Spatially inhomogeneous confinement/deconfinement transition in rotating gluodynamics and QCD

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in collaboration with

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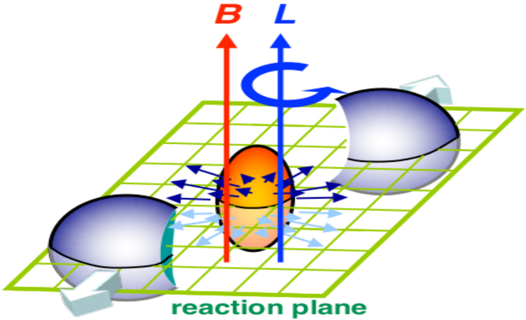
XXVI International Baldin Seminar on High Energy Physics Problems
"Relativistic Nuclear Physics and Quantum Chromodynamics"

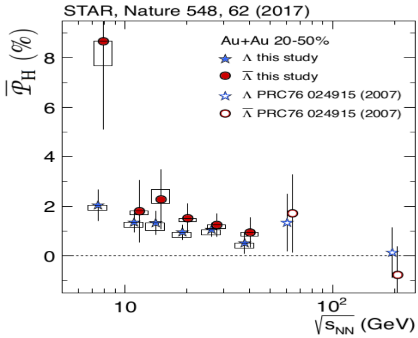
16 September 2025

QCD under extreme conditions



- High temperatures
- Large baryon density
- Intense magnetic fields
- Strong acceleration (up to $a \sim 1$ GeV)
  talk of Jayanta Dey
- **Relativistic rotation**

Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar\Lambda})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 10^{22}\, s^{-1}$)
- Relativistic rotation of QGP

How relativistic rotation influences QCD?

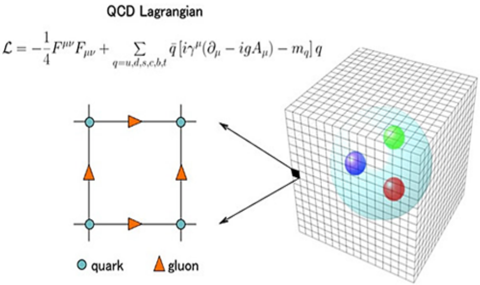
Quantum Chromodynamics(QCD)

- Degrees of freedom
  - Quarks q
  - Gluons A
- The QCD Lagrangian is well known

$$L = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_{f=u,d,s,\ldots} \bar{q}_f (i\gamma^\mu \partial_\mu - m) q_f + g \sum_{f=1}^{N_f} \bar{q}_f \gamma^\mu \hat{A}_\mu q_f$$

- Non-linear equations of motion with $g \sim 1$
- The main problem: calculation of observables based on the QCD Lagrangian
- Theoretical approaches contain assumptions with systematic errors which are difficult to estimate

Building lattice QCD



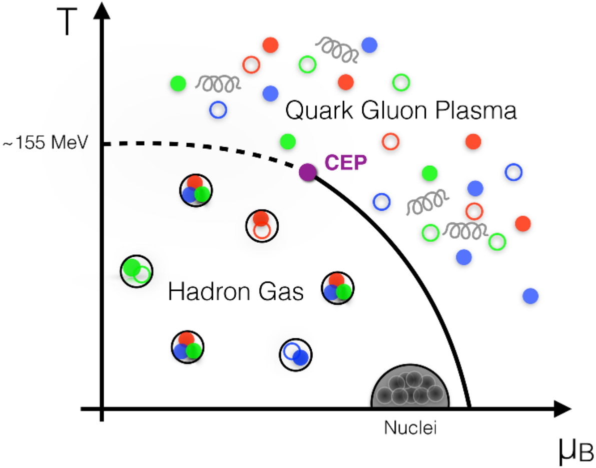
- Introduce regular cubic four dimensional lattice $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- Lattice spacing–a
- Degrees of freedom
  - **Gluon fields:** 3x3 matrices $U \in SU(3)$, live on links
  - **Quarks fields:** column $q, \bar{q}$, live on sites

Lattice QCD

- QCD partition function (thermodynamic equilibrium!)

  $Z = \int DU D\psi D\bar{\psi} \exp\left(-S_G - S_Q\right) = \int DU e^{-S_{eff}(U)}$
- Hybrid Monte Carlo simulations (generation of gluon configurations with weight $\sim e^{-S_{eff}(U)}$)
- In continuum lattice partition function exactly reproduces QCD partition function
  - Gluon contribution: $S_G\big|_{a\to 0} = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a$
  - Quark contribution: $S_Q\big|_{a\to 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$
- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: coupling constant $g(a)$ and masses of quarks $m_q(a)$
- The millennium problem can be solved for one hour

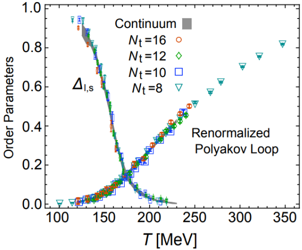
Phase transitions in QCD

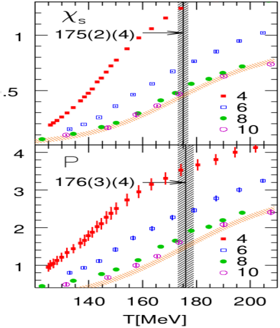


- Confinement/deconfinement transtion
- Chiral symmetry breaking/restoration transition

Confinement/deconfinement transition in QCD

- Order parameter Polyakov line:
  $P(\vec{x}) = \langle Tr P \exp\left(ig\int_0^T dx^4 \hat{A}_4(\vec{x}, x^4)\right)\rangle$
  $P = e^{-F_Q/T}$
  - Confinement: $F_Q = \infty \Rightarrow P = 0$
  - Deconfinement: $F_Q < \infty \Rightarrow P \neq 0$
- Z_3 symmetry of gluodynamics
  but $P \to e^{2\pi k/3i}P$, $k = 0, 1, 2$
- P is similar to magnetization in ferromagnetic
- First order phase transition in SU(3) gluodynamics
- Quarks violate Z_3 symmetry
- Confinement/deconfinement is crossover
  $T_c = (176 \pm 5)$ MeV
  Z. Fodor, Phys.Lett.B 643 (2006)





Lattice studies of rotating QCD

- The first lattice study

  A. Yamamoto and Y. Hirono, Phys. Rev. Lett. 111, 081601 (2013)

- Critical temperature of gluodynamics

  V. Braguta, A. Kotov, D. Kuznedelev, A. Roenko, JETP Lett. 112 (2020) 1, 6
  V. Braguta, A. Kotov, D. Kuznedelev, A. Roenko, Phys.Rev.D 103 (2021) 9, 094515

- Critical temperatures in QCD

  V. Braguta, A. Kotov, A. Roenko, D. Sychev, PoS LATTICE2022 (2023) 190
  Ji-Chong Yang, Xu-Guang Huang, e-Print: 2307.05755

- Equation of state and moment of inertia
  talks of Egor Eremeev and Dmitrii Sychev

  V. Braguta, M. Chernodub, A. Roenko, D. Sychev, Phys.Lett.B 852 (2024) 138604
  V. Braguta, M. Chernodub, I. Kudrov, A. Roenko, D. Sychev, JETP Lett. 117 (2023) 9
  V. Braguta, M. Chernodub, I. Kudrov, A. Roenko, D. Sychev, Phys.Rev.D 110 (2024) 1, 014511

- Inhomogeneous phase transition

  V. Braguta, M. Chernodub, A. Roenko, Phys.Lett.B 855 (2024) 138783
  V. Braguta, M.N. Chernodub, Ya. Gershtein, A. Roenko, JHEP 09 (2025) 079

Study of rotating QGP

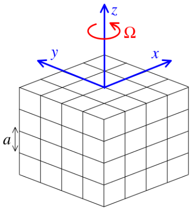
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**
  Causality condition: $v<c$

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \text{Tr}\,\exp\left[-\beta \hat{H}\right] = \int DU D\psi D\bar{\psi}\ \exp\left[-S\right]$$

- Euclidean action(in the cylindrical coordinates)

$$\begin{aligned} S &= S_0 + S_1 \Omega + S_2 \Omega^2 \\ S_1 &= \int d^4x\ r \left(\frac{i}{g^2} \left[F^a_{r\hat{\varphi}} F^a_{\tau r} - F^a_{\hat{\varphi} z} F^a_{\tau z} \right] + \bar{\psi} \gamma_4 D_{\hat{\varphi}} \psi + \frac{i}{2} \bar{\psi} \gamma_4 \sigma_{12} \psi \right) \end{aligned}$$

talk of Artem Roenko

$$S_2 = -\frac{1}{2g^2} \int d^4x\ r^2 \left[(F^a_{\hat{\varphi} z})^2 + (F^a_{r\hat{\varphi}})^2 \right]$$

- S_1 total momentum, S_2 centrifugal force
- Competition of S_1 and S_2

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$
- The dependence on boundary conditions is the property of all approaches
- One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening

Details of the simulations

Sign problem

$$
\begin{aligned}
S_G &= \frac{1}{2g^2}\int d^4x\Big[(F^a_{\tau r})^2+(F^a_{\tau\hat{\varphi}})^2+(F^a_{\tau z})^2+\\
&+(F^a_{rz})^2+\left(1-(\Omega r)^2\right)(F^a_{\hat{\varphi}z})^2+\left(1-(\Omega r)^2\right)(F^a_{r\hat{\varphi}})^2+\\
&+2ir\Omega(F^a_{r\hat{\varphi}}F^a_{\tau r}-F^a_{\hat{\varphi}z}F^a_{\tau z})\Big]
\end{aligned}
$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω
- Sometimes instead of Ω^2 we use $v^2=(\Omega R)^2$ and $v_I^2=(\Omega_I R)^2$

Ehrenfest–Tolman law

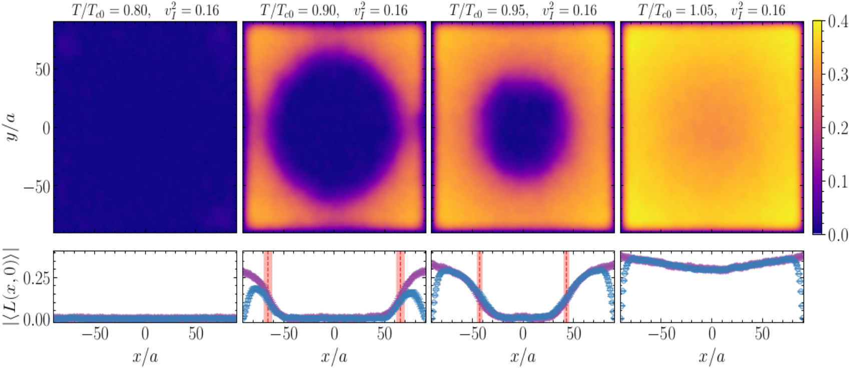
- In gravitational field the temperature is not constant in space at thermal equilibrium

$$T(r)\sqrt{g_{00}} = const = T_0$$

$$T(r) = \frac{T_0}{\sqrt{1-(\Omega r)^2}} = \frac{T_0}{\sqrt{1+(\Omega_I r)^2}}$$

- We use the designation $T = T(r=0) = T_0$
- Rotation effectively heats the system: $T(r) > T(r=0)$
- Inhomogeneous phase: confinement in the center and deconfinement in the periphery
- For imaginary rotation: deconfinement/confinement in the center/periphery
- We observe this phenomenon for accelerated observer talk of Jayanta Dey

Inhomogeneous phase transition in GP

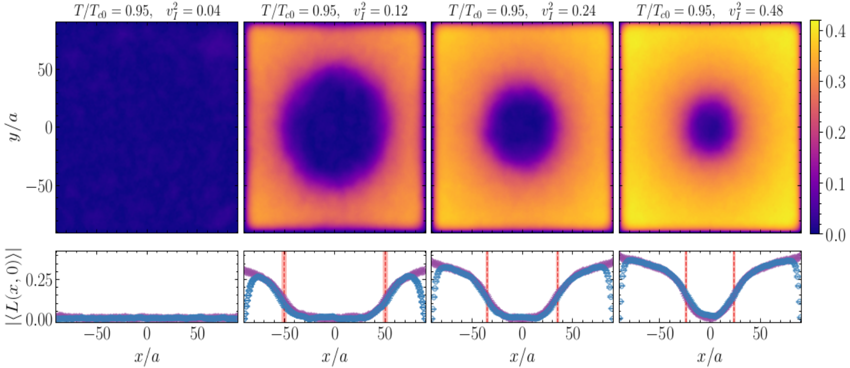


- Huge lattices are required for simulations
- Cylindrical Symmetry is restored
- The results for PBC and OBC coincides in the bulk
- Confinement in the center and deconfinement in the periphery

  In disagreement with Ehrenfest–Tolman law
- Inhomogeneous phase takes place below T_c

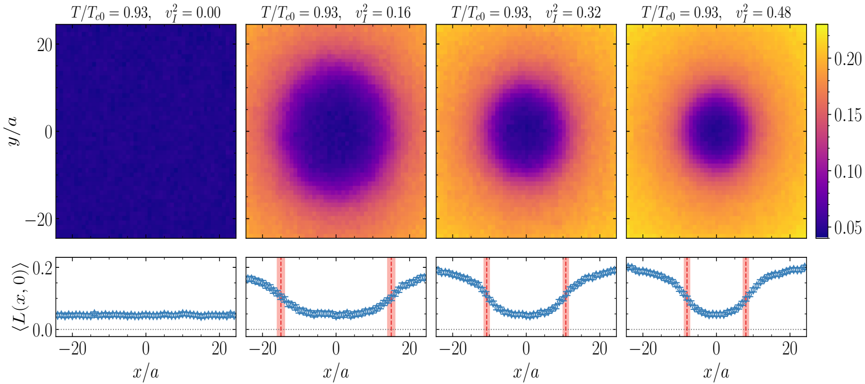
  Confinement/deconfinement phase transition as a vortex?

Inhomogeneous phase transition in QGP



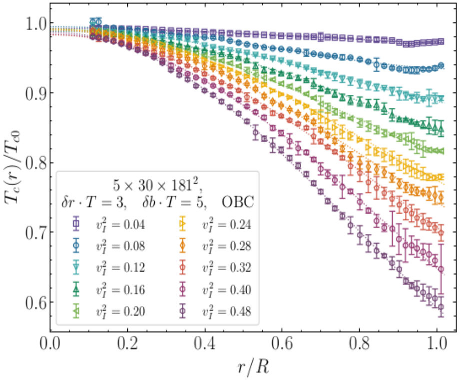
- The phase transition is induced by rotation

Inhomogeneous phase transition in QCD



- It remains to be true for quarks (Preliminary results!)

Local critical temperature $T_c(r, \Omega_I)$

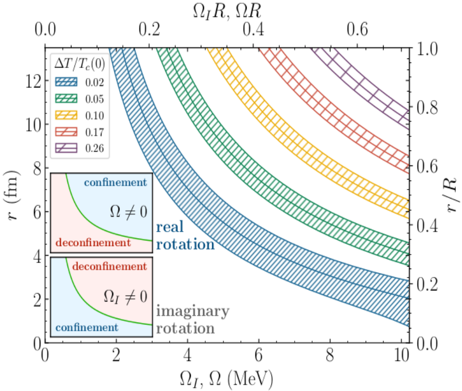


- Our results can be well described by the formula

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa_2 (\Omega_I r)^2$$

- Within the uncertainty $\frac{T_c(r=0,\Omega_I)}{T_{c0}} = 1$
- Weak dependence on the simulation parameters

Analytical continuation to real rotation



- Analytical continuation $\Omega_I^2 \to -\Omega^2$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2$$

- Inhomogeneous phase can be realised for $T > T_{c0}$
- Deconfinement in the center and confinement in the periphery
- Asymmetric QGP

Decomposition of the action

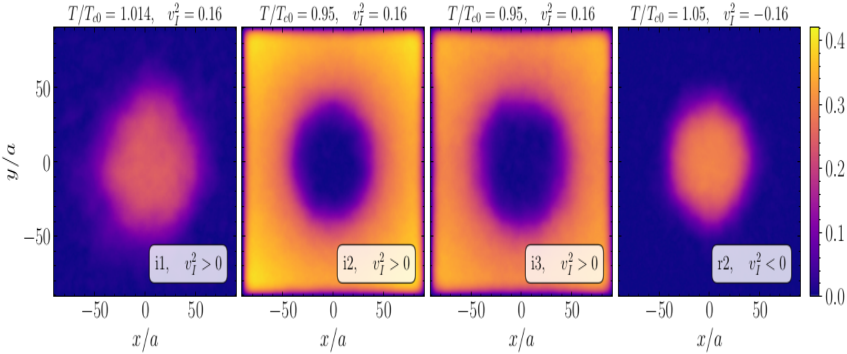
- Rotating action in the cylindrical coordinates

$$S = S_0 + S_1\Omega_I + S_2\Omega_I^2$$

  - $S_1 = -\frac{1}{g^2}\int d^4x\ r\left[F^a_{r\hat{\varphi}}F^a_{\tau r} - F^a_{\hat{\varphi}z}F^a_{\tau z}\right]$
  - $S_2 = \frac{1}{2g^2}\int d^4x\ r^2\left[(F^a_{\hat{\varphi}z})^2 + (F^a_{r\hat{\varphi}})^2\right]$
- S_1 is the total angular momentum and gives $I > 0$
- S_2 is the centrifugal force and gives $I < 0$

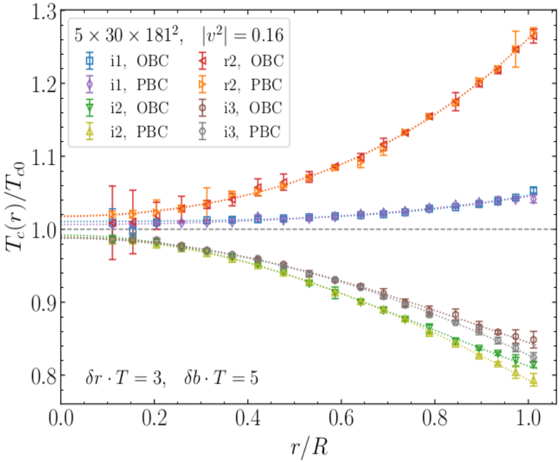
How S_1 and S_2 influence on the inhomogeneous phase transition?

Decomposition of the action



- S_2 is similar to the total acton and gives the dominant contribution
- S_1 effect is the opposite to the the total acton

Decomposition of the action



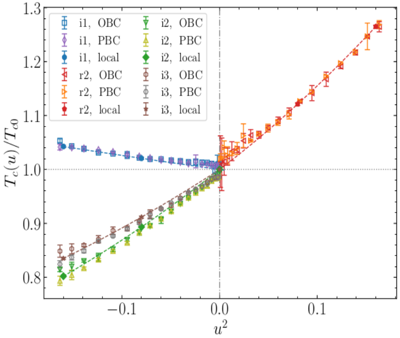
- S_1 increases the local critical temperature
- S_2 decreases the local critical temperature
- The contribution of S_2 is dominant

Local thermalization hypothesis

$$S = \frac{1}{2g^2} \int d^4x \Big[(F^a_{\tau r})^2 + (F^a_{\tau\hat{\varphi}})^2 + (F^a_{\tau z})^2 + (F^a_{rz})^2 + \\ + \left(1 - (\Omega r)^2\right) (F^a_{\hat{\varphi} z})^2 + \left(1 - (\Omega r)^2\right) (F^a_{r\hat{\varphi}})^2 + \\ + 2ir\Omega (F^a_{r\hat{\varphi}} F^a_{\tau r} - F^a_{\hat{\varphi} z} F^a_{\tau z}) \Big]$$

- For slow rotation $\Omega\zeta \ll 1$ the coefficients vary slowly
- Local thermalization approximation: study the action with the coefficients freezed at $r = r_0$

Local thermalization hypothesis



- Good agreement with the full action for sufficiently small Ω
- A lot of advantages
  - The higher order coefficients can be found
    $T_c(r,\Omega)/T_{c0} = 1 + \sum_n c_n(\Omega r)^{2n}$, $T_c(r=0,\Omega)/T_{c0} = 1$
  - Weak dependence on the BC
  - One can study small lattices
  - Allows to understand inhomogeneous phase transition

Origin of the inhomogeneous phase transition

$$S_G = \int d^4x \, \Big[\frac{1}{2g^2}\left((E_x^a)^2 + (E_y^a)^2 + (E_z^a)^2 + (H_y^a)^2\right) + \\ + \frac{1}{2\tilde{g}^2}\left((H_x^a)^2 + (H_z^a)^2\right)\Big]$$

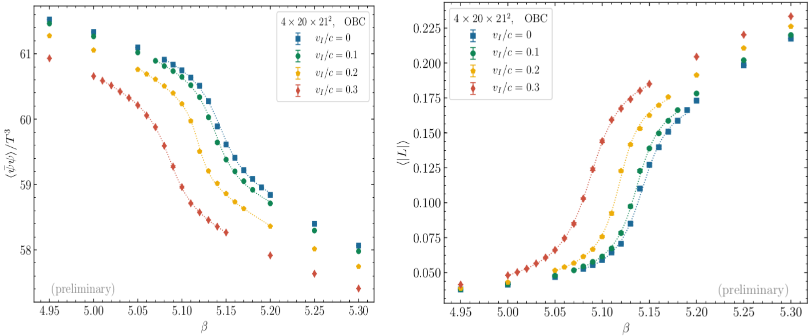
- Linear in Ω term can be neglected
- External gravitational field leads to the asymmetric action $\frac{g^2}{\tilde{g}^2} = 1 - (\Omega r)^2$
- The asymmetry $g^2/\tilde{g}^2$ is larger in the periphery region leading to the shift of the critical temperature
- GR effect!

Conclusion

- Lattice studies of rotating gluodynamics and QCD have been carried out
- We observed inhomogeneous phase transition in GP: deconfinement in the central and confinement in the periphery regions
- External gravitational field leads to asymmetryc action and shift of the critical temperature in the periphery regions
- We believe that all the observed effects remain in QCD

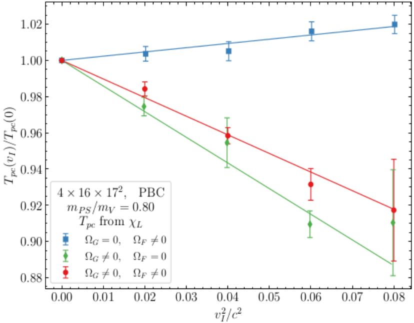
THANK YOU!

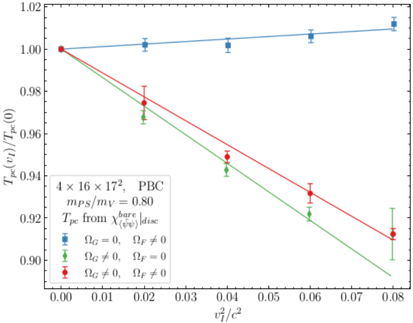
Backup slides: Simulation with fermions



- Lattice simulation with Wilson fermions
- Critical couplings of both transitions coincide
- Critical temperatures are increased

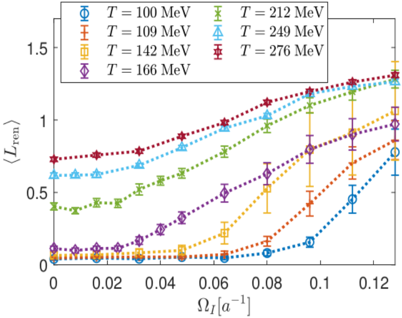
Backup slides: Simulation with fermions

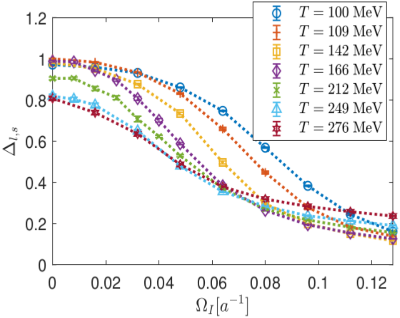




- QCD action: $S = S_f(\Omega_F) + S_g(\Omega_G)$
- One can introduce velocities for gluons Ω_G and fermions Ω_F
- $\Omega_F \neq 0, \Omega_G = 0$ decreases critical temperatures
- $\Omega_F = 0, \Omega_G \neq 0$ increases critical temperatures
- The gluon sector gives the dominant contribution

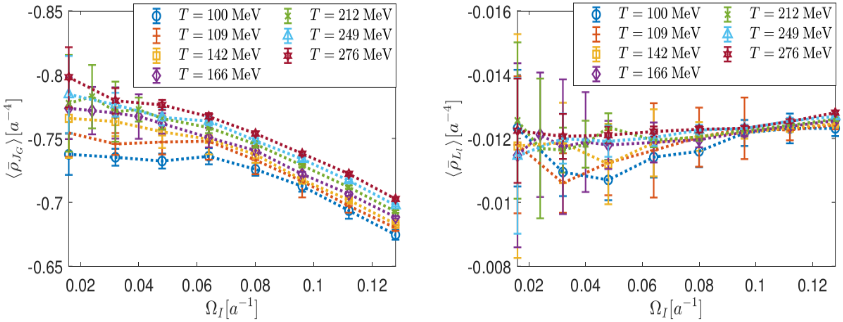
Backup slides: Simulation with fermions (e-Print: 2307.05755)





- Increase of the bulk average critical temperatures of both transitions

Backup slides:
Simulation with fermions (e-Print: 2307.05755)



- Rotational rigidities: $\rho_{J_G} = \frac{J_G}{\Omega R^2}, \quad \rho_{L_f} = \frac{L_f}{\Omega R^2}$
- Spin susceptibility: $\zeta_f = \frac{s}{\Omega}$
- Negative moment of inertia