# Gluodynamics in accelerated frames using lattice simulation

Preliminary

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# Overview

# Motivations and Objectives: QCD matter in an accelerated frame

- Fundamental study of properties of $SU_C(3)$, QCD matter under **Acceleration** or **Gravity**
- Huge acceleration up to $\sim 1$ GeV in HICs.
  (Ref.1) D. Kharzeev and K. Tuchin, *Nucl. Phys. A 753, 316 (2005)*.
- Such systems are also relevant for Black hole physics.
- Properties of SU(3) Yang-Mill theory, thermalized at high temperature and in uniformly accelerated frame.
- Weak acceleration, $\alpha \ll \Lambda_{\text{QCD}}$, and far from Rindler horizon, $\alpha z < 1$. Beyond the scope of Unruh effect.
- Studied in Rindler coordinate, observer is in a co-accelerating frame of reference.

# Rindler geometry and modification of Gluon action

$$\text{Minkowski observer: } T(t) = \frac{c}{\alpha}\left(\sinh(\alpha t/c)\right)$$

$$Z(t) = \frac{c^2}{\alpha}\left(\cosh(\alpha t/c) - 1\right)$$

$$\boxed{Z = \sqrt{(c^2/\alpha)^2 + c^2 T^2} - c^2/\alpha}$$

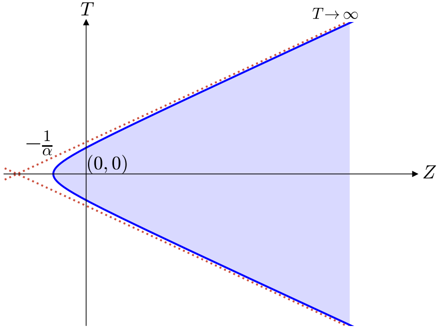
$$\text{Boost for co-acceleration: } V_{\text{inst}} = c\tanh(\alpha t/c)$$

$$\text{Interval: } ds^2 = -(1 + z\alpha/c)^2 c^2 dt^2 + dz^2 + dx^2 + dy^2$$

$$\text{Singularity/ Horizon at: } g_{tt} = -(1 + z\,\alpha/c^2)^2 = 0$$

$$\boxed{z_{\mathbf{H}} = -c^2/\alpha}$$

$$\text{local acceleration: } \alpha(z) = \frac{\alpha}{1 + \alpha z/c^2}$$




$$Z \to Z + c^2/\alpha$$

# Gluon action in Rindler coordinate (Euclidean)

Wick rotation: $t \to -i\tau$;  In natural unit:

$$\text{Metric: } \mathrm{diag}(g_E^{\mu\nu}) = \left[\frac{1}{(1+z\alpha)^2},\ 1,\ 1,\ 1\right]$$

$$S^E = \int d^4x \frac{1}{4g_{\text{YM}}^2}\sqrt{g^E}\, g_E^{\mu\alpha} g_E^{\nu\beta}\mathrm{Tr}(\mathrm{F}_{\mu\nu}\mathrm{F}_{\alpha\beta}) \tag{1}$$

$$S^E = \int d^4x \frac{1}{4g_{\text{YM}}^2}\Big\{\frac{1}{(1+az)}\mathrm{Tr}[\mathrm{F}_{\tau\mathrm{x}}^2 + \mathrm{F}_{\tau\mathrm{y}}^2 + \mathrm{F}_{\tau\mathrm{z}}^2] + (1+az)\ \mathrm{Tr}[\mathrm{F}_{\mathrm{xy}}^2 + \mathrm{F}_{\mathrm{xz}}^2 + \mathrm{F}_{\mathrm{yz}}^2]\Big\} \tag{2}$$

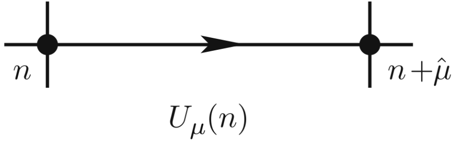
$$= \int d^4x \frac{1}{4g_{\text{YM}}^2}\left(\frac{1}{1+\alpha z}(\vec{E}^a)^2 + (1+\alpha z)(\vec{H}^a)^2\right) \tag{3}$$

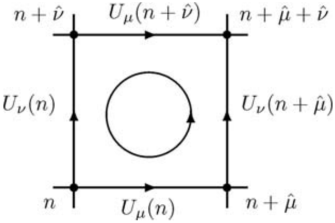
Recent lattice study implement acceleration by means of Tolman-Ehrenfest law.

Ref.(2) M. N. Chernodub *et al.*, *PRL 134, 111904 (2025)*

# Lattice setup and the observable

# Lattice setup: Discretization




- Continuous space-time $\rightarrow$ $4D$ Euclidean lattice with spacing '**a**'
- Link variable: $U_\mu(n) = \exp(iaA_\mu)$; $A_\mu \equiv$ Lattice gauge field.
- $U_\mu$'s are element of gauge group $SU(3)$
- Plaquette: $U_p = U_\mu(n)U_\nu(n+\hat{\mu}) + U_\mu(n+\hat{\nu})^\dagger + U_\nu(n)^\dagger$; smallest loop

$$S^E = \int d^4x \frac{1}{g_{YM}^2} \mathrm{Tr}[F_{\mu\nu}F_{\mu\nu}] \rightarrow \quad S^E = \sum_P \beta \left(1 - \frac{1}{N_c} \mathrm{Re}\,\mathrm{Tr}\, U_p\right)$$

# Action in Rindler coordinate and the observable

$$\text{Wilson Action: } S_E = \beta \sum_x \left[ \frac{1}{1+\alpha z} \sum_i \left( 1 - \frac{1}{N_c} \mathrm{Re\,Tr}\, \bar{U}_{0i} \right) + (1+\alpha z) \sum_{j>k} \left( 1 - \frac{1}{N_c} \mathrm{Re\,Tr}\, \bar{U}_{jk} \right) \right.$$

- Tree-level improved Symanzik action is used for measurement.
- Polyakov loop: Order parameter for confinement - deconfinement transition.

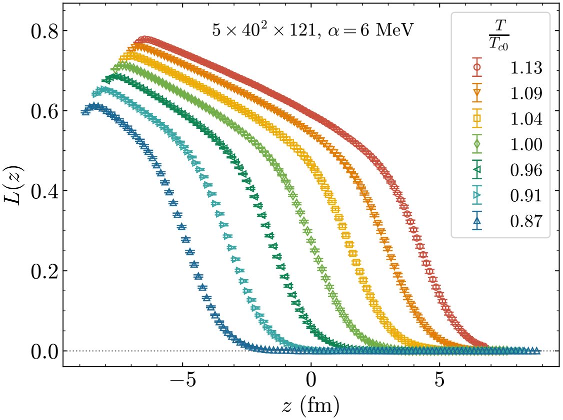
$$L(\boldsymbol{r}) = \mathrm{Tr}\, \mathcal{P} \exp \left( \oint_0^{1/T} d\tau A_4(\tau, \boldsymbol{r}) \right) \to L^b(\boldsymbol{r}) = \frac{1}{N_c} \mathrm{Tr} \left[ \prod_{\tau=0}^{N_t-1} U_4(\tau, \boldsymbol{r}) \right], \quad (4)$$

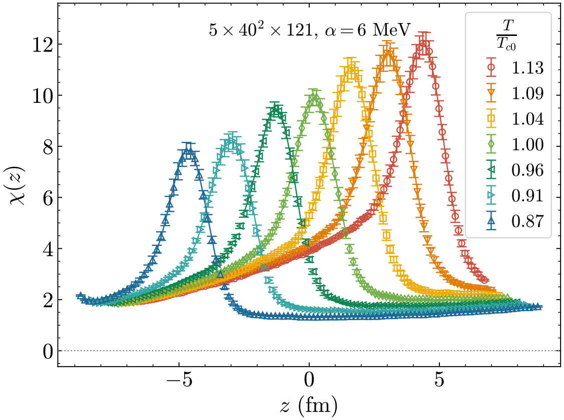
- $\langle L \rangle = \exp\left(-F_q/T\right)$, In confinement, $\langle L \rangle = 0$, $F_q \to \infty$

$$\boxed{\chi(z) = N_s^2 \left( \langle |L(z)|^2 \rangle - \langle |L(z)| \rangle^2 \right) \quad \text{where, } L(z) = \left( Z(g^2) \right)^{N_t(1+\alpha z)} L^b(z)} \quad (5)$$

# Results

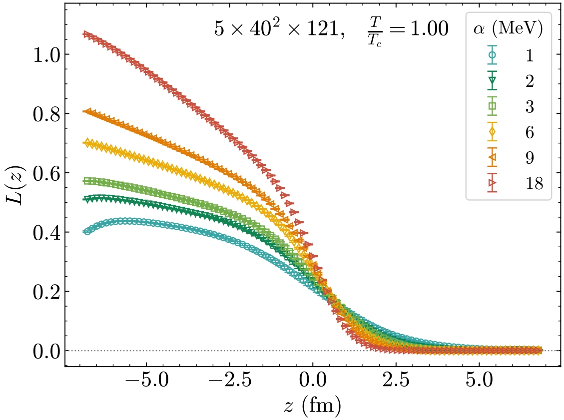
# Spatial transition

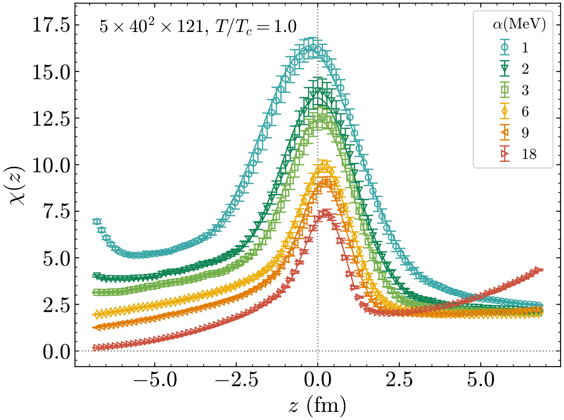


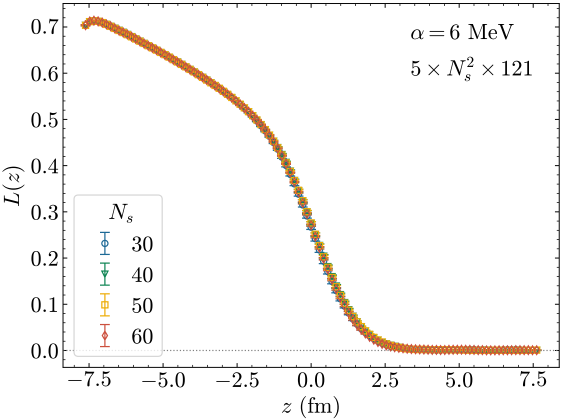

- Fixed $\alpha = 6$ MeV, vary $T$
- Spatial confinement to de-confinement transition.
- With increasing $T$, transition point shift in the direction of $\alpha$.

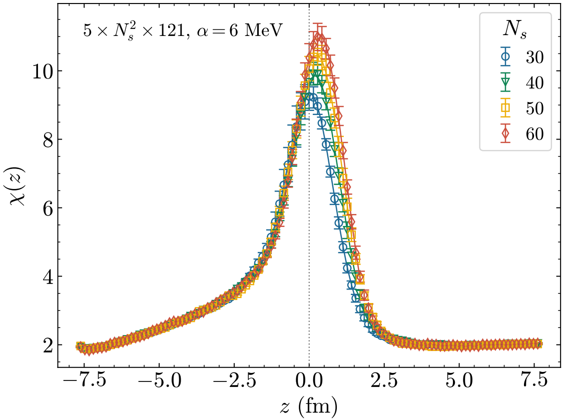
# Spatial crossover transition



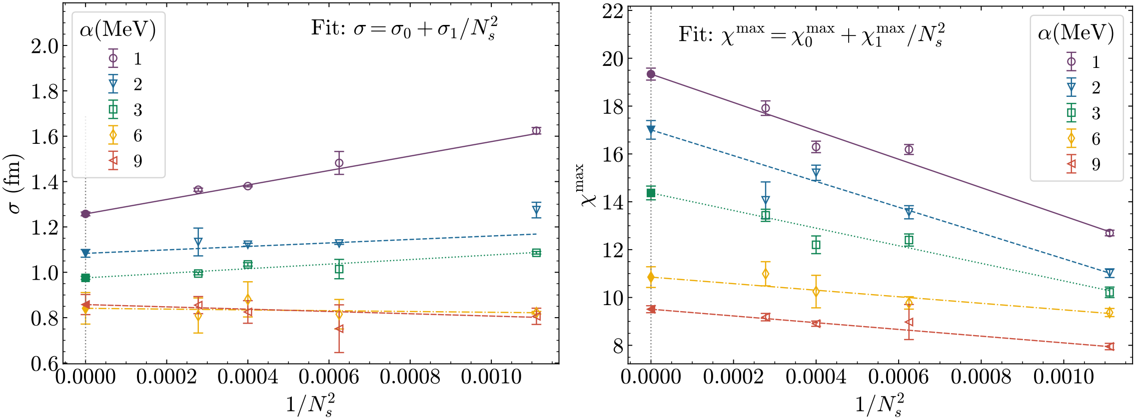

- Fixed $T = T_{c0}$, vary $\alpha$
- Position of transition almost independent of $\alpha$.
- Width and height decreases with temperature- suggesting crossover transition.




- Fixed $T = T_{c0}$ and $\alpha = 6$ MeV, vary transverse length $N_S$.
- Small, finite effect of $N_s$ is noticeable
- $\chi(z) = A_0 \exp\left(-\frac{(z-z_c)^2}{2\sigma^2}\right) + A_1 + A_2 z$

# Infinite volume limit




- Left: Width, Right: Height of transitions vs $1/N_s^2$ at $T = T_c$
- For $N_s \to \infty$, $\sigma = \sigma_0$, $\chi^{\max} \to \chi_0^{\max}$. Finite width and height: **crossover** transition.
- Width of transition decrease with $\alpha$, in contradiction with **Ref. (2)**

- Bare lattice results are $N_t$ dependent. For interpretation of correct physics we need $N_t \to \infty$, which is very costly.
- Here, we use the geometry to renormalize the polyakov loop:

Renormalized Polyakov: $\boxed{L = Z(\beta)^{N_t} L^b}$ (6)

**Gupta** et al. *Phys. Rev. D 77, 034503 (2008)*

Temperature, $T = \frac{1}{N_t a}$; $a =$ is lattice spacing

TE: $T(z) = \frac{T_0}{1 + \alpha z}$

$N_t \to N_t(1 + \alpha z)$

$$\boxed{L(z) = Z(\beta)^{N_t(1+\alpha z)} L(z)^b} \quad (7)$$

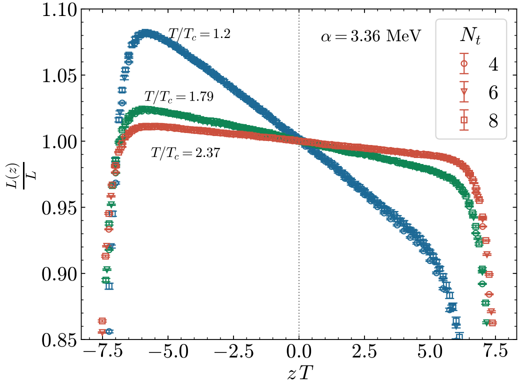
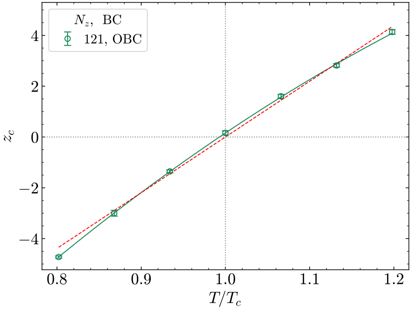


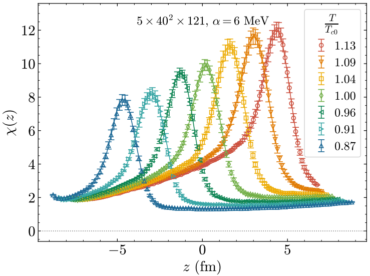

Figure: Ratio of renormalised spatial Polyakov in Rindler space to Polyakov of static homogeneous system

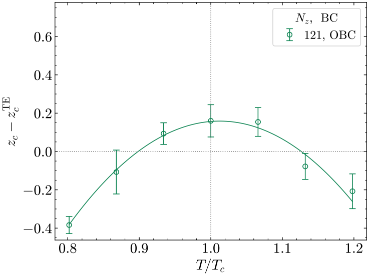
# Validity of Tolman-Ehrenfest law?

$$T(z) = \frac{T}{1+\alpha z}, \qquad (8)$$

$$\text{at, } z = z_c,\ T(z) = T_{c0},\ \boxed{z_c = \frac{1}{\alpha}\left(\frac{T}{T_{c0}} - 1\right)} \qquad (9)$$



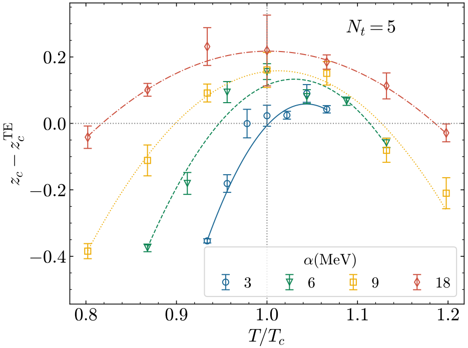




Fit: $z_c = \frac{K}{\alpha}\left(\frac{T}{T_{c0}} - C\right)$, $k = k_0 + k_1 \frac{T}{T_{c0}}$

# Validity of Tolman-Ehrenfest law?

- Small finite deviation from TE, $k \neq 1$
- Verified with different boundary conditions: Open and Dirichlet boundary conditions along $z$.
- Verified with different spatial length $N_z = 81, 121, 151, 181$
- Some geometric effect from observer's perspective ?



# Summary

- We study gluon plasma in accelerated frame
- Acceleration leads to crossover transition
- Width and height of transition decrease with increasing acceleration
- We renormalized the Polyakov loop
- Validity of TE law is not very clear yet.

Thank you for attention!