



Parton energy loss in Cu+Au and U+U collisions at
 $\sqrt{s_{NN}} = 200$ and 193 GeV, respectively

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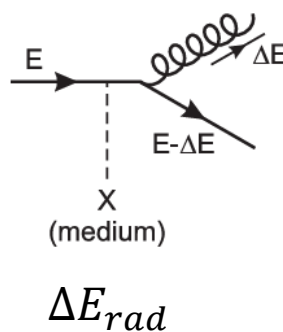
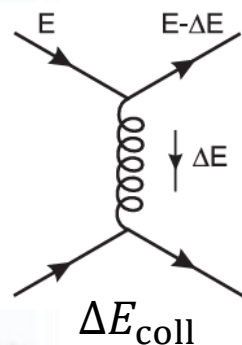
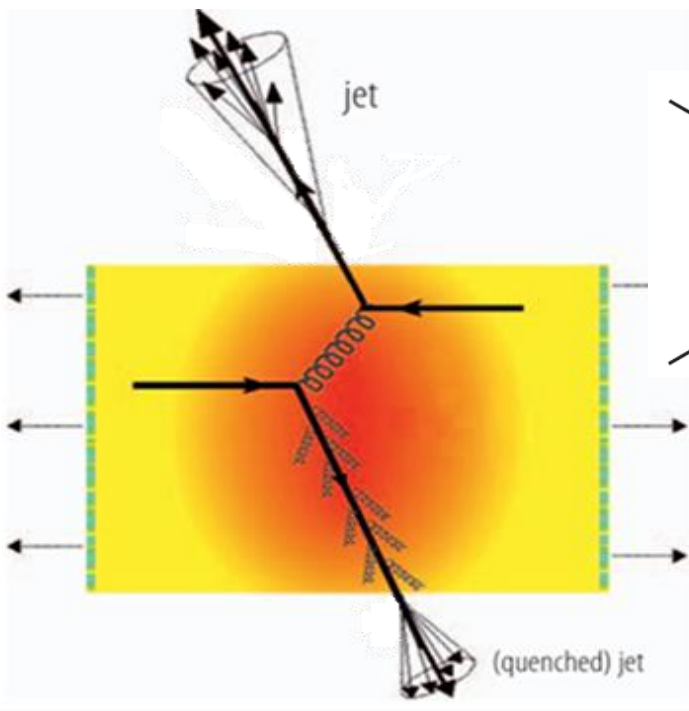
We acknowledge support from the Ministry of Science and Higher Education of the Russian Federation, state assignment for fundamental research (code FSEG-2025-0009).

Quark-gluon plasma. Jet quenching.

The production of partons (quarks and gluons) as a result of hard scattering of nucleons:

$$p_T, m, \varepsilon \gtrsim Q_0 (= \mathcal{O}(1 \text{ GeV})) \gg \Lambda_{QCD} (\approx 0.2 \text{ GeV})$$

Integral energy loss : $\Delta E = f(E_{in}, M, T, \alpha_s, L)$



1. Nuclear modification factor:

$$R_{AB}(p_T) = \frac{1}{\langle N_{coll} \rangle} \frac{d^2 N_{AB}^{\pi^0} / dp_T dy}{d^2 N_{pp}^{\pi^0} / dp_T dy}$$

$\langle N_{coll} \rangle$ – average number of inelastic binary nucleon-nucleon interactions for the given centrality

2. Effective fractional parton energy loss

$$S_{loss} = \frac{\Delta E}{E_{in}} \Rightarrow S_{loss} \approx \frac{p_T^{pp} - p_T}{p_T^{pp}} = \frac{\Delta p_T}{p_T^{pp}}$$

$p_T(p_T^{pp})$ – average transverse momentum of hadrons in A+B (p+p) collisions at fixed invariant yield

QGP effects => jet quenching => a parton loses its energy while going through QGP
=> decreasing of the yields of the leading hadrons in a jet

$$\Delta E = \Delta E_{coll} + \Delta E_{rad}$$

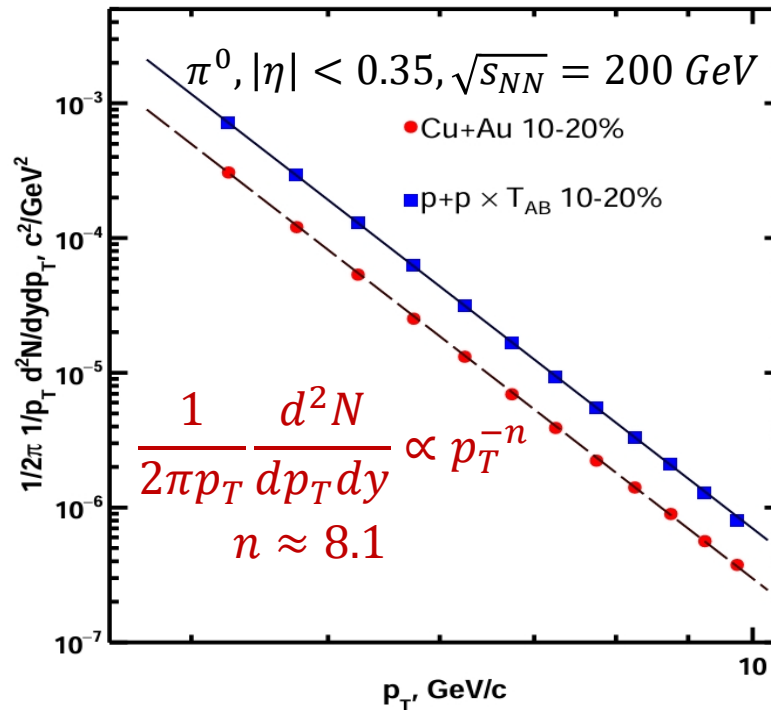
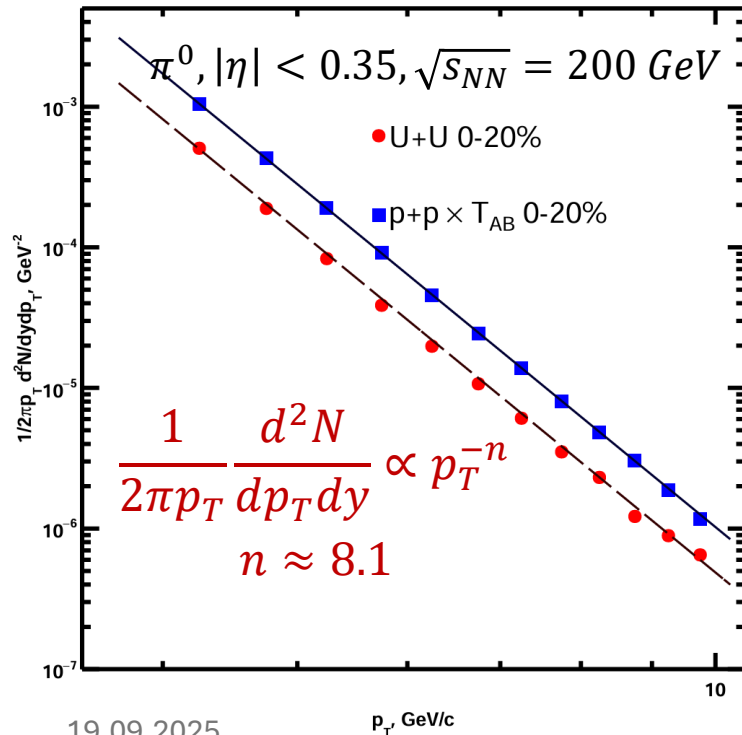
[d'Enterria, D. (2010). 6.4 Jet quenching. In: Stock, R. (eds) Relativistic Heavy Ion Physics. Landolt-Börnstein – Group I Elementary Particles, Nuclei and Atoms, vol 23. Springer, Berlin, Heidelberg]

Relationship between R_{AB} and S_{loss}

If the spectra of π^0 in nucleus-nucleus collisions are parallel to the spectra of π^0 in proton-proton collisions (at least in some area of transverse momentum):

$$\frac{\Delta p_T}{p_T} \approx \text{const} \quad \frac{d\Delta p_T}{dp_T} \approx \text{const} \quad (p_T > 4 \text{ GeV}/c)$$

$$S_{loss}(p_T) = 1 - R_{AB}^{\frac{1}{n-2}}(p_T) \approx \text{const}(p_T) \quad [\text{PRC 76, 034904 (2007)}]$$

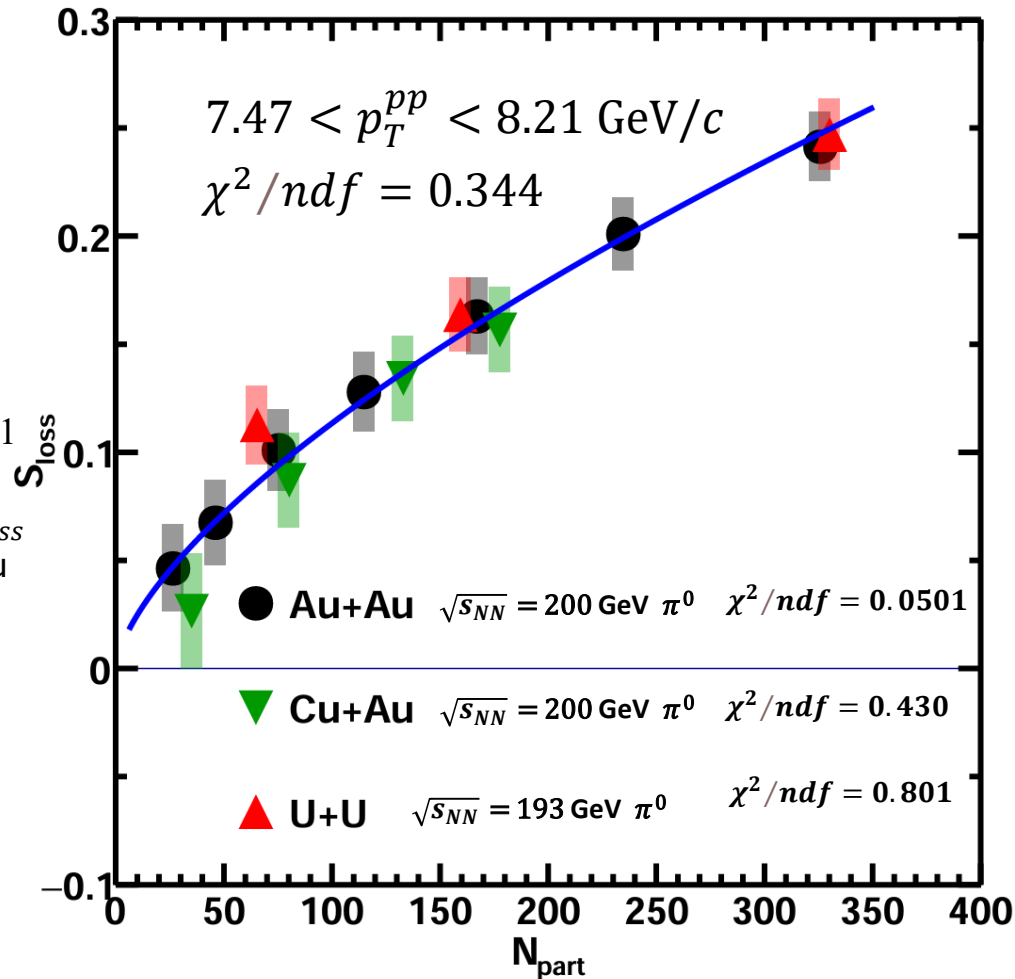


In many centrality classes, the nuclear modification factor is a constant with good accuracy at $p_T > 4 \text{ GeV}/c$. Spectra are parallel.

Based on [PRC 102, 064905 (2020)] and [PRC 98, 054903 (2018)]

Dependence of S_{loss} on centrality for π^0 in Cu+Au and U+U collisions

The values of R_{AB} for Au+Au are taken from [PRC 87, 034911 (2013)] to calculate the S_{loss} value for Au+Au



$$S_{loss}(N_{part}) = 1 - \left(R_{AB}(N_{part})\right)^{\frac{1}{n-2}}$$

N_{part} – the average number of nucleons that have experienced at least one inelastic collision.

Approximation:

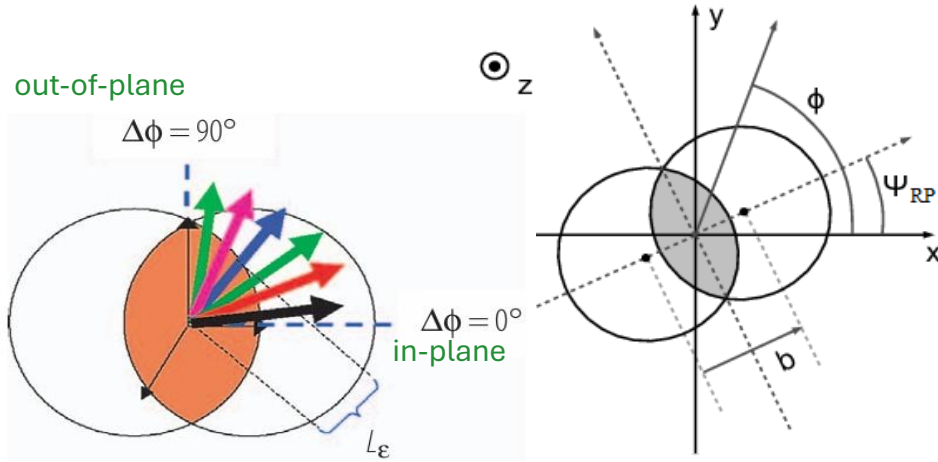
$$S_{loss} \approx k N_{part}^{2/3}$$

k – free parameter

$$\Delta E \propto \frac{1}{\underbrace{A_{\perp}}_{\propto N_{part}^{2/3}}} \times \frac{dN^g}{dy} \times \underbrace{L}_{\propto N_{part}^{1/3}} \propto N_{part}^{2/3}$$

$\propto \frac{dN_{ch}}{dy} \propto N_{part}$

Anisotropy of colliding nuclei



- The region of nuclear overlap is azimuthally asymmetric with respect to the angle Ψ_{RP}
- The path-length of a parton in QGP, color charge density and other characteristics are anisotropic => different energy losses
- Angular distribution of particles:

$$\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\phi - \Psi_{RP}))$$

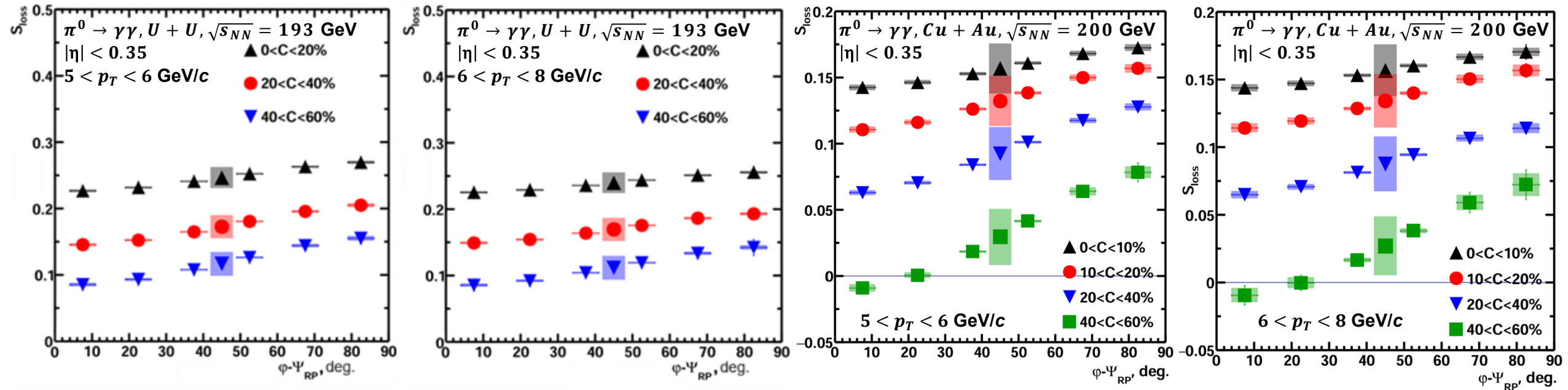
- The main contribution to the anisotropy – second harmonic – elliptic flow v_2
- Energy losses and particle yields depend on the azimuthal angle [PRC 76, 034904 (2007)] :

$$R_{AB}(p_T, \Delta\phi) \approx (1 + 2v_2(p_T) \cos(2\Delta\phi)) R_{AB}(p_T)$$

$$S_{loss}(p_T, \Delta\phi) = 1 - R_{AB}^{\frac{1}{n-2}}(p_T, \Delta\phi)$$

- Naïve expectations:
 - Losses should be minimal in the reaction plane ($\Delta\phi = 0^\circ$)
 - If $\Delta\phi = 90^\circ$ losses should be maximal

Dependence of S_{loss} of π^0 on $\Delta\varphi$ in U+U and Cu+Au collisions



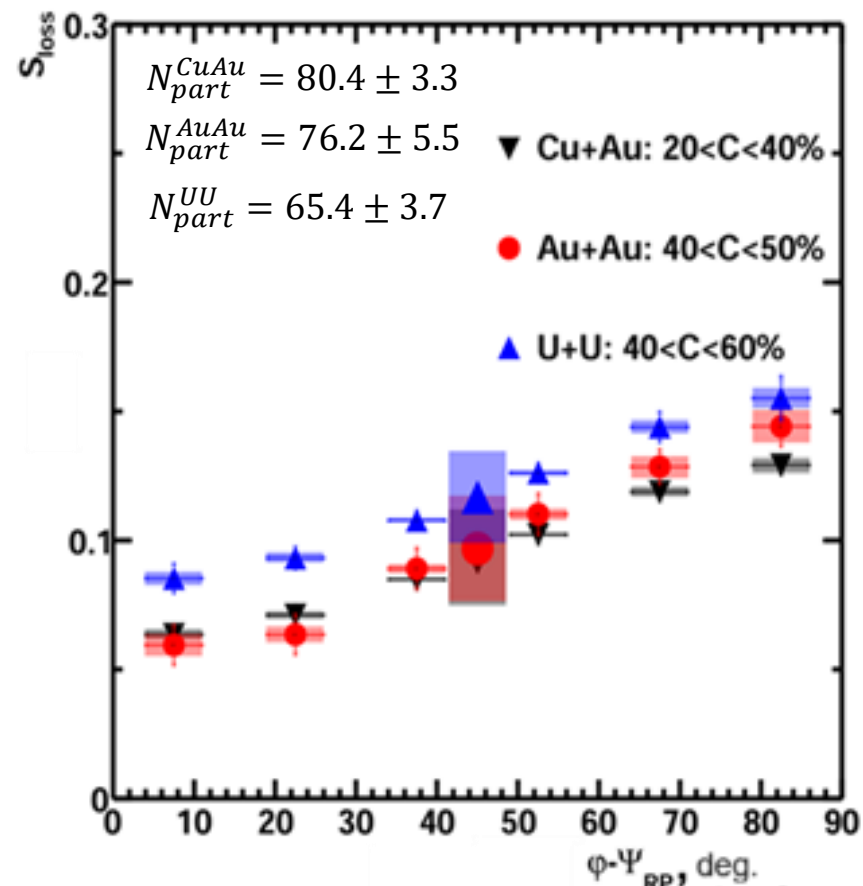
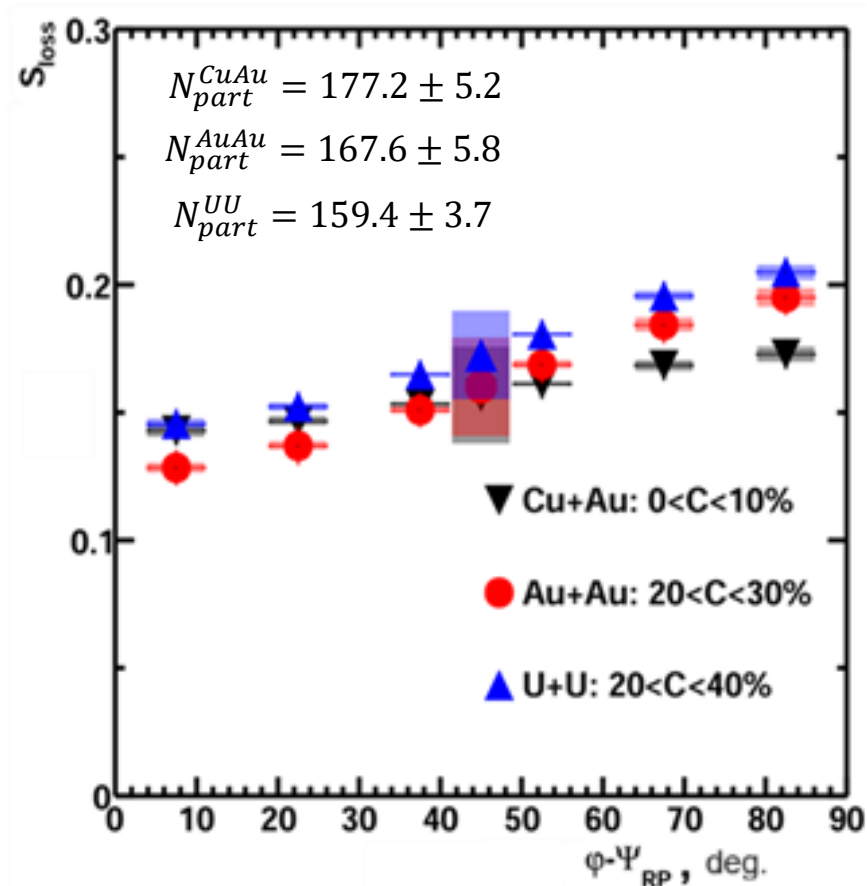
$S_{loss} \uparrow$ if $N_{part} \uparrow$

$S_{loss} \uparrow$ if $\Delta\varphi \uparrow$ as expected

The anisotropy of $S_{loss} \uparrow$ if $N_{part} \downarrow$

Comparison of $S_{loss}(\Delta\varphi)$ in different systems at close N_{part}

$$5 < p_T < 6 \text{ GeV}/c$$



Dependence of energy losses on the parton path length

- The parton path-length in the QGP and the various characteristics of the QGP itself are anisotropic => there is $S_{loss}(\Delta\varphi)$ => it's possible to express $S_{loss}(L)$
- At high p_T , radiative losses prevail
- For thin media ($L \ll \lambda$) – Bete-Heitler spectrum:

$$\omega \frac{dI_{rad}}{d\omega} \approx \frac{\alpha_S \hat{q} L^2}{\omega} \quad \Delta E_{rad}^{BH} \approx \alpha_S \hat{q} L^2 \ln \left(\frac{E}{m_D^2 L} \right)$$

- Thick media ($L \gg \lambda$) – Landau-Pomeranchuk-Migdal spectrum:

$$\omega \frac{dI_{rad}}{d\omega} \approx \alpha_S \begin{cases} \sqrt{\frac{\hat{q} L^2}{\omega}} & (\omega < \omega_c) \\ \frac{\hat{q} L^2}{\omega} & (\omega > \omega_c) \end{cases} \quad \Delta E_{rad}^{LPM} \approx \alpha_S \begin{cases} \hat{q} L^2 & (\omega < \omega_c) \\ \hat{q} L^2 \ln \left(\frac{E}{m_D^2 L} \right) & (\omega > \omega_c) \end{cases}$$

First expectation:
 $\Delta E \propto L^2$

$\hat{q}$ – the transport coefficient, m_D – the Debye mass , ω – the energy of a gluon

[d'Enterria, D. (2010). 6.4 Jet quenching. In: Stock, R. (eds) Relativistic Heavy Ion Physics. Landolt-Börnstein – Group I Elementary Particles, Nuclei and Atoms, vol 23. Springer, Berlin, Heidelberg]

The transition to the effective parton path-length

- The efficiency of parton energy loss may vary along the way
- Parton may not be created in the center of the QGP
- QGP is expanding
- Therefore, the effective (rather than the real) parton path-length in a homogenized effective QGP is considered [Eur. Phys. J. C 38, 461–474 (2005)]

$$\langle \hat{q} \rangle(b, \Delta\varphi) = \frac{2}{L^2} \int_{\tau_0}^{\tau_0+L} (\tau - \tau_0) \hat{q}(\tau) d\tau$$

$$I_0(b, \Delta\varphi) = \langle \hat{q} \rangle L_{eff}(b, \Delta\varphi) = \int_0^{\infty} \hat{q}(\tau) d\tau$$

$$I_1(b, \Delta\varphi) = \omega_{c_{eff}}(b, \Delta\varphi) = \frac{1}{2} \langle \hat{q} \rangle L_{eff}^2 = \int_0^{\infty} \hat{q}(\tau) \tau d\tau$$

$$L_{eff} = \frac{2I_1}{I_0}$$

Effective path-length in Glauber model

$$\hat{q}(\tau, b) \propto \rho_c(\tau, b) \propto \rho_{part}(\tau, b)$$

- Parton creation point (x_0, y_0) and its direction of movement φ_0 are generated in Glauber model

$$\rho_{part}(\tau, b) = \rho_{part}(x_0 + \beta\tau \cos(\varphi_0 - \psi_2), y_0 + \beta\tau \sin(\varphi_0 - \psi_2), b)$$

- ρ_{part} is maximal in the center of overlap area and decreases towards the edges
- Longitudinal QGP expansion – Bjorken expansion [Phys. Rev. C 80, 054907 (2009)]:

$$\rho_c(\tau) = \rho_c(\tau_0) \frac{\tau_0}{\tau}$$

- Regularized form:

$$\rho_c(\tau) = \rho_{c0} \frac{\tau / \tau_0}{1 + \tau^2 / \tau_0^2}$$

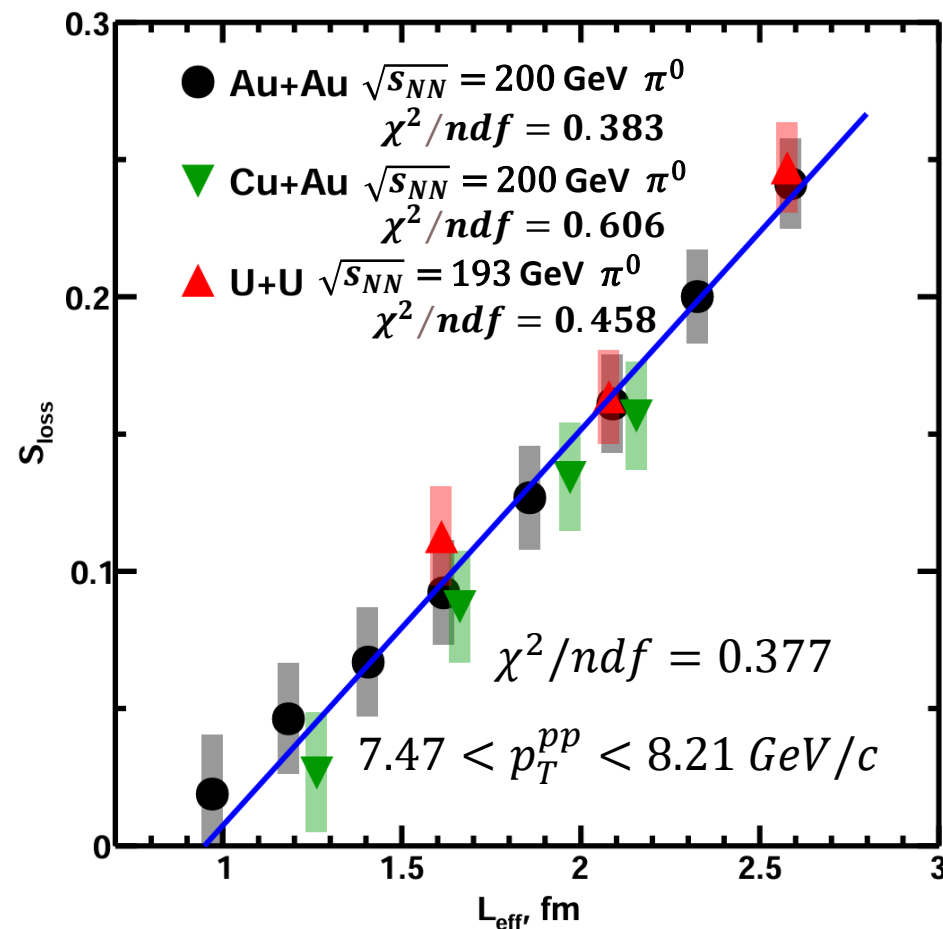
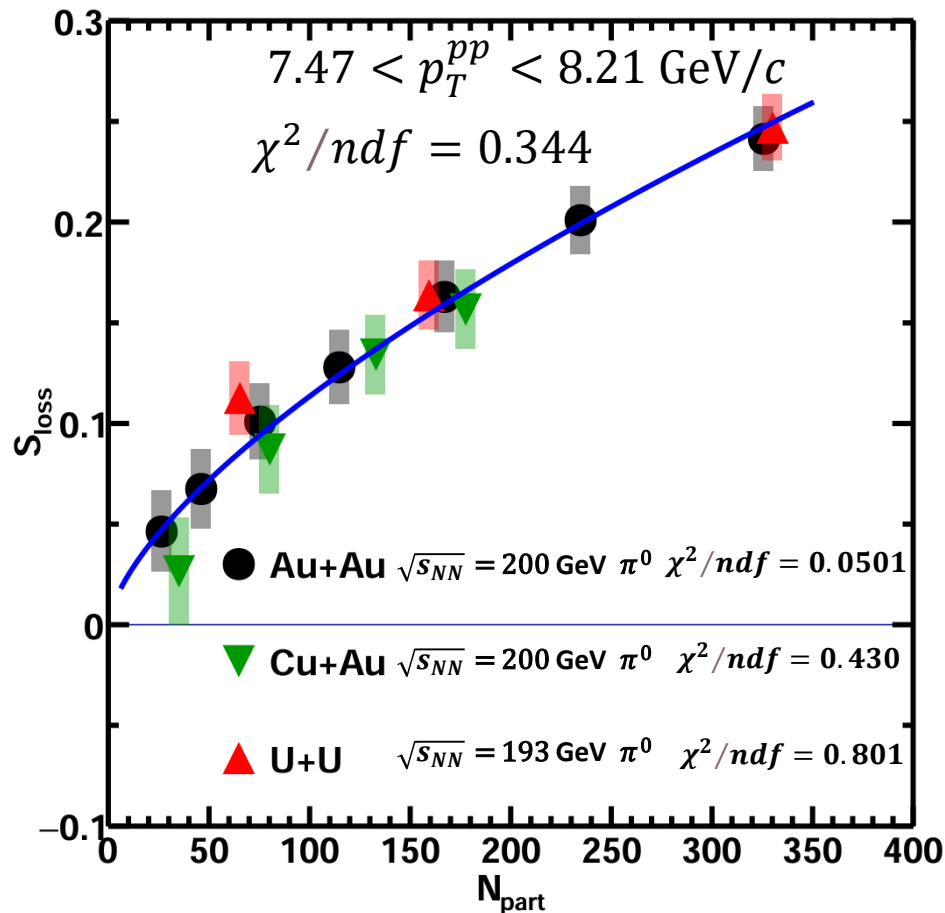
- Losses:

$$\Delta E \propto L$$

$$\text{Not } \Delta E \propto L^2!$$

$$L_{eff}(b, \Delta\varphi) = 2\beta \frac{\int_0^\infty \rho_{part} \left(\frac{\tau / \tau_0}{1 + \tau^2 / \tau_0^2} \right) \tau d\tau}{\int_0^\infty \rho_{part} \left(\frac{\tau / \tau_0}{1 + \tau^2 / \tau_0^2} \right) d\tau}$$

Dependence $S_{loss}(L_{eff}) \pi^0$ in Cu+Au and U+U collisions



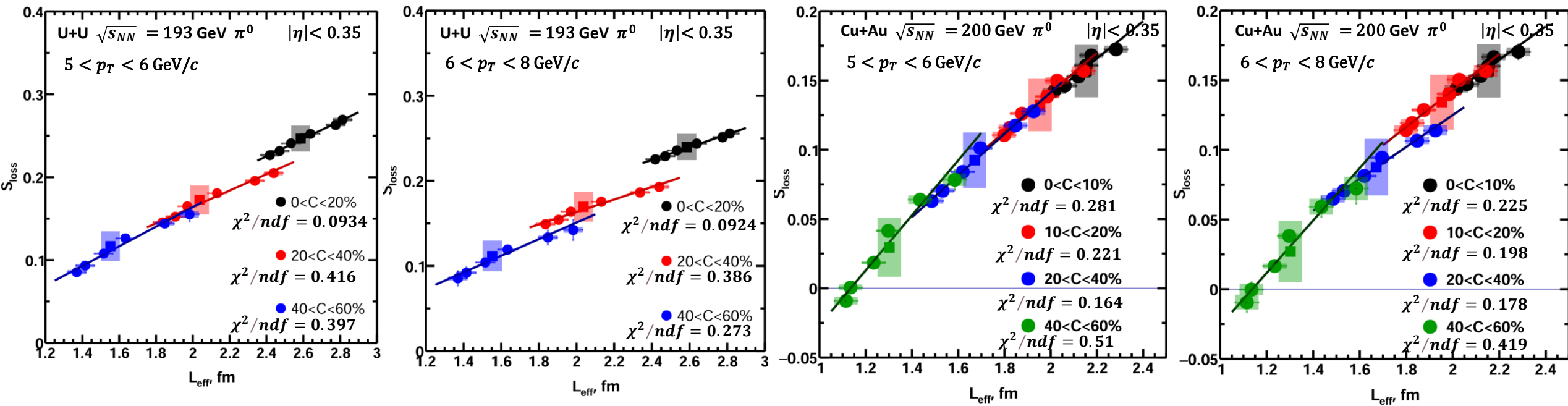
$$\Delta E \propto L$$

$$S_{loss} = kL_{eff} + b$$

The points of different systems fall on 1 straight line

The values of R_{AB} for Au+Au are taken from [PRC 87, 034911 (2013)] to calculate the S_{loss} value for Au+Au

Dependence $S_{loss}(L_{eff})$ for different $\Delta\varphi$ of π^0 in Cu+Au and U+U collisions

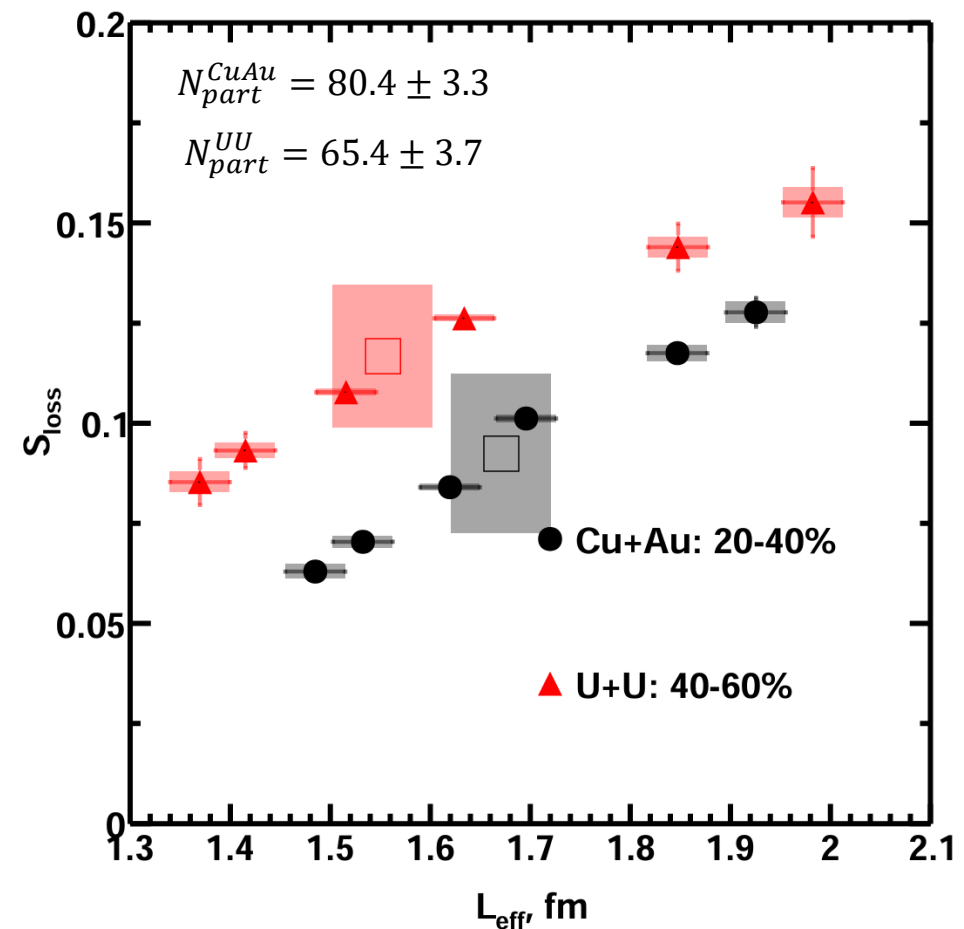
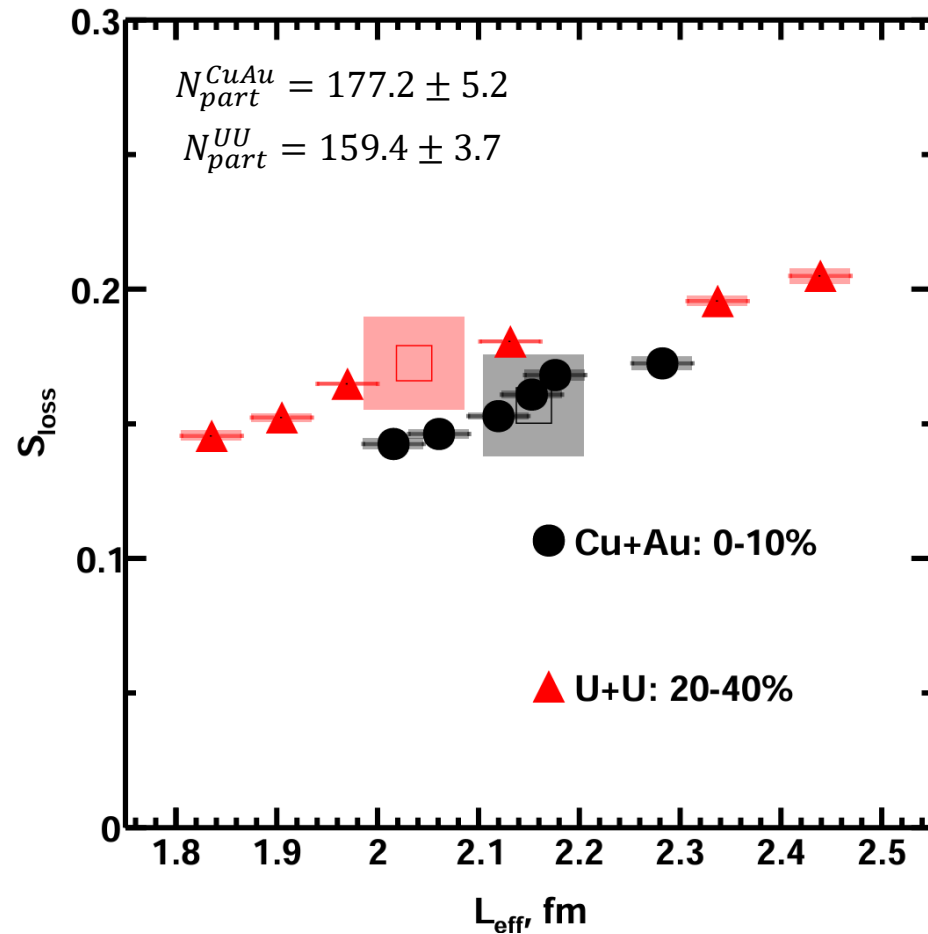


$S_{loss} \uparrow$ if $L_{eff} \uparrow$;

$L_{eff} \uparrow$ if $\Delta\varphi \uparrow$

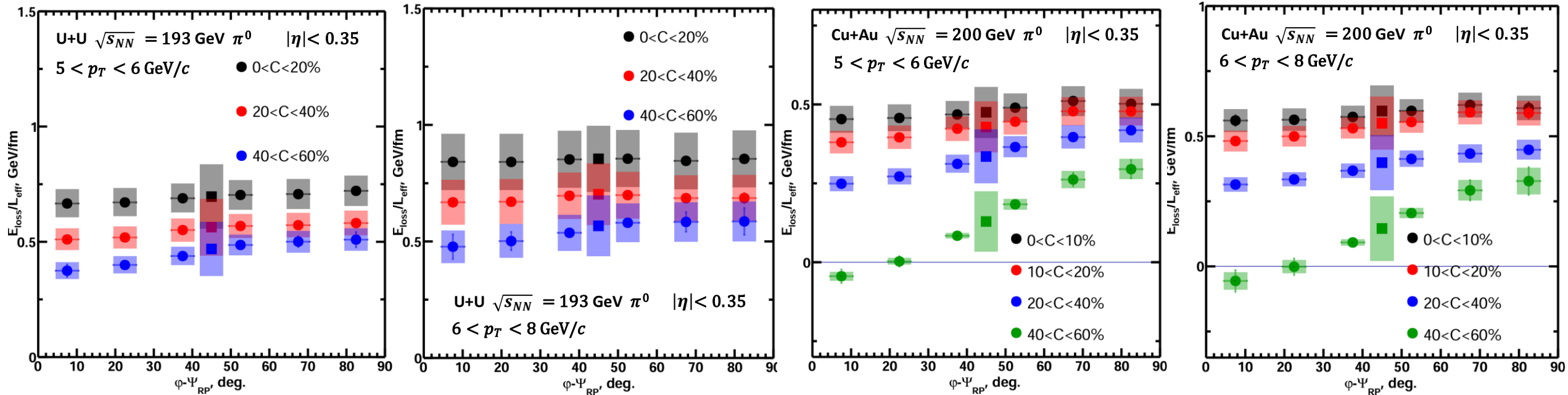
Comparison of $S_{loss}(L_{eff}(\Delta\phi))$ in Cu+Au and U+U collisions at close N_{part}

$5 < p_T < 6 \text{ GeV}/c$



Dependence $E_{loss}/L_{eff}(\Delta\varphi)$ of π^0 in U+U and Cu+Au collisions

$$\Delta E = E_{loss} = S_{loss} p_T'$$



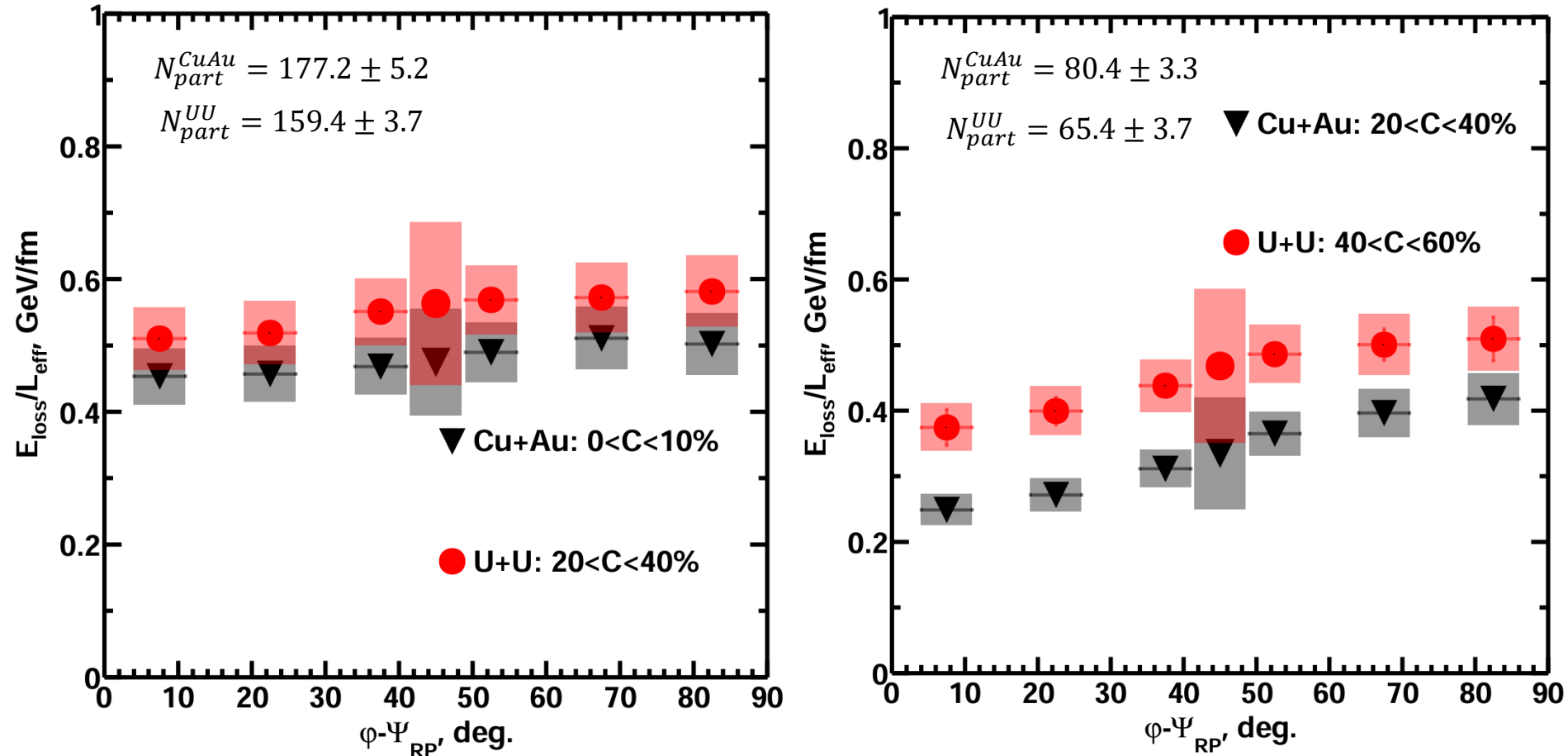
$E_{loss}/L_{eff} \approx \text{constant}(\Delta\varphi)$

$E_{loss}/L_{eff} \uparrow \text{ if } N_{part} \uparrow$

$E_{loss}/L_{eff} \uparrow \text{ if } p_T \uparrow$

Comparison of $E_{loss}/L_{eff}(\Delta\varphi)$ in different systems at close N_{part}

$5 < p_T < 6 \text{ GeV}/c$

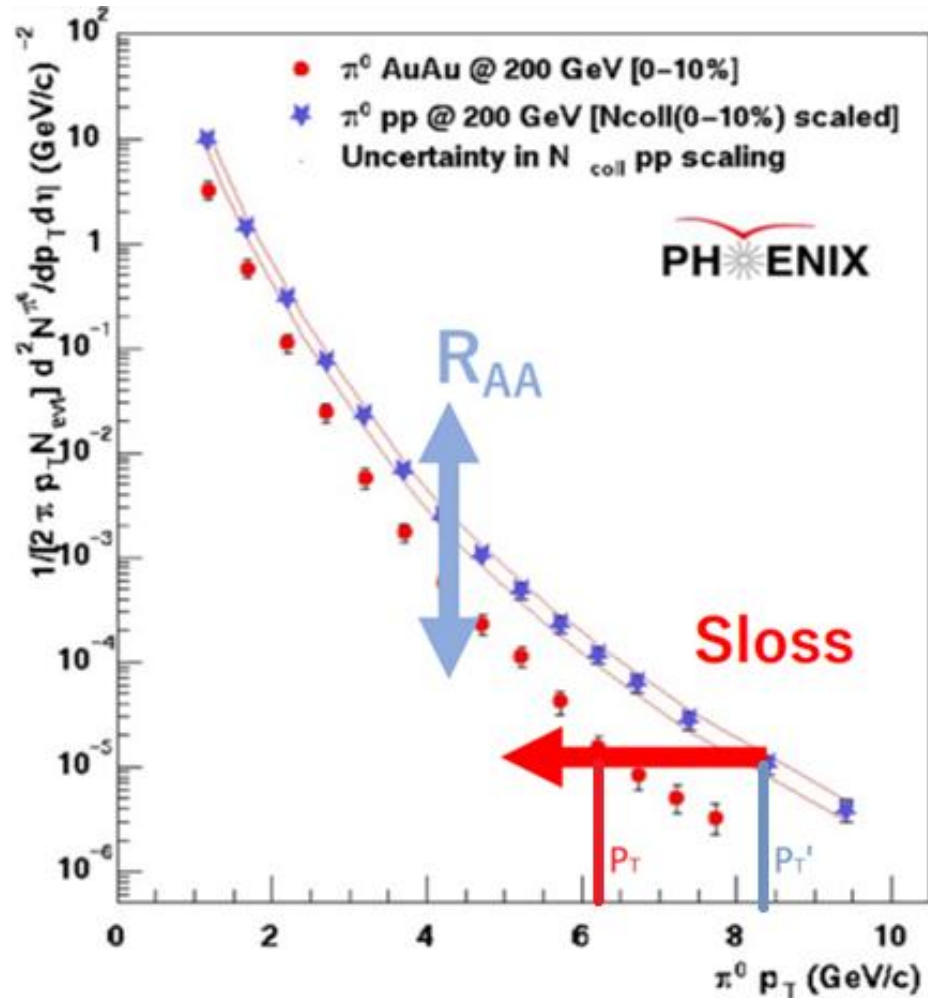


Conclusion

- There are explicit dependencies $S_{loss}(N_{part})$, $S_{loss}(L_{eff})$, $S_{loss}(\Delta\varphi)$ and $S_{loss}(L_{eff}(\Delta\varphi))$
- Angle-inclusive losses $S_{loss}(L_{eff})$ and $S_{loss}(N_{part})$ dependencies have the same behavior for different collision systems (U+U, Cu+Au):
 - $S_{loss} = kL_{eff} + b$
 - $S_{loss} = kN_{part}^{2/3}$
- Similar inclusive losses for different systems at close N_{part} and p_T^{pp}
- Different anisotropy of losses for different systems at close N_{part} and p_T^{pp}
- $S_{loss}(L_{eff}(\Delta\varphi))$ are linear almost everywhere as expected
- $S_{loss}(L_{eff}(\Delta\varphi))$ for different systems are almost matching
- $E_{loss}/L_{eff}(\Delta\varphi)$ is an almost constant value almost everywhere
- $E_{loss}/L_{eff}(\Delta\varphi)$ for different systems are almost matching
- The results of this work can possibly be expanded for use in the MPD experiment of the NICA project

Thank you for
your attention!

Nuclear modification factors and effective fractional energy loss S_{loss}



- The change in particle yields in heavy ion collisions compared to the pp-collisions is characterized by nuclear modification factors R_{AB}
- $$R_{AB} = \frac{1}{\langle N_{coll} \rangle} \frac{d^2 N_{AB}}{dp_T dy} / \frac{d^2 N_{pp}}{dp_T dy}$$
- $\langle N_{coll} \rangle$ - the average number of binary inelastic nucleon-nucleon collisions
- The energy loss of particles in QGP is characterized by S_{loss}
- $$S_{loss} = \frac{\Delta E}{E_{initial}} \approx \frac{p_T' - p_T}{p_T'} = \frac{\Delta p_T}{p_T'}$$
- p_T' - transverse momentum of π^0 -mesons in p+p collisions
- p_T - transverse momentum of π^0 -mesons in A+B collisions

$$\frac{1}{N_{AB}^{evt}} \frac{d^2 N_{AB}}{dp_T dy} = \frac{d^2 \sigma_{pp}}{dp'_T dy} \Big|_{p'_T = p_T + \Delta p_T} \cdot \frac{dp'_T}{dp_T}$$

$$\frac{d^2 \sigma_{pp}}{dp'_T dy} \Big|_{p'_T = p_T + \Delta p_T} \cdot \frac{dp'_T}{dp_T} = \frac{d^2 \sigma_{pp}}{dp'_T dy} \Big|_{p'_T = p_T + \Delta p_T} \cdot \left[1 + \frac{d\Delta p_T}{dp_T} \right]$$

$$E \frac{d^3 \sigma}{dp^3} \sim p_T^{-n} \quad (n = 8.1 \pm 0.05) \text{ for } p_T \gtrsim 4 \text{ GeV}/c$$

$$\frac{d^3 p}{E} = dp_x dp_y \frac{dp_z}{E}$$

$$p_z = m_T \sinh y$$

$$dp_z = m_T \cosh y dy$$

$$E = m_T \cosh y$$

$$dp_z = E dy$$

$$\frac{d^3 p}{E} = dp_x dp_y dy$$

$$E \frac{d^3 \sigma}{dp^3} = \frac{d^3 \sigma}{dp_x dp_y dy} = \frac{1}{p_T} \frac{d^3 \sigma}{dp_T d\phi dy}$$

$$\int_0^{2\pi} E \frac{d^3 \sigma}{dp^3} d\phi = \int_0^{2\pi} \frac{1}{p_T} \frac{d^3 \sigma}{dp_T d\phi dy} d\phi = \frac{1}{2\pi p_T} \frac{d^2 \sigma}{dp_T dy}$$

$$\begin{aligned}
\int_0^{2\pi} E \frac{d^3\sigma}{dp^3} d\varphi &= \frac{1}{2\pi p_T} \frac{d^2\sigma}{dp_T dy} \sim \frac{p_T^{-n}}{p_T} \\
\frac{d^2\sigma}{dp_T dy} &= 2\pi p_T \int_0^{2\pi} E \frac{d^3\sigma}{dp^3} d\varphi \\
\frac{d^2\sigma}{dp_T dy} &= A p_T^{-n+1} \\
\frac{1}{\langle T_{AB} \rangle} \frac{1}{N_{AB}^{evt}} \frac{d^2 N_{AB}}{dp_T dy} &= \frac{d^2 \sigma_{pp}}{dp'_T dy} \Big|_{p'_T = p_T + \Delta p_T} \cdot \left[1 + \frac{d\Delta p_T}{dp_T} \right] \\
\frac{1}{\langle T_{AB} \rangle} \frac{1}{N_{AB}^{evt}} \frac{d^2 N_{AB}}{dp_T dy} &= A p_T'^{-n+1} \cdot \left[1 + \frac{d\Delta p_T}{dp_T} \right] \\
R_{AB}(p_T) &= \frac{\frac{1}{N_{AB}^{evt}} \frac{d^2 N_{AB}}{dp_T dy}}{\langle T_{AB} \rangle \frac{d^2 \sigma_{pp}}{dp_T dy}} (p_T) = \frac{A p_T'^{-n+1} \cdot \left[1 + \frac{d\Delta p_T}{dp_T} \right]}{\frac{d^2 \sigma_{pp}}{dp_T dy}} = \frac{A p_T'^{-n+1} \cdot \left[1 + \frac{d\Delta p_T}{dp_T} \right]}{A p_T^{-n+1}} \\
R_{AB} &= \frac{(p_T + \Delta p_T)^{-n+1}}{p_T^{-n+1}} \left[1 + \frac{d\Delta p_T}{dp_T} \right] = \left[1 + \frac{\Delta p_T}{p_T} \right]^{-n+1} \left[1 + \frac{d\Delta p_T}{dp_T} \right] \\
\frac{\Delta p_T}{p_T} &= S_0 = \text{const} = \frac{d\Delta p_T}{dp_T} \text{ for parallel spectra} \\
R_{AB}(p_T) &= \left[1 + \frac{\Delta p_T}{p_T} \right]^{-n+1} \left[1 + \frac{d\Delta p_T}{dp_T} \right] = [1 + S_0]^{-n+1} [1 + S_0] = [1 + S_0]^{-n+2}
\end{aligned}$$

$$\begin{aligned}
R_{AB}(p_T) &= \text{const} \\
R_{AB}^{n-2} &= 1 + S_0 \\
\frac{1}{1 + S_0} &= R_{AB}^{\frac{1}{n-2}}(p_T) \\
S_{loss} &= \frac{\Delta p_T}{p'_T} \\
S_{loss} &= \frac{\frac{\Delta p_T}{p_T}}{\frac{p'_T}{p_T}} = \frac{\frac{\Delta p_T}{p_T}}{\frac{p_T + \Delta p_T}{p_T}} = \frac{\frac{\Delta p_T}{p_T}}{1 + \frac{\Delta p_T}{p_T}} = \frac{S_0}{1 + S_0} = 1 - \frac{1}{1 + S_0} \\
S_{loss}(p_T) &= 1 - R_{AB}^{\frac{1}{n-2}}(p_T)
\end{aligned}$$

$$\langle \hat{q} \rangle(b, \Delta\varphi) = \frac{2}{L^2} \int_{\tau_0}^{\tau_0+L} (\tau - \tau_0) \hat{q}(\tau) d\tau$$

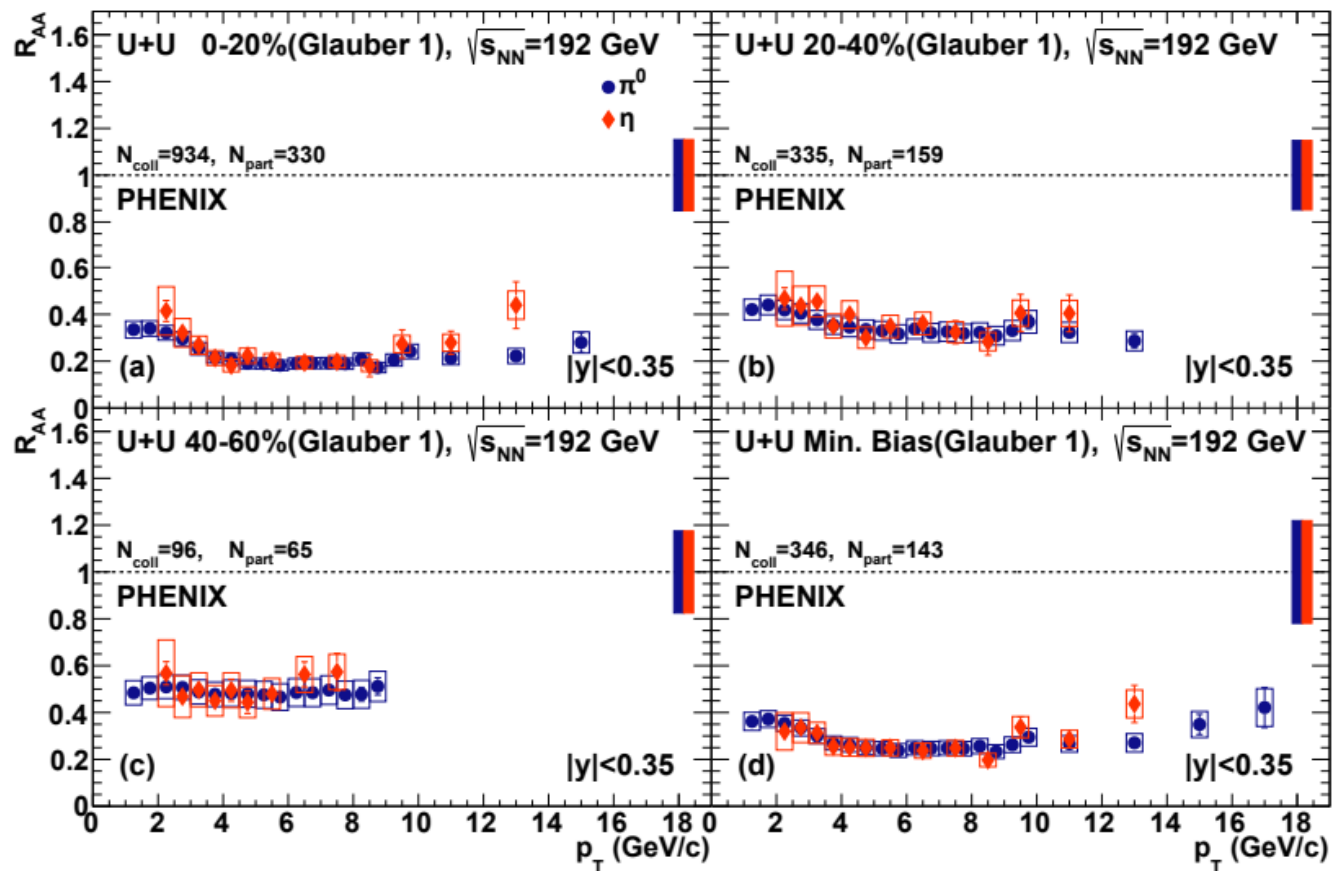
$$\hat{q}(\tau) \Rightarrow \rho_0 \frac{\tau/\tau_0}{1 + (\tau/\tau_0)^2}$$

$$\begin{aligned} \frac{2}{L^2} \int_{\tau_0}^{\tau_0+L} (\tau - \tau_0) \hat{q}(\tau) d\tau &\Rightarrow \frac{2}{L^2} \int_{\tau_0}^{\tau_0+L} (\tau - \tau_0) \rho_0 \frac{\tau/\tau_0}{1 + (\tau/\tau_0)^2} d\tau \\ \int_{\tau_0}^{\tau_0+L} (\tau - \tau_0) \rho_0 \frac{\tau/\tau_0}{1 + (\tau/\tau_0)^2} d\tau &= \int_{\tau_0}^{\tau_0+L} \rho_0 \frac{\tau^2/\tau_0}{1 + (\tau/\tau_0)^2} d\tau - \int_{\tau_0}^{\tau_0+L} \rho_0 \frac{\tau}{1 + (\tau/\tau_0)^2} d\tau \\ \int_{\tau_0}^{\tau_0+L} \rho_0 \frac{\tau^2/\tau_0}{1 + (\tau/\tau_0)^2} d\tau &= \rho_0 \tau_0^2 \int_{\tau_0}^{\tau_0+L} \frac{\tau^2/\tau_0^2}{1 + (\tau/\tau_0)^2} d\frac{\tau}{\tau_0} = \rho_0 \tau_0^2 \int_{\tau_0}^{\tau_0+L} \left(1 - \frac{1}{1 + (\tau/\tau_0)^2} \right) d\frac{\tau}{\tau_0} \\ &= \rho_0 \tau_0^2 \int_{\tau_0}^{\tau_0+L} d\frac{\tau}{\tau_0} - \rho_0 \tau_0^2 \int_{\tau_0}^{\tau_0+L} \frac{1}{1 + (\tau/\tau_0)^2} d\frac{\tau}{\tau_0} = \rho_0 \tau_0 L - \rho_0 \tau_0^2 \int_1^{1+L/\tau_0} \frac{1}{1 + x^2} dx \\ &= \rho_0 \tau_0 L - \rho_0 \tau_0^2 \arctan \left(1 + \frac{L}{\tau_0} \right) + \rho_0 \tau_0^2 \frac{\pi}{4} \end{aligned}$$

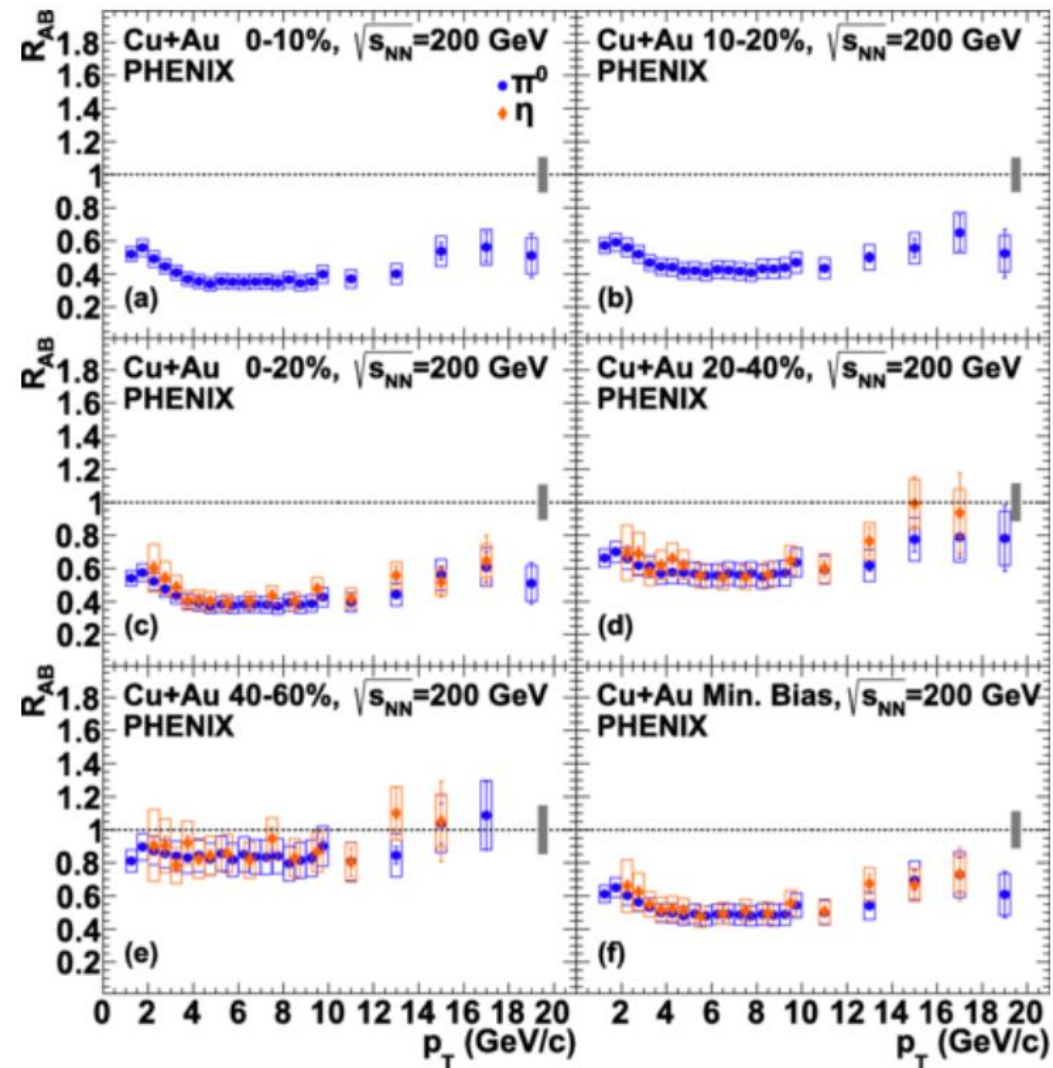
$$\begin{aligned}
\int_{\tau_0}^{\tau_0+L} \rho_0 \frac{\tau}{1 + (\tau/\tau_0)^2} d\tau &= \rho_0 \tau_0^2 \int_{\tau_0}^{\tau_0+L} \frac{\tau/\tau_0}{1 + (\tau/\tau_0)^2} d\frac{\tau}{\tau_0} = \rho_0 \tau_0^2 \int_{\tau_0}^{\tau_0+L} \frac{\tau}{\tau_0^2 + \tau^2} d\tau = \frac{1}{2} \rho_0 \tau_0^2 \int_{\tau_0}^{\tau_0+L} \frac{1}{\tau_0^2 + \tau^2} d(\tau_0^2 + \tau^2) \\
&= \frac{1}{2} \rho_0 \tau_0^2 \ln(\tau_0^2 + (\tau_0 + L)^2) - \frac{1}{2} \rho_0 \tau_0^2 \ln(2\tau_0^2) = \\
&\quad \frac{1}{2} \rho_0 \tau_0^2 \ln\left(\frac{\tau_0^2 + (\tau_0 + L)^2}{2\tau_0^2}\right) \\
\langle \hat{q} \rangle(b, \Delta\varphi) &= \frac{2}{L^2} \left(\rho_0 \tau_0 L - \rho_0 \tau_0^2 \arctan\left(1 + \frac{L}{\tau_0}\right) + \rho_0 \tau_0^2 \frac{\pi}{4} - \frac{1}{2} \rho_0 \tau_0^2 \ln\left(\frac{\tau_0^2 + (\tau_0 + L)^2}{2\tau_0^2}\right) \right)
\end{aligned}$$

Main contribution: $\frac{2\rho_0\tau_0}{L} \Rightarrow \langle \hat{q} \rangle \sim 1/L$

$$\Delta E \sim \hat{q} L^2 \Rightarrow \Delta E \sim L$$



[PRC 102, 064905 (2020)]



[PRC 98, 054903 (2018)]