Lattice study of total momentum and free energy of rotating gluon plasma

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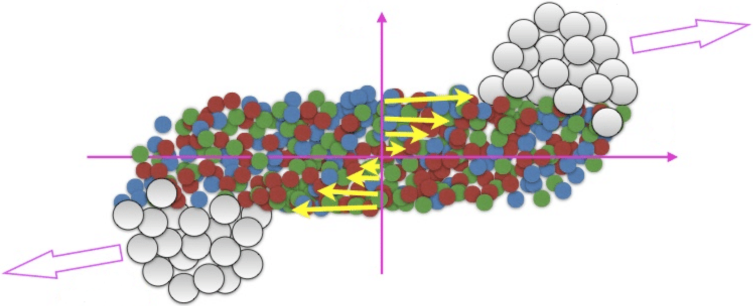
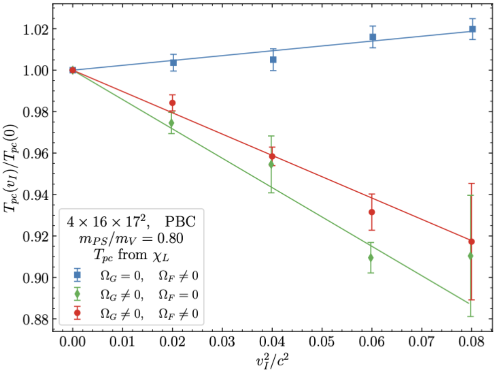
Motivation

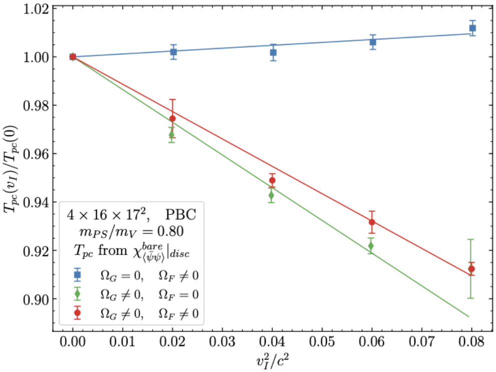
- In non-central heavy ion-collisions droplets of QGP with angular momentum $\sim 10^3\hbar$ might be produced.



Gluodynamics motivation

- The shift in pseudo-critial temperature of rotating QCD is dominated by the influence of rotation on gluons.



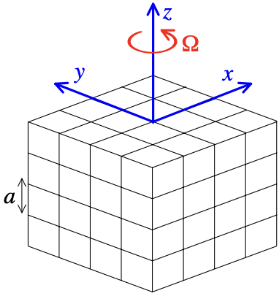


Gluodynamics in Minkowski space

$$t = t_{\text{lab}}, \quad r = r_{\text{lab}}, \quad z = z_{\text{lab}}, \quad \varphi = [\varphi_{\text{lab}} - \Omega t]_{2\pi}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathcal{L}_G = -\frac{1}{4g_{\text{YM}}^2} g^{\mu\mu'} g^{\nu\nu'} F^a_{\mu'\nu'} F^a_{\mu\nu}$$



Gluodynamics in Euclidean space

After Wick rotation ($t = -i\tau$), action can be rewritten as follows:

$$\mathcal{L}_G = \mathcal{L}_0 - (i\Omega)\mathcal{L}_1 - \frac{\Omega^2}{2}\mathcal{L}_2$$

$$\mathcal{L}_0 = \frac{1}{4g_{\mathrm{YM}}^2} F^a_{\mu\nu} F^a_{\mu\nu},$$

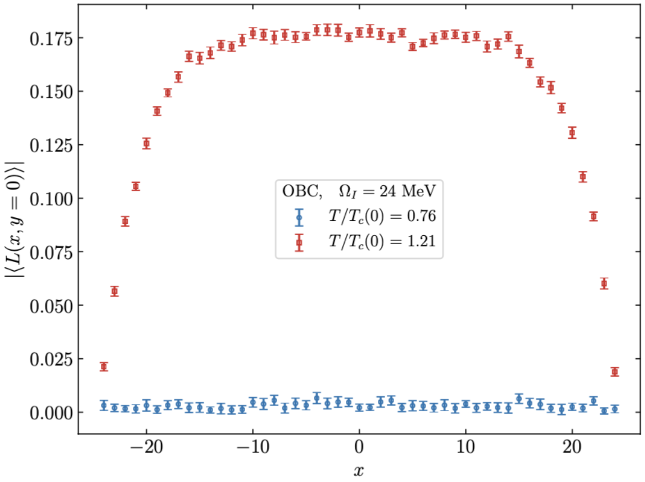
$$\mathcal{L}_1 = \frac{1}{g_{\mathrm{YM}}^2}\left[-y F^a_{\mathrm{xy}} F^a_{\mathrm{y}\tau} - y F^a_{\mathrm{xz}} F^a_{\mathrm{z}\tau} + x F^a_{\mathrm{yx}} F^a_{\mathrm{x}\tau} + x F^a_{\mathrm{yz}} F^a_{\mathrm{z}\tau}\right]$$

$$\mathcal{L}_2 = \frac{1}{g_{\mathrm{YM}}^2}\left[r^2 (F^a_{\mathrm{xy}})^2 + x^2 (F^a_{\mathrm{yz}})^2 + y^2 (F^a_{\mathrm{xz}})^2 - 2xy F^a_{\mathrm{xz}} F^a_{\mathrm{yz}}\right]$$

To avoid the sign problem ($P \propto \exp(-S_E)$) for Monte-Carlo simulations, one can:

- Either perform analytical continuation $\Omega_I = -i\Omega$
- Or calculate expansion coefficients at $\Omega = 0$

Boundary conditions



Local thermalization approximation

For $SU(3)$ gluodynamics the phase transition is of the first order.

$$\xi \ll L_x, L_y, L_z$$

Then in the vicinity of the $(x,y) = (x_0, y_0)$ point:

$$\mathcal{L}_G \;\approx\; \mathcal{L}_G^{(loc)} \equiv \mathcal{L}_0^{(loc)} - (i\Omega)\mathcal{L}_1^{(loc)} - \frac{\Omega^2}{2}\mathcal{L}_2^{(loc)},$$

where

$$\mathcal{L}_0^{(loc)} = \frac{1}{4g_{\mathrm{YM}}^2} F^a_{\mu\nu} F^a_{\mu\nu},$$

$$\mathcal{L}_1^{(loc)} = \frac{1}{g_{\mathrm{YM}}^2} \left[-y_0 F^a_{\mathrm{xy}} F^a_{\mathrm{y}\tau} - y_0 F^a_{\mathrm{xz}} F^a_{\mathrm{z}\tau} + x_0 F^a_{\mathrm{yx}} F^a_{\mathrm{x}\tau} + x_0 F^a_{\mathrm{yz}} F^a_{\mathrm{z}\tau} \right],$$

$$\mathcal{L}_2^{(loc)} = \frac{1}{g_{\mathrm{YM}}^2} \left[r_0^2 (F^a_{\mathrm{xy}})^2 + x_0^2 (F^a_{\mathrm{yz}})^2 + y_0^2 (F^a_{\mathrm{xz}})^2 - 2x_0 y_0 F^a_{\mathrm{xz}} F^a_{\mathrm{yz}} \right].$$

Angular momentum density

$$J_{i,\,\mathrm{lab}} = \frac{1}{2}\epsilon_{ijk}\int d^3\bar{x}\left(\bar{x}^j T^{0k}_{\mathrm{lab}} - \bar{x}^k T^{0j}_{\mathrm{lab}}\right)$$

$$T^{\mu\nu}_{\mathrm{lab}} = -\frac{1}{g^2_{YM}}\left(\bar{F}^a_{\mu\alpha}\bar{F}^{a,\alpha}{}_{\nu} - \frac{1}{4}g_{\mu\nu}\bar{F}^a_{\alpha\beta}\bar{F}^{a,\alpha\beta}\right)$$

$$J_{\mathrm{lab}} \equiv J_{z,\,\mathrm{lab}} = -\frac{1}{g^2_{YM}}\int d^3\bar{x}\big(-\bar{x}\bar{F}^a_{\mathrm{yx}}\bar{F}^a_{\mathrm{xt}} - \bar{x}\bar{F}^a_{\mathrm{yz}}\bar{F}^a_{\mathrm{zt}} + \bar{y}\bar{F}^a_{\mathrm{xy}}\bar{F}^a_{\mathrm{yt}} + \bar{y}\bar{F}^a_{\mathrm{xz}}\bar{F}^a_{\mathrm{zt}}\big)$$

$$\bar{F}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \bar{x}^\mu}\frac{\partial x^\beta}{\partial \bar{x}^\nu}F_{\alpha\beta}$$

$$J \equiv J_z = \int d^3x\,(\mathcal{L}_1 + 2\Omega\mathcal{L}_2)$$

Angular momentum density

$$R = L_s/2 \qquad \text{— characteristic size.}$$

$$J = \Omega \cdot I = \Omega \cdot V R^2 \left(\mathsf{i}_2 + \mathsf{i}_4 (\Omega R)^2 + \mathsf{i}_6 (\Omega R)^4 + \dots \right)$$

$$\frac{j_I(T)}{\Omega_I R^2 T^4} = -\frac{\mathsf{i}_2(T)}{T^4} + \frac{\mathsf{i}_4(T)}{T^4} v_I^2 + \dots \qquad (\text{where } v_I = \Omega_I R)$$

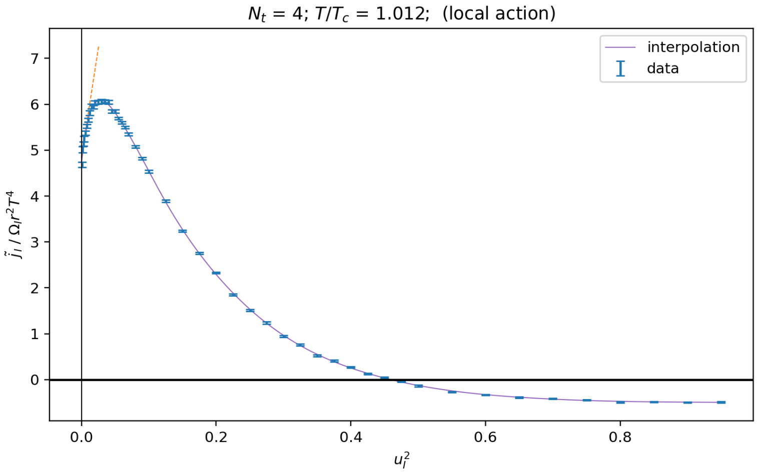
$$\frac{\tilde{j}_I(T)}{\Omega_I r^2 T^4} = -\frac{\tilde{\mathsf{i}}_2(T)}{T^4} + \frac{\tilde{\mathsf{i}}_4(T)}{T^4} u_I^2 + \dots \qquad (\text{where } u_I = \Omega_I r)$$

$$j(T, R, \Omega) = \frac{J}{V} = \frac{1}{V} \int_V d^3x \, \tilde{j}(T, r, \Omega)$$

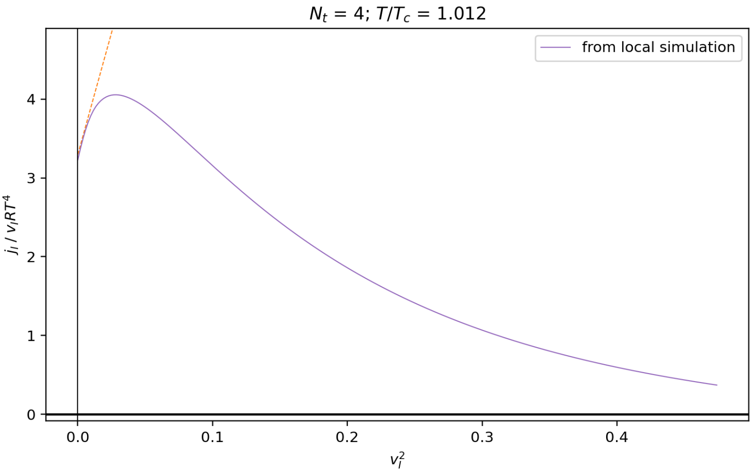
$$\mathsf{i}_n = \tilde{\mathsf{i}}_n \alpha_n$$

$$\alpha_n = \frac{1}{V R^n} \int_V d^3x r^n$$

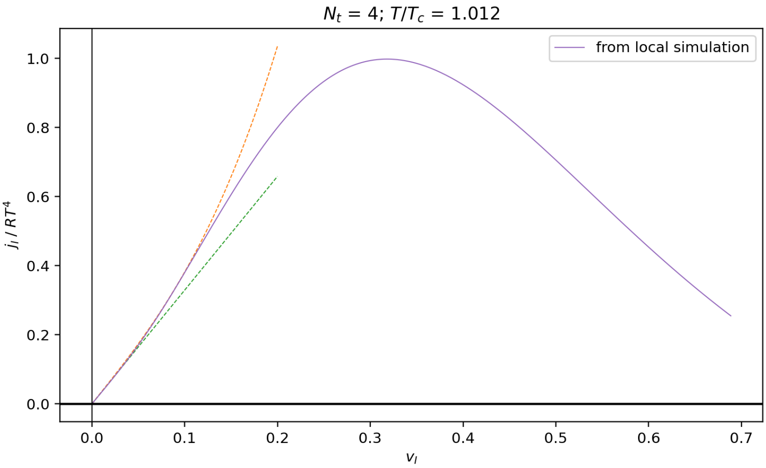
Local thermalization



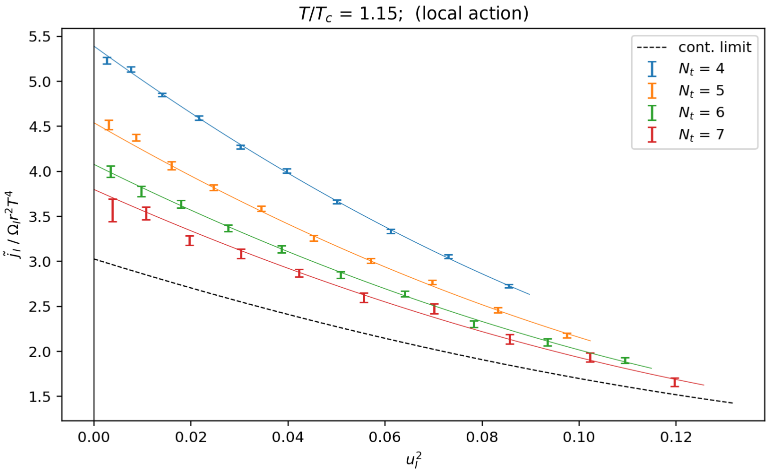
Local thermalization



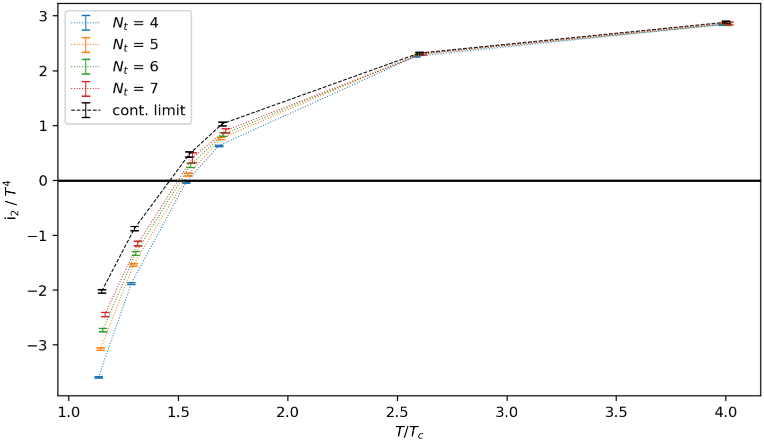
Local thermalization



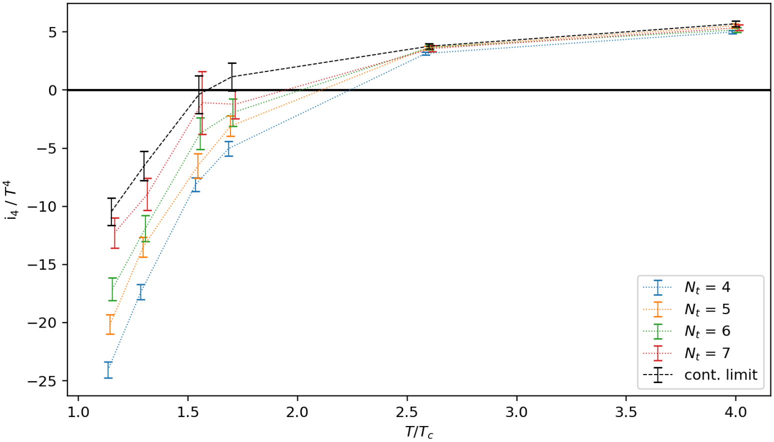
Local thermalization



Local thermalization

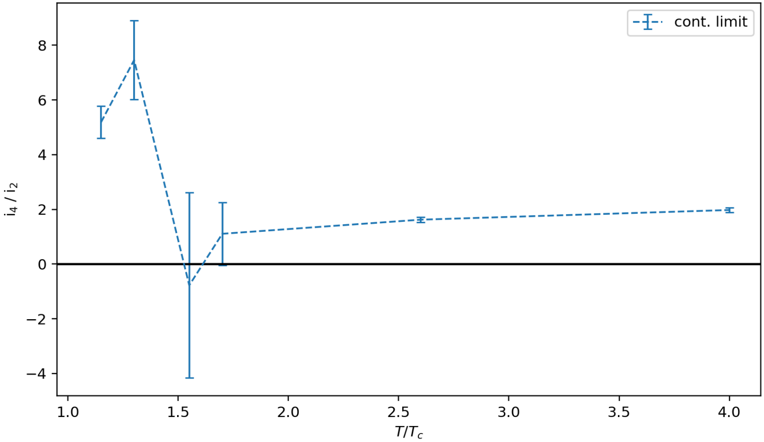


Local thermalization



Local thermalization

Near T_c gluon plasma is more deformable:



Magnetic gluon condensate

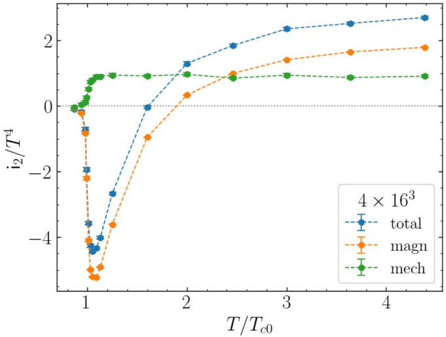
In previous works it was shown that one can split $\mathfrak{i}_2$ into the following parts:

$$\mathfrak{i}_2 = \mathfrak{i}_2^{\text{mech}} + \mathfrak{i}_2^{\text{magn}},$$

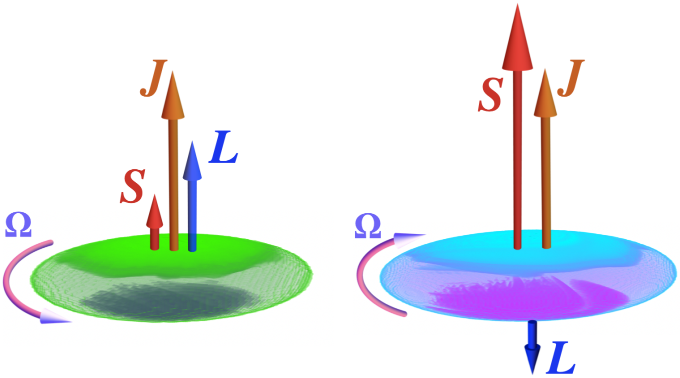
$$\mathfrak{i}_2^{\text{mech}} = \frac{1}{T}(\langle J^2 \rangle - \langle J \rangle^2) \geq 0,$$

$$\mathfrak{i}_2^{\text{magn}} = \frac{\alpha}{3} R^2 \langle (G_{\text{magn}})^2 \rangle.$$

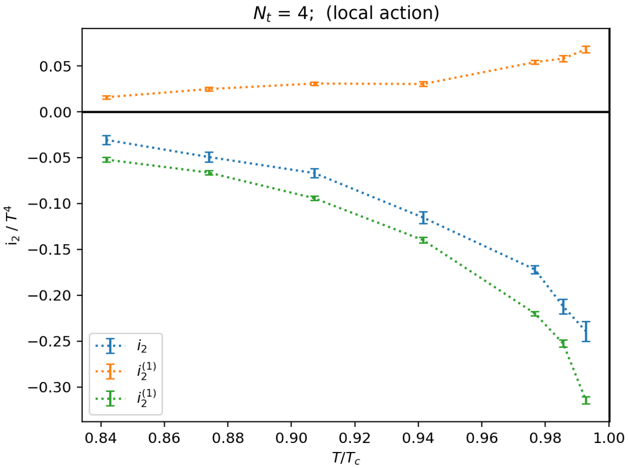
Then magnetic gluon condensate G_{magn} is solely responsible for negative sign of $\mathfrak{i}_2$.



Negative Barnett effect



Negative Barnett effect at $T < T_c$



Decomposition of i_4

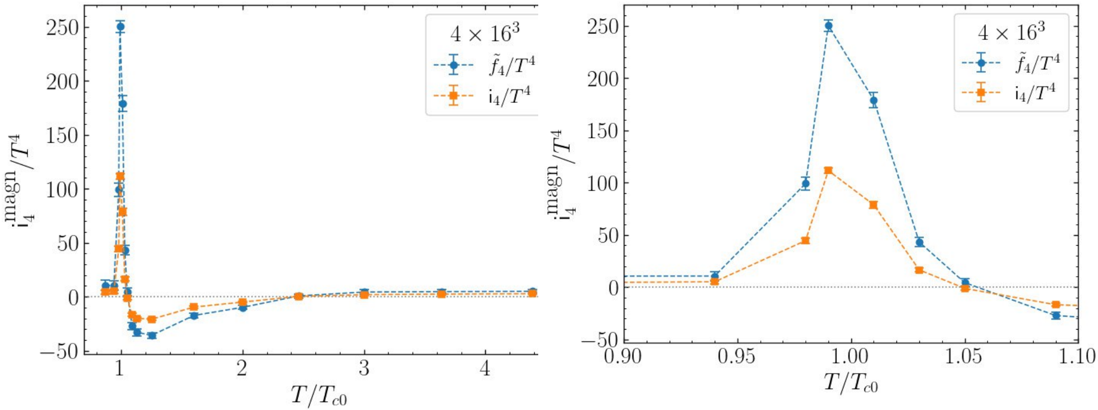
$$\mathrm{i}_4 = \mathrm{i}_4^{\text{mech}} + \mathrm{i}_4^{\text{mix}} + \mathrm{i}_4^{\text{magn}}$$

$$\mathrm{i}_4^{\text{mech}} = \frac{T}{6VR^4}\left(\langle S_1^4\rangle - 3\langle S_1^2\rangle^2\right)$$

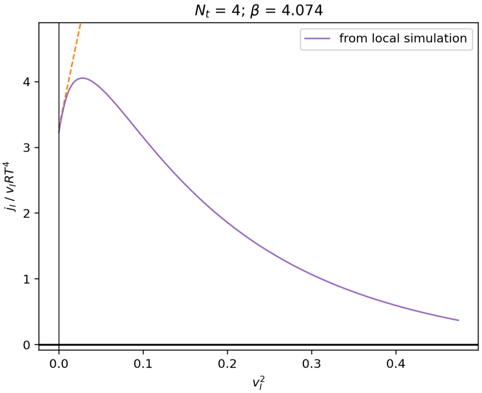
$$\mathrm{i}_4^{\text{mix}} = \frac{T}{VR^4}\left(2\langle S_1^2\rangle\langle S_2\rangle - 2\langle S_2 S_1^2\rangle\right)$$

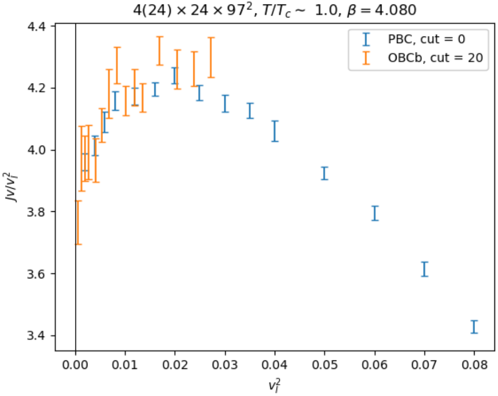
$$\mathrm{i}_4^{\text{magn}} = \frac{T}{VR^4}\left(2\langle S_2^2\rangle - 2\langle S_2\rangle^2\right)$$

Behaviour near T_c



Behaviour near T_c





Summary

- The moment of inertia seems to be negative at $T < T_c$. The negative Barnett effect might occur;
- The i_4 coefficient seems to change sign at $T \sim 1.05(2) * T_c$ and $T \sim 1.70(12) * T_c$;
- Local thermalization approximation holds for large enough volumes;
- Near T_c gluon plasma is more deformable than at high temperatures.

Summary

Thanks for attention!