Study of the running coupling constant $\alpha_s(q^2)$ of π^- mesons and protons from $p+p$ and $p+C$ interactions at 4.2 GeV/c

XXVIth International Baldin Seminar

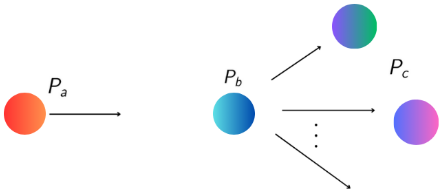
Urangua.M
Baatar.Ts, Malakhov.A.I, Sovd.M, Khishigbuyan. N, Sharkhuu.G, Otgongerel.B, Batgerel.B

Institute of Physics and Technology, Ulaanbaatar, Mongolia
Joint Institute of Nuclear Research, Dubna, Russia

16 Sep 2025

CONTENTS

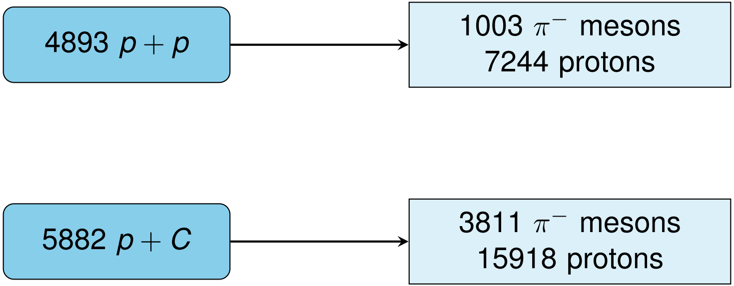
Introduction



In this work following reactions at 4.2 GeV/c were studied

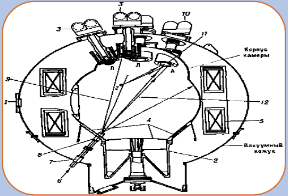
1. $p + p \rightarrow \pi^- + X$
2. $p + C \rightarrow \pi^- + X$
3. $p + p \rightarrow p + X$
4. $p + C \rightarrow p + X$

The numbers of events and secondary particles of the reactions studied here



Experimental method

JOINT INSTITUTE JINR FOR NUCLEAR RESEARCH



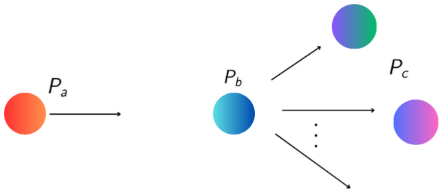
ROOT Data Analysis Framework

$p + (C_3H_8)$

$p + p$

$p + C$

Cumulative number



It is well established that, in hadron-nucleus and nucleus-nucleus interactions, secondary particles can be produced in the regions of phase space that are kinematically forbidden for simple hadron-nucleon collisions. These unusual particles arise due to multinucleon interactions inside the nucleus.
To study such particles effectively, use a variable known as the cumulative number.

Cumulative number

Cumulative number determined by following formula:

$$n_c = \frac{P_a \cdot P_c}{P_a \cdot P_b} = \frac{E_c - \beta_a P_c^{||}}{m_p} \tag{1}$$

Where P_a, P_b, P_c - the four-dimensional momenta of the incident, target and secondary particles under consideration, $E_c, P_c^{||}$ - energy and longitudinal momentum of the secondary particle, β_a - velocity of the incident particle, m_p - mass of proton
Cumulative number and transferred momentum are connected by following formula:

$$q^2 = -(P_a - P_b)^2 = 2E_a \cdot m_p \cdot n_c - (m_a^2 + m_c^2) \tag{2}$$

Where E_a - energy of the incident particle, m_a - mass of the incident particle, m_c - mass of the secondary particle.

Figure 1 a,b. Cumulative number distributions of secondary π^- mesons

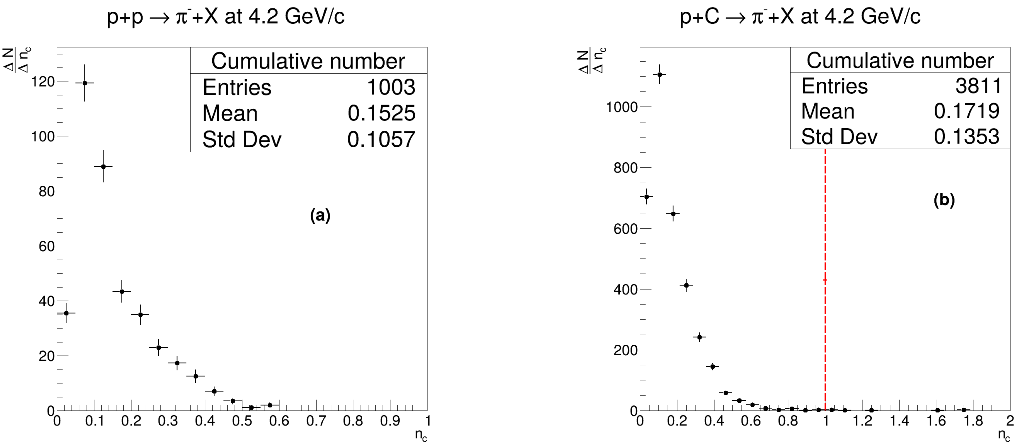
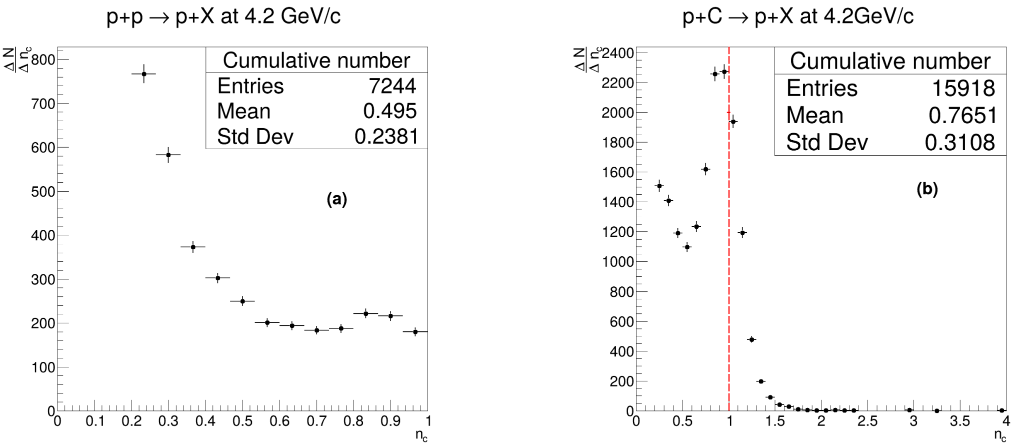


Figure 2 a,b. Cumulative number distributions of secondary protons



We choose QCD cut parameter that $\Lambda_{QCD} = (c\hbar)GeV = 0.197GeV$ [1]. QCD running coupling constant in leading order determined by following formula:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2}{\Lambda^2_{QCD}}\right)} \tag{3}$$

Where:

$$q^2 = 2E_a \cdot m_p \cdot n_c - (m_a^2 + m_c^2)$$

$$\beta_0 = 11 - \frac{2}{3}n_f$$

β_0 - beta function coefficient (first approximation)
n_f - number of quark flavors

[1] Baatar.Ts, Khishigbuyan.N, Malakhov.A.I, Batgerel.B, Otgongerel.B, Sovd.M, Sharkhuu.G, Urangua.M, Study of the strong coupling constant $\alpha_s(q^2)$ of π^- and K^0 mesons and protons from $\pi^- + p$, $\pi^- + C$ interactions at 40 GeV/c, PEPAN letters, 2025 (submitted).

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2}{\Lambda^2_{QCD}}\right)}$$

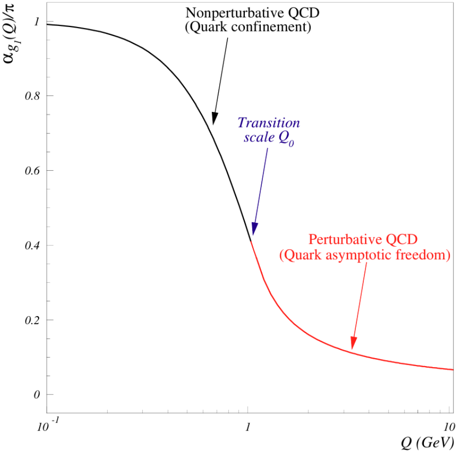
- Confinement

$$q^2 \approx \Lambda^2_{QCD}, \qquad \alpha_s \to \infty$$

- Asymptotic freedom

$$q^2 \to \infty, \qquad \alpha_s \to 0$$

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction"



Deur, S.J.Brodsky, G.F. de Teramond, "QCD running coupling", Progress in Particle and Nuclear Physics, vol.90, p-1-74, 2016. https://doi.org/10.48550/arXiv.1604.08082

Figure 3 a,b. CERN ROOT class - TFUMILI

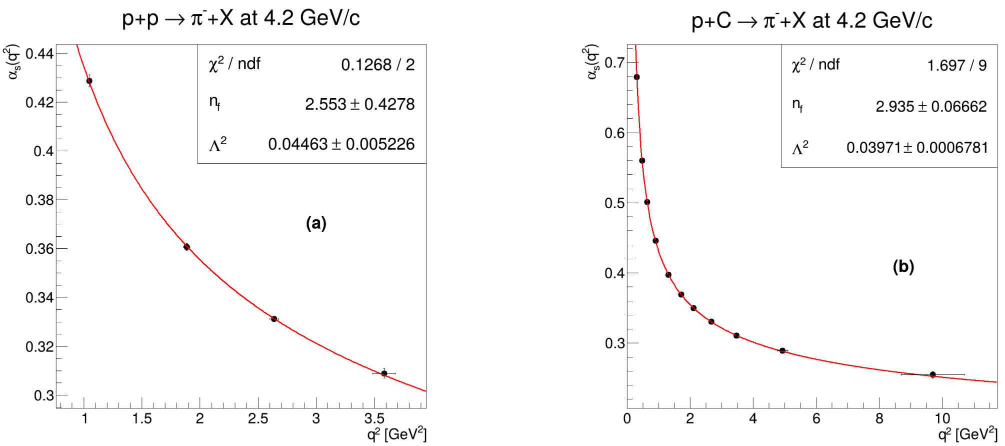


Figure 3 c,d. CERN ROOT class - TFUMILI

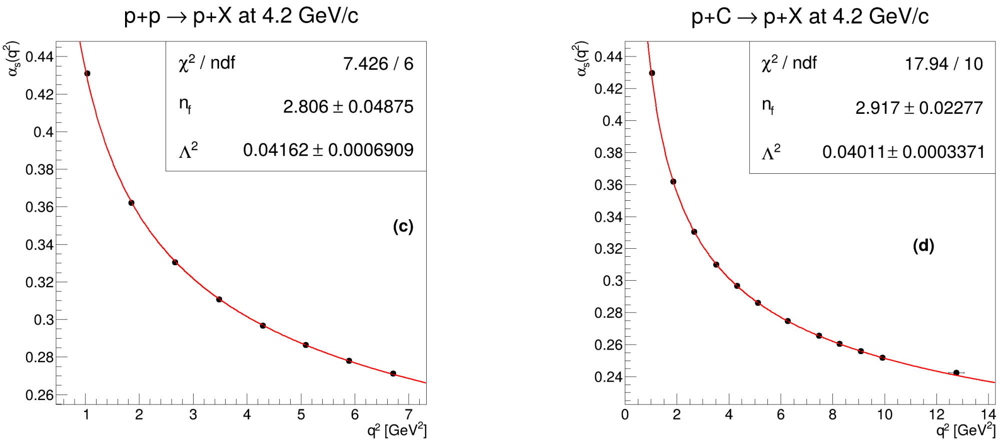
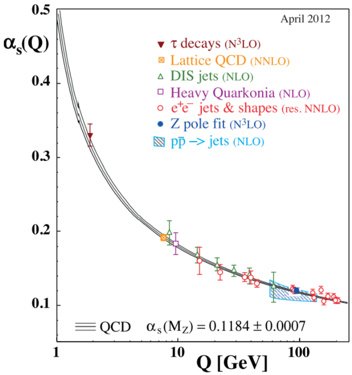


Figure 4 a,b. QCD running coupling constant of secondary π^- mesons



- *J.Beringer et al. (PDG), Phys.Rev. D 86, 010001(2012)*

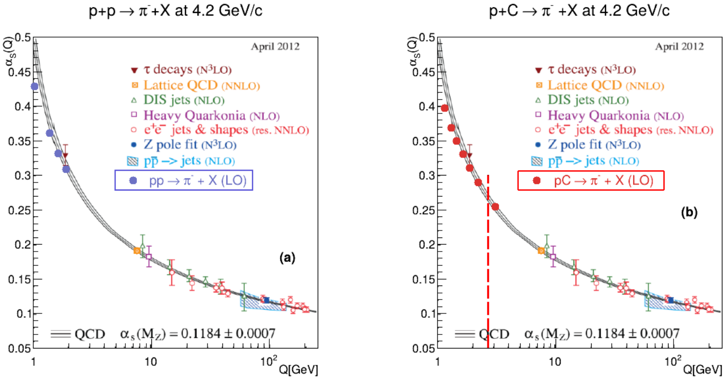
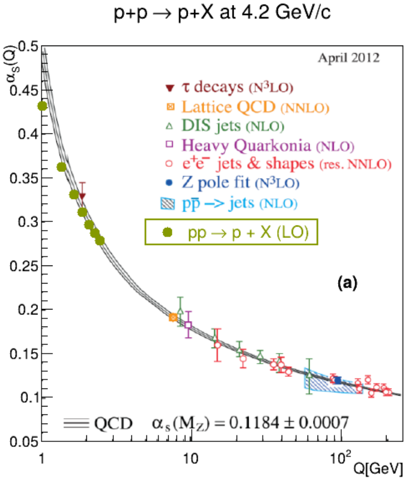


Figure 5 a,b. QCD running coupling constant of secondary protons



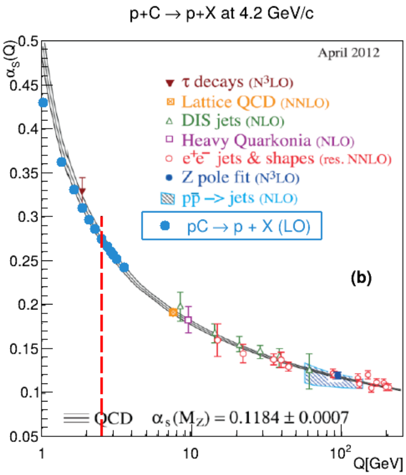


Figure 6 a,b. QCD running coupling constant

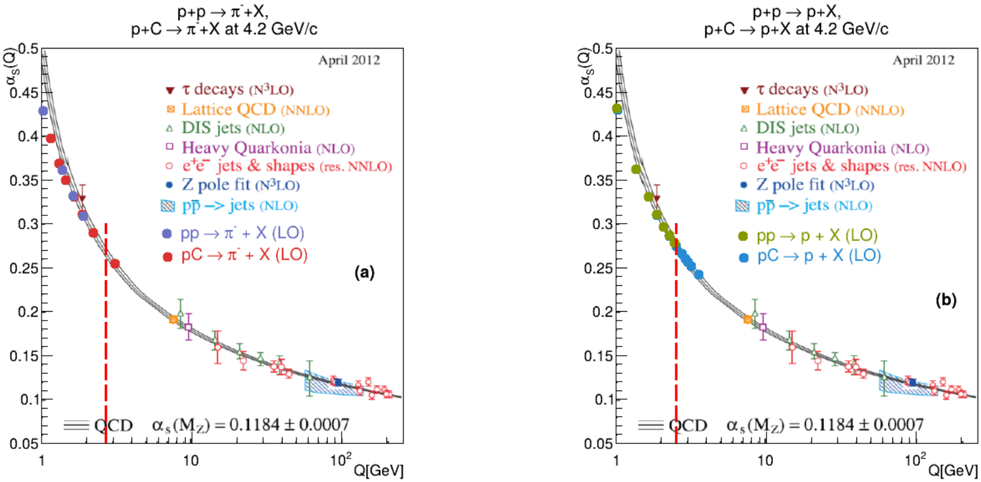
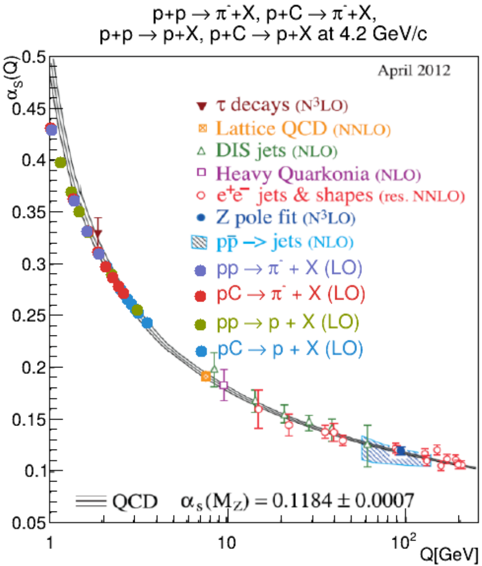


Figure 7. QCD running coupling constant



Conclusion

- The values of $\alpha_s(q^2)$ with cut parameter $\Lambda_{QCD} = (c\hbar)GeV = 0.197GeV$ are compared with the QCD predictions and the agreement between the experimental data and theory are satisfactory.
- The values of the $\alpha_s(q^2)$ for the cumulative π^- mesons and protons produced in $p + C$ interactions are also in agreement with QCD predictions, but these values of $\alpha_s(q^2)$ correspond to the cumulative region or high q^2 values not allowed for $p + p$ interaction at the same incident energy.

REFERENCES

- Baatar.Ts, Khishigbuyan.N, Malakhov.A.I, Batgerel.B, Otgongerel.B, Sovd.M, Sharkhuu.G, Urangua.M, Study of the strong coupling constant $\alpha_s(q^2)$ of π^- and K^0 mesons and protons from $\pi^- + p, \pi^- + C$ interactions at 40 GeV/c, PEPAN letters, 2025 (submitted).
- Baatar.Ts, Khishigbuyan.N, Malakhov.A.I, Batgerel.B, Otgongerel.B, Sovd.M, Sharkhuu.G, Urangua.M, Study of the strong coupling constant $\alpha_s(q^2)$ of π^- mesons and protons from $p + p$ interactions at 205 GeV/c, PEPAN letters, 2025 (submitted).
- Baldin A. M. Particles and Nuclei, 1977, V.8 p.429.
- Perkins D. Introduction to high energy physics, 2000, 4th ed.
- A. Deur, S. J. Brodsky, G. F. de Teramond, QCD running coupling, Prog.Part.Nucl.Phys.90:1-74,2016. https://doi.org/10.48550/arXiv.1604.08082
- G. M. Prosperi, M. Raciti, C.Simolo, On the running coupling constant in QCD, Prog.Part.Nucl.Phys.58:387-438,2007. https://doi.org/10.48550/arXiv.hep-ph/0607209
- J.Beringer et al. (PDG), Phys.Rev. D 86, 010001(2012)

Thank you for your attention!

$$q^2 = -(P_a - P_c)^2 = 2E_a(E_c - \beta_a P_c^{||}) - (m_a^2 + m_c^2) = 2E_a m_p n_c - (m_a^2 + m_c^2) \qquad (4)$$

$$\frac{q^2}{m_a^2 + m_c^2} = \frac{2E_a m_p n_c - (m_a^2 + m_c^2)}{m_a^2 + m_c^2} = \frac{2E_a m_p n_c}{m_a^2 + m_c^2} - 1 \qquad (5)$$

$$\ln\left(\frac{q^2}{m_a^2 + m_c^2}\right) = \ln\left(\frac{2E_a m_p n_c - (m_a^2 + m_c^2)}{m_a^2 + m_c^2}\right) = \ln\left(\frac{2E_a m_p n_c}{m_a^2 + m_c^2} - 1\right) \qquad (6)$$

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2}{\Lambda^2}\right)} = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2}{m_a^2 + m_c^2}\right)} \qquad (7)$$

$$m_a^2 + m_c^2 = \frac{(c\hbar)^2[GeV^2 fm^2]}{(\lambda_C^a)[fm^2]} + \frac{(c\hbar)^2[GeV^2 fm^2]}{(\lambda_C^c)[fm^2]} = (c\hbar)^2\left[\frac{(\lambda_C^a)^2 + (\lambda_C^c)^2}{(\lambda_C^a)^2 \cdot (\lambda_C^c)^2}\right] = k(c\hbar)^2[GeV^2] \qquad (8)$$

$$k = \left[\frac{(\lambda_C^a)^2 + (\lambda_C^c)^2}{(\lambda_C^a)^2 \cdot (\lambda_C^c)^2}\right] \qquad (9)$$

The main part of events on the carbon target is separated by the following criteria

- The total charge of the all detected secondary particles $2_{n_{z\geq 2}} + n_{z=1} - n_{z=-1} > Z_{Ai} + 1$,
- The number of slow ($P \leq 0.75 GeV/c$) protons $n_p > 1$,
- The number of protons emitted in the backward hemisphere in the laboratory system $n_p^b > 0$,
- $n_{z=-1} > 1$ for pC events, $n_{z=-1} > 2$ for $dC-, \alpha C-, uCC$ events
- The number of the charged particles in the event $n_{z=\pm 1}$ is odd for pC and dC events,
- Criterion of the target mass. $M_t > 1.5 \cdot m_p$, where $M_t = \Sigma(E_i - P_i \cos\theta_i)$, E_i is the energy, $P_i \cos\theta$ is the longitudinal momentum, m_p is the proton mass, the summation is performed by all pions and protons, excluding stripping fragments from the projectile ($p_p > 3GeV/c, \theta < 4^\circ$) and evaporated fragments from the target ($p_p < 0.3GeV/c$).