



ПРОЯВЛЕНИЕ АНИЗОТРОПНОЙ ГИДРОДИНАМИКИ НА ОСНОВЕ ПОСЛЕДНИХ ДАННЫХ ПО ИСПУСКНИЮ ПИОНОВ И КАОНОВ В СТОЛКНОВЕНИЯХ ТЯЖЕЛЫХ ИОНОВ В BM@N ЭКСПЕРИМЕНТЕ

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*MANIFESTATION OF ANISOTROPIC HYDRODYNAMICS
BASED ON THE LATEST DATA ON PION AND KAON
EMISSION IN HEAVY-ION COLLISIONS IN THE BM@N
EXPERIMENT*

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In the **BM@N** collaboration experiment, double differential distributions of transverse momentum and rapidity of positively charged pions and kaons have recently been found for collisions of ^{40}Ar ions with different targets at 3.2 GeV per nucleon [1]. The distributions of secondary particles in transverse momentum can be described using a Boltzmann distribution by fitting different temperatures at different rapidities [1]. One can say that the "transverse temperature" is approximately half the "longitudinal temperature"

In our opinion, this can be considered on the basis of anisotropic nonequilibrium hydrodynamics. In our work a quantum nonequilibrium hydrodynamic approach is developed, in which the nonequilibrium state of the nuclear medium is described by a joint solution of the kinetic equation and the hydrodynamic equations. We established a connection between the hydrodynamic equations and the effective Schrödinger and Klein-Fock-Gordon equations. In such an approach, many features of inclusive differential spectra of emitted secondary particles in heavy ion collisions can be described, including the cumulative effect. In this description, the selection of the resulting hot spot is significant and an advantage was found in comparison with standard cascade calculations in describing the available experimental data.

In our nonequilibrium hydrodynamic approach applied to the data of [1], it turns out to be possible to represent the nucleon distribution function in the form depending on an ellipsoid in momentum space with semi-axes proportional to the longitudinal and transverse temperatures in the proper frame of reference. Thus, in accordance with the solution of the kinetic equation, the effective temperature can depend on the particle emission angle. Such anisotropic hydrodynamics with the selection of a hot spot is in agreement with the experimental data [1] for pions and kaons. Calculations using cascade models, as shown in [1], give an overestimated yield of kaons. The first comparison of our approach with experimental data obtained for the differential yields of protons, deuterons and tritons in the BM@N collaboration experiment with the same nuclei as in [1] turned out to be quite successful.

"... важно сконцентрировать имеющиеся ресурсы на основных прорывных направлениях..." В.В. Путин.

The Schrödinger equation and Hydrodynamics

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + U\Psi \quad \Psi = \Phi \exp(imS / \hbar)$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \rho \vec{v} = 0$$

$$\frac{\partial m\rho \vec{v}}{\partial t} + \vec{\nabla} (m\rho \vec{v}) \vec{v} = -\rho \vec{\nabla} U - \rho \vec{\nabla} \left(-\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$$

$$\rho = \Phi^2 \quad \vec{v} = \vec{\nabla} S$$

- ✓ *To take into account dissipation in the process of collisions of complex systems, in addition to equations (2)-(3), a dissipative function related to the temperature of the system should be introduced into these equations*



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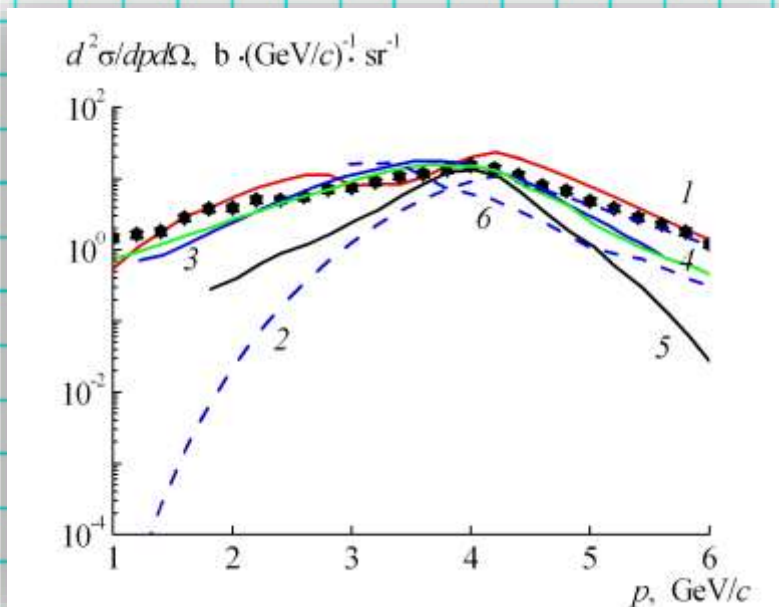


GRIDNEV Konstantin Alexandrovich
1938-2015

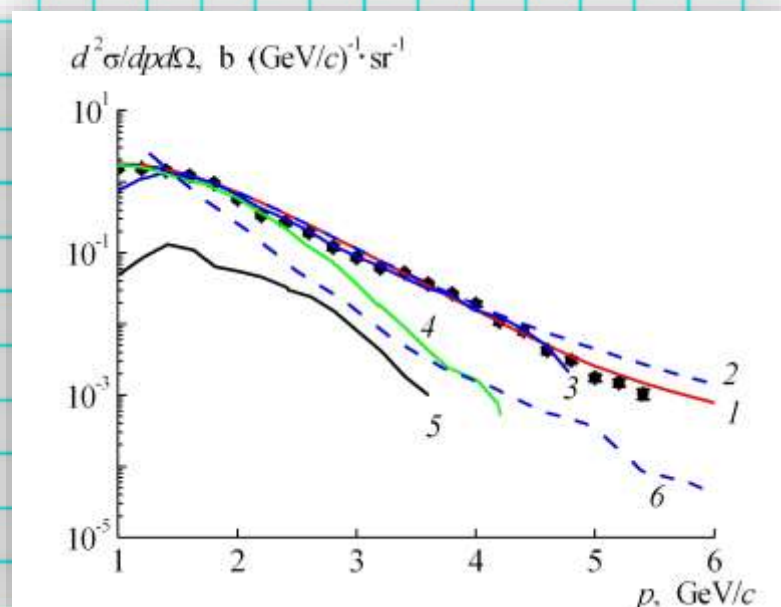
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$^{12}\text{C}+^9\text{Be}$ 3.2 GeV/nucl.
(protons) 3,5⁰



$^{12}\text{C}+^9\text{Be}$ 3.2 GeV/nucl.
(negative pions) 3,5⁰



Б.М. Абрамов, М. Базнат, Ю.А. Бородин, ..., В.В. Куликов, и др., ЯФ **84**, 331 (2021)

1,2 Our approach
3 Sobolevsky
4 Mashnik
5 GEANT4
6 HSD

EQUATIONS OF QUANTUM RELATIVISTIC HYDRODYNAMICS

The equations of relativistic hydrodynamics can be related to the effective Klein-Fock-Gordon equation.

It contains the effective mean field $U_f = mc^2 / (1 - U / mc^2)^2 / 2 - mc^2 / 2$, where $U(\rho)$ is the effective density-dependent mean nucleon potential, m is the nucleon mass, c is the speed of light, \hbar is the Planck constant, and also the dissipative term $J = \kappa \frac{I}{\rho}$ where I is the thermal energy density, $\kappa = 3q + \frac{5}{3}(1 - q)$ is the adiabatic index.

$$-\frac{\partial^2 \Psi}{\partial x^0 \partial x_0} - \frac{\partial^2 \Psi}{\partial x^l \partial x_l} = \frac{2m}{\hbar^2} (U_f + J + mc^2 / 2) \Psi$$

In addition to the Klein-Gordon equation with the dissipative term, or the equations of hydrodynamics (the continuity equation and the Euler equation), an equation for the thermal term should be added

$$\frac{\partial E}{\partial t} + \frac{\partial E v_l}{\partial x_l} = -\nabla^l (v_l P), \quad E = \frac{\rho mc^2 + e + P}{1 - (v/c)^2} - P, \quad e = e_{kin} + e_{int}, \quad e_{int} = \int_0^\rho U_f d\rho, \quad P = -\frac{d(e/\rho)}{d(1/\rho)} \quad \frac{df}{dt} = \frac{f_0 - f}{\tau}$$

From first equation follow the equations of relativistic quantum hydrodynamics, containing the Madelung quantum potential

$$\frac{\partial \rho u_0}{\partial x_0} + \frac{\partial \rho u_l}{\partial x_l} = 0$$

$$\frac{\partial (mc^2 \rho u_0 u_k)}{\partial x_0} + \frac{\partial (mc^2 \rho u_k u_l)}{\partial x_l} = -\rho \frac{\partial (U_f + J)}{\partial x_k} - \rho \frac{\partial}{\partial x_k} \frac{\hbar^2}{2m} \left(\frac{\partial^2 \rho^{1/2}}{\rho^{1/2} c^2 \partial t^2} - \frac{\Delta \rho^{1/2}}{\rho^{1/2}} \right)$$

EQUATIONS OF ANISOTROPIC HYDRODYNAMICS

The equations of anisotropic hydrodynamics can be obtained on the basis of our quantum hydrodynamic approach, associated with the Klein-Fock-Gordon equation. The nonequilibrium state of a nuclear system is described as a result of solving the kinetic equation

$$\frac{df}{dt} = \frac{f_0 - f}{\tau} + v(f_0 - f)^2, \quad (1)$$

where $f(r, p, t)$ is the sought nucleon distribution function, f_0 is the equilibrium distribution function, τ is the relaxation time, v is the parameter in the expansion of the collision term to the second order for the deviation of f from f_0 . If we seek a solution to equation (1) in the form of a function that depends only on time $f=f(t)$, then we find the solution

$$f(t) = f_1 q + f_0(1 - q) \quad (2)$$

where $q = \frac{2e^{t/\tau}}{e^{t/\tau} + 1}$ is the relaxation factor, f_1 is the initial distribution function, and here, $v(f_1 - f_0) = \frac{1}{2\tau}$

which follows from the solution of equation (1) in the form (2). From here, for the single-particle excitation energy we find

$$E_i = E_{\parallel} q + E_{\perp} (1 - q) \quad (3)$$

where E_{\parallel}, E_{\perp} are the longitudinal and transverse energies, respectively. If we put $E_i = E_{\parallel} + E_{\perp}$ then from (3) we can express $E_{\perp} = E_{\parallel} \frac{q-1}{q}$. That is, at $t=0$ $E_{\perp} = 0$, and at $t \rightarrow \tau$ $E_{\perp} = E_{\parallel} / 2$. Also, the Boltzmann temperatures T_{\parallel}, T_{\perp} are related by the relation

$$T_{\perp} = T_{\parallel} \frac{q-1}{q} \quad (4)$$

when $T_{\perp} = T_{\parallel} / 2$ at $t \rightarrow \tau$, which is observed in the experiment [1].

HYDRODYNAMIC MODEL OF FRAGMENTATION

- L.D. Landau drew attention to the propagation of a shock wave as the only possibility of realizing the first stage of the process of collisions of protons and nuclei with nuclei, and proposed a one-dimensional solution of the hydrodynamic equations for the subsequent adiabatic stage of expansion of the system during the collision of high-energy protons. In our hydrodynamic approach, after the passage of a shock wave with a changing front, a hot spot is formed – a source of high-energy particles. In the

approximation $\rho = \rho(t)$ and pressure $P = P(t)$ for the Hubble law for 4 velocities $c\mathbf{u}(\mathbf{r}, t) = \frac{\mathbf{v}}{\sqrt{1-(v/c)^2}} = H\mathbf{r}$ where $H(t) = d(R)/dt / R$ the hydrodynamic equations are reduced to the equations

$$\frac{d\rho}{dt} + 3H\rho = 0, \quad \frac{dH}{dt} + H^2 = 0 \quad \text{these equations have a simple solution} \quad H = 1/t \quad \rho = \rho_1 t_0^3 / t^3$$

Taking into account equation (2), we have for adiabatic expansion $I = I_1 \left(\frac{\rho}{\rho_1} \right)^\kappa$

Using this approximate solution in the next order approximation, we can obtain the linear equations

$$\frac{\partial \delta}{\partial t} + H\mathbf{r}\nabla\delta + \nabla c\mathbf{u} \ll 0,$$

$$\frac{\partial c\mathbf{u}}{\partial t} + Hc\mathbf{u} + H\mathbf{r}\nabla c\mathbf{u} = -c_s^2 \nabla\delta,$$

$$\delta = \frac{\delta\rho}{\rho}, \quad c_s^2 = \frac{dP}{\rho d\rho}$$

An approximate analytical solution

- The variables are separated here $\delta = \delta_r(r)\delta_1(t)$, and for the time-dependent $\delta_1(t)$, we obtain a linear differential equation of the second order

$$\frac{d^2\delta_1}{dt^2} + \frac{2}{t} \frac{d\delta_1}{dt} - \frac{c_s^2}{a^2} \delta_1 = 0$$

That is, for $\delta_1(t)$ we find an approximate analytical solution

$$\delta_1 = \exp(-\ln(t/t_0)) / \sqrt{K(t)} (C_1 \exp(\int_{t_0}^t K(t) dt) + C_2 \exp(-\int_{t_0}^t K(t) dt)), \quad K(t) = \sqrt{1/t^2 + (c_s/a)^2}$$

From this formula we can see the first term growing exponentially with time t

and the onset of fragmentation for an expanding fireball – hot spot at $t > 2t_0 = 2a/c_s$

This hydrodynamic consideration has an analogy with the expanding early Universe during the formation of stars and galaxies [35] (the well-known Jeans instability).

If we take into account the quantum terms in equation, this leads to an additional term in equation:

$$\frac{\hbar^2}{2m^2 a^4} \delta_1 \quad \text{on its left-hand side, then there will be} \quad K(t) = \sqrt{1/t^2 + (c_s/a)^2 - 1/2(\hbar/ma^2)^2}$$

$a < \hbar/\sqrt{2}mc_s \approx$ fm the expression under the square root for K becomes negative

i.e. there is no growing solution with increasing time

This means that for small hot spot radii, the quantum terms in hydrodynamics lead to stability with respect to fragmentation

NUMERICAL IMPLEMENTATION. COMPARISON WITH EXPERIMENTAL DATA

In the previous section, an approximate analytical solution for the onset of fragmentation in the hydrodynamic approach was obtained. In our approach, after the formation of a hot spot and its subsequent hydrodynamic evolution, fragmentation occurs when the system reaches a critical density, determined from the condition $\frac{\partial U}{\partial \rho} = 0$.

In this case, the double differential cross section in the $A+B \rightarrow f+X$ reaction has the form

$$E_f \frac{d^2 \sigma}{p_f^2 dp_f d\Omega} = \frac{2\pi(2s+1)}{(2\pi\hbar)^3} \int G b db \gamma N(E - \mathbf{p}\mathbf{v}) F \exp\left(\gamma \frac{\mu N + vZ - N(E - \mathbf{p}\mathbf{v})}{T}\right) d\mathbf{r}$$

$$F = \frac{(2\pi\hbar)^3}{(2\pi\sigma_N^2)^{3/2}} \exp\left(-\frac{(\mathbf{p}_f - \mathbf{p}_{0f})^2}{2\sigma_N^2}\right) \rho^*$$

$$Y_{N,Z} \sim \left(\frac{2\pi\hbar^2}{mT}\right)^{-3/2} N^{3/2} \exp\left(\frac{\mu N + vZ}{T}\right)$$

$$\sigma_N^2 = \sigma_0^2 \frac{N(A - N(1 - U/mc^2))}{A - 1}$$

DOUBLE DIFFERENTIAL DISTRIBUTION. COMPARISON WITH EXPERIMENTAL DATA

We compared our calculations with experimental data obtained in the work [1] of the **BM@N** collaboration for double differential distributions in transverse momentum and rapidity of positively charged pions and kaons in reactions of incident ^{40}Ar nuclei at an energy of 3.2 GeV per nucleon with different target nuclei. The differential distribution in transverse momentum and rapidity has the form (b is the impact parameter)

$$\frac{dN}{dy d^2 p_t} = \frac{m_t}{(2\pi\hbar)^3} \int 2\pi b db G(b) ch(y - y^*) f(y - y^*, p_t, r, t) dr, \quad (5)$$

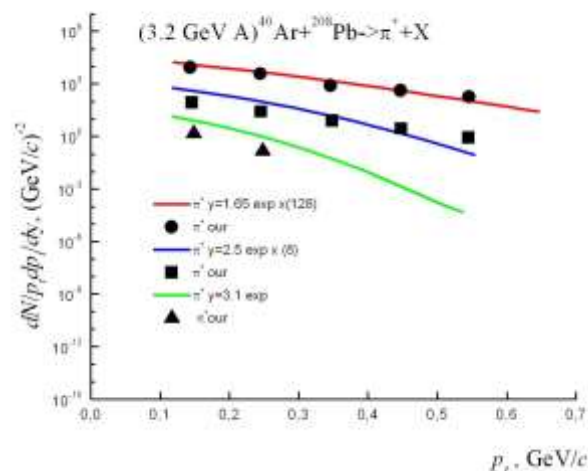
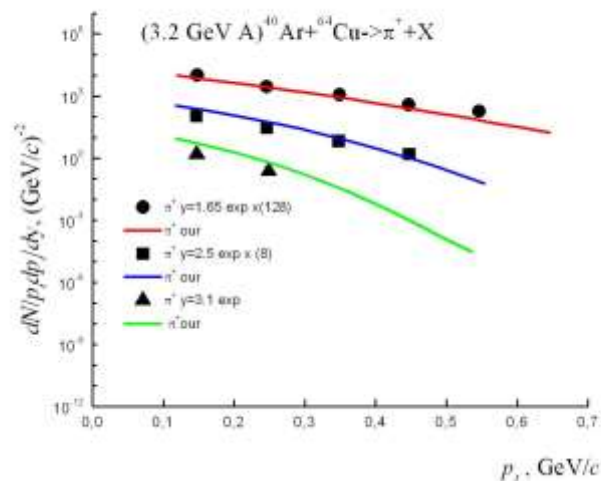
where the meson distribution function, neglecting the nonequilibrium component, has the form of a Bose-Einstein distribution

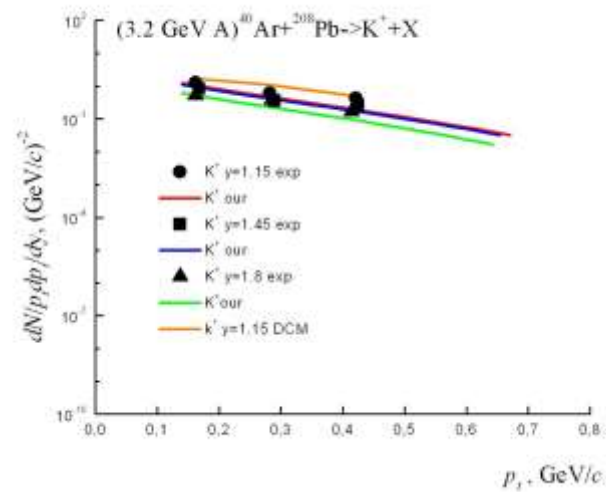
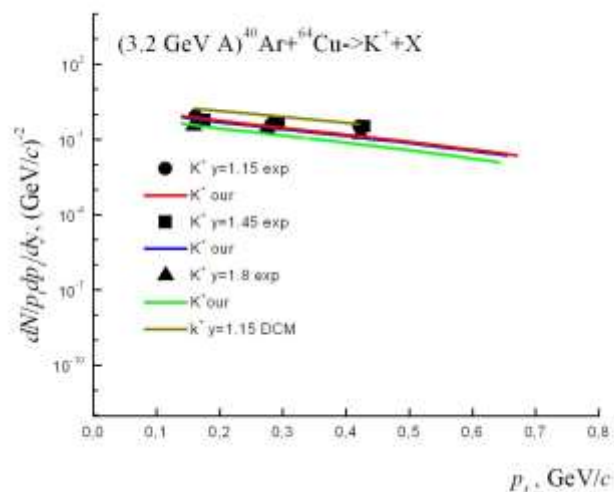
$$f(y, p_t, r, t) = g \left[\exp \left(\frac{m_t}{T} ch(y - y^*) \right) - 1 \right]^{-1} \quad (6)$$

m_t is the transverse mass, y is rapidity of the emitted particle, y^* is rapidity of the expanding substance element moving with velocity at the moment of time t of reaching the critical density, found from the condition $\frac{dW}{dp} = 0$, $T(r, t, y)$ is effective temperature, depending on the angle of particle emission (velocity) and the coordinate of the substance element. Here the spin factor $g=1$, the chemical potential of pions and positively charged kaons was assumed to be zero, the factor $G(b) = \sigma_t / \sigma_g / (\pi(b_{max})^2)$ takes into account the difference between the total cross section of hot spot formation and the geometric one and identifies the same centrality for all nuclei under consideration.

COMPARISON WITH EXPERIMENTAL DATA

We compared the transverse momentum distributions (5)-(6) with temperature (4) for pions in the reactions $^{40}\text{Ar} + ^{64}\text{Cu} \rightarrow \pi^+ + X$ (Fig. 1) and $^{40}\text{Ar} + ^{208}\text{Pb} \rightarrow \pi^+ + X$ (Fig. 2) at an energy of 3.2 GeV per nucleon for incident argon nuclei with the corresponding experimental data obtained in [1]. Here $T_{ij} = 120$ MeV, in the calculations, the chemical potential of pions was assumed to be zero, since the number of pions is not specified. The calculations performed at rapidities $y = 1.65, 2.5, 3.1$ for two reactions are in agreement with the experimental data [1], which are shown in the figures as dots.

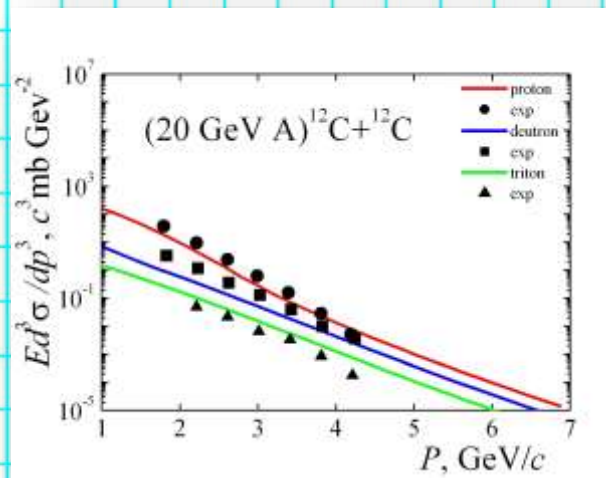




- In Figs. 3, 4 the distributions of the transverse momentum of kaons calculated by formulas (5)-(6) are compared with the experimental data [1] in the reactions $^{40}\text{Ar} + ^{64}\text{Cu} \rightarrow \text{K}^+ + \text{X}$ (Fig. 3) and $^{40}\text{Ar} + ^{208}\text{Pb} \rightarrow \text{K}^+ + \text{X}$ (Fig. 4) at an energy of 3.2 GeV per nucleon for incident argon nuclei at rapidities $y=1.15, 1.45$ and 1.8 . The temperature T_{ij} is 120 MeV, the chemical potential is also small here. It can be seen that the calculation with an effective temperature changing with the angle (rapidity) satisfactorily reproduces the experimental data [1] (points). The calculations for kaons carried out in [1] using cascade models somewhat overestimate the kaon yields compared to these experimental data. Although there are few experimental points.

Cumulative fragments (IHEP)

Figure 5 shows the invariant double differential cross sections for the emission of protons, deuterons and tritons at an angle of 40° in the reaction $^{12}\text{C}+^{12}\text{C} \rightarrow f+X$ at an energy of 19.6 GeV per nucleon for incident ^{12}C nuclei. The results of calculations using formula (15) (red line – protons, blue line – deuterons, green line – tritons), dots – experimental data from work [30], obtained at the U-70 accelerator (IHEP). One can see agreement with the experiment in the emission spectra of these fragments and the presence of scaling – the same slope of these curves. In this case, good agreement was obtained for tritons, for deuterons there is an agreement closer to the end of the experimental spectrum. These are the spectra of cumulative fragments, the average temperature in our approach is 150 MeV, as in our previous work [29], devoted to the description of the emission of cumulative protons, pions, kaons and antiprotons at the U-70 accelerator at the same energy of incident carbon nuclei.

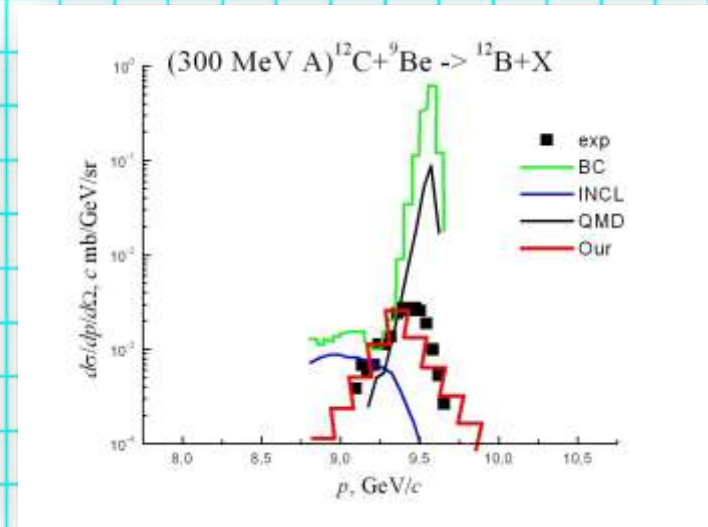
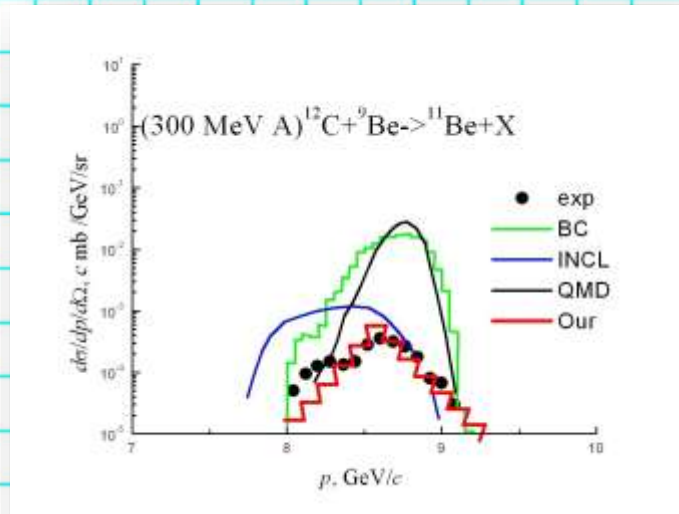


FRAGM(ITEP)

Fig. 6 shows the double differential cross sections of the yield of ^{11}Be fragments emitted at an angle of 3.5° in the reaction $^{12}\text{C}+^9\text{Be} \rightarrow ^{11}\text{Be}+X$ at an energy of 300 MeV per nucleon for incident carbon nuclei .

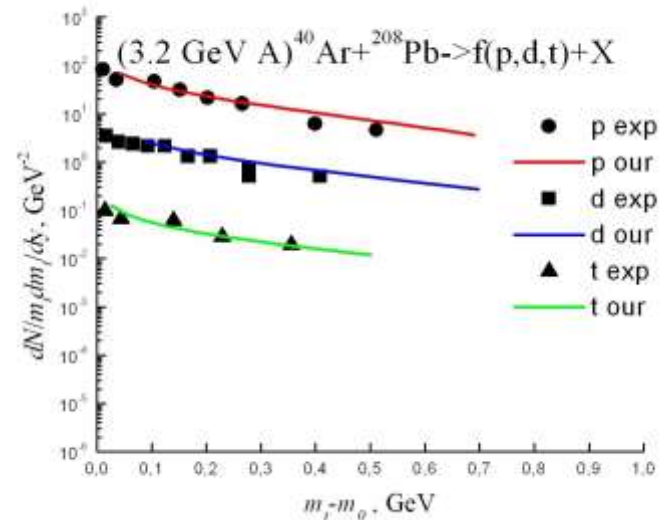
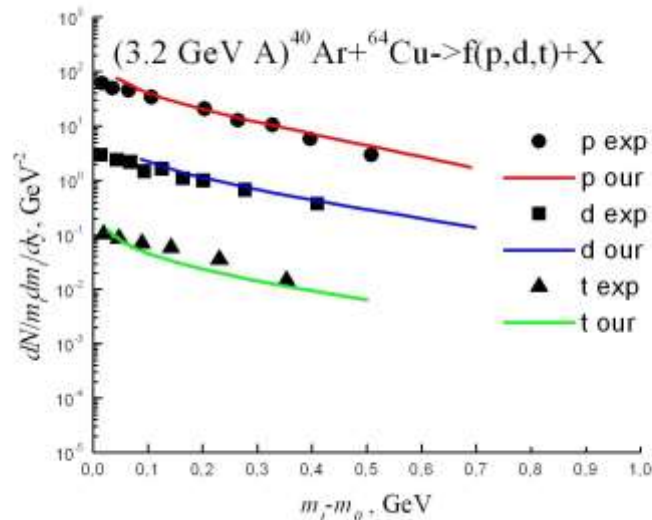
Fig. 7 shows the double differential cross sections of the yield of ^{12}B for the incident carbon nuclei.

The dots correspond to the experimental data obtained in the FRAGM (ITEP) experiment [24] Our calculated curves red and blue in these figures correspond to the average 40 MeV temperature, and red curves were obtained in the calculation taking into account the quantum terms, as in our works, and blue curves – without taking these terms into account. As can be seen from Figs. 6 and 7, there is agreement between our calculations and the available experimental data. The calculated curves obtained in work [24] for cascade models and for quantum molecular dynamics [16] differ significantly from each other and from these experimental data.



FRAGMENTATION(BM@N)

- **Fig. 8** shows the double differential cross sections of the yield of p,d,t fragments emitted at the rapidity of $y=1.4$ in the reaction $^{40}\text{Ar}+^{64}\text{Cu} \rightarrow f(p,d,t)+X$ at an energy of 3.2 GeV per nucleon for incident argon nuclei **Fig. 9** shows the double differential cross sections of the yield of p,d,t for the incident argon nuclei in the reaction $^{40}\text{Ar}+^{208}\text{Pb} \rightarrow f(p,d,t)+X$ at the centrality of 40%.
- The dots correspond to the experimental data obtained in the BM@N (NICA) experiment As can be seen from Figs. 4 and 5, there is agreement between our calculations and the available experimental data.



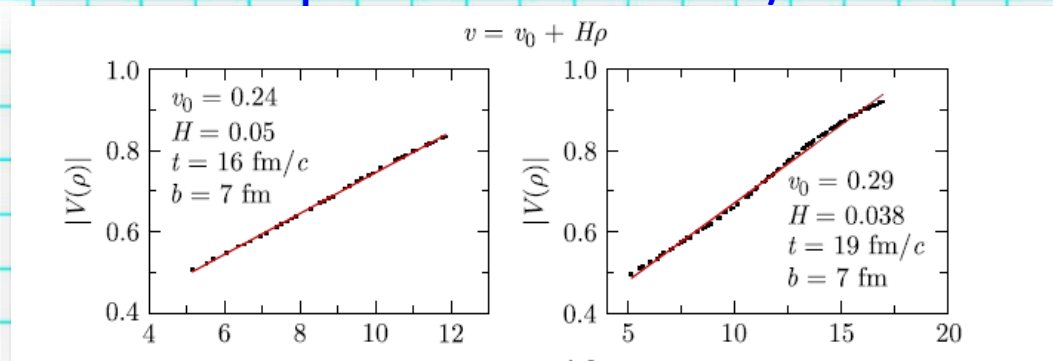
ABOUT HUBBLE'S LAW AND POLARIZATION

Equations admit a simple solution - Hubble's law with the Hubble constant

$$H = 1/t \quad \text{or} \quad H = 1/(t + t_0)$$

For $H = 1/(t + t_0)$ in the work [Baznat, Teryaev and Zinchenko] the results were obtained by the PHSD model for the collision of Au+Au nuclei at the energy of $\sqrt{s} = 7.7 \text{ GeV}$. For the impact parameter $b = 7 \text{ fm}$ it turned out that $H = 0.05 \text{ fm}^{-1}$ at $t = 16 \text{ fm}/c$ and $H = 0.04 \text{ fm}^{-1}$ at $t = 19 \text{ fm}/c$. This agrees with the experimental data of STAR (RHIC).

That is, the Hubble law is approximately satisfied for peripheral collisions. It turns out better when choosing $t_0 = -4 \text{ fm}/c$. Such a choice of t_0 means, in our opinion, that the agreement with the Hubble law and hydrodynamics is achieved at earlier times than in the PHSD model, i.e. when a hot spot is formed after the initial shock-wave compression. In this case, for everything to converge correctly, for example for the speed of light, the Hubble constant should be equal to 0.08 fm^{-1} at $t = 16 \text{ fm}/c$.



Dependence of particle velocity on transverse radius $\rho = \sqrt{x^2 + y^2}$ at $z = 0$ and energy $\sqrt{s} = 7.7 \text{ GeV}$ with impact parameter $b = 7 \text{ fm}$

On the polarization of Λ -hyperons in Au+Au collisions at energy of $\sqrt{s} = 7.7 \text{ GeV}$

In equations, we can distinguish the radial Hubble velocity component $v_r / \sqrt{1 - (v/c)^2} = Hr$ and the tangential v_τ one during the expansion of the hot spot. Neglecting the expansion, we obtain

$$\left(v_\tau / \sqrt{1 - (v/c)^2} \right)^2 / r = \frac{c_s^2}{\rho} \frac{\partial \rho}{\partial r}$$

From equation (2) in this approximation for the hydrodynamic equations, we can obtain

$$v_\tau / \sqrt{1 - (v/c)^2} = \sqrt{3} \tilde{n}_s$$

From here, moving to the reduced thermodynamic velocity, we find the polarization for the impact parameter $b=8 \text{ fm}$ and temperature $T=150 \text{ MeV}$

$$P \approx \frac{\hbar \omega}{2} = \hbar v_\tau / \sqrt{1 - (v/c)^2} / 2b \approx \frac{\hbar \sqrt{3} c_s}{2bT} \approx 5\%$$

This is in agreement with the experimental data of STAR We can calculate for taking into account the Hubble expansion. In this case, we obtain

$$v_\tau / \sqrt{1 - (v/c)^2} = \sqrt{3} c_s \left(\frac{1}{6} + \sqrt{1 + \frac{1}{36}} \right)$$

which leads to $P \approx 6\%$ in agreement with the experimental data

Conclusions

- Thus, in developing the nonequilibrium hydrodynamic approach to describing the yields of fragments, light and close in mass to the incident nucleus, agreement has been achieved with the available experimental data obtained in the IHEP (Serpukhov) experiments at an energy of 19.6 GeV per nucleon and FRAGM (ITEP, Moscow) at an energy of 300 MeV per nucleon for incident carbon nuclei.
- Thus, in this paper we have further developed the nonequilibrium anisotropic hydrodynamics model for describing high-energy heavy ion collisions. As a result, we have explained the first experimental results of the BM@N collaboration at the NICA accelerator in collisions of argon nuclei with various targets at an energy of 3.2 GeV per nucleon. The transverse momentum distributions for different rapidities for pions and kaons reveal a change in the effective temperature with the emission angle of secondary particles in accordance with the features of our anisotropic hydrodynamics.

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THANK YOU !

