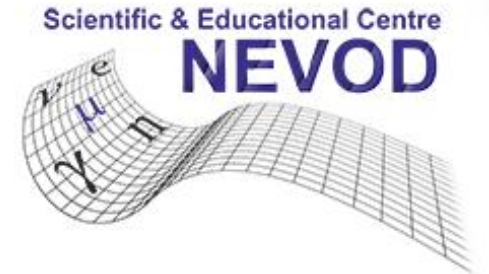
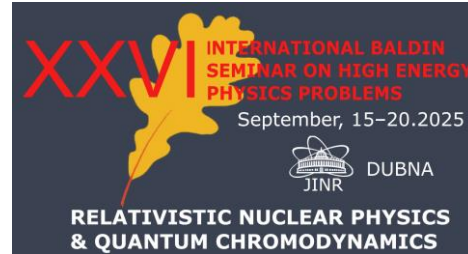
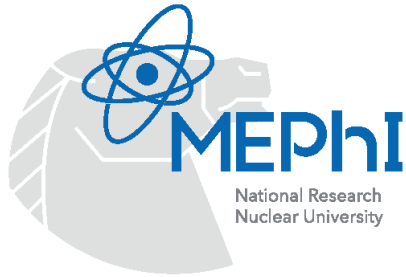


The XXVIth International Baldin Seminar on High Energy Physics Problems
“Relativistic Nuclear Physics and Quantum Chromodynamics”
(ISHEPP-2025)



Particles production in ATROPOS string fragmentation model

R. V. Nikolaenko

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute)

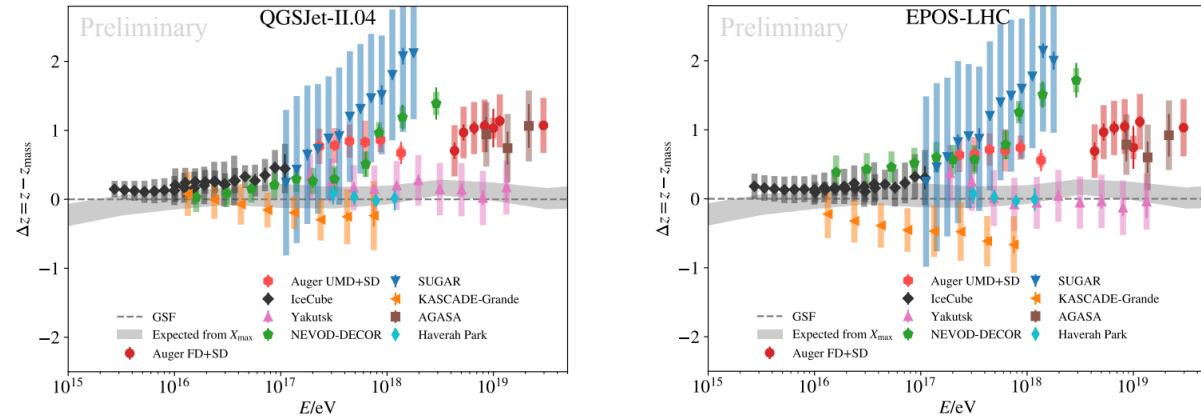
rvnikolaenko@mephi.ru

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15-20 September, 2025

Introduction: a role of hadronization in EAS hadronic interactions physics

The “**Muon Puzzle**”: an excess of muons in EAS induced by UHECR in comparison to predictions from simulations

From J.C. Arteaga-Velázquez, ICRC-2023



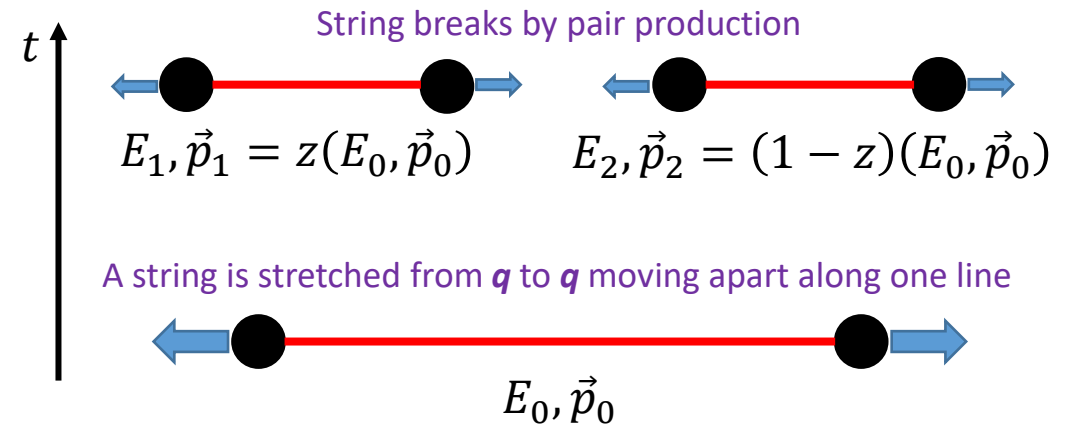
- Need to increase muon number, but not e^\pm, γ
 - An important parameter of hadronic interactions:

$$R = \frac{\langle E_{e/m} \rangle}{\langle E_{\text{hadr}} \rangle}$$

- To decrease R :
 - Enhance heavier quarks production
 - Enhance baryon/meson resonances production
 - Suppress leading π^0 formation

- Modify **hadronization** process, but how?

- Usually, string fragmentation is seen *universal* for all colliding systems and energies:



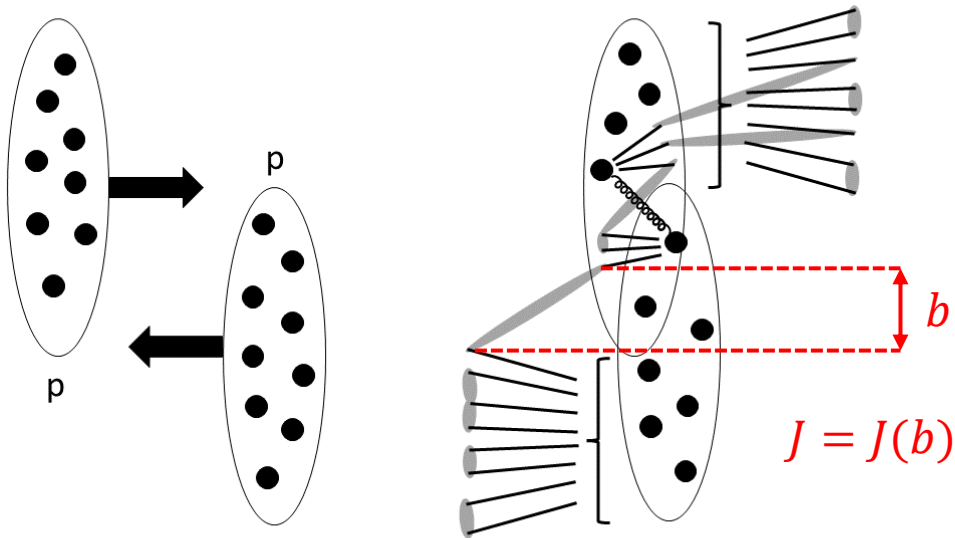
- z is sampled universally for all collisions

- *Break* string fragmentation universality:
 - Collective effects (used in EPOS core-corona approach)

➤ Consider **angular momentum** + more general rules for string-to-hadron transition

General idea

- Add angular momentum of the string to account for the impact parameter of color-connected partons

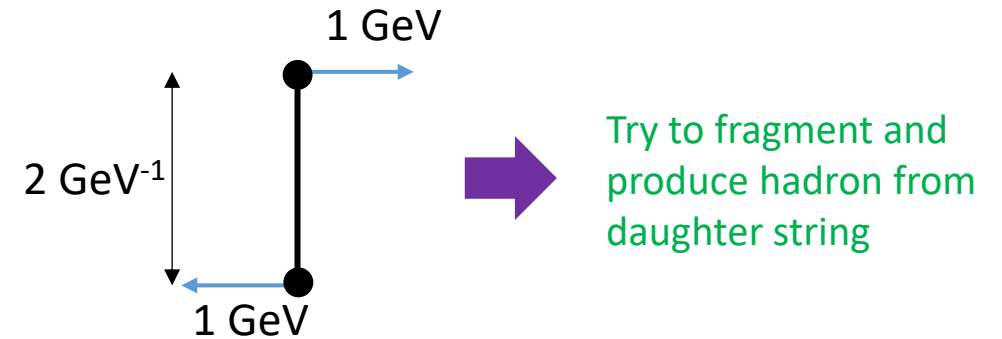


- A difference between colliding systems emerges:

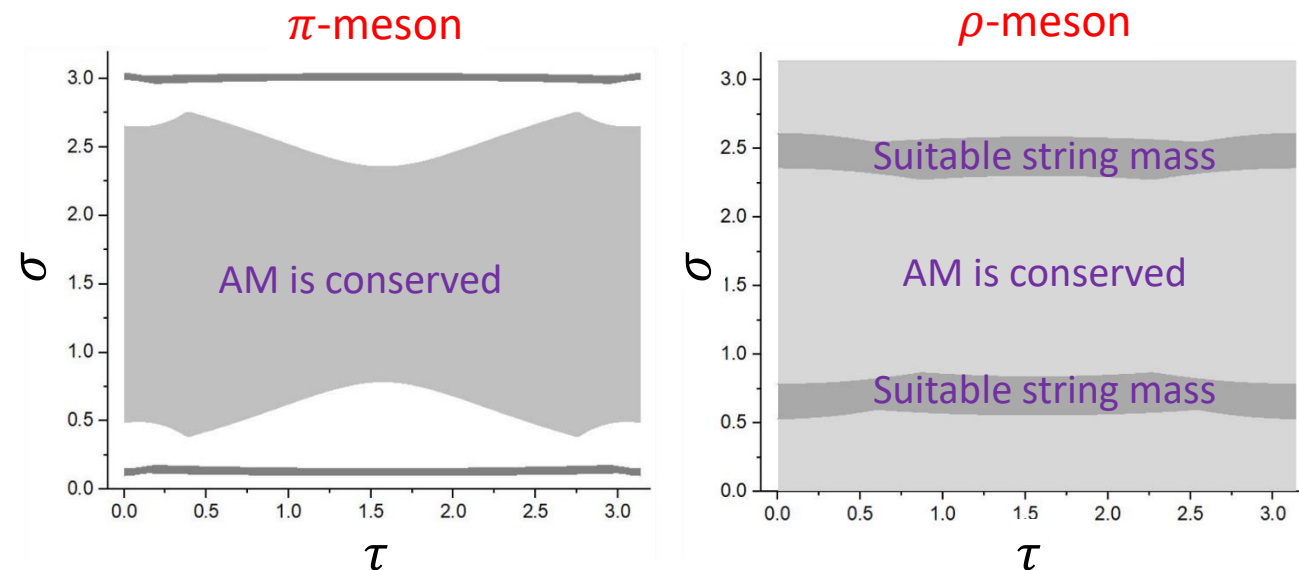
$$J_{e^+e^-} < J_{hh} < J_{hA} < J_{AA}$$

*note that, as only classical string model is used for hadron production, it forces considering angular momentum for ee system too

- A simple **example** of hadron production change from angular momentum conservation:



A parameter configuration space of the string break point:



New string hadronization model: ATROPOS

- Use the **Nambu-Goto string** theory:

$$S_{\text{string}} = -\kappa \int_{\tau_1}^{\tau_2} d\tau \int_{\sigma_1(\tau)}^{\sigma_2(\tau)} d\sigma \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}$$

- Main task: find a way to *properly* define the initial conditions ...
 $x_\mu(\tau = 0, \sigma) \equiv \rho_\mu(\sigma), \quad \partial x_\mu / \partial \tau(\tau = 0, \sigma) \equiv v_\mu(\sigma)$
- ... and develop a model for fragmenting this string.

- Use the **Virasoro** conditions for that

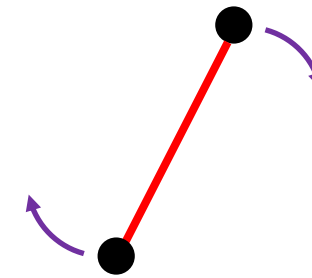
*** Skip the long and boring mathematics ***

For the strict and detailed derivation of the basics of the ATROPOS string model, see

<https://doi.org/10.48550/arXiv.2504.08968>

Result:

- A string is modeled as a rotating rigid rod

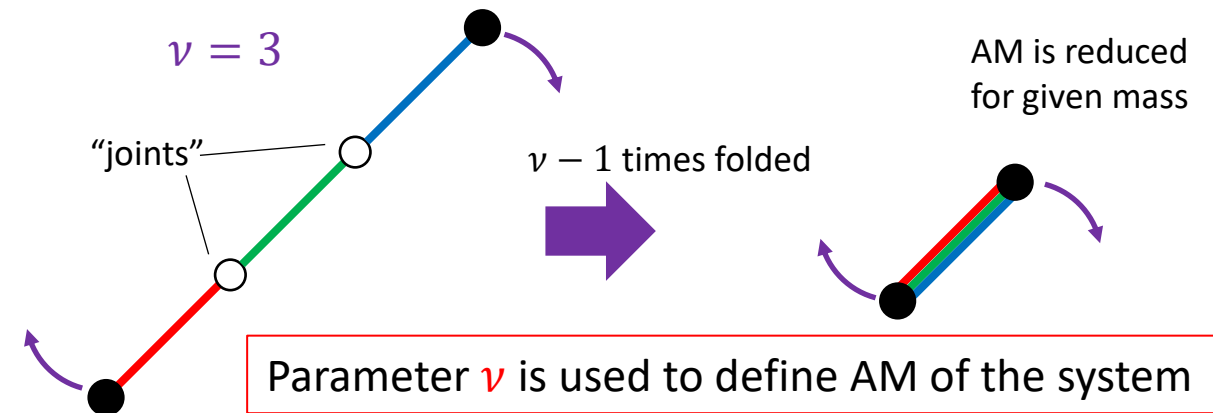


$$\rho_\mu(\sigma), v_\mu(\sigma) \sim \cos(v\sigma)$$

v : eigenharmonic of the string

$$J = \frac{1}{v} \frac{M^2}{2\kappa\pi}$$

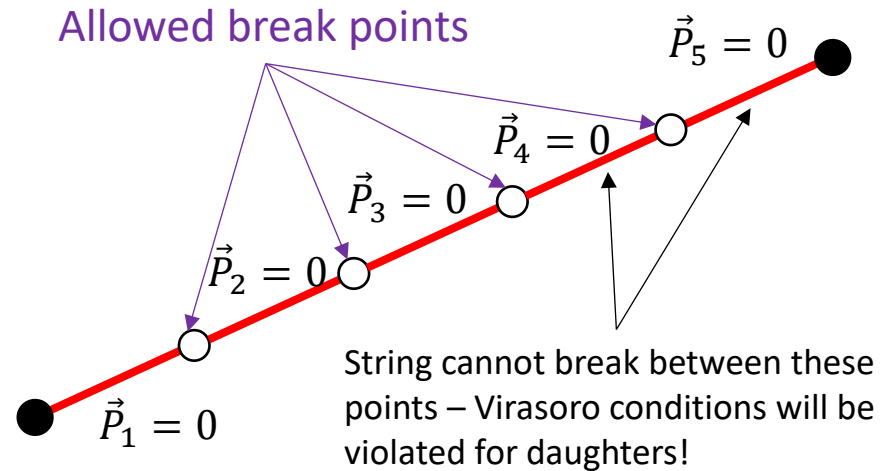
- Unable to find other possible configurations!
- Taking $v > 1$ can be seen as “folding” of the string:



* colors are used to highlight different segments of the string; the choice of colors is arbitrary

String fragmentation

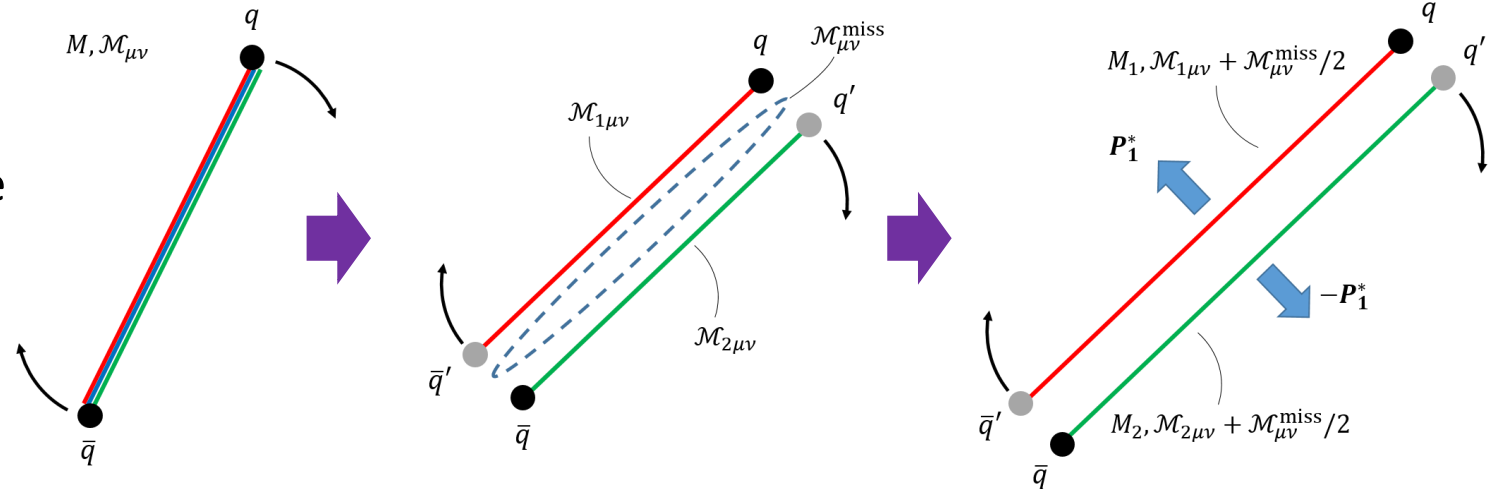
- Daughter strings must satisfy the Virasoro conditions too
- **Countable set of break points!!**
- Since each segment of the string has a total momentum of 0, new strings are produced at rest in the CM system
- Unrealistic particle production for e^+e^- collisions
- Need to add energy release to the fragmentation scheme
- Make a chunk of the string disappear and then redistribute energy and momentum between string pieces



$$\sigma_{\text{break}} = \frac{r\pi}{\nu}$$

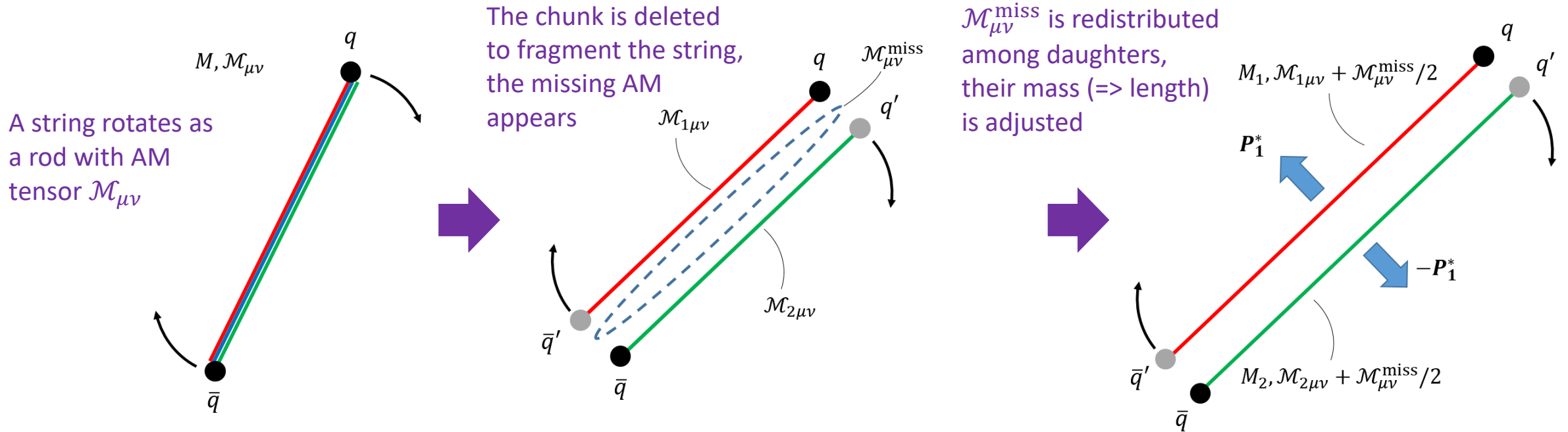
r is an integer cut factor

- The length of the daughter strings remains the same as those for the mother



- Note that eigenharmonic ν is inherited by the daughters!

Angular momentum conservation



- For a string of arbitrary generation, the angular momentum is defined by its mass: $J = M^2/(2\kappa v\Delta\sigma)$
- The missing chunk of the string carries AM $J^{\text{miss}} = M^2/(2\kappa v[\sigma_{\text{br}2} - \sigma_{\text{br}1}])$, where $\sigma_{\text{br}1,2}$ are sampled break points
- J^{miss} redistributes between string fragments: their masses M_1, M_2 are adjusted to match the total AM

- Different proportion laws may be proposed to calculate the fraction of J^{miss} taken by each fragment

- For $J \propto M^2$:

$$M_1 = \frac{r_1 - l_1}{\sqrt{(l_2 - l_1)(l_2 - l_1 - r_2 + r_1)}} M,$$

$$M_2 = \frac{l_2 - r_2}{\sqrt{(l_2 - l_1)(l_2 - l_1 - r_2 + r_1)}} M.$$

$$\sigma = \frac{l_1\pi}{v}$$

$$\sigma = \frac{r_1\pi}{v}$$

$$\sigma = \frac{r_2\pi}{v}$$

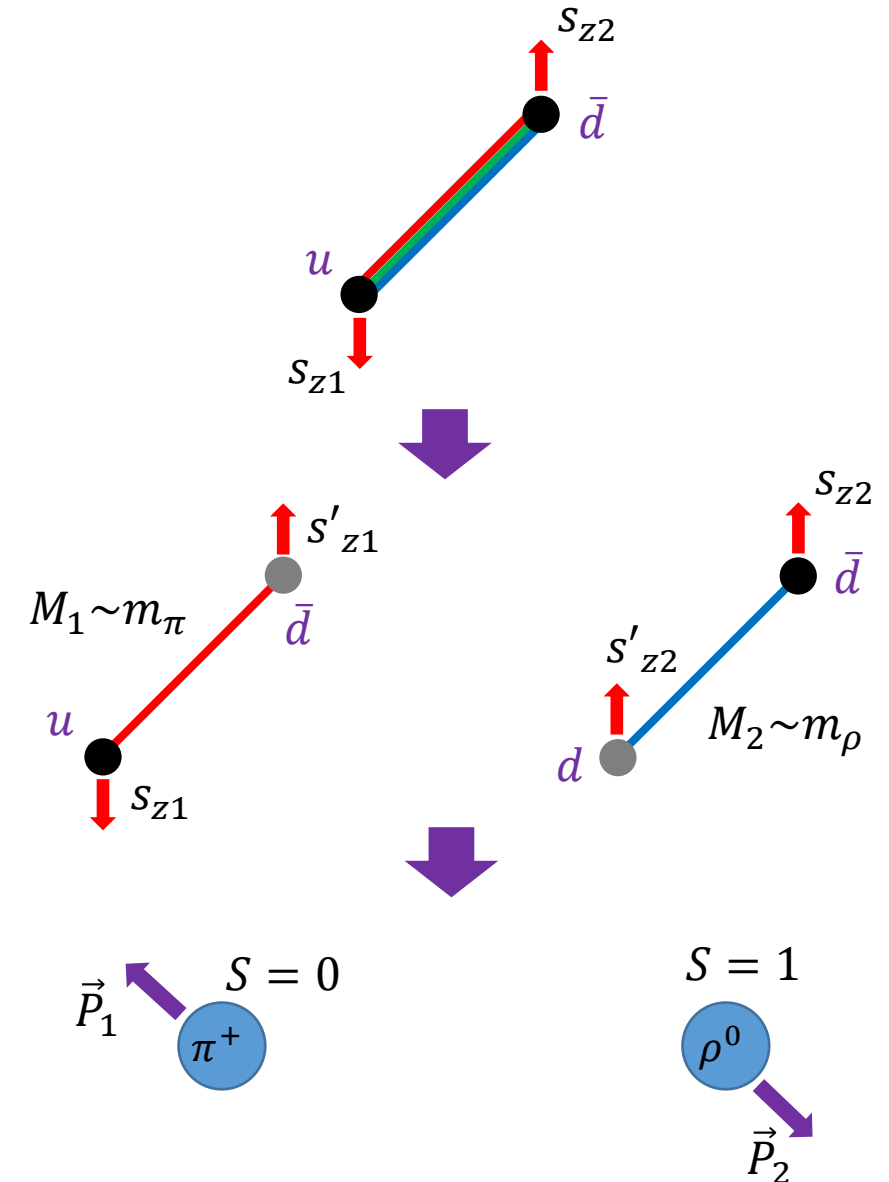
$$\sigma = \frac{l_2\pi}{v}$$

String-to-hadron transition

We adopt the following rules to model the transition from string to hadron:

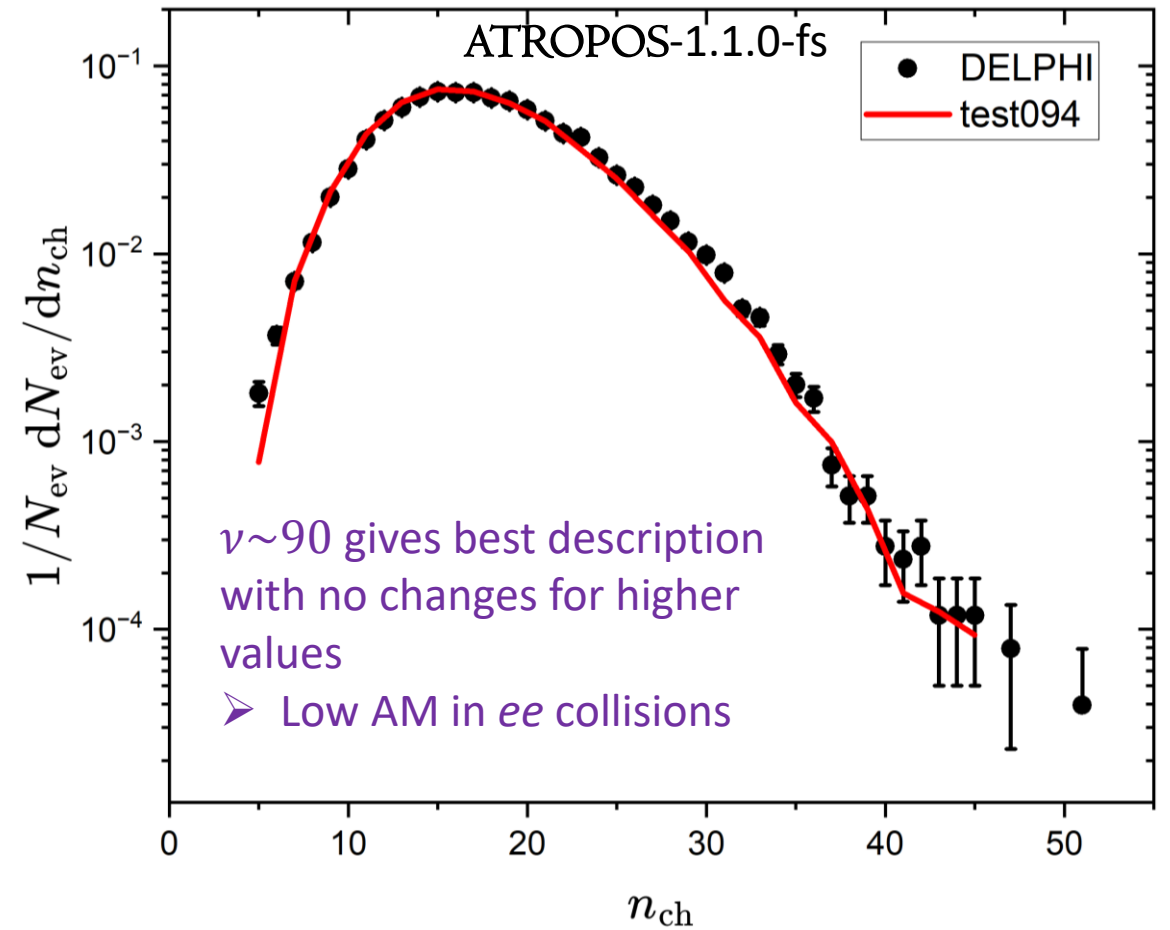
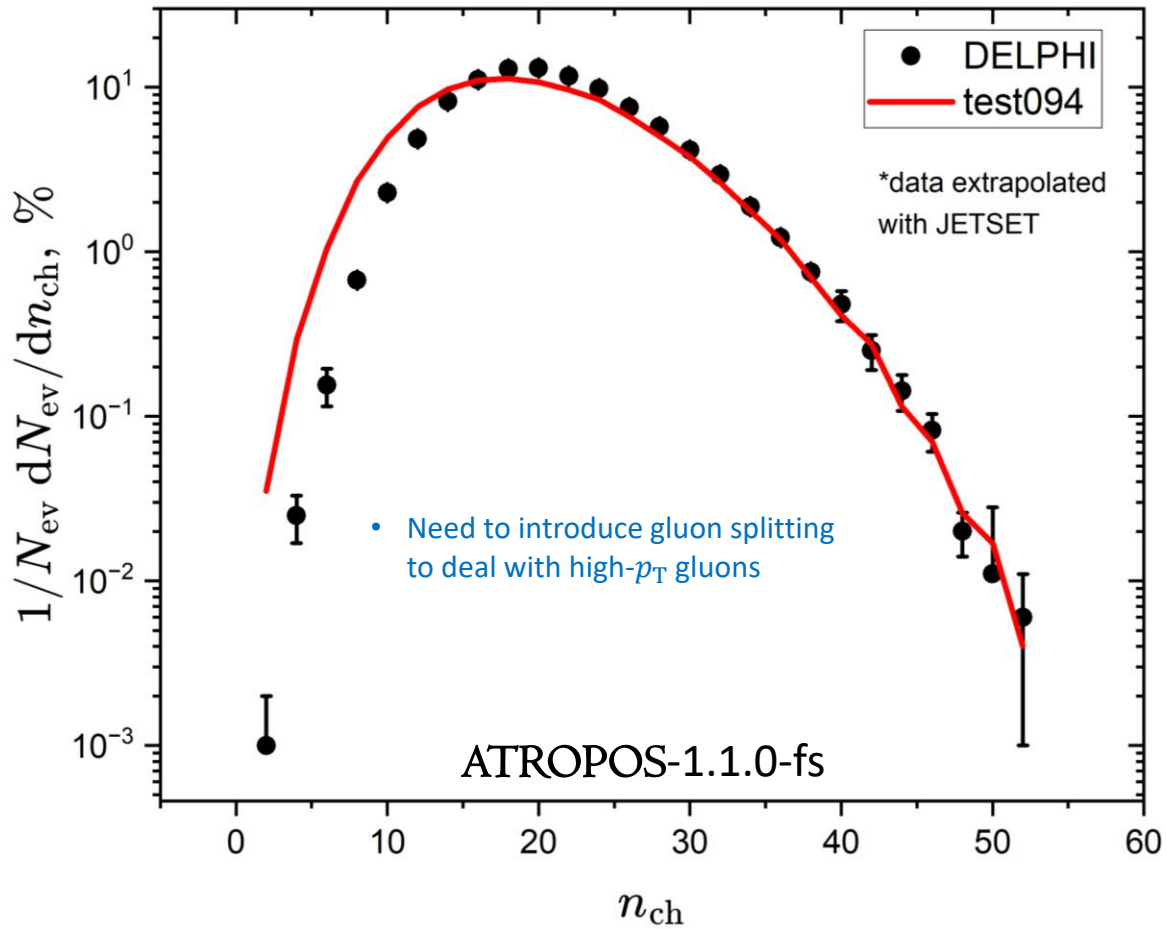
1. The string may or may not become hadron after it is produced
2. The string must have the same flavor content as a potential hadron
 - The type of the parton pair produced in the break point is sampled after the coordinates of the break point are selected according to Area Law
 - Each pair type is assigned a relative probability of production
3. The end-point partons of the string must have the spin projection values that combine to the total spin of the hadron
 - The spin projections for partons are sampled after the flavor
 - The free parameter is used to define the relative probability for possible spin states
4. The value of the string mass must be close enough to the mass of hadron
 - Another parameter is used to define the allowed relative difference
 - To produce hadrons on-shell, the inter-string interaction is used to redistribute energy and momentum

+ additional criteria?



First results: e^+e^- collisions

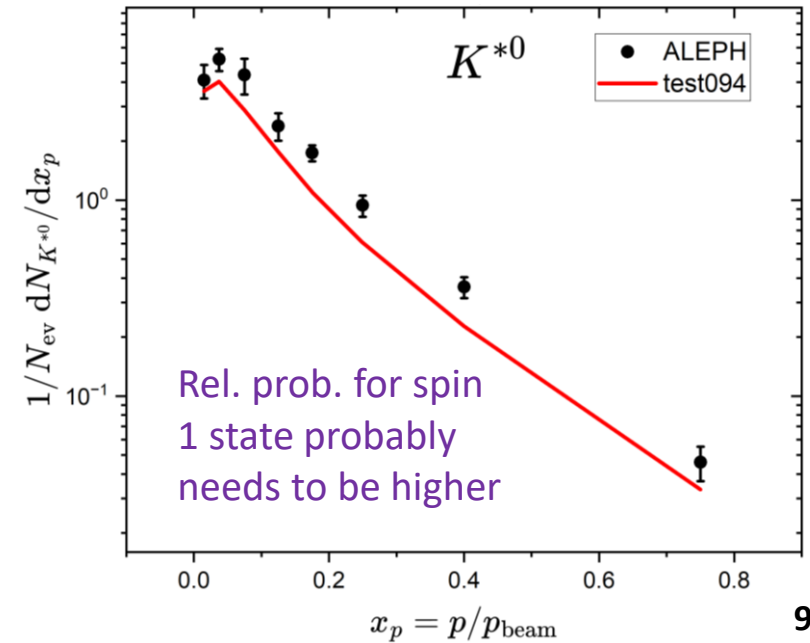
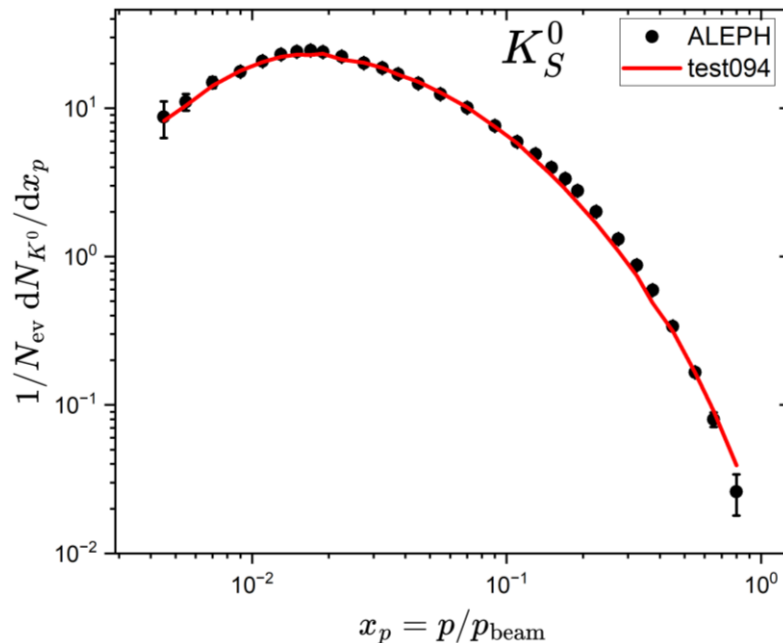
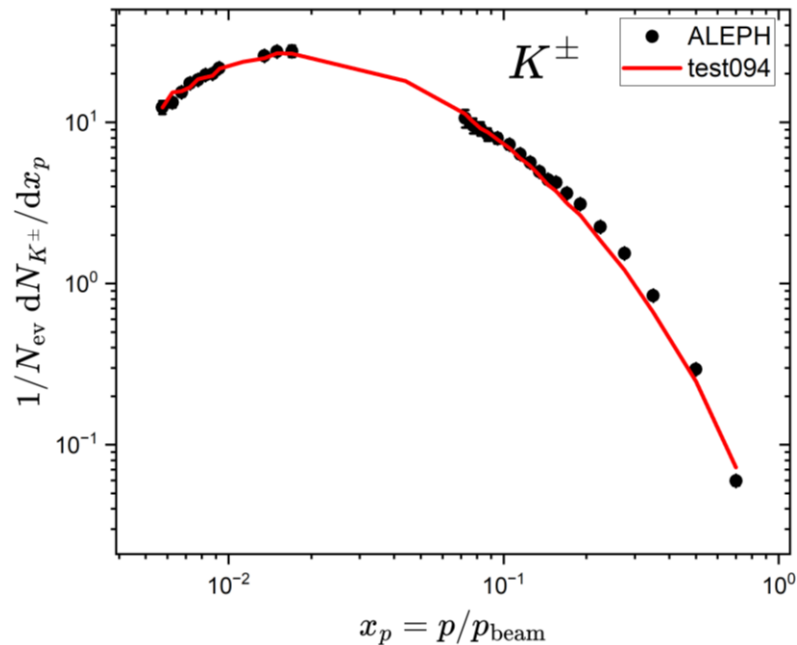
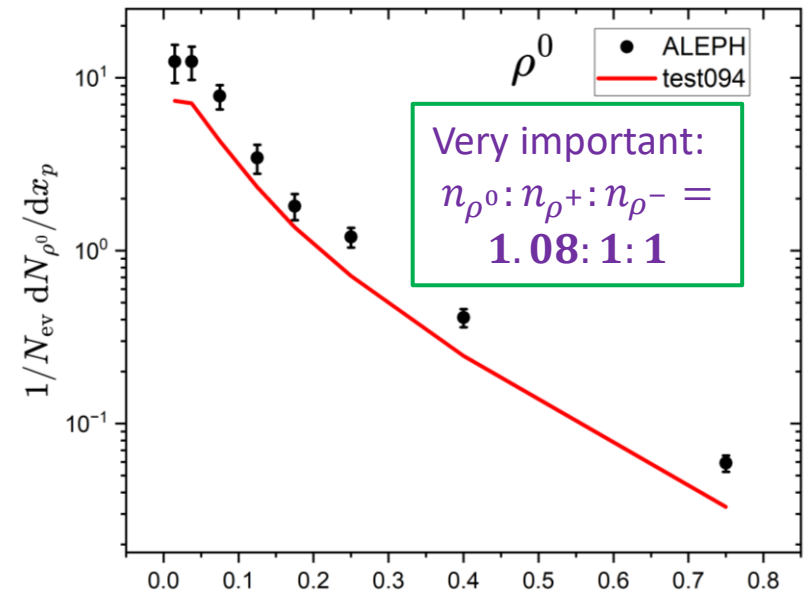
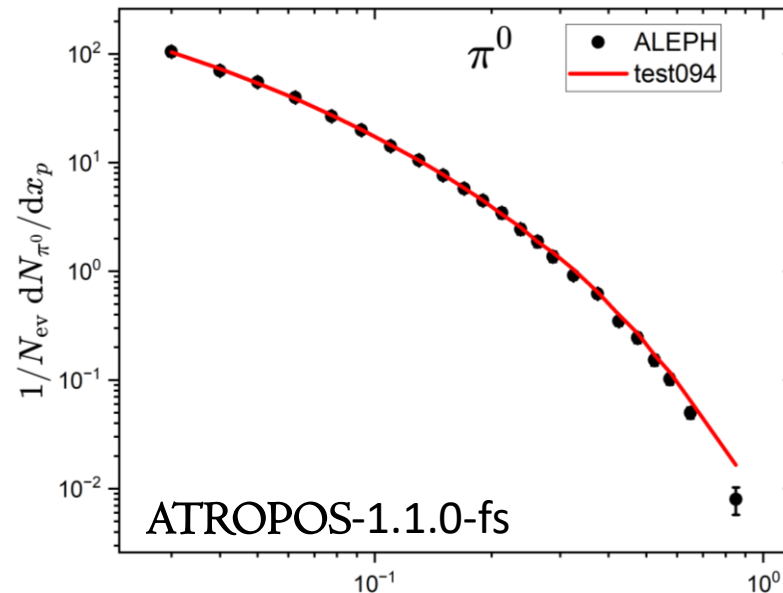
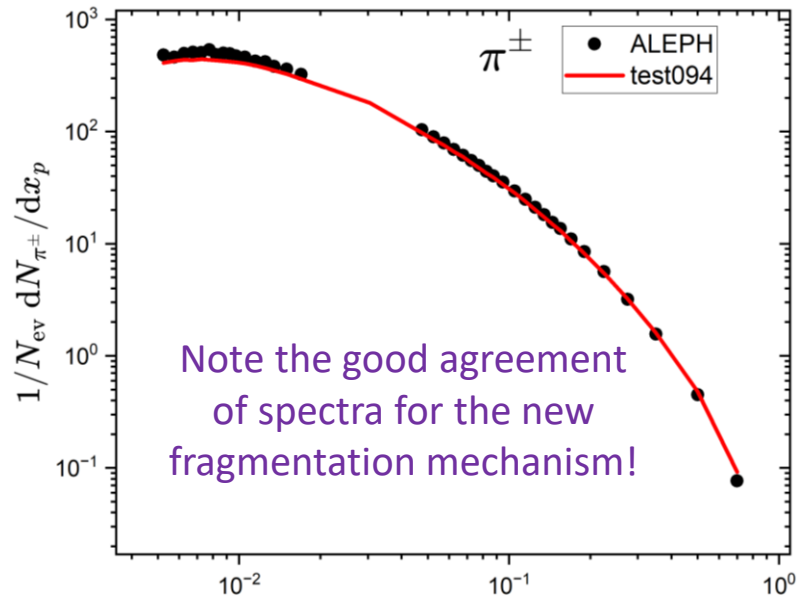
$e^+e^- \rightarrow \gamma^*, Z \rightarrow \text{hadronic}, \sqrt{s} = 91.2 \text{ GeV}$
Pythia 8.315 Parton Level



- More low-multiplicity events than in data, but note that data is extrapolated with JETSET (Pythia)

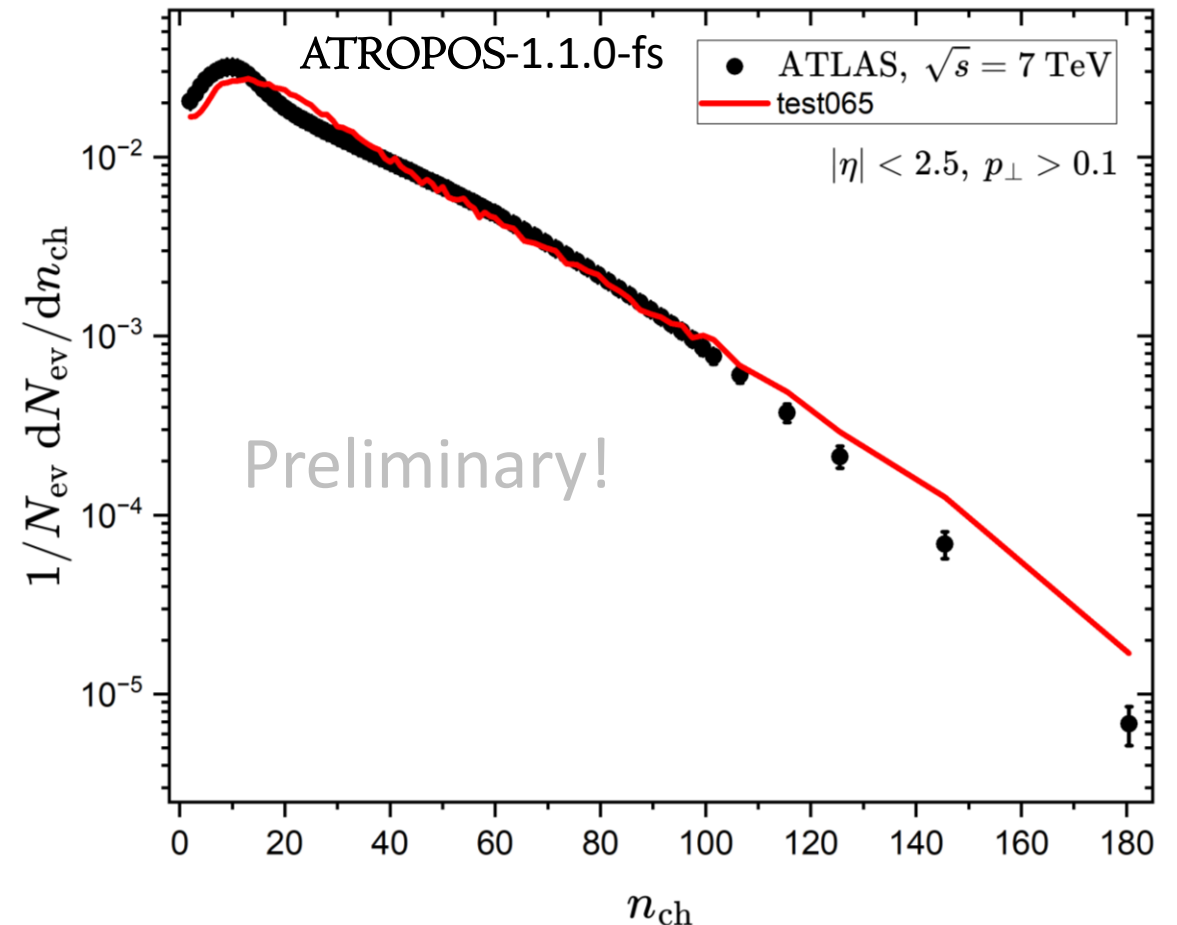
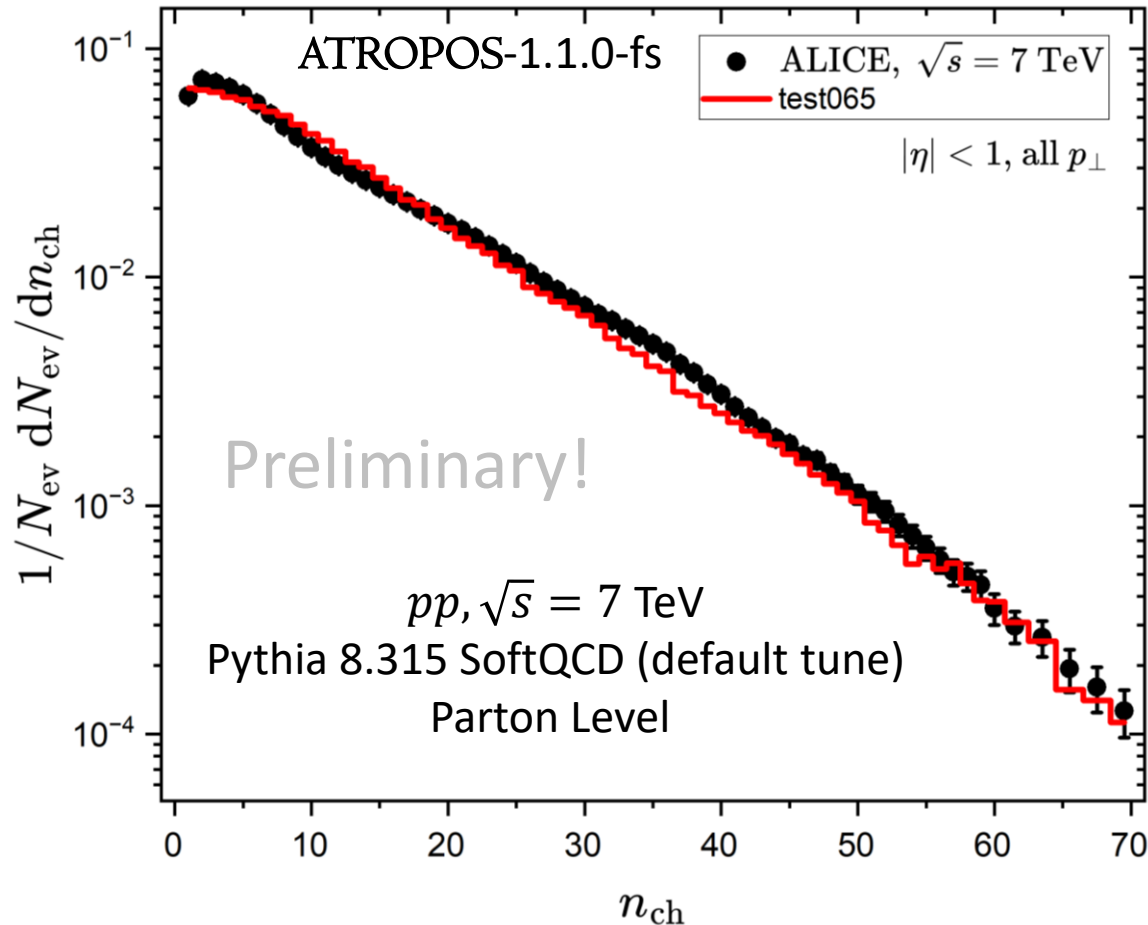
- Raw data is in much better agreement with the model

First results: e^+e^- collisions, tuning the particles yield (preliminary!)

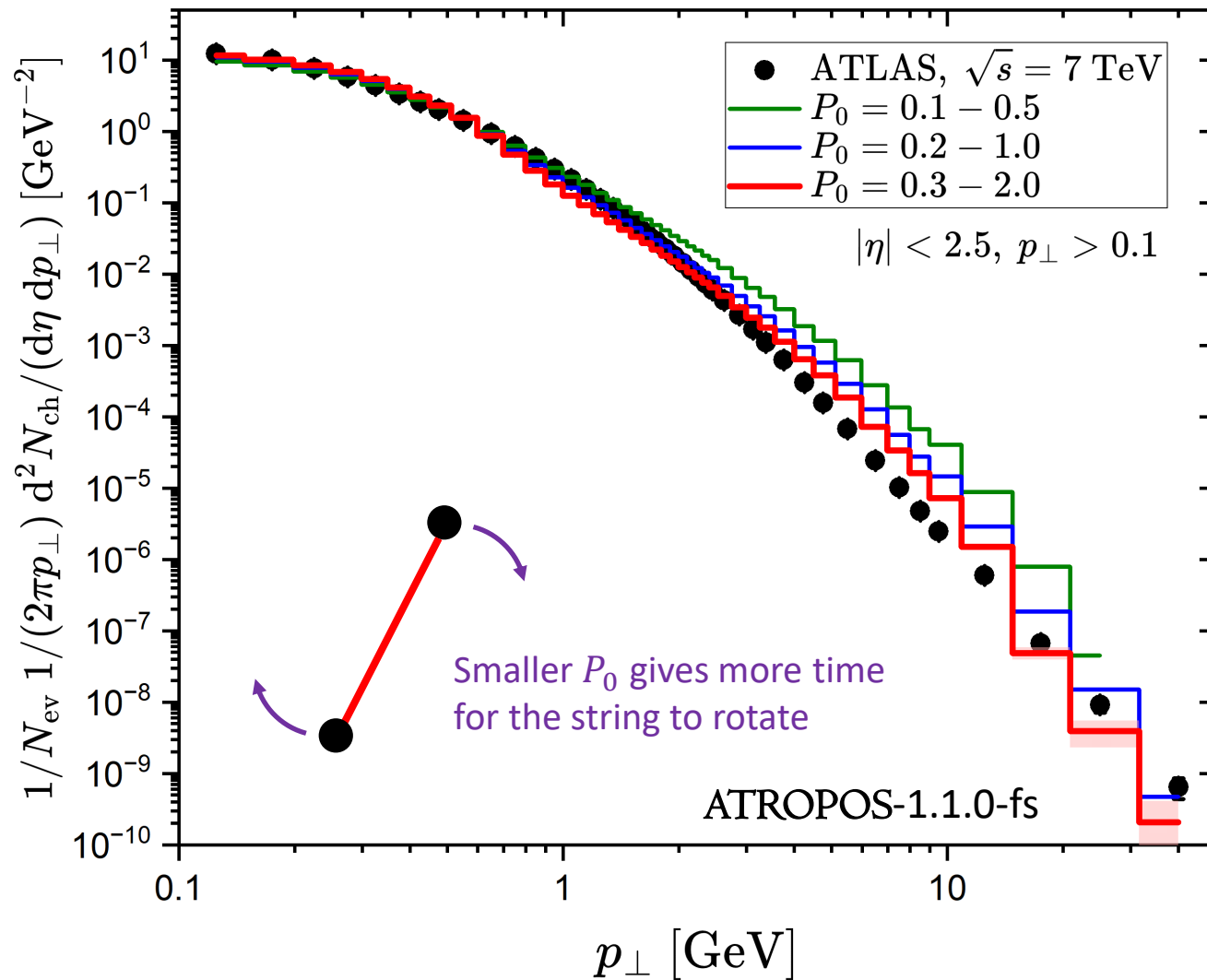


First results: pp collisions at LHC energies

- Multiplicities are sensitive to the low-limit mass of the string and to the eigenharmonic value ν :
 - Best agreement for $M_{\text{low}} \sim 7$ GeV (but may change with proper hard gluons treatment)
 - Eigenvalue $\nu \approx 80 - 90$, but not higher (or too hard multiplicity spectra)!
 - Rotation for pp is sensible at this energies



First results: pp collisions at LHC energies



- String decay is governed by the Area Decay Law (Artru and Mennesieur):

$$\frac{dP}{\kappa dA} = \text{const} \equiv P_0,$$

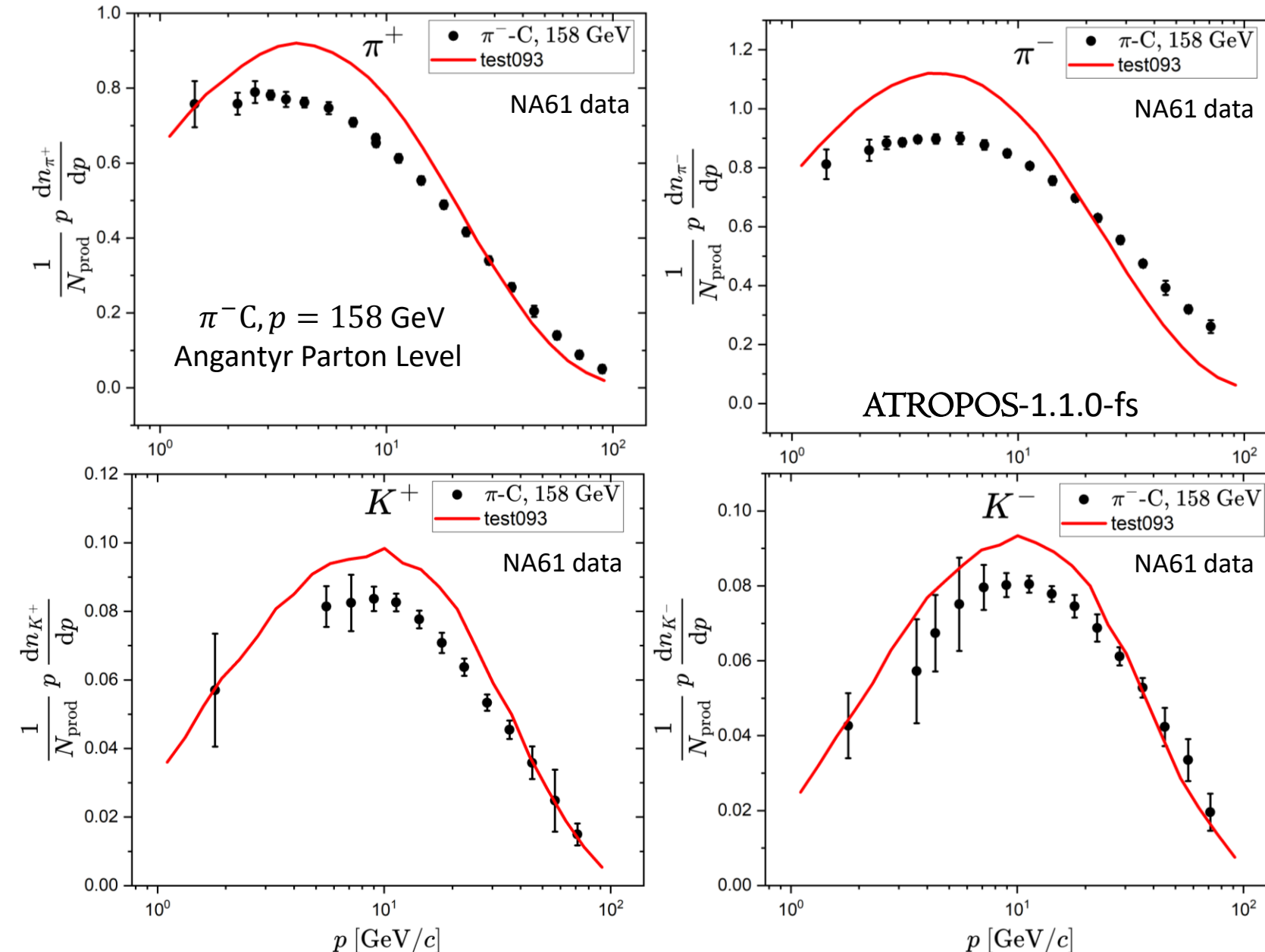
where A is an invariant area of the world sheet of the string.

- In ATROPOS:

$$P(\tau) \propto \exp\left(-\frac{P_0 M^2 \tau}{2\kappa(\sigma_2 - \sigma_1)}\right)$$

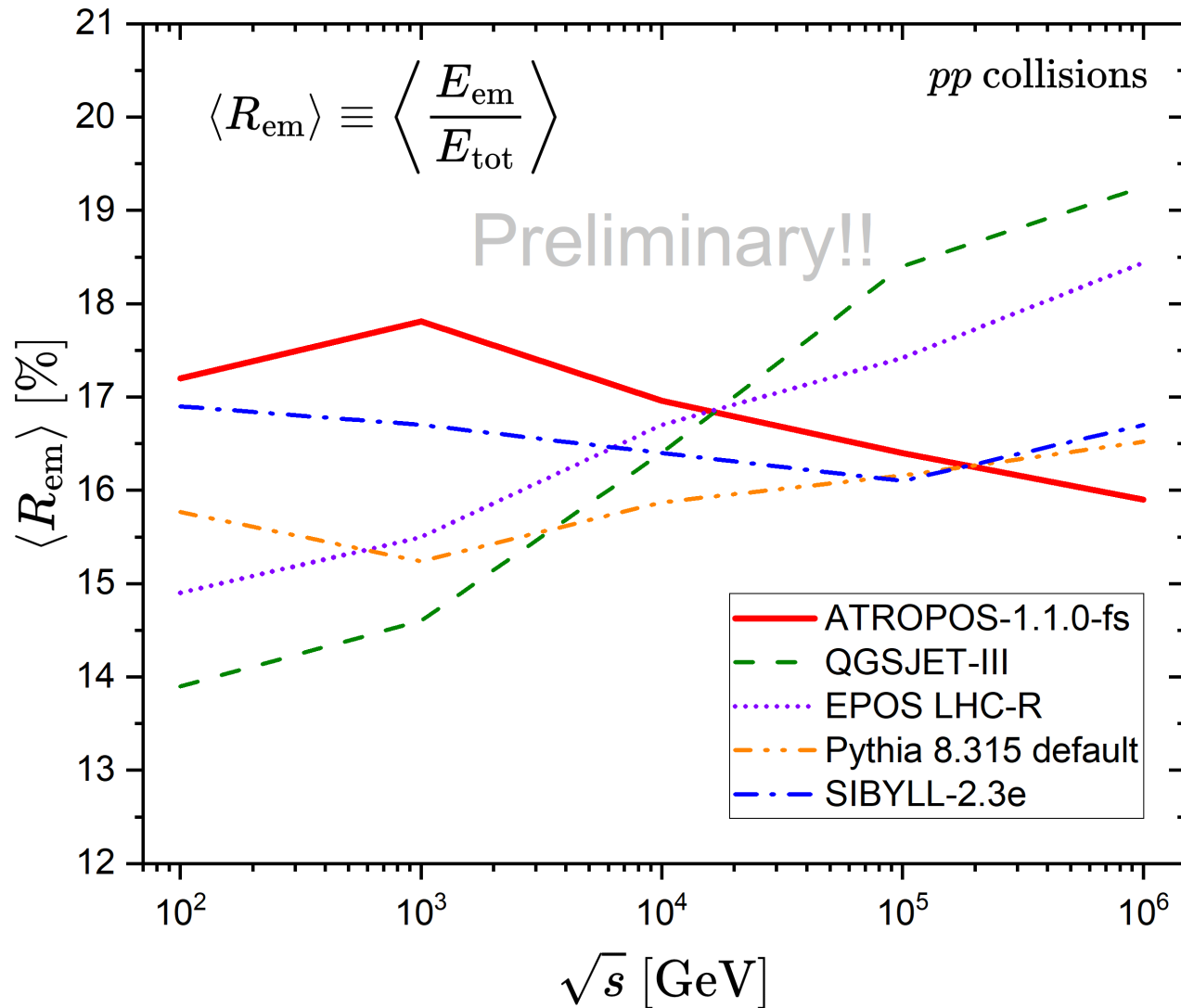
- It seems that the best fit is obtained when P_0 is taken to be slowly increasing with string mass
- Small values of P_0 lead to too high transverse momentum of final hadrons
 - **A reliable way to tune P_0 !**
- Increases forward particle production for heavier strings!**

First results: π^-C collisions



- Qualitatively good agreement, but not ideal
 - Low-energy physics needs improvement
- Best fit for eigenvalue $\nu \approx 20$!
 - Already noticeable AM
 - But also driven by need to keep low-mass limit
- Light-string behavior is important (mini-string in Pythia)

Impact on the EAS physics



- Electromagnetic energy ratio is an essential parameter for muon production in EAS (H. Dembinski, T. Pierog)
- Affected by leading π^0 fraction and neutral-to-charged pions ratio
- ATROPOS shows **decreasing** $\langle R_{\text{em}} \rangle$ at $\sqrt{s} > 1$ TeV
 - That is due to the hadron production mechanism: hadron selection is based on mass and spin sampling, so at higher energies more heavy resonances are produced
 - Suppressed direct production of light mesons
- But more tests and tuning are required

Summary

- ATROPOS is the first string hadronization model to implement **angular momentum conservation** to the fragmentation process.
- A string in ATROPOS is seen as a rigidly rotating “folded” rod with “joints” being the only permitted points for string breaking
 - Forward particle production is enhanced for large-mass strings
- The fragmentation in ATROPOS is **non-universal** for different collision systems: different AM values favor *ee*, *pp* and *hA* collisions.

Plans for future:

- Integrate ATROPOS in **CRMC** package (R. Ulrich, T. Pierog and C. Baus) to allow interface with CR models
- Integration into **EPOS.LHC-R** (with Tanguy Pierog): **work in progress**
 - First test runs in coming weeks...
- Optimize computation time: use pre-generated tables of string fragmentation modes

Thank you for your attention!

BACK UP SLIDES

Orthonormal gauge in the Nambu-Goto theory

Nambu-Goto string action:

$$S_{\text{string}} = -\kappa \int_{\sigma_1}^{\sigma_2} d\sigma \int_{\tau_1(\sigma)}^{\tau_2(\sigma)} d\tau \sqrt{(x' \dot{x})^2 - x'^2 \dot{x}^2}$$

It produces the following equations of motion (EM):

$$\frac{\partial}{\partial \tau} \left(\frac{(\dot{x} x') x'_\mu - x'^2 \dot{x}_\mu}{\sqrt{(\dot{x} x')^2 - \dot{x}^2 x'^2}} \right) + \frac{\partial}{\partial \sigma} \left(\frac{(\dot{x} x') \dot{x}_\mu - \dot{x}^2 x'_\mu}{\sqrt{(\dot{x} x')^2 - \dot{x}^2 x'^2}} \right) = 0.$$

To simplify the EM, a special gauge is selected to define two relations between τ and σ :

$$\dot{x}^2 + x'^2 = 0, \quad \dot{x} x' = 0.$$

It is called an orthonormal gauge and allows to simplify the EM:


$$\ddot{x}_\mu - x''_\mu = 0.$$

$x_\mu(\tau, \sigma)$ is a 2-parameter definition of the string world sheet, where σ numerates the points of the string, and τ defines the evolution in time.


$$\dot{x}_\mu \equiv \frac{\partial x_\mu(\tau, \sigma)}{\partial \tau}$$
$$x'_\mu \equiv \frac{\partial x_\mu(\tau, \sigma)}{\partial \sigma}$$

How Virasoro conditions restrict the string motion

Substitute the solution to the EM into the orthonormal gauge expressions:

$$x_\mu(\tau, \sigma) = Q_\mu + P_\mu \frac{\tau}{\pi\kappa} + \frac{i}{\sqrt{\pi\kappa}} \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} e^{-in\tau} \frac{\alpha_{n\mu}}{n} \cos(n\sigma)$$

$$\begin{cases} \dot{x}^2 + x'^2 = 0 \\ \dot{x}x' = 0. \end{cases}$$

The resulting set of equalities is called the **Virasoro conditions**:


$$\sum_{m=-\infty}^{+\infty} \alpha_{n-m} \alpha_m = 0, \quad n = 0, \pm 1, \pm 2, \dots$$

Here $\alpha_{n\mu}$ are Fourier amplitudes defined as

$$\alpha_{n\mu} = \sqrt{\frac{\kappa}{\pi}} \int_0^\pi d\sigma \cos(n\sigma) \left(\boldsymbol{v}_\mu(\sigma) - in\boldsymbol{\rho}_\mu(\sigma) \right), \quad n \neq 0, \quad \alpha_{0\mu} = \frac{P_\mu}{\sqrt{\kappa\pi}}.$$

The functions $\boldsymbol{v}_\mu(\sigma)$ and $\boldsymbol{\rho}_\mu(\sigma)$ define velocity and coordinates of the string at the initial moment in time.

- Thus, the Virasoro conditions restrict the initial data of the boundary-value problem for the string motion.

FOEE method to define the initial conditions of the string

The problem:

Most of the functions do not satisfy the Virasoro conditions if the string is massive ($M \neq 0$)

A new method:

- Let us express the initial data functions as a finite series over the Sturm-Liouville boundary problem eigenfunctions (**F**inal-**O**rders **E**igenfunction **E**xpansion, **FOEE**):

$$v_{\mu}(\sigma) = a_{0\mu}u_0(\sigma) + \sum_{k=1}^N a_{k\mu}u_k(\sigma), \quad \rho_{\mu}(\sigma) = b_{0\mu}u_0(\sigma) + \sum_{k=1}^N b_{k\mu}u_k(\sigma).$$

- For a free string:

$$v_{\mu}(\sigma) = a_{0\mu} + \sum_{k=1}^N a_{k\mu} \cos(k\sigma), \quad \rho_{\mu}(\sigma) = b_{0\mu} + \sum_{k=1}^N b_{k\mu} \cos(k\sigma).$$

Constructing the FOEE system

- The eigenfunctions of the S.-L. problem are orthogonal, so the system is finite:

$$\left\{ \begin{array}{l} \sum_{\substack{m=\max(n-N, -N) \\ m \neq 0, m \neq n}}^{\min(n+N, N)} (a_{n-m}a_m - m(n-m)b_{n-m}b_m) + \frac{4}{\kappa\pi} P a_n = 0 \\ \sum_{\substack{m=\max(n-N, -N) \\ m \neq 0, m \neq n}}^{\min(n+N, N)} (m a_{n-m}b_m + (n-m)a_m b_{n-m}) - \frac{4n}{\kappa\pi} P b_n = 0, \end{array} \right. \quad n \neq 0, \quad \left\{ \begin{array}{l} \sum_{\substack{m=-N \\ m \neq 0}}^N (a_{-m}a_m + m^2 b_{-m}b_m) = -\frac{2P^2}{(\kappa\pi)^2} \\ \sum_{\substack{m=-N \\ m \neq 0}}^N m(a_{-m}b_m - a_m b_{-m}) = 0. \end{array} \right.$$

- Add conservation laws:

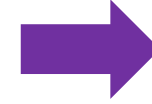
$$\kappa \int_0^\pi d\sigma v_\mu(\sigma) = \kappa \int_0^\pi d\sigma \dot{x}_\mu(0, \sigma) = P_\mu,$$

$$\kappa \int_0^\pi d\sigma [\rho_\mu(\sigma)v_\nu(\sigma) - \rho_\nu(\sigma)v_\mu(\sigma)] = \kappa \int_0^\pi d\sigma [x_\mu(0, \sigma)\dot{x}_\nu(0, \sigma) - x_\nu(0, \sigma)\dot{x}_\mu(0, \sigma)] = M_{\mu\nu}.$$

The FOEE system: 1st order

The initial data functions:

$$\begin{aligned}v_\mu(\sigma) &= a_\mu + b_\mu \cos(\sigma) \\ \rho_\mu(\sigma) &= c_\mu + d_\mu \cos(\sigma)\end{aligned}$$



System of the Virasoro conditions:

$$\begin{cases} b^2 - d^2 = 0 \\ bd = bP = dP = 0 \\ b^2 + \frac{2P^2}{(\kappa\pi)^2} = 0 \end{cases}$$

$$\begin{cases} \alpha_{0\mu} = \frac{P_\mu}{\sqrt{\kappa\pi}} \\ \alpha_{1\mu} = \frac{\sqrt{\kappa\pi}}{2} (b_\mu - id_\mu) \\ \alpha_{-1\mu} = \frac{\sqrt{\kappa\pi}}{2} (b_\mu + id_\mu) \end{cases}$$

Fourier amplitudes

4-momenta conservation gives: $a_\mu = \frac{P_\mu}{\kappa\pi}$

AM tensor conservation: $c_\mu P_\nu - c_\nu P_\mu + \frac{\kappa\pi}{2} (d_\mu b_\nu - d_\nu b_\mu) = M_{\mu\nu}.$

15 equations, 16 variables

- **Very difficult to solve the system due to its non-linearity.**

FOEE(1)-string in the center-of-mass

- Define the string with:

$$P_0 \equiv M, P_i = 0, i = 1, 2, 3.$$

- Rotate the coordinate system so that string rotation occurred in a XZ-plane:

$$\mathcal{M}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{M}_{13} \\ 0 & 0 & 0 & 0 \\ 0 & -\mathcal{M}_{13} & 0 & 0 \end{pmatrix}.$$

Get the following initial data functions:

$$v_{\mu}^*(\sigma) = (\kappa\pi)^{-1}M[\delta_{0\mu} + \delta_{1\mu} \cos(\sigma)], \quad \rho_{\mu}^*(\sigma) = -(\kappa\pi)^{-1}\xi M \delta_{3\mu} \cos(\sigma),$$

wher $\delta_{\mu\nu}$ is the Kronecker delta, ξ is a string rotation signature:

$$\xi = \text{sign}\mathcal{M}_{13}.$$

FOEE(1)-string in the center-of-mass

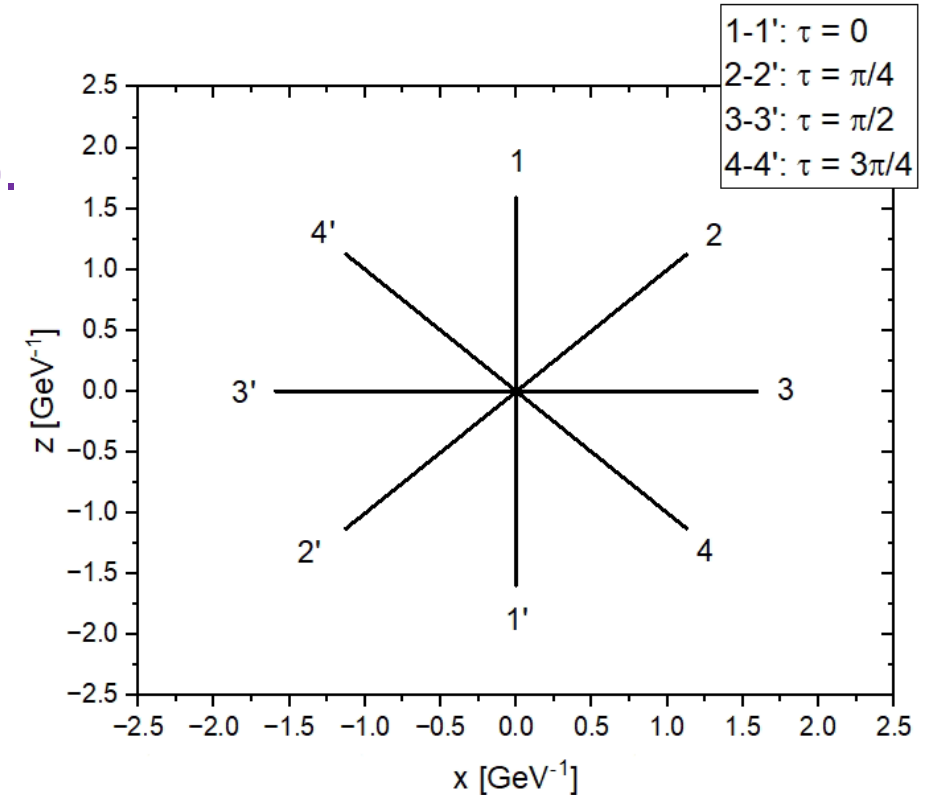
- Obtain the following formula for string coordinates:

$$x_\mu(\tau, \sigma) = (\kappa\pi)^{-1} M (\delta_{0\mu} \tau + [\delta_{1\mu} \sin(\tau) - \xi \delta_{3\mu} \cos(\tau)] \cos(\sigma)).$$

- An important relation:

$$2\kappa\pi |\mathcal{M}_{13}| = M^2.$$

- String must rotate (have spin) to be massive!
- important: FOEE(1)-string satisfies the conditions for the tangent vectors to the string world sheet.



FOEE(1)-string rotates as a rigid rod in CM

FOEE(1)-string in arbitrary system

Lorentz boost to the system where string has momentum \vec{P} :

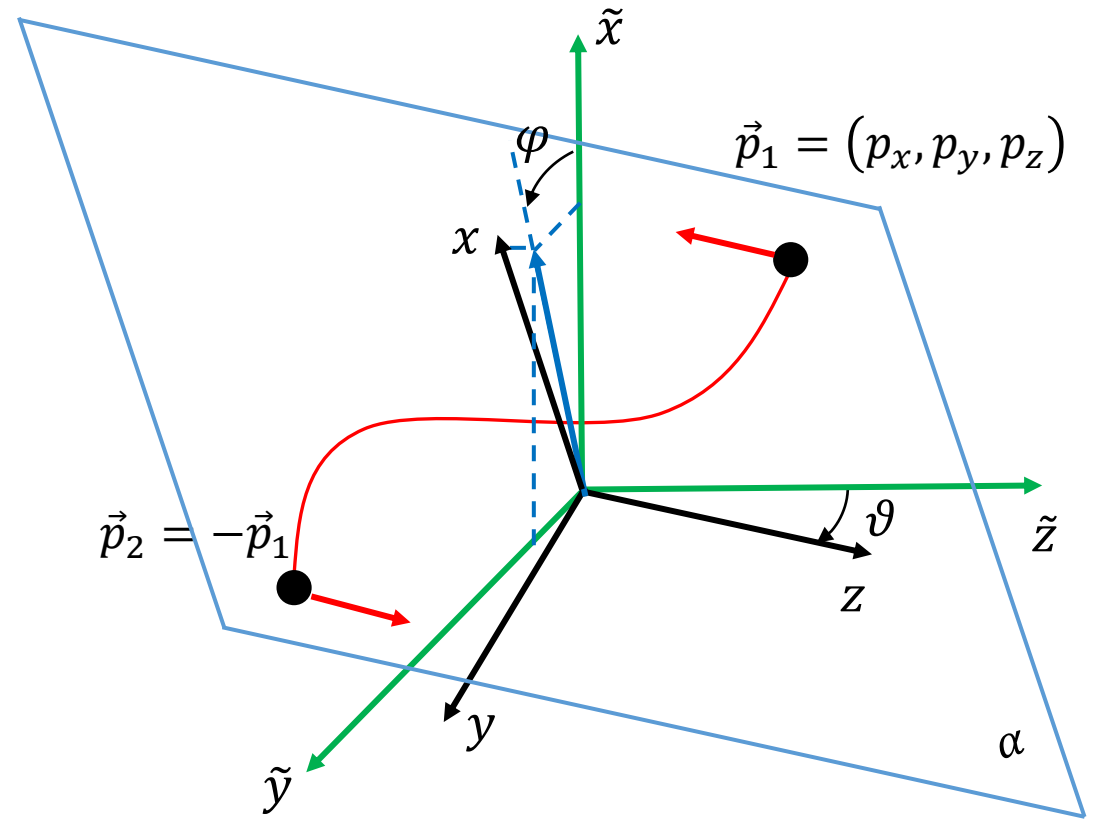
$$v_0(\sigma) = \frac{P_0 v_0^*(\sigma) + \vec{P} \vec{v}^*(\sigma)}{M},$$

$$\vec{v}(\sigma) = \vec{v}^*(\sigma) + \vec{P} \frac{v_0(\sigma) + v_0^*(\sigma)}{P_0 + M}.$$

Rotate the coordinate axis:

$$R(\vartheta, \varphi) = \begin{pmatrix} \cos \varphi & -\sin \varphi \cos \vartheta & \sin \varphi \sin \vartheta \\ \sin \varphi & \cos \varphi \cos \vartheta & -\cos \varphi \sin \vartheta \\ 0 & \sin \vartheta & \cos \vartheta \end{pmatrix},$$

$$\cos \vartheta = \frac{p_z}{\sqrt{p_x^2 + p_y^2 + p_z^2}}, \quad \cos \varphi = \frac{p_y}{\sqrt{p_x^2 + p_y^2}}.$$



The scheme of the coordinate axis rotation in the CM of the string

FOEE(1)-string in arbitrary system

Initial conditions:

$$v_\mu(\sigma) = (\kappa\pi)^{-1} [P_\mu + \xi(M\psi_\mu - (P\psi)\chi_\mu) \cos(\sigma)], \quad \rho_\mu(\sigma) = (\kappa\pi)^{-1} (M\lambda_\mu - (P\lambda)\chi_\mu) \cos(\sigma),$$

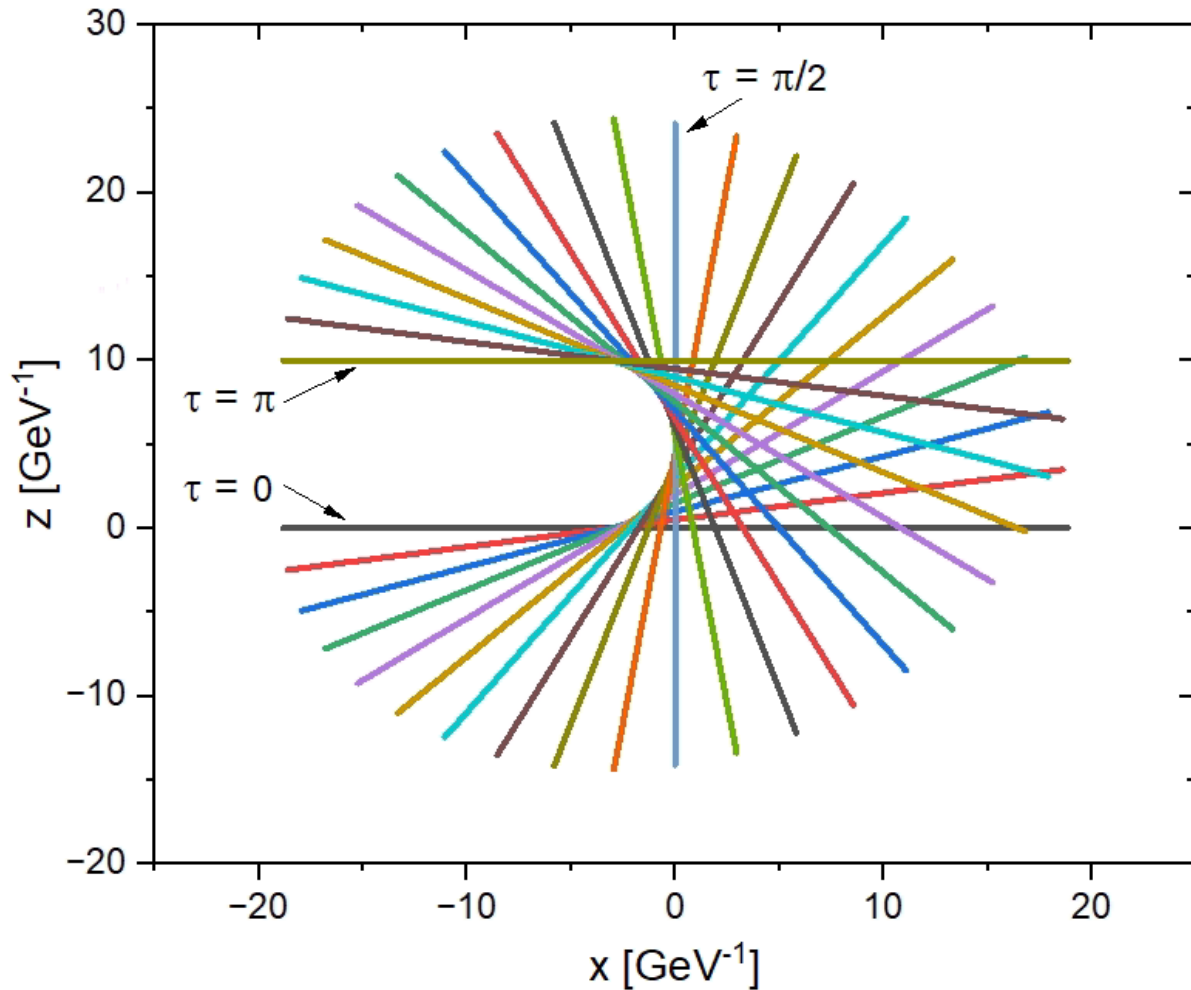
$$\psi_\mu = \begin{pmatrix} 0 \\ \sin \varphi \sin \vartheta \\ -\cos \varphi \sin \vartheta \\ \cos \vartheta \end{pmatrix}, \quad \lambda_\mu = \begin{pmatrix} 0 \\ \cos \varphi \\ \sin \varphi \\ 0 \end{pmatrix}, \quad \chi_0 \equiv 1, \quad \vec{\chi} = \frac{\vec{P}}{P_0 + M}.$$

The formula for the coordinates of the string:

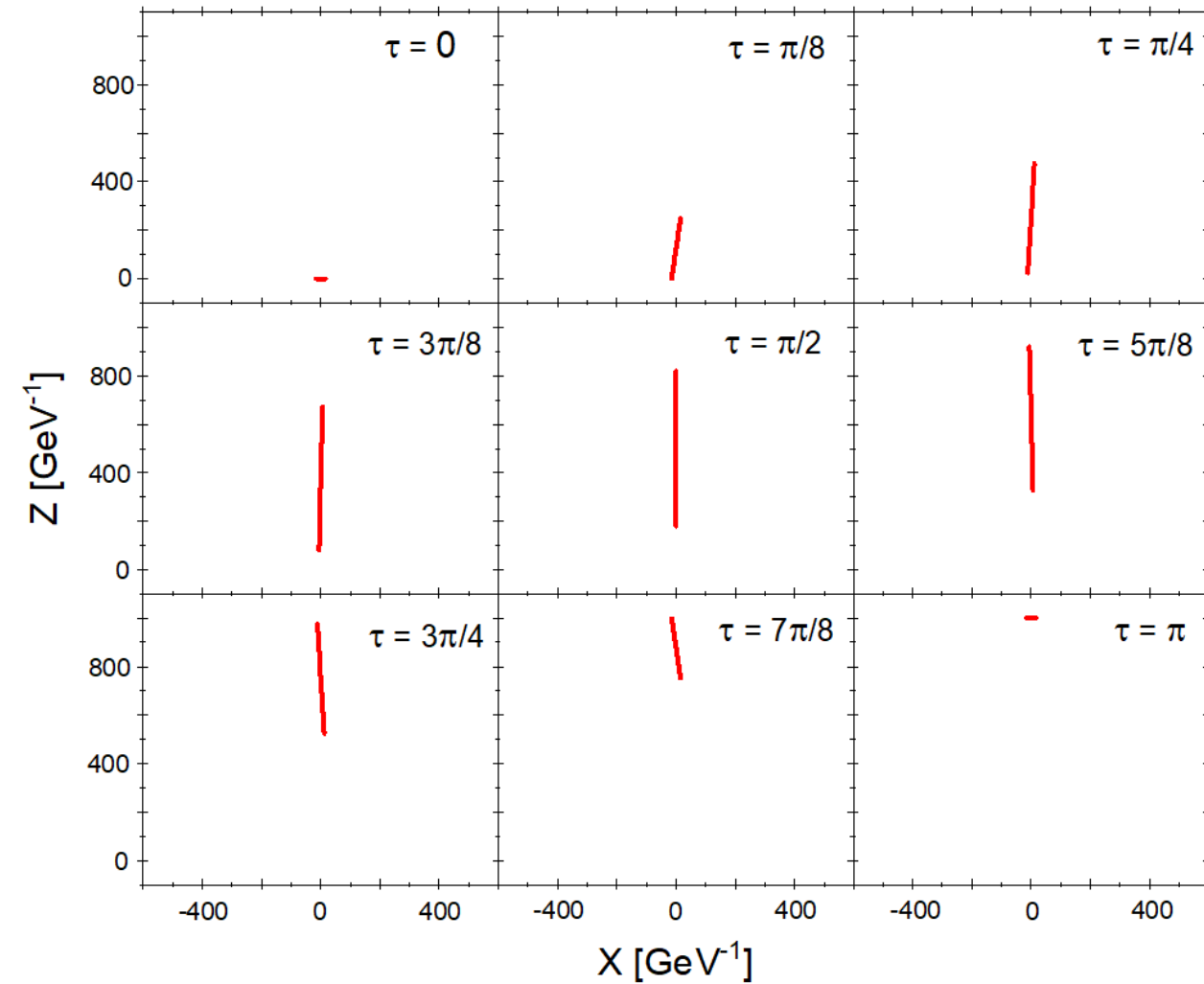
$$x_\mu(\tau, \sigma) = (\kappa\pi)^{-1} (P_\mu \tau + [M\Omega_\mu(\tau) - (P\Omega(\tau))\chi_\mu] \cos(\sigma)),$$

$$\Omega_\mu(\tau) = \lambda_\mu \cos(\tau) + \xi \psi_\mu \sin(\tau), \quad \Lambda_\mu(\tau) = \psi_\mu \cos(\tau) - \xi \lambda_\mu \sin(\tau).$$

Examples of the FOEE(1)-string motion



Motion of FOEE(1)-string of mass 12 GeV and $P_z = 2$ GeV



Motion of FOEE(1)-string of mass 10 GeV and $P_z = 100$ GeV

Generalization for the case of the higher order eigenharmonic

- FOEE(1)-string can have eigenharmonic with non-zero amplitude of arbitrary order:

$$\begin{cases} v_\mu(\sigma) = a_\mu + b_\mu \cos(\sigma) \\ \rho_\mu(\sigma) = c_\mu + d_\mu \cos(\sigma) \end{cases} \quad \longrightarrow \quad \begin{cases} v_\mu(\sigma) = a_\mu + b_\mu \cos(\boldsymbol{v}\sigma) \\ \rho_\mu(\sigma) = c_\mu + d_\mu \cos(\boldsymbol{v}\sigma) \end{cases} \quad \boxed{\boldsymbol{v} \text{ is natural number}}$$

- The resulting equation of motion is similar to the case $\boldsymbol{v} = \mathbf{1}$:

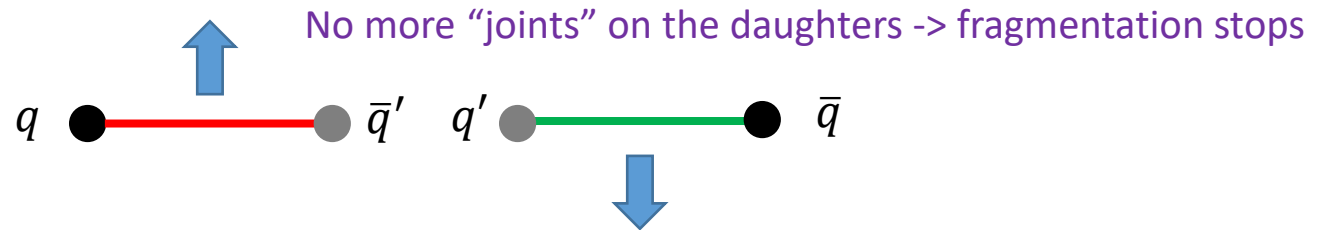
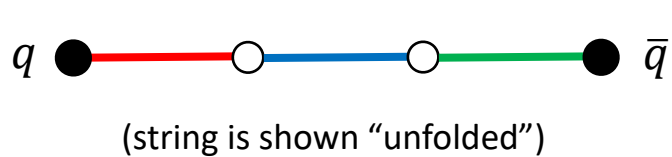
$$x_\mu(\tau, \sigma) = (\kappa\pi)^{-1} (P_\mu \tau + \boldsymbol{v}^{-1} [M\Omega_\mu(\tau, \boldsymbol{v}) - (P\Omega(\tau, \boldsymbol{v}))\chi_\mu] \cos(\sigma)),$$

$$\Omega_\mu(\tau, \boldsymbol{v}) = \lambda_\mu \cos(\boldsymbol{v}\tau) + \xi\psi_\mu \sin(\boldsymbol{v}\tau).$$

Some features of the model

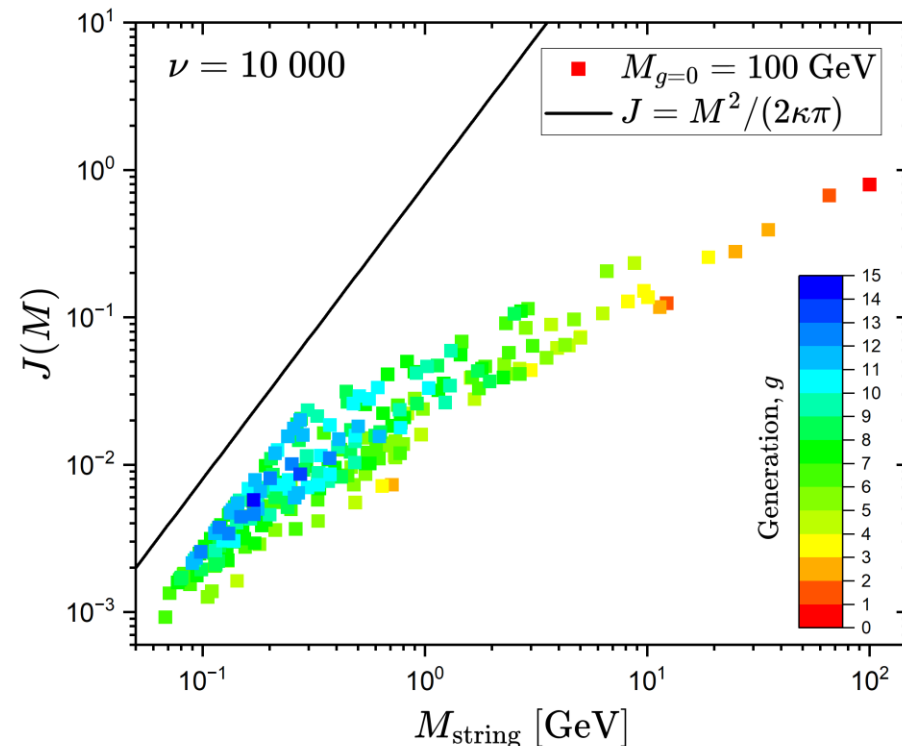
1. The string fragmentation process is *naturally* limited: there is always a last “fragmentable” string composed of the 3 segments (2 in case the production at rest is permitted)

The “shortest” string that can fragment



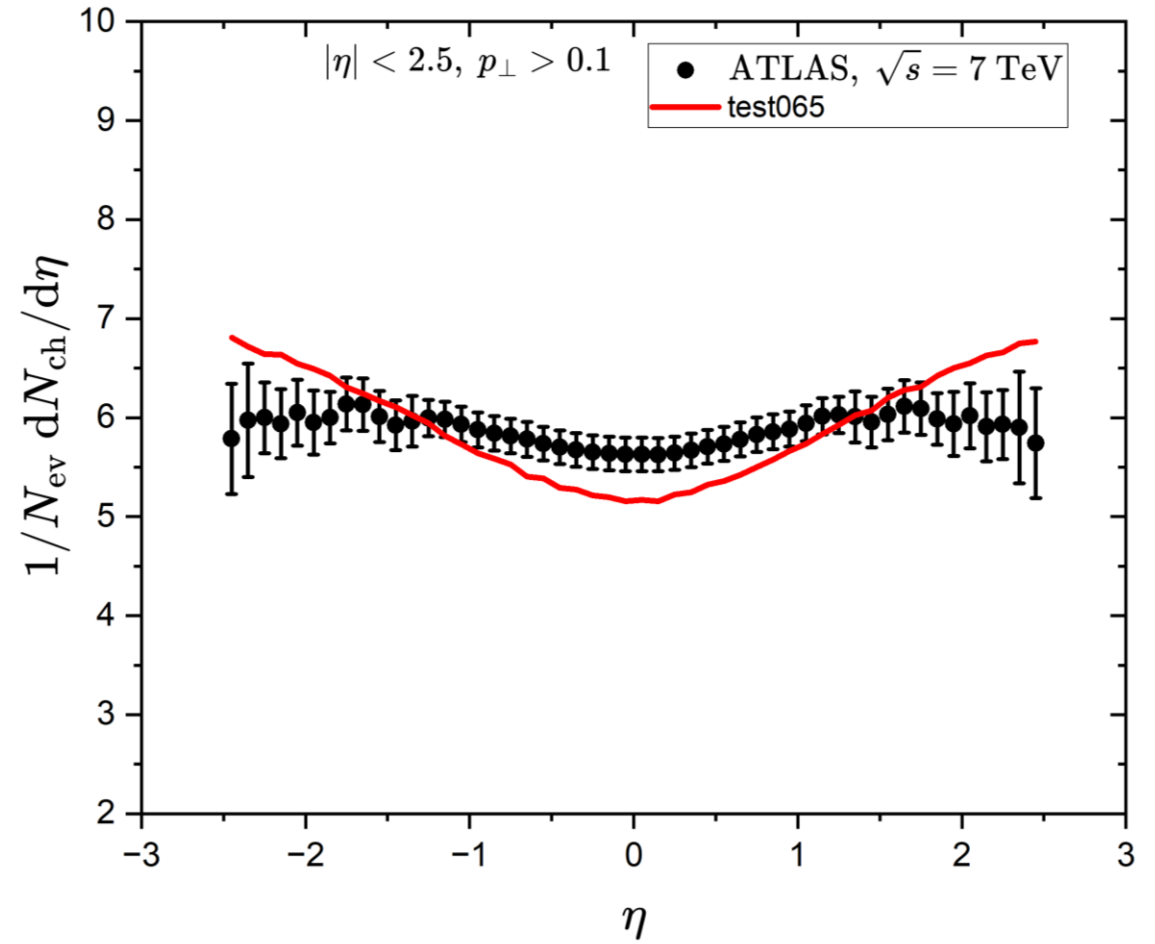
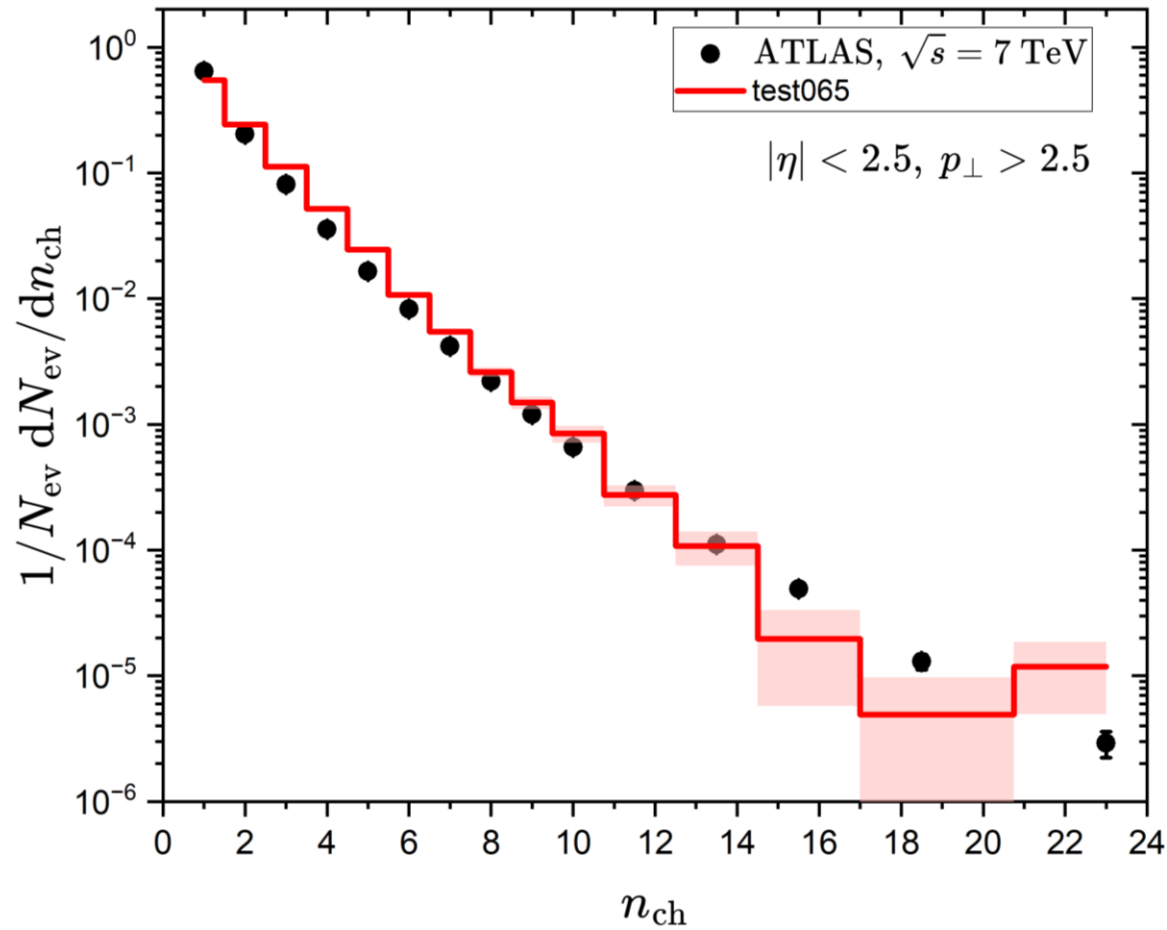
2. Close-to-Regge behavior of the spin-mass relation for light daughter strings
 - The connection between the slope of the Regge trajectory α and the string tension κ is derived in the string theory based on $J(M)$ dependence:

$$\alpha = (2\kappa\pi)^{-1}$$
 - This would lead to huge AM for heavy strings, but in the ATROPOS model, the proportionality coefficient between J and M^2 decreases with the daughter string generation.

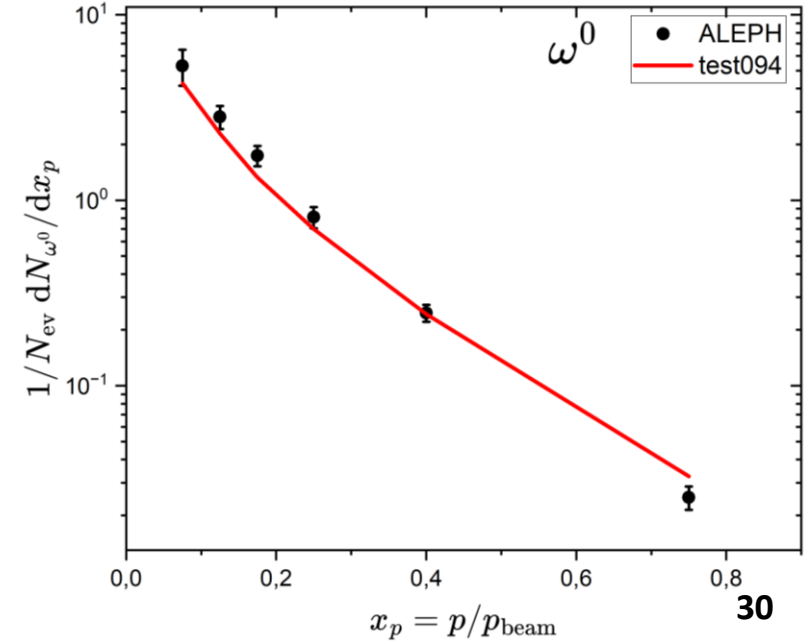
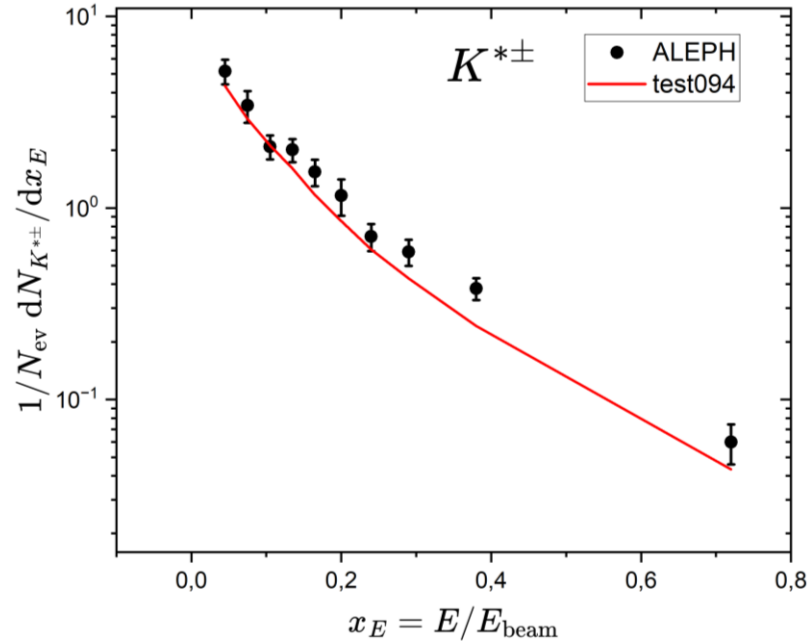
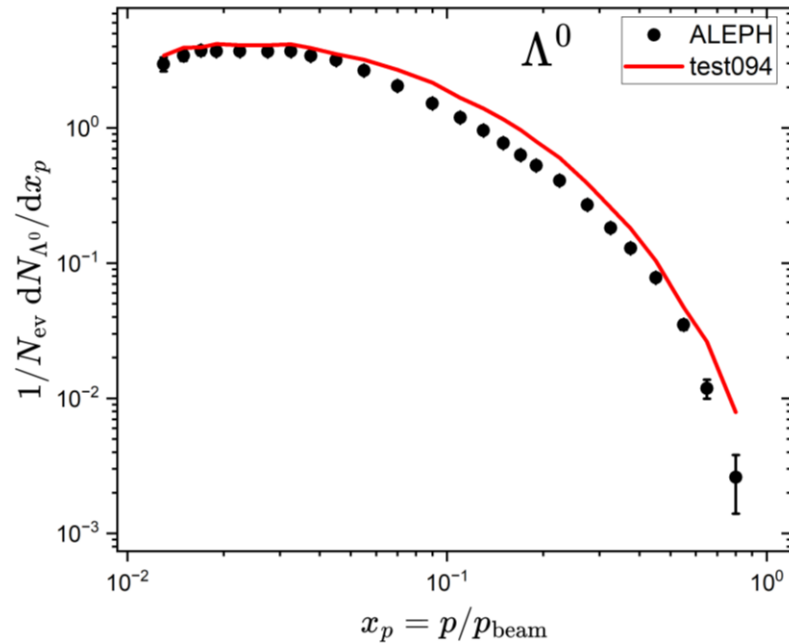
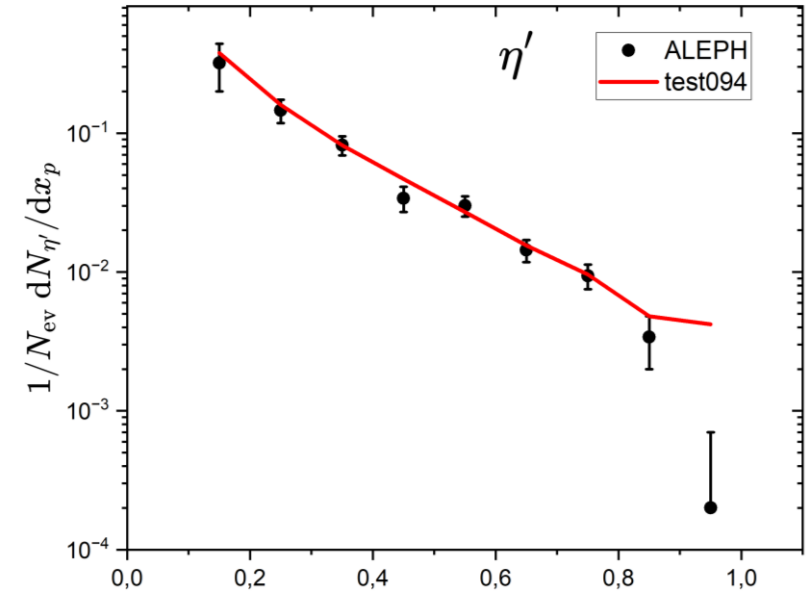
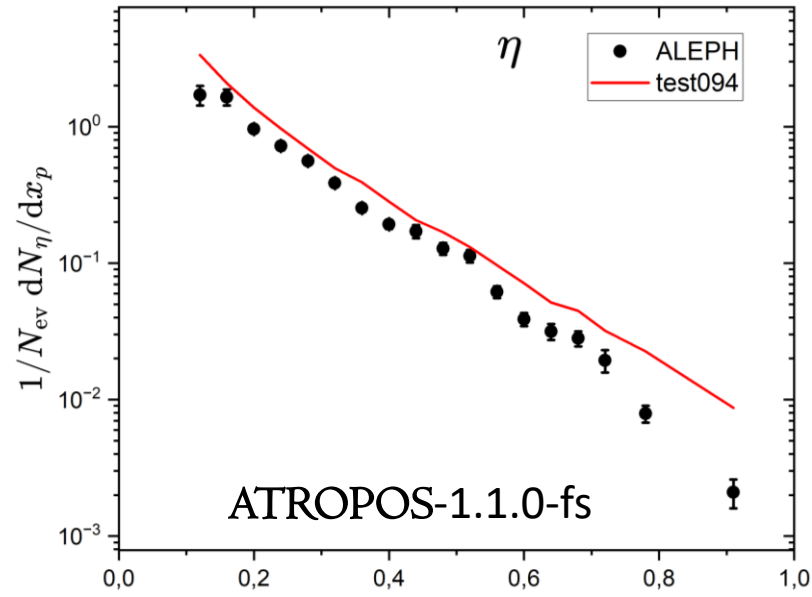
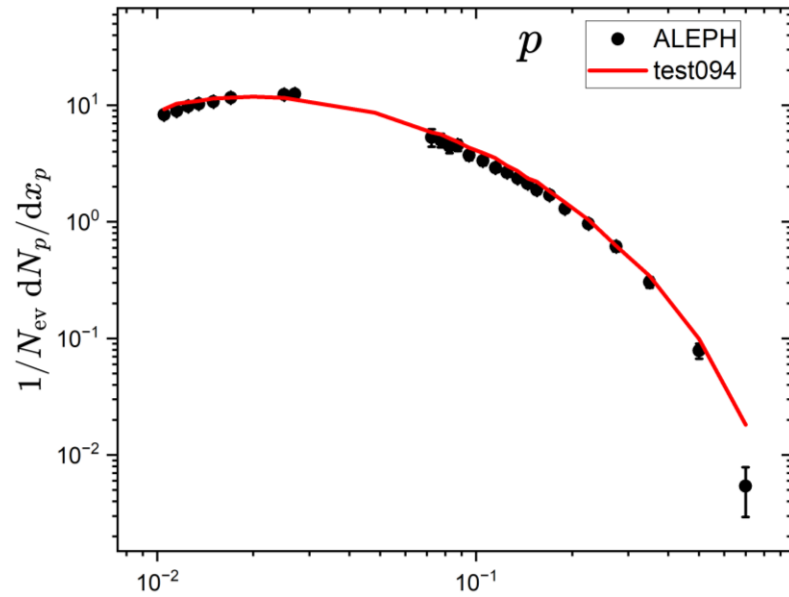


An example of sampled values of the daughter strings AM for a primary string of mass $M = 100 \text{ GeV}$ and with $\nu = 10^4$

First results: pp collisions at LHC energies



First results: e^+e^- collisions, additional



Free parameters tuning: set FPS-55

Parameter	Physical meaning	Value
κ	String tension	0.2 GeV^2
P_0	Area Decay constant	$P_0 = 0.3 + 1.7 M_{\text{string}}/(7 \text{ TeV})$
$P_{u\bar{u}} = P_{d\bar{d}}$	Relative pair production probability	0.3645
$P_{s\bar{s}}$	Relative pair production probability	0.12
$P_{uu\bar{u}\bar{u}} = P_{dd\bar{d}\bar{d}} = P_{ud\bar{u}\bar{d}}$	Relative pair production probability (diquark)	0.04
$P_{us\bar{u}\bar{s}} = P_{ds\bar{d}\bar{s}}$	Relative pair production probability (diquark)	0.015
$P_{ss\bar{s}\bar{s}}$	Relative pair production probability (diquark)	0.001
SHMT	String-to-hadron transition mass tolerance (relative to hadron mass)	0.1
DIQ01S	Suppression of spin 1 state to spin 0 for diquarks in pairs (if diff. flavor)	0.3
HFSS	Suppression of co-directional spin projections of string end-point partons	0.5 (probably too strong suppression)
M_{indiv}	Indivisible string mass (low-mass limit -> phase decay)	11 GeV for ee and πC , 7 GeV for pp (hard gluons are important)