Study of the strong coupling constants $\alpha_s(q^2)$ of π^- mesons and protons from $p+p$, $p+C$ interactions at 10 GeV/c

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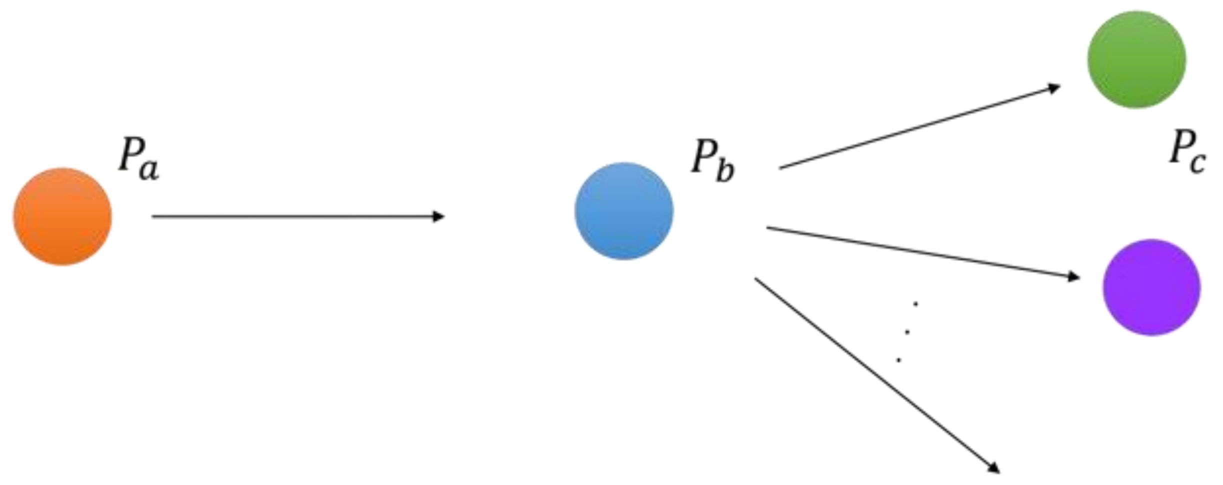
INTRODUCTION



In this work following reactions at 10 GeV/c were studied

- $p + p \rightarrow \pi^- + X$ (1)
- $p + p \rightarrow p + X$ (2)
- $p + C \rightarrow \pi^- + X$ (3)
- $p + C \rightarrow p + X$ (4)

The numbers of events and secondary particles of the interactions studied.

$p + C$ interaction

16428 events

17299 π^- mesons

49703 protons

$p + p$ interaction

12275 events

7106 π^- mesons

21740 protons

EXPERMENTAL METHODS

Accelerator:	JINR synchrophasotron
Projectile:	Proton (p)
Incident momentum :	10 GeV/c
Detector:	Dubna 2-meter propane (C_3H_8) bubble chamber
The advantage of the bubble chamber :	The distribution are obtained under the condition of 4π geometry of the secondary particles
Average error :	momentum measurements $\sim 12\%$ angular measurements $\sim 0.6^0$.

CUMULATIVE NUMBER AND THE SQUARE OF 4-MOMENTUM TRANSFER q^2

Cumulative number determined by following formula:

$$n_c = \frac{P_a \cdot P_c}{P_a \cdot P_b} = \frac{E_c - \beta_a P_c^{||}}{m_p} \qquad (1)$$

where P_a, P_b and P_c are the four-dimensional momenta of energy incident, target and secondary particles under consideration. E_c and $P_{||}^c$ are the energy and longitudinal momentum the secondary particle, $\beta_a = \frac{P_a}{E_a}$ is the velocity of the incident particle. At high energy experiments $\beta_a = 1$ and m_p is the proton mass.

Cumulative number and transferred momentum are connected by following formula:

$$q^2 = -(P_a - P_b)^2 = 2E_a\left(E_c - \beta_a P_c^{||}\right) - (m_a^2 + m_c^2) = 2E_a \cdot m_p \cdot n_c - (m_a^2 + m_c^2) \qquad (2)$$

where E_a and m_a are the energy and mass of incident energy particle, m_c mass of the secondary particles under consideration

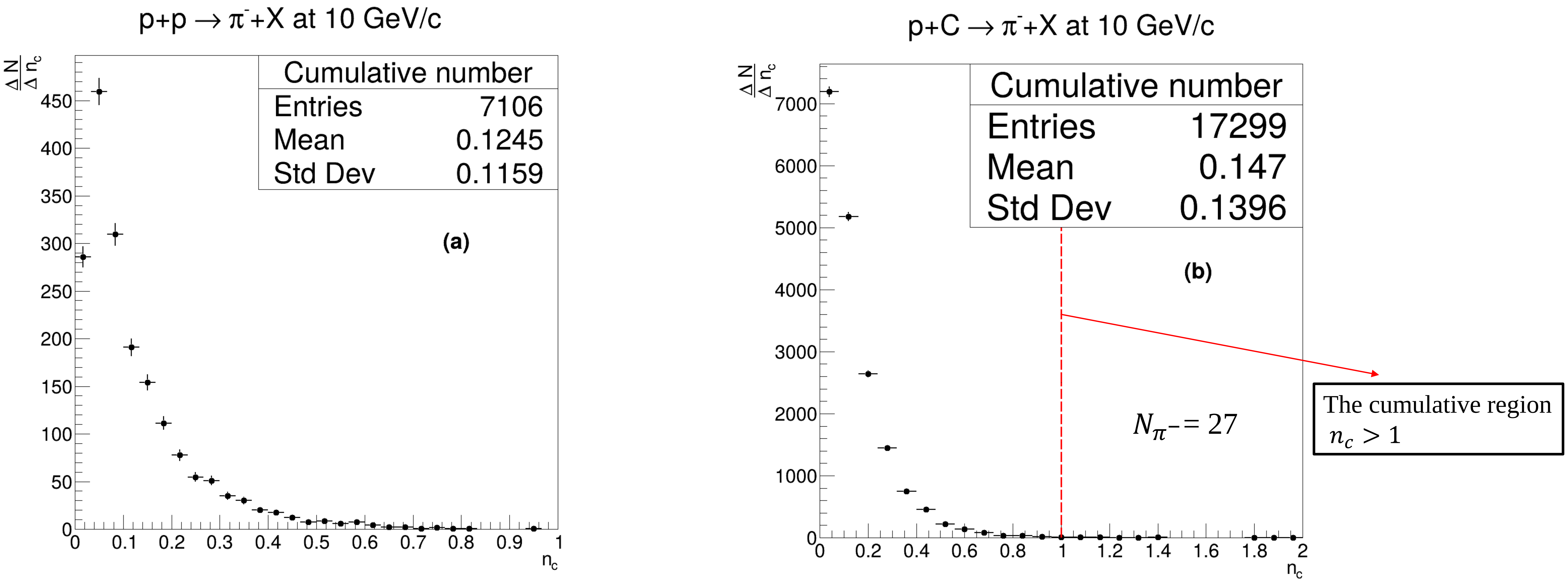


Figure 1 a,b. *The n_c distribution of π^- mesons from $p + p$ interaction (a) and $p + C$ interaction (b).*

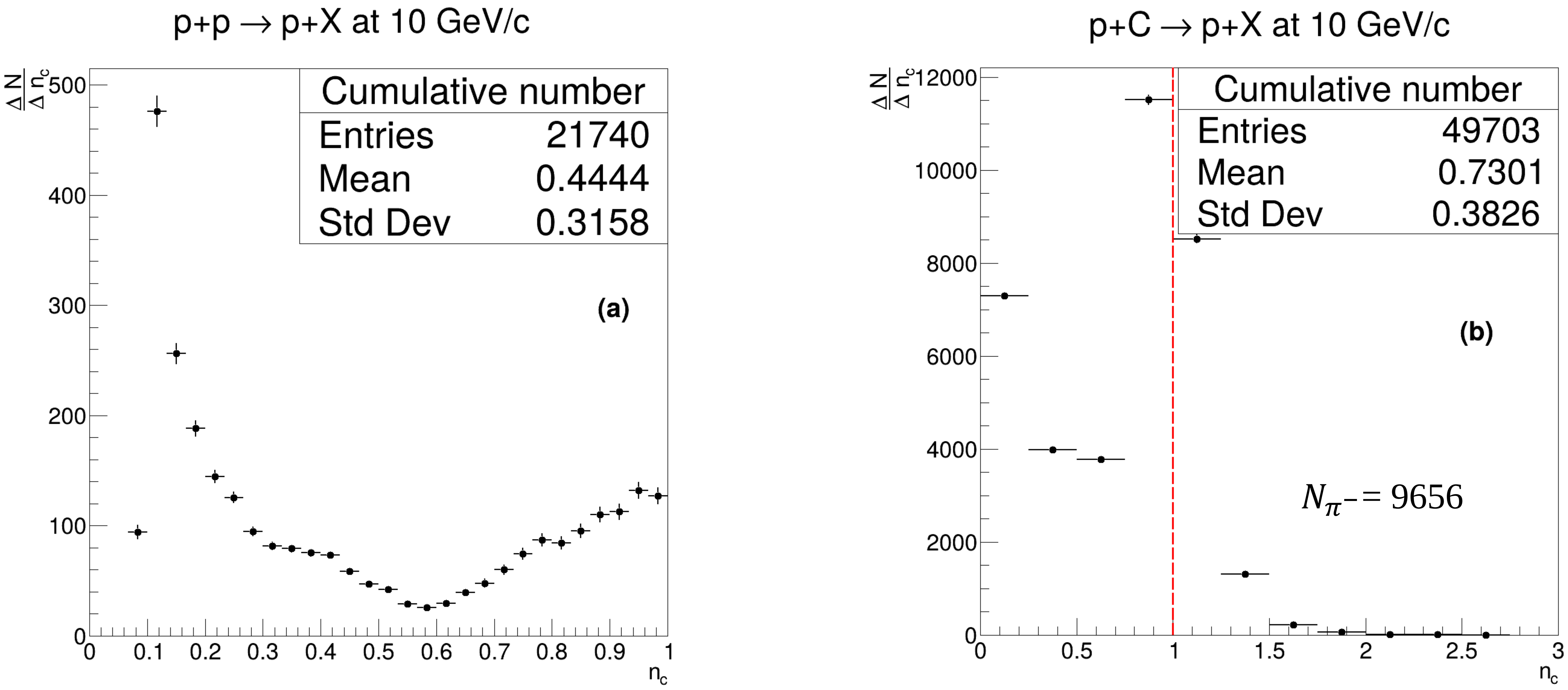


Figure 2 a,b. *The n_c distribution of protons from $p + p$ interaction (a) and $p + C$ interaction (b).*

From these figures, we can see the results:

- In high-energy hA and AA interactions, the secondary particles are produced in a region kinematically forbidden for hN interactions.
- Cumulative particles are produced as a result of multinucleon interactions.

QCD running coupling constant

The strong coupling constant in the LO (leading order) approximation is determined by following formula:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{\Lambda^2_{QCD}}\right)} \qquad (3)$$

$$\beta_0 = 11 - \frac{2}{3} * n_f$$

where β_0 is the one-loop QCD β-function and the coefficients obtained when calculating the different radiation corrections. n_f is the number of quark flavor actively participating in the interaction.

Where, $q^2 = 2E_a m_p n_c - (m_a^2 + m_c^2)$

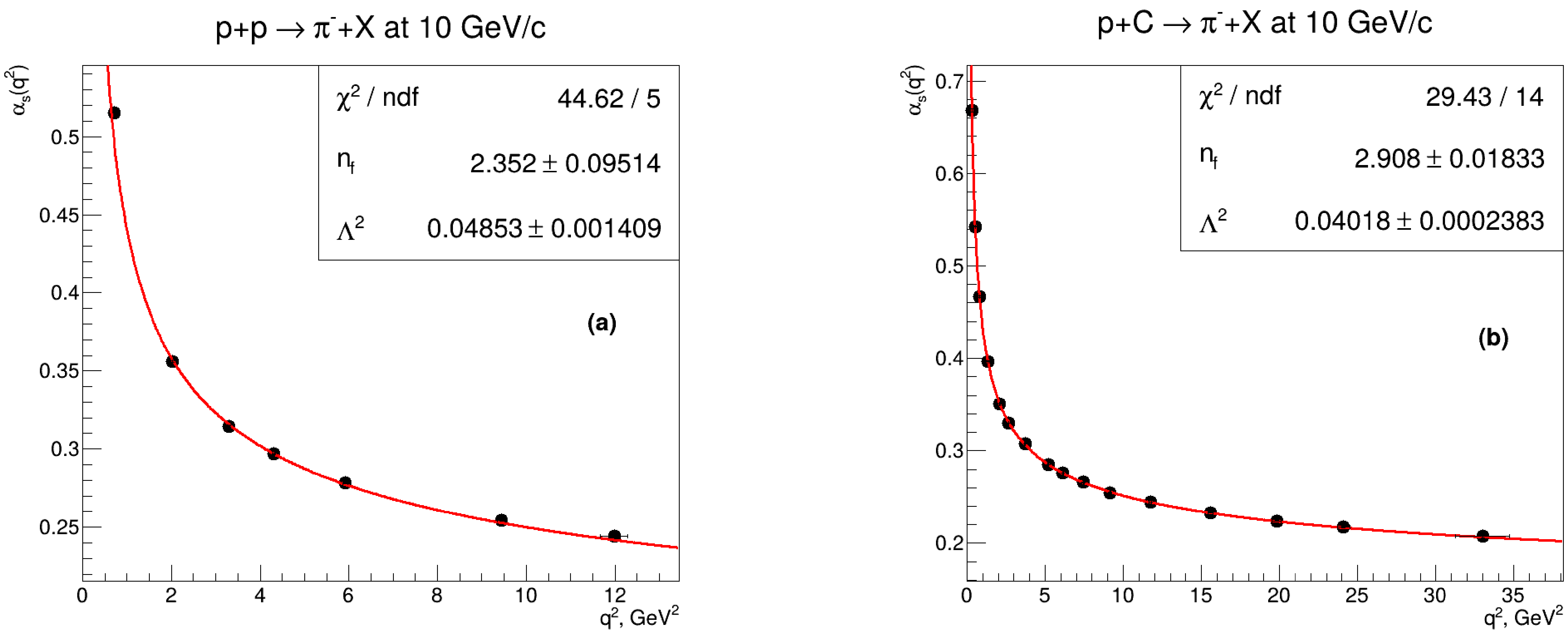
So, we chose this constant as a cut parameter of the strong coupling constant $\alpha_s(q^2)$.

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{(c\hbar)^2}\right)} \qquad (4)$$

1. Baatar.Ts, Khishigbuyan.N, Malakhov.A.I, Batgerel.B, Otgongerel.B, Sovd.M, Sharkhuu.G, Urangua.M, Study of the strong coupling constant $\alpha_s(q^2)$ of π^-, K^0 mesons and protons from $\pi^- + p, \pi^- + C$ interactions at 40 GeV/c, PEPAN letters, 2025.

QCD cut parameter: $\Lambda_{QCD}^2 = (c\hbar)^2[GeV^2]$=0.0388 GeV^2 or $\Lambda_{QCD} = 0.197\ [GeV]$ and number of quark flavor n_f=3.

The cut parameter Λ_{QCD} and n_f of $\alpha_s(q^2)$ was taken as in paper [1,2].



Fitting by CERN Root program class “Fumili”

Figure 3a, b. Fitting results for secondary π^- mesons

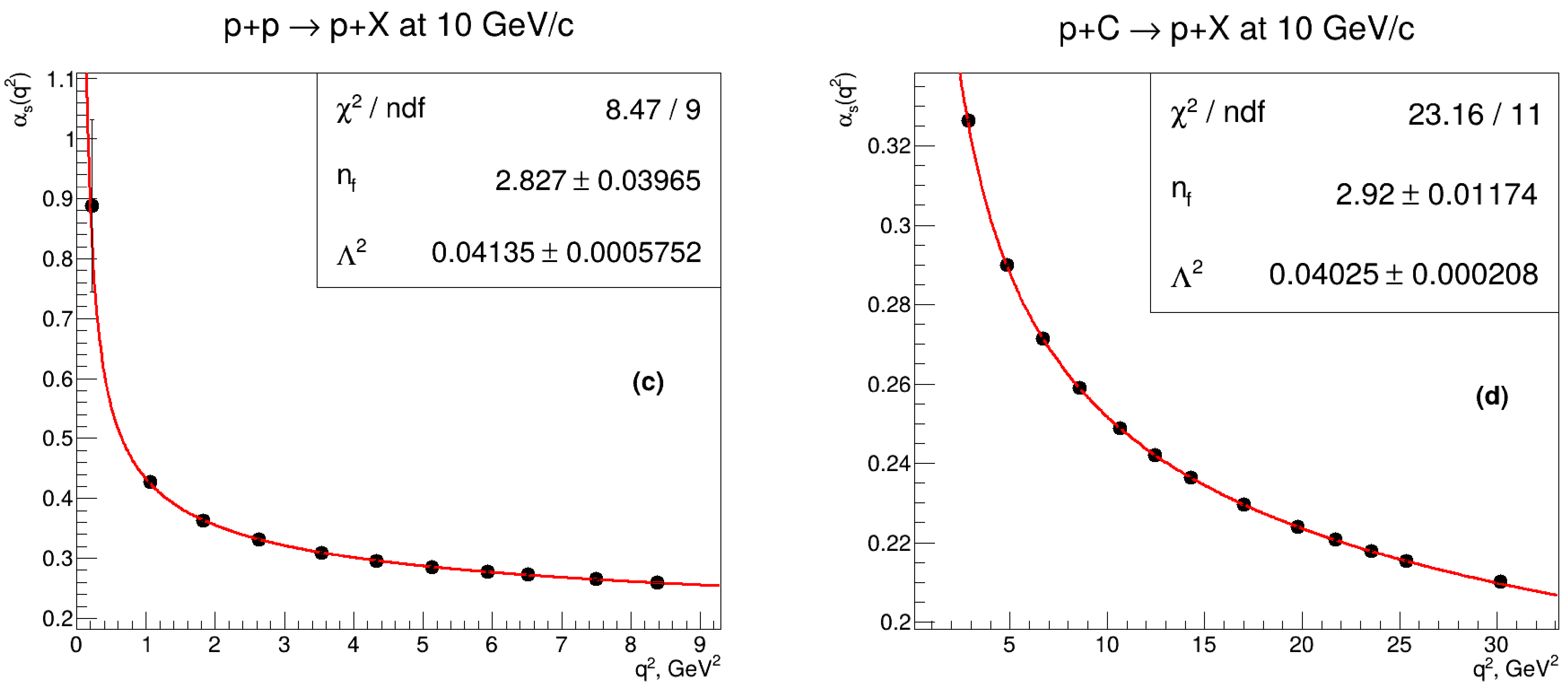


Figure 3c, d. Fitting results for secondary protons

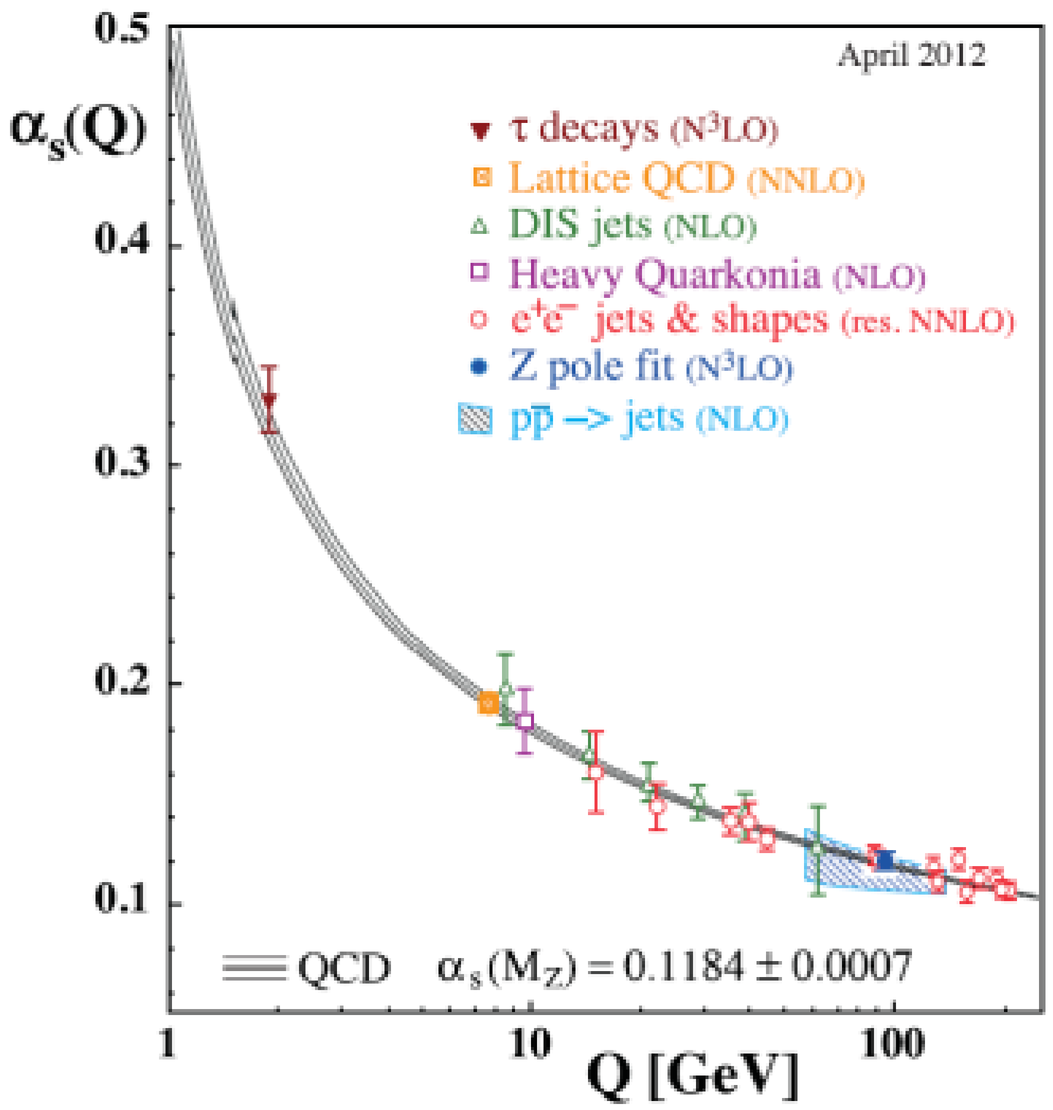
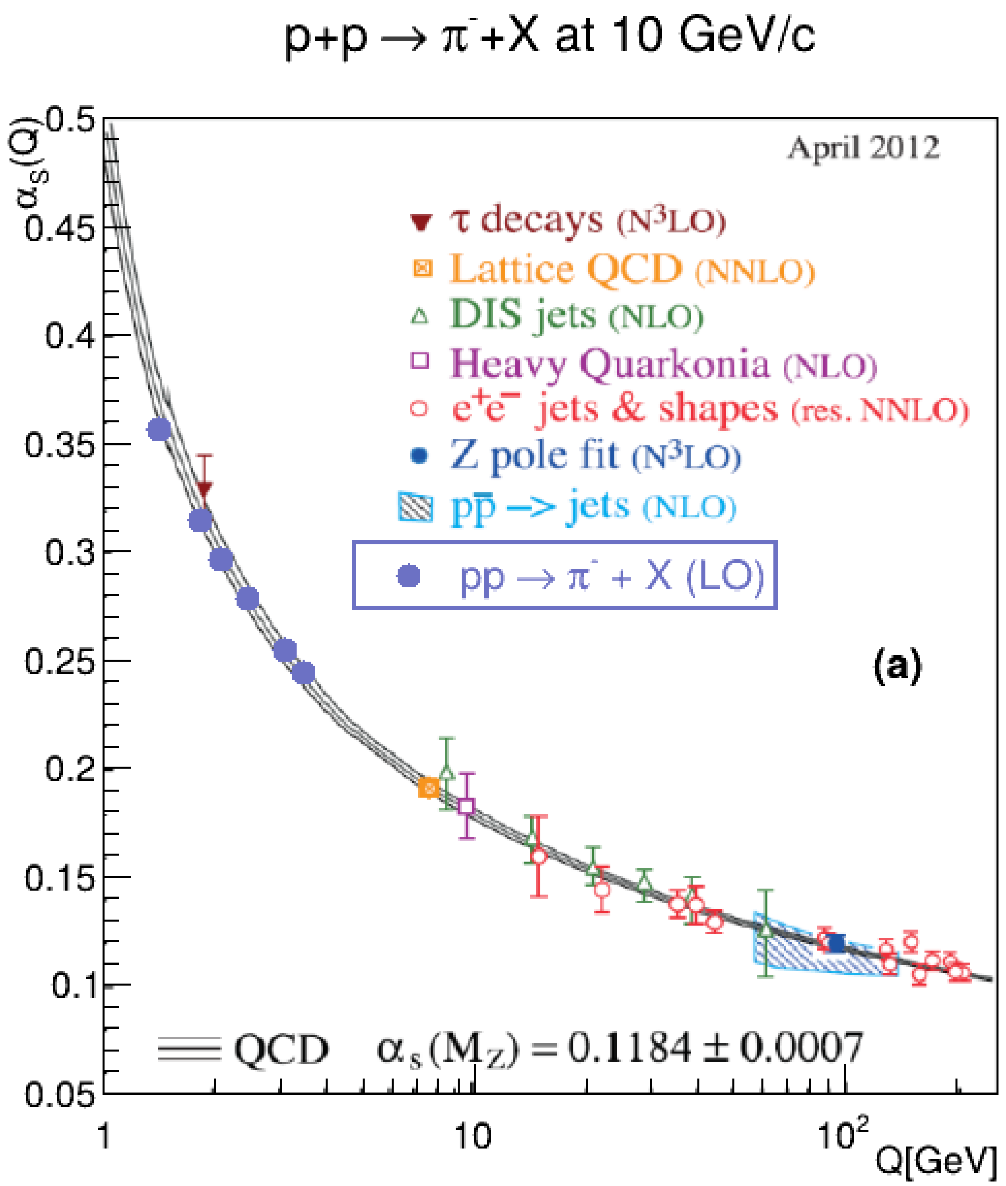


Figure adapted from Particle Data Group, 2012

[J. Beringer et al. (PDG), Phys. Rev. D 86, 010001 (2012)]



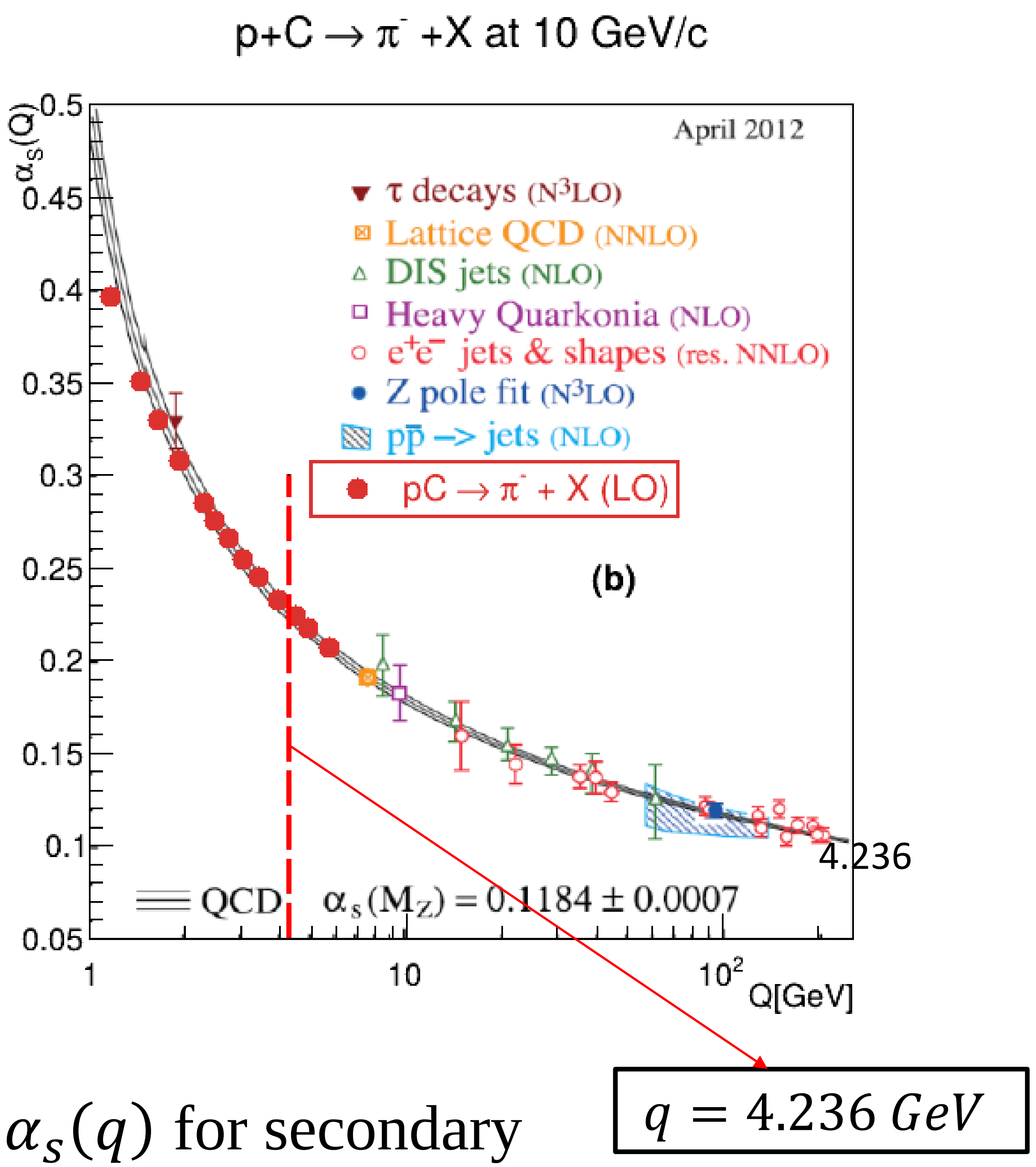


Figure 4a, b. Dependence of $\alpha_s(q)$ for secondary π^- mesons

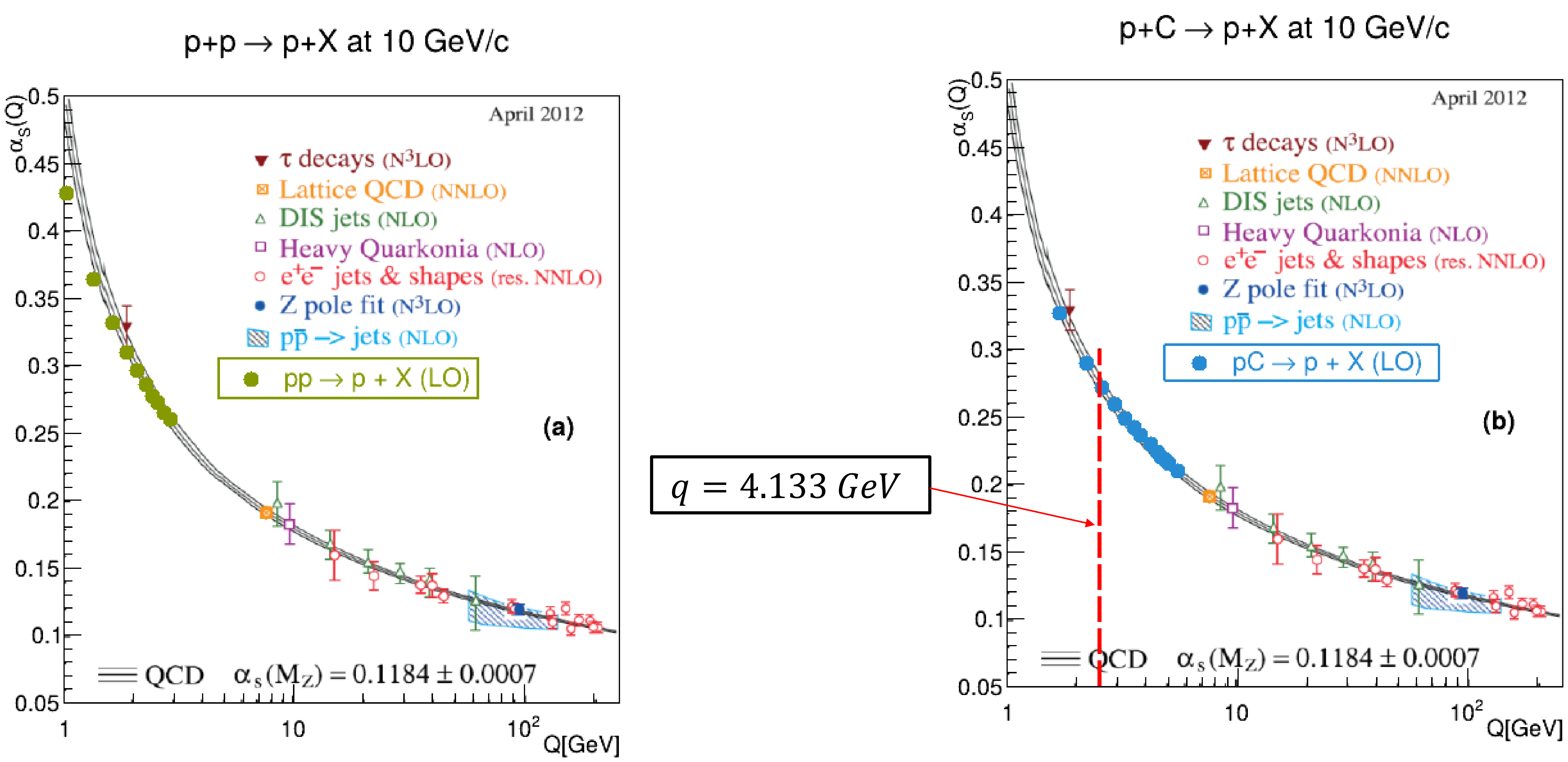


Figure 5a, b. Dependence of $\alpha_s(q)$ for secondary protons

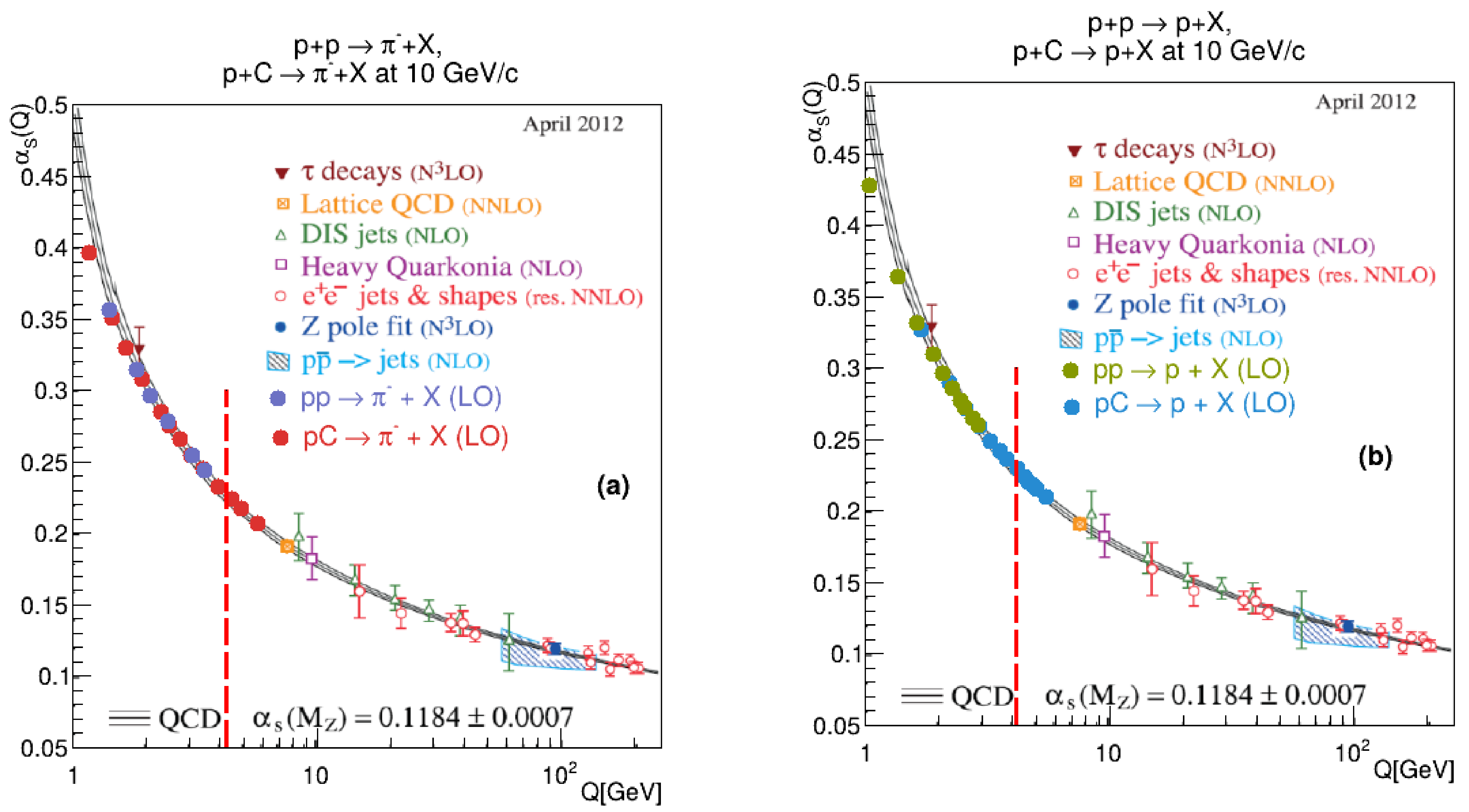
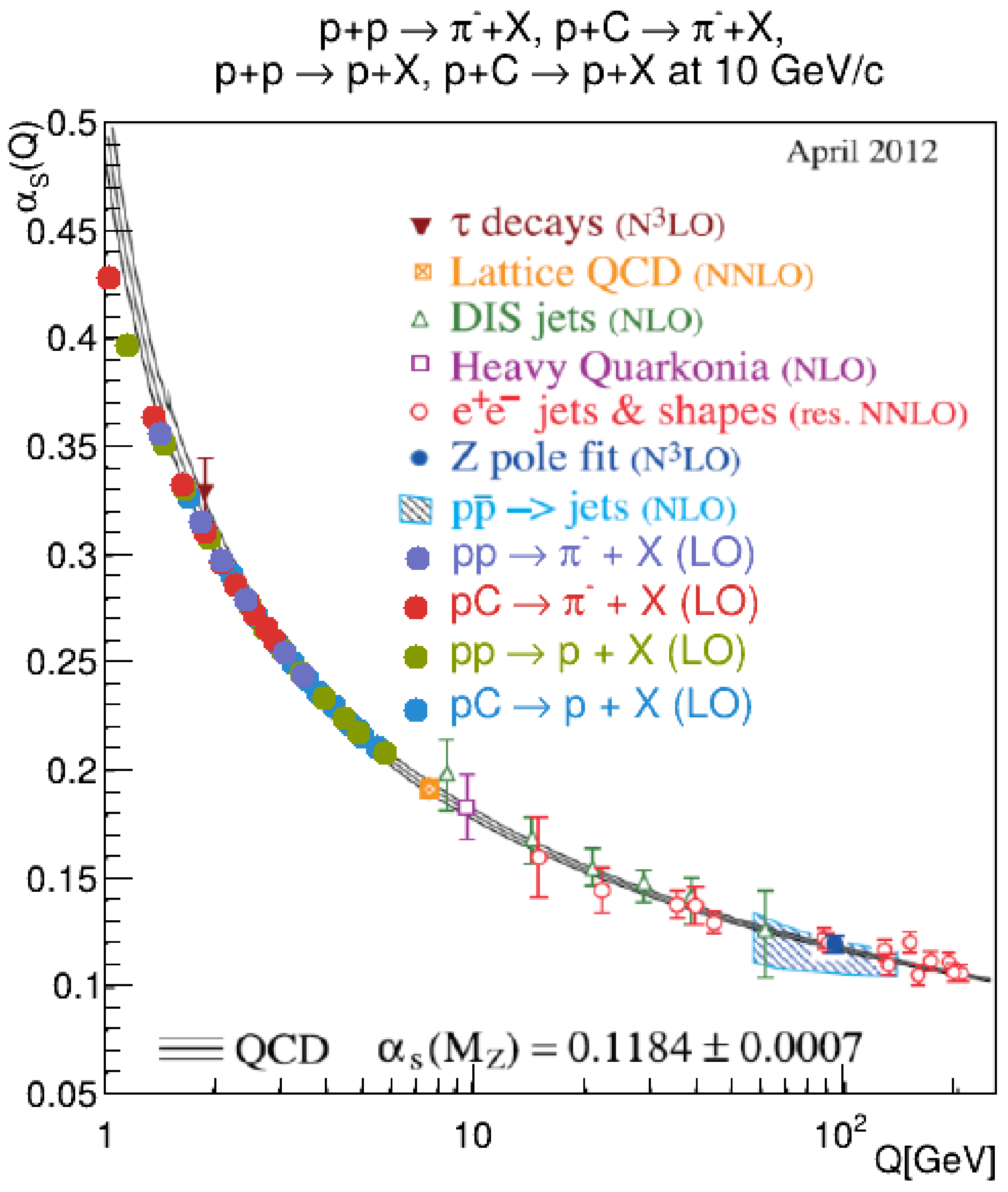


Figure 6a, b. Dependence of $\alpha_s(q)$ for secondary π^- mesons and secondary protons

The cumulative numbers demonstrate that their production is governed by the mechanisms of QCD



From this figure, we can see the results

- As the value of Q approaches 1, our experimental values for different reactions at low energy deviate from the theoretical curve.
- In the region above q = 1.5 GeV, our experimental values are in good agreement with the QCD predictions.

Figure 7. *Dependence of* $\alpha_s(q)$ *for all secondary particles*

CONCLUSION

- The experimental values of the running coupling constants $\alpha_s(q^2)$ of π^- mesons and protons obtained in this experiment are good agreement with QCD predictions.
- The values of the $\alpha_s(q^2)$ for the cumulative π^- mesons and protons produced in $p + C$ interactions are also in agreement with QCD predictions, but these values of $\alpha_s(q^2)$ correspond to the cumulative region or high q^2 values not allowed for $p + p$ interaction at the same incident energy.

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THANK YOU FOR YOUR ATTENTION

formula extraction

- The variable n_c called the cumulative number in the fixed target experiment is determined by the following formula [1.2];

- $$n_c = \frac{P_a P_c}{P_a P_b} = \frac{E_c - \beta_a P_{||}^c}{m_p} \qquad (5)$$

- where P_a, P_b and P_c are the four-dimensional momenta of energy incident, target and secondary particles under consideration. E_c and $P_{||}^c$ are the energy and longitudinal momentum the secondary particle, $\beta_a = \frac{P_a}{E_a}$ is the velocity of the incident particle. At high energy experiments $\beta_a = 1$ and m_p is the proton mass.

- As mentioned in the introduction, the square of the momentum q^2 plays a very important role in the multiparticle production process at high energies. The square of the transferred momentum q^2 is determined by the following formula:

- $$q^2 = -(P_a - P_c)^2 = 2E_a\left(E_c - \beta_a P_c^{||}\right) - (m_a^2 + m_c^2) = 2E_a m_p n_c - (m_a^2 + m_c^2) \qquad (6)$$

- where E_a and m_a are the energy and mass of incident energy particle, m_c mass of the secondary particles under consideration, the other notations are the same as in subsection 3.

Formula (5) maybe written in the following form using formula (4);

$$\frac{q^2}{m_a^2+m_c^2} = \frac{2E_a m_p n_c-(m_a^2+m_c^2)}{m_a^2+m_c^2} = \frac{E_a m_p n_c}{m_a^2+m_c^2} - 1 \qquad (7)$$

Now we can take the natural logarithm from both sides of this formula,

$$ln\left(\frac{q^2}{m_a^2+m_c^2}\right) = ln\left(\frac{2E_a m_p n_c-(m_a^2+m_c^2)}{m_a^2+m_c^2}\right) = ln\left(\frac{E_a m_p n_c}{m_a^2+m_c^2} - 1\right) \qquad (8)$$

From the other hand side, the coupling constant of the strong interaction in the LO approximate as a function of the 4-momentum transfer q^2 is determined by the following formula:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{\Lambda^2}\right)} = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{m_a^2+m_c^2}\right)} \qquad (9)$$

$m_a^2 + m_c^2$ is Formula (9) can be expressed by the Compton wave length of the corresponding particles λ_c and the conversion coefficient $(c\hbar) = 0.197$ [GeV·fm] [5].

$$m_a^2 + m_c^2 = \frac{(c\hbar)^2[GeV^2 fm^2]}{(\lambda_C^a)^2[fm^2]} + \frac{(c\hbar)^2[GeV^2 fm^2]}{(\lambda_C^c)^2[fm^2]} = (c\hbar)^2\left[\frac{(\lambda_C^a)^2+(\lambda_C^c)^2}{(\lambda_C^a)^2*(\lambda_C^c)^2}\right] = k*(c\hbar)^2[[GeV^2] \qquad (10)$$

Where $k = \left[\frac{(\lambda_C^a)^2+(\lambda_C^c)^2}{(\lambda_C^a)^2*(\lambda_C^c)^2}\right]$ is the dimensionless parameter and this parameter k in our case plays the role of the gauge parameter.

From Formula (10) we obtain the following formula,

$$(c\hbar)^2 = \frac{m_a^2 + m_c^2}{k} \tag{11}$$

The change of the renormalized point $m_a^2 + m_c^2$ is compensated the corresponding change of the gauge parameter k and as a result of this picture we obtained the universal constant $(c\hbar)^2$.
We note that this result (Formula(11)) is consistent with the theoretical result obtained in paper [6]. So, we choosed this constant as a cut parameter of the strong coupling constant $\alpha_s(q^2)$.

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 ln\left(\frac{q^2}{(c\hbar)^2}\right)} \tag{12}$$

From Formula (12) we see that as a result of our analysis carried out in this paper the cut parameter of the strong coupling constant $\alpha_s(q^2)$ is determined by the following formula,

$$\Lambda_{QCD}^2 = (c\hbar)^2\left[[GeV^2]\right] \text{ or } \Lambda_{QCD} = 0.197\ [GeV] \tag{13}$$

Note that Formula (13) gives the exact value of the cut parameter Λ_{QCD}.

The experimental methods of separating of $p+p$ and $p+C$ events obtained with the help of the Dubna 2 meter propane bubble chamber experiment exposed to 10 GeV/c proton beams are considered in paper [3].

The main part of events on the carbon target is separated by the following criteria,

1) The total charge of the all detected secondary particles $2n_{z\geq 2}+n_{z=1}-n_{z=-1}>Z_{Ai}+1$,
2) The number of slow $(P\leq 0.75\ GeV/c)$ protons $n_p>1$,
3) The number of protons emitted in the backward hemisphere in the laboratory system $n_p^b>0$
4) $n_{z=-1}>1$ for pC events , $n_{z=-1}>2$ for $dC-$, $\alpha C-$, $u\ CC-$ events
5) The number of the charged particles in the event $n_{z=\pm 1}$ is odd for pC and dC events
6) Criterion of the target mass. $M_t>1.5\ \cdot m_p$, where $M_t=\sum(E_i-P_i\cos\theta_i)$, E_i is the energy, $P_i\cos\theta$ is the longitudinal momentum, m_p is the proton mass, the summation is performed by all pions and protons, excluding stripping fragments from the projectile $\left(p_p>3\ GeV/c\,,\theta<4^{\circ}\right)$ and evaporated fragments from the target $\left(p_p<0.3\ GeV/c\right)$.

If the event under consideration is satisfied at least one condition from the above mentioned criteria, then this event belongs to the interaction on the carbon target. But with the help of these criteria are separated approximately 70% of the total $p+C$ interactions. The unseparated ~30% of $p+(C_3H_8)$ events are separated using the statistical weights. In this case we used the number of events corresponding to the known cross sections on the hydrogen and carbon targets