$(\mathbf{A}|Q_{\mathbf{v}}\mathbf{B}) = -(Q_{\mathbf{v}}\mathbf{A}|\mathbf{B}), \quad (\mathbf{A}|\widetilde{Q}_{\mathbf{v}}\mathbf{B}) = -(\widetilde{Q}_{\mathbf{v}}\mathbf{A}|\mathbf{B}).$$

Now our goal is to represent the general covariant Dirac equations for the spin field.

General Covariant Dirac Equations

As in the case of the electromagnetic field the Bilateral symmetry defined above provides the natural way to formulate the dynamical general covariant equations for the spin field. In our case we need to give an exact realization of the bilateral symmetry for the spin field. We define that left $\overline{\mathbf{A}}$ and right $\mathbf{A}$ spinor fields are connected by the transformation

$$\overline{\mathbf{A}} = R\mathbf{A} = -Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}\mathbf{A}, \quad R^2 = 1.$$

We consider real spinor field because there are no sufficient reason to introduce complex spin field. But simple algebra shows that since for the imaginary unit $i^2 = -1$ then

$$R == -Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}} = i^2 Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}} = (iQ_{\mathbf{t}})(i\widetilde{Q}_{\mathbf{t}}).$$

We conclude that for the complex spin fields $\mathbf{\Psi} = \mathbf{A} + i\mathbf{B}$ the bilateral symmetry can be realized as two different transformations of the form

$$\overline{\mathbf{\Psi}} = R\mathbf{\Psi} = iQ_{\mathbf{t}}\mathbf{\Psi},$$

$$R^2 = 1; \quad \overline{\mathbf{\Psi}} = R\mathbf{\Psi} = i\widetilde{Q}_{\mathbf{t}}\mathbf{\Psi}, \quad R^2 = 1.$$

In view of this, we at first formulate a dynamical theory of a real spin field and after that consider a dynamical theory of a complex spin field. To formulate the

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Lagrange formalism, we introduce the natural derivative of spin field as follows

$$\mathrm{D_e}\mathbf{A} = \Big(0,\, \partial_i a,\, \partial_i a_j - \partial_j a_i,\, \partial_{[i} a_{jk]},\, \partial_{[i} a_{jkl]}\Big).$$

We consider the Lagrangian

$$\mathcal{L}_{\mathbf{A}} = -\frac{1}{2}(Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}\mathbf{A}|\mathrm{D_e}\mathbf{A}) + \frac{m}{2}(Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}\mathbf{A}|\mathbf{A})$$

The Lagrangian $\mathcal{L}_{\mathbf{A}}$ is invariant with respect to the transformations

$$\overline{\mathbf{A}} = \frac{\Lambda}{|\Lambda|}\mathbf{A}, \quad \Lambda = \Lambda_1 + \Lambda_2\widetilde{Q}_{\mathbf{t}}, \quad |\Lambda|^2 = {\Lambda_1}^2 + {\Lambda_2}^2.$$

The operator $\widetilde{Q}_{\mathbf{t}}$, defined by the gradient of time, is the operator of the electric charge that provides the law of conservation of the electric charge .

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Further, we consider the antisymmetrical tensor field e_{ijkl} with the essential component $e_{1234} = \sqrt{g}$ and an operator $\widetilde{H}$ that is defined by this tensor field as follows:

$$\widetilde{H}\mathbf{A} =$$

$$(\frac{1}{(4)!}e_{ijkl}a^{ijkl}, -\frac{1}{(3)!}e_{ijkl}a^{jkl}, -\frac{1}{(2)!}e_{ijkl}a^{kl}, e_{ijkl}a^{l}, e_{ijkl}a)$$

and introduce the operator

$$\widetilde{J} = -\widetilde{H}Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}.$$

The following relations are fulfilled:

$$\widetilde{J}^2 = -1, \quad \widetilde{J}\widetilde{Q}_{\mathbf{t}} + \widetilde{Q}_{\mathbf{t}}\widetilde{J} = 0, \quad Q_{\mathbf{t}}\widetilde{J} - \widetilde{J}Q_{\mathbf{t}} = 0;$$

however, the Lagrangian $\mathcal{L}_{\mathbf{A}}$ is not invariant with respect to the transformations

$$\overline{\mathbf{A}} = \frac{\Lambda}{|\Lambda|}\mathbf{A}, \quad \Lambda = \Lambda_1 + \Lambda_2\widetilde{J}, \quad |\Lambda|^2 = {\Lambda_1}^2 + {\Lambda_2}^2,$$

but in the case of the complex spin field, it is the case. The fundamental Lagrangian of the complex spin field, which is associated with the new representation of bilateral symmetry, takes the form

$$\mathcal{L}_{\mathbf{\Psi}} = -\frac{i}{2}(Q_{\mathbf{t}}\overset{*}{\mathbf{\Psi}}|\mathrm{D_e}\mathbf{\Psi}) + \frac{i}{2}(Q_{\mathbf{t}}\mathbf{\Psi}|\mathrm{D_e}\overset{*}{\mathbf{\Psi}}) + im(Q_{\mathbf{t}}\overset{*}{\mathbf{\Psi}}|\mathbf{\Psi}).$$

This Lagrangian is invariant with respect to the transformations

$$\mathbf{\Psi}' = \Lambda/|\Lambda|\mathbf{\Psi},$$

where

$$\Lambda = \Lambda_1 + \Lambda_2\widetilde{I} + \Lambda_3\widetilde{J} + \Lambda_4\widetilde{K},\ |\Lambda|^2 = {\Lambda_1}^2 + {\Lambda_2}^2 + {\Lambda_3}^2 + {\Lambda_4}^2,$$

$$\widetilde{I} = \widetilde{Q}_{\mathbf{t}},\ \widetilde{K} = \widetilde{I}\widetilde{J}$$

The operator $\widetilde{J}$ is tightly connected with the concept of orientation. The parity non conservation gives right to

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call this operator the operator of the neutrino charge. The algebra of the self-conjugated operators of the electric charge operators $Q_e = i\widetilde{Q}_{\mathbf{t}}$ and the operator of neutrino charge. $Q_\nu = i\widetilde{J}$ is very simple

$$Q_e{}^2 = 1, \quad Q_\nu{}^2 = 1, \quad Q_e\, Q_\nu + Q_\nu\, Q_e = 0.$$

With this algebra we can give the following interpretation of the general covariant complex Dirac field. Let us introduce the projection operators:

$$P_+ = \frac{1}{2}(E + Q_e), \quad P_- = \frac{1}{2}(E - Q_e),$$

$$R_+ = \frac{1}{2}(E + Q_\nu), \quad R_- = \frac{1}{2}(E - Q_\nu).$$

For the identity operator we have the following expansions:

$$E = P_+ R_+ + P_+ R_- + P_- R_+ + P_- R_-,$$

$$E = R_+P_+ + R_+P_- + R_-P_+ + R_-P_-.$$

Hence, the complex spin field can be in for different states:

$$\mathbf{\Psi}_{++} = P_+R_+\mathbf{\Psi},\ \mathbf{\Psi}_{+-} = P_+R_-\mathbf{\Psi},$$

$$\mathbf{\Psi}_{-+} = P_-R_+\mathbf{\Psi},\ \mathbf{\Psi}_{--} = P_-R_-\mathbf{\Psi},$$

which are the eigenstates of the operator of the electric charge. In these states, the operator of the neutrino charge shows itself in the hidden form.
We evidently have as well four states

$$\mathbf{\Phi}_{++} = R_+P_+\mathbf{\Psi},\ \mathbf{\Phi}_{+-} = R_+P_-\mathbf{\Psi},$$

$$\mathbf{\Phi}_{-+} = R_-P_+\mathbf{\Psi},\ \mathbf{\Phi}_{--} = R_-P_-\mathbf{\Psi},$$

which are the eigenstates of the operator of the neutrino charge and where the electric charge shows itself in the

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hidden form. These states show itself, for example, in the form of proton and antiproton, neutron and antineutron.

Now we show the general covariant Dirac equation for the spin field in the form suitable for applications

$$\begin{aligned}(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\kappa &= \operatorname{div}\mathbf{K} - m\,\mu\\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\lambda &= \operatorname{div}\mathbf{L} - m\,\nu\\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mu &= \operatorname{div}\mathbf{M} + m\,\kappa\\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\nu &= \operatorname{div}\mathbf{N} + m\,\lambda\end{aligned} \tag{6}$$

$$\begin{aligned}(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{K} &= -\operatorname{rot}\mathbf{L} + \operatorname{grad}\kappa + m\,\mathbf{M}\\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{L} &= \operatorname{rot}\mathbf{K} + \operatorname{grad}\lambda + m\,\mathbf{N}\\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{M} &= \operatorname{rot}\mathbf{N} + \operatorname{grad}\mu - m\,\mathbf{K}\\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{N} &= -\operatorname{rot}\mathbf{M} + \operatorname{grad}\nu - m\,\mathbf{L}.\end{aligned} \tag{7}$$

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Conclusion.

Taking into account the duality of natural Time and the bilateral symmetry, we can distinguish four levels of organization of matter at the quantum level in the framework of the general covariant Dirac theory of the particles with Spin.

1 1. The real spin field and Time of translation motion. In this case the spin field carries the electric charge and can explain the so called "pairing of electrons" in the physics of atoms and molecules.

2 2. The complex spin field and Time of translation motion. In this case the spinor field carry the pseudo-charge, the electric charge and the neutrino charge. On the phenomenological level this state of affair corresponds to the so–called electroweak interactions but idea of generations is realized hear as

internal property. But in this case the number of generations is strictly fixed and should be equal four. Parity is not conserved.

3. The real spin field and Time of rotational motion. In this case in nuclear reactions parity is conserved.

4. The complex spin field and Time rotational motion. In this case the spin field carry the pseudo-charge, the electric charge and the neutrino charge. This level of organization corresponds to the physics of nuclei where parity is not conserved.

Time of rotational motion naturally explains the confinement and the baryon number conservation. Indeed, the baryon number conservation simply expresses in a symbolic form the existence of the second Time in the Nature and nothing more.

THANK YOU FOR ATTENTION