Target dependence of fragmentation reaction cross-sections at Fermi energies and its calculation in the relativistic and low-energies oriented models.

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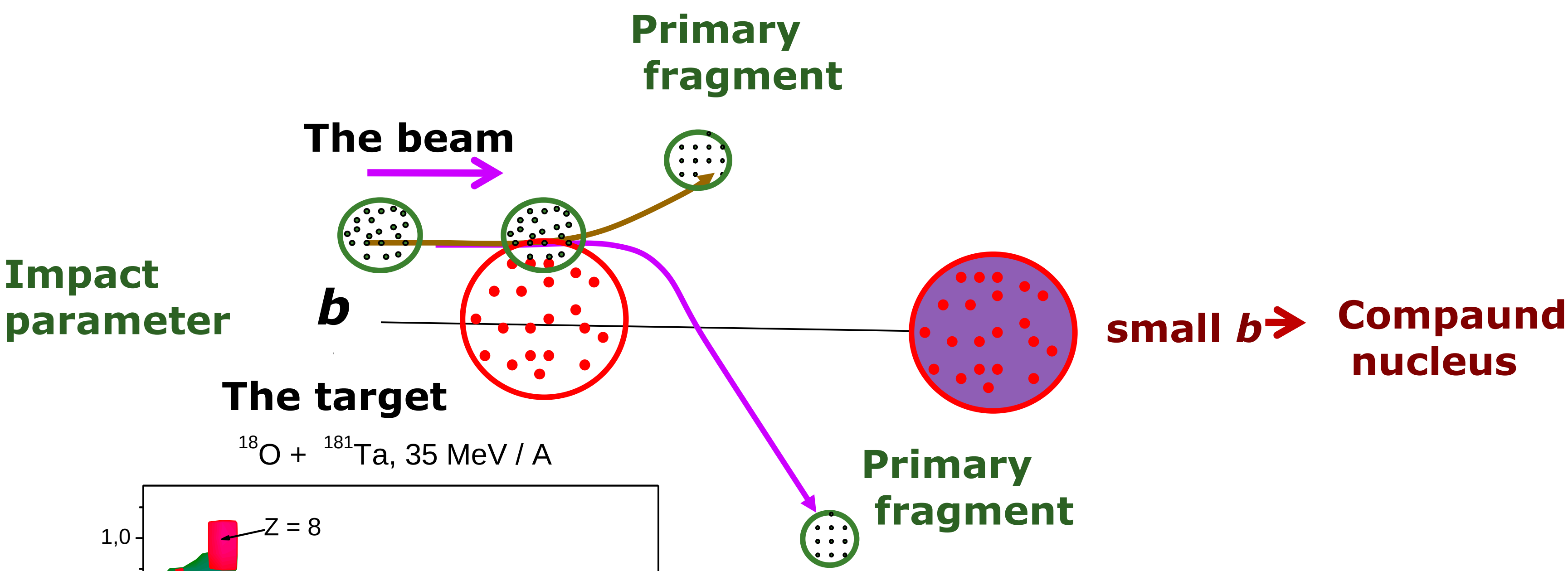
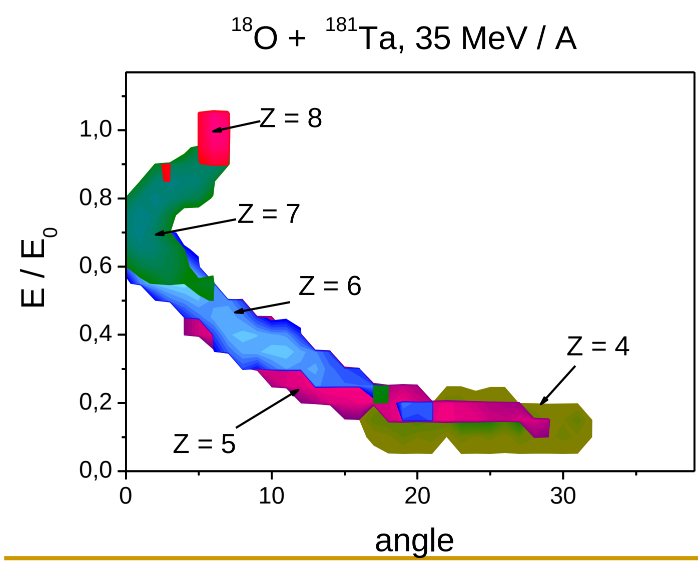
Outline

- Evolution off ragmentation reaction mechanism from deep-inelastic to direct reactions.
- Description of heavy-ion collisions with transport (BNV)-statistical (SMM) approach . Comparison with other model calculations (EPAX, Abrasion-Ablation and Fracs).
- Explanation of target ratio dependence on the fragment mass number *A* from the point of view of transport model calculations.
- Comparison with the experimental data for reactions with four different projectiles ^{18}O (35 MeV per nucleon) , ^{86}Kr (64 MeV per nucleon), ^{40}Ca and ^{48}Ca (140 MeV per nucleon) on two targets : ^{181}Ta and ^{9}Be
- Conclusion

Nuclear collisions at low energies

$E_{projectile}$<20 MeV/nucleon





Deep-inelastic collisions: Wilczynski plot

J. Wilczynski, Phys. Lett. B 47, 484 (1973).

Вильчинский Я., Волков В. В., Дедовски И. Ядерная физика и исследования с многозарядными ионами // ЯФ. 1967. Т. 5. С. 942.

First observation of direct component in nuclear collisions at Fermi energies

Gelbke et al 1977, $^{16}O+^{208}Pb$ at 27 *A* MeV

Energy spectra of reaction products N, C, B, Be, and Li at the laboratory angle 15 °.

VC – exit channel Coulomb barrier,

EF – the model predicted fragment energy

EP --the energy of a fragment with the projectile velocity.

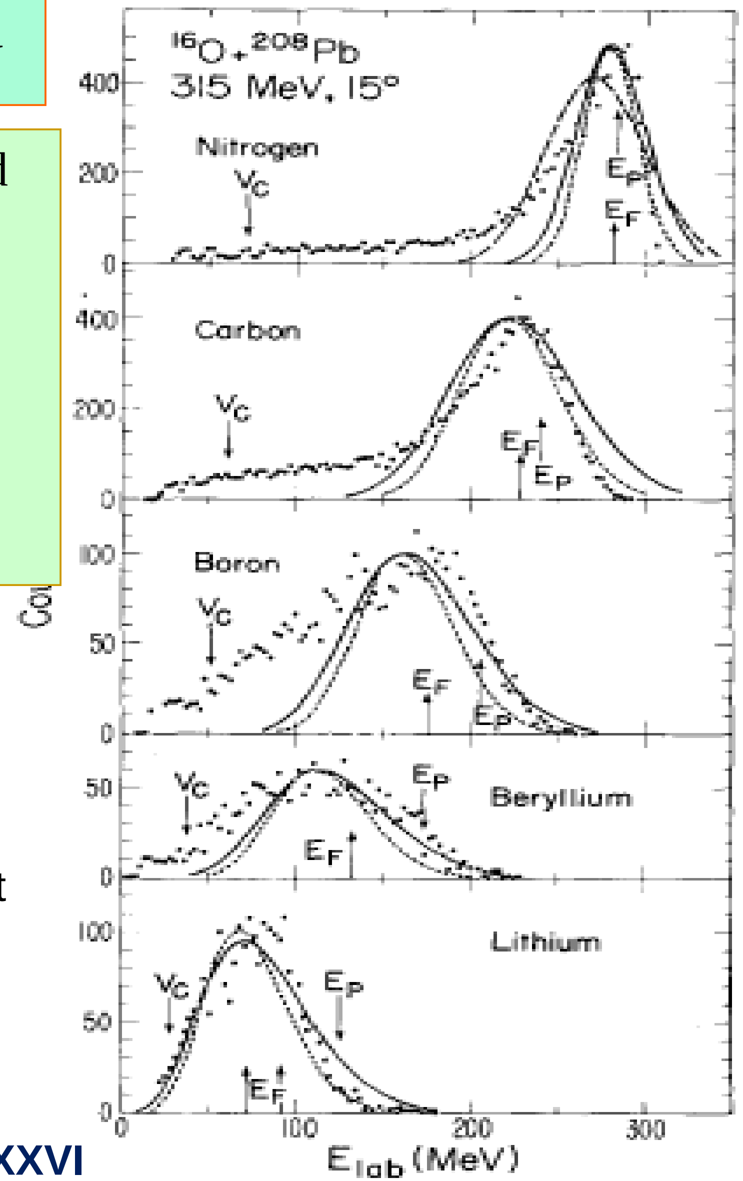
Goldhaber, Phys. Lett. V53B (1974) p306

$$\sigma^2 = \sigma_0^{\,2} \frac{A_F(A_P - A_F)}{A_P - 1},$$

$$\sigma_0 \approx 90 MeV/c$$

P – projectile, F – fragment

$$\boxed{\sigma_0^{\,2} = 80 - 100 MeV/c^2}$$

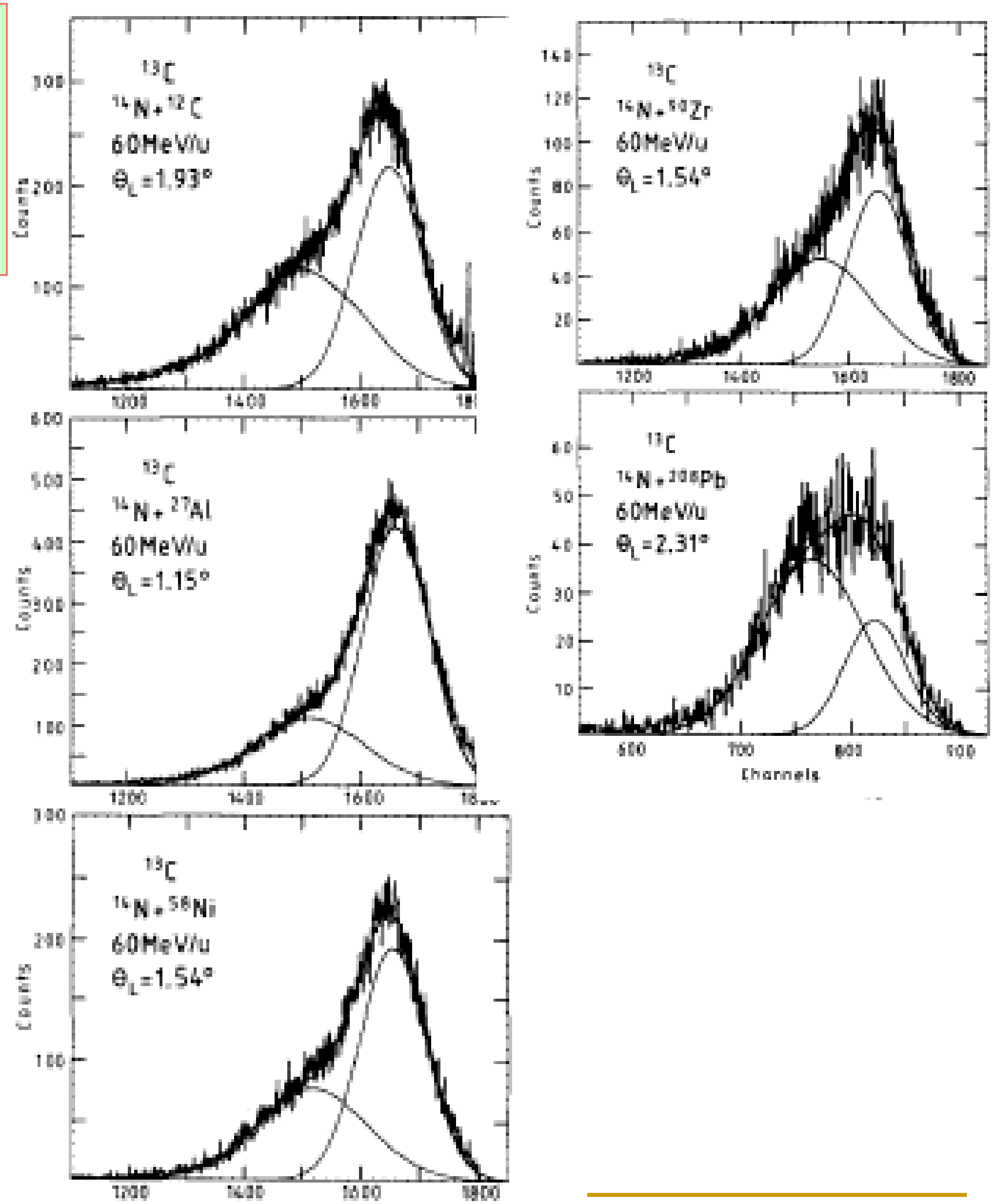


Transfer and fragmentation reactions of ^{14}N at 60 MeV/u , Lahmer et al, Z. Phys. A - Atomic Nuclei 337, 425-437 (1990)

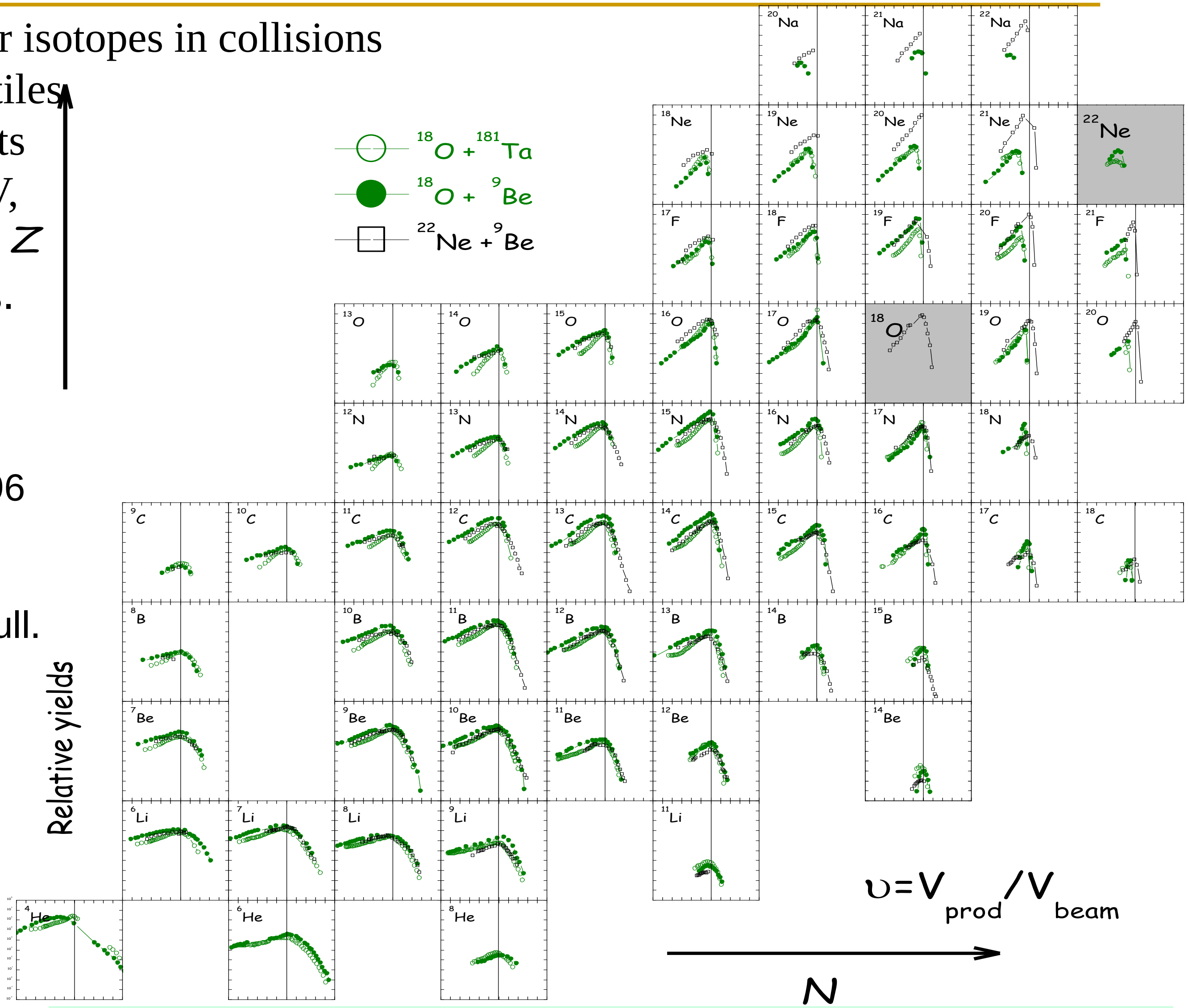
Two-component fits to ^{13}C spectra, measured for 60 MeV/u ^{14}N on various target nuclei

High energy component also interpreted as direct break-up.

$$\sigma_0{}^2 = 61 MeV/c^2$$



Velocity distributions for isotopes in collisions of ^{18}O and ^{22}Ne projectiles with ^{181}Ta and ^{9}Be targets at energies 35-40 *A* MeV, COMBASS set-up, FLNR, 2002-2003 years.



Artukh A.G., Gridnev G.F., Gruszecki M. *et al.*// Nucl. Phys. A. 2002. V. 701. P. 96 .

Erdemchimeg B., Artukh A.G., Klygin S.A. *et al.*// Bull. Russ. Acad. Sci. Phys. 2021. V.85. P.1457.

The velocities of fragments with $Z < Z_{beam}$ and $N < N_{beam}$ have maximum at beam velocity

Approaches commonly used to predict isotope distributions produced in nuclear collisions at Fermi energies, implemented in code LISE++

Bazin D., Tarasov O., Lewitowicz M. et al. // Nucl. Instrum. Meth. A. 2002. V. 482. P. 307

EPAX (an Empirical PArametrization of fragmentation CROSS sections) *K. Summerer and B. Blank, Phys. Rev. C. 61, 034607 (2000)*

"… the fragmentation yields are no longer energy dependent, This is certainly true for incident energies considerably above the Fermi energy in nuclei (40A MeV), in particular for the typical SIS energies of (500–1000)A MeV."

FRACS (Mei B. Improved empirical parameterization for projectile fragmentation cross sections // Phys. Rev. C. 2017. V. 95. P. 034608.

" Comparing to the EPAX 2.1 and EPAX 3.1, the FRACS formula can better predict the fragments production cross sections by further incorporating the target and incident energy dependence, as well as the odd–even staggering effect."

Abrasion-Ablation model (A-A), *Bowman J.D., Swiatecki W.J., Tsang C.F. // LBL Report. 1973. LBL-2908.*

One of the efficient models to describe projectile fragmentation is the abrasion–ablation (AA) model. Bowman et al. developed the geometric abrasion–ablation model for heavy-ion collisions at high bombarding energies (E > 200MeV/u)

Transport theory: Boltzmann-Nordheim-Vlasov (BNV) approach

time evolution of the one-body phase space density: f(r,p;t)

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla} f - \vec{\nabla} U \vec{\nabla}_p f = I_{coll}[f, \sigma]$$

Where:

U(ρ) is the mean field potential and σ in-medium elastic cross section

U(ρ(r)) = Nuclear Mean Field + Symmetry terms + Coulomb

$$U(\rho) = A\left[\frac{\rho}{\rho_0}\right] + B\left[\frac{\rho}{\rho_0}\right]^d + C(-1)^k\left(\rho_n - \rho_p\right)/(\rho_n + \rho_p) + U_{\text{coul}}$$

$$A = -356 MeV, B = 303 MeV, d = 7/6, k = 1(p), 2(n), C = 36 MeV$$

F. Bertsch, S. Das Gupta, Phys. Rep. **160** (1988) 189

V. Baran, M. Colonna, M. Di Toro, Phys. Rep., **410** (2005**)** 335

The computational code was developed on the basis of the code developed in LNS-INFN Catania in collaboration with M. Di Toro(LNS-INFN, Catania) and H.H. Wolter (LMU, Muenchen)

Mikhailova T.I., Erdemchimeg B., Di Toro M., Wolter H.H. // Phys. Part. Nucl. 2023. V. 54. No. 3. P. 510.

Solution of transport equation

This partial integro-differential equation for *f(r,p;t)* is solved by simulation with the test particle method:
N -finite element test particles (TP) per nucleon.
Each TP carries charge and isospin number.
A – number of nucleons in the system
g – the shape of the TP in space
ρ – the density

$$f(\vec{r},\vec{p},t)=\frac{1}{N}\sum_{i}^{NA} g(\vec{r}-\vec{r}_i(t))\,\bar{g}(\vec{p}-\vec{p}_i(t))$$

$$g=\mathrm{e}^{-(\vec{r}-\vec{r}_i(t))^2/L^2}\ \ldots;\ \bar{g}=\mathrm{e}^{-(\vec{p}-\vec{p}_i(t))^2/l^2}$$

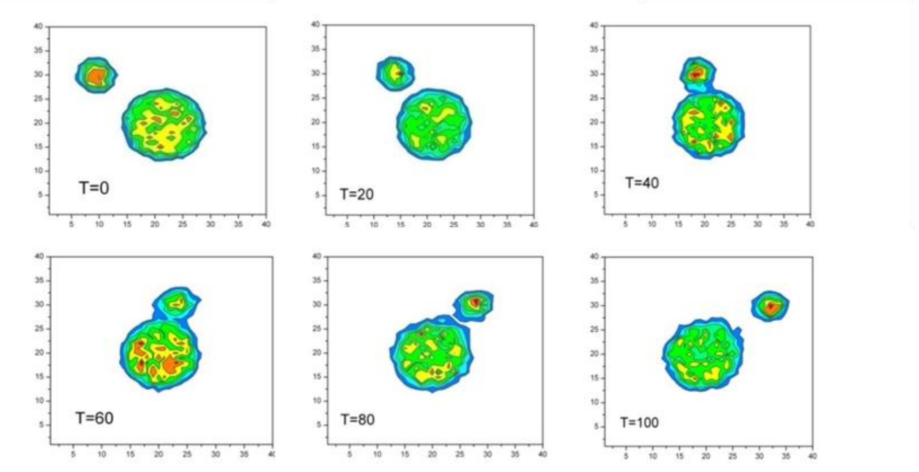
$$\rho(r;t)=\int d\vec{p}\ f(\vec{r},\vec{p};t)$$

Equations of motion of TP

(Newton equation of motions:):

$$\frac{\partial\vec{p}_i(t)}{\partial t}=-\vec{\nabla}_r U(r_i,t) \qquad \frac{\partial\vec{r}_i(t)}{\partial t}=\frac{\vec{p}_i(t)}{m}$$

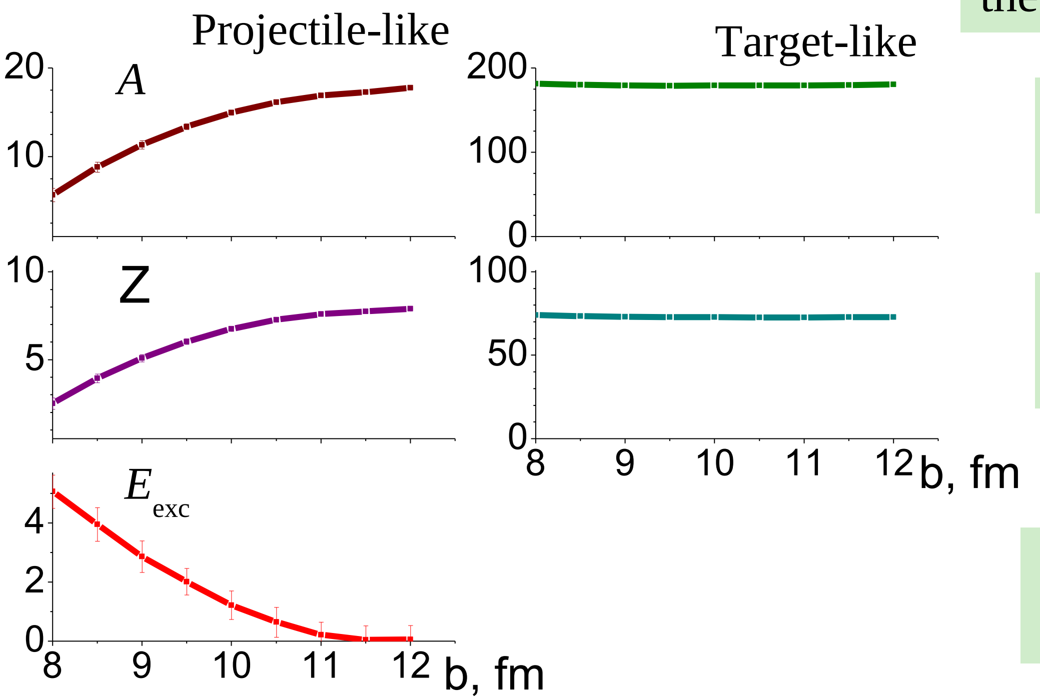
These differential equations are calculated numerically with the leapfrog algorithm, N =200 and collision term is calculated stochastically.



Density contour plots in the reaction
^{18}O(35 *A* MeV) + ^{181}Ta at *b* = 9 fm
(t=0, 20, 40, 60, 80, 100 fm / *c* (10 fm/*c*=3.3*10-23 c))

Calculations with 200 TP for nucleon charachteristics of forward moving fragments

$^{18}O(35\ A\ Mev)+^{181}Ta$



The results smoothly depends on the value of impact parameter *b*

Heavier ion stays practically the unchanged!

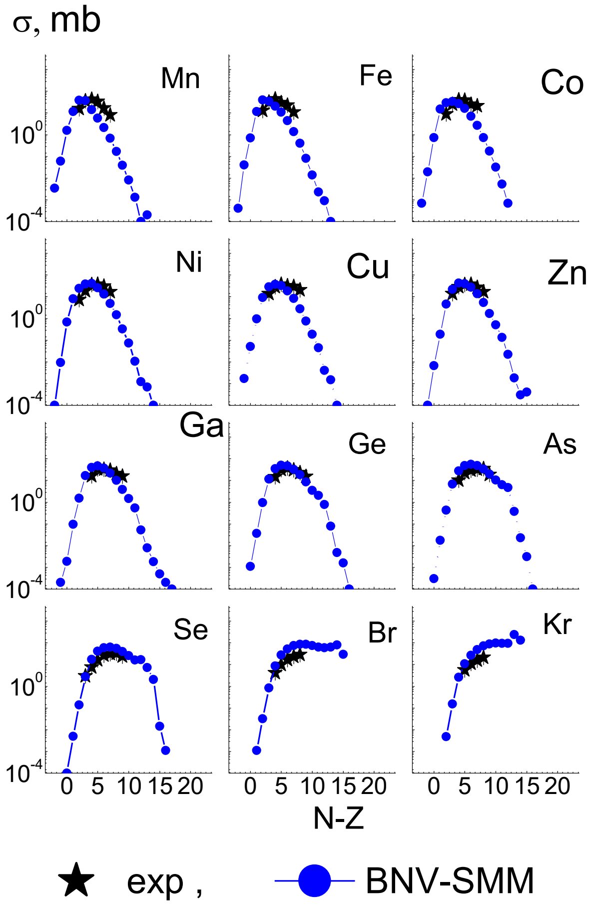
Heavier ion stays practically the unchanged!

Projectile like fragments are excited.

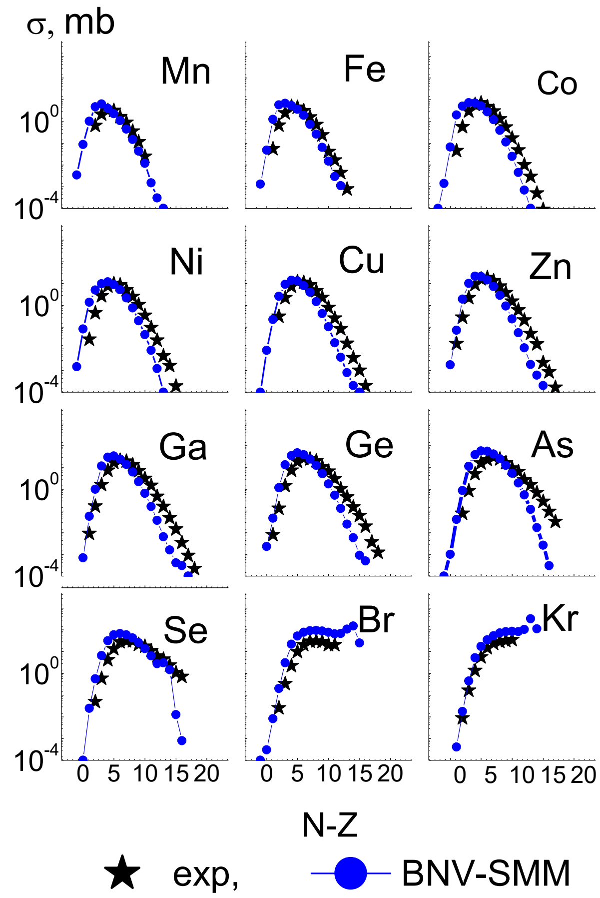
To de-excite the primary fragments the statistical code is used **SMM** code, P. Bondorf, et al.,Phys. Rep. 257, 133 (1995)

BNV-SMM calculations of isotope distributions for reactions ^{86}Kr (64 *A* MeV) on two targets ^{181}Ta and ^{9}Be and comparison with experimental data

^{181}Ta

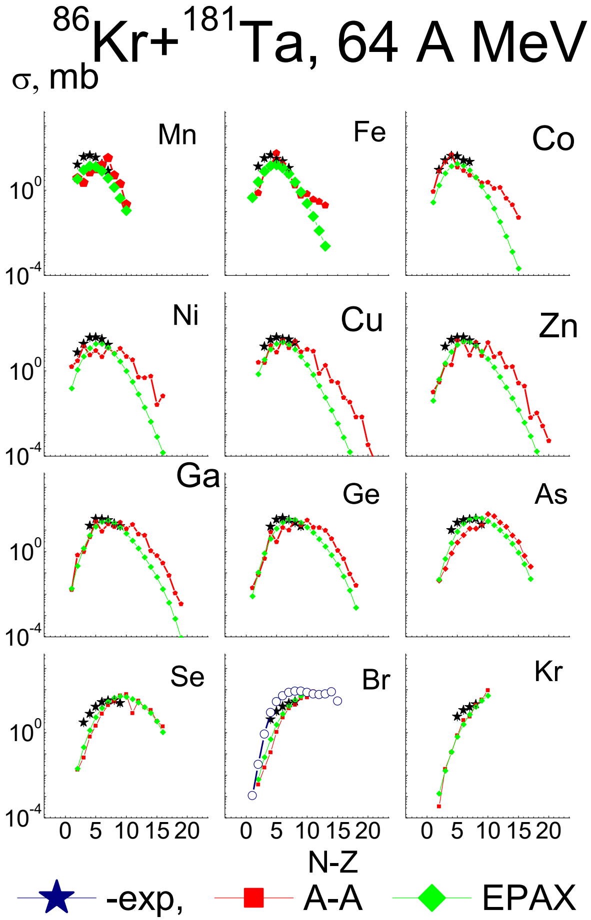


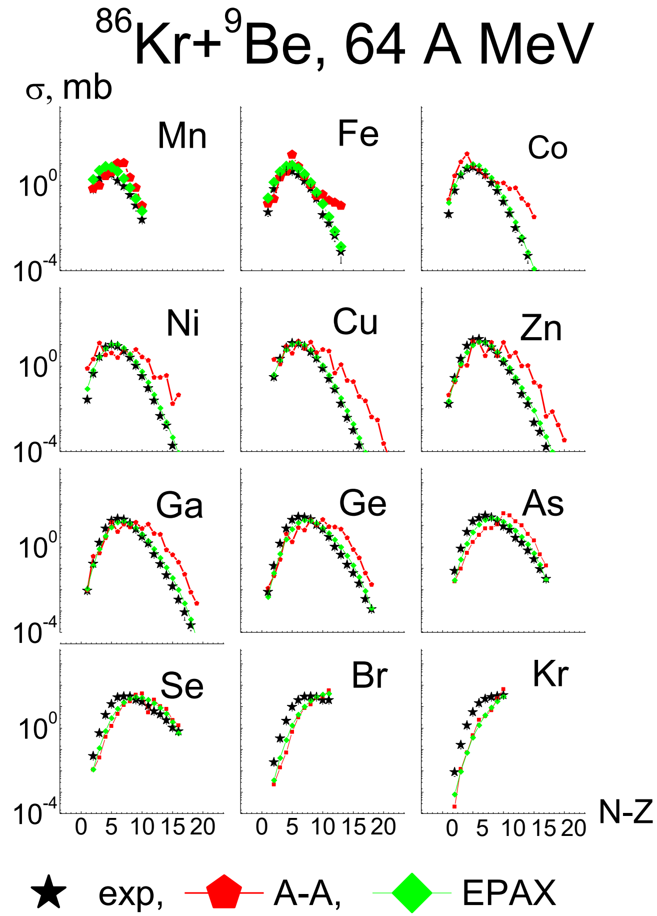
^{9}Be



M. Mocko et.al Phys. Rev. C76, 014609(2007)

EPAX, Abrasion-Ablation models calculations for the same reactions as in 11



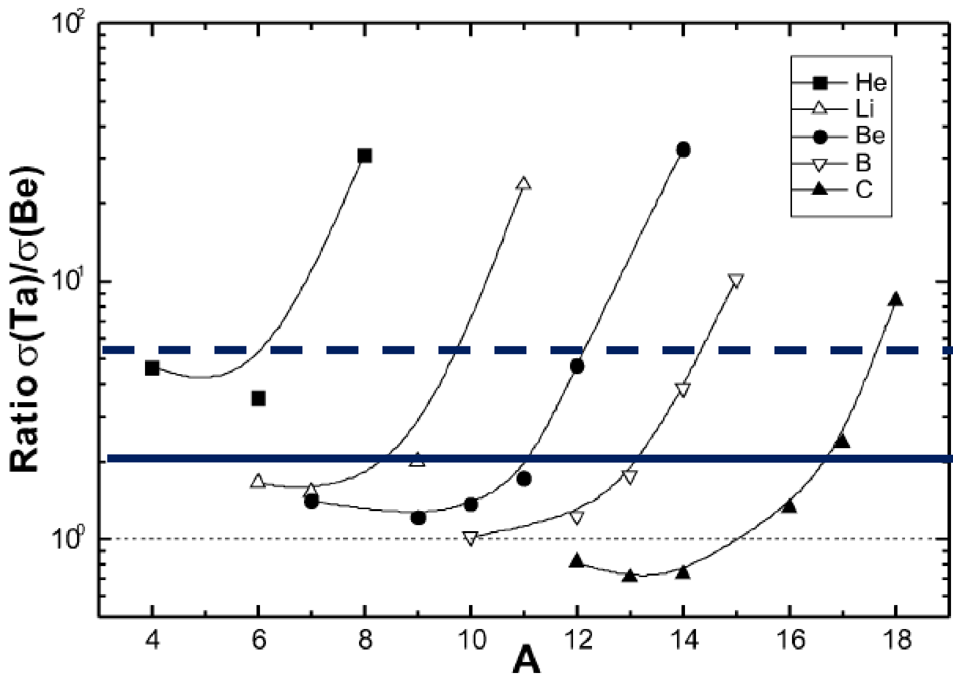


M. Mocko et.al Phys. Rev. C76, 014609(2007)

Target ratio of forward emitted fragments

σ(^{18}O+^{181}Ta) /σ(^{18}O+^{9}Be), 35 *A* MeV

A.G. Artukh et al. / Nuclear Physics A 701 (2002) 96c–99c 99c



$$R(A) = \frac{(A_{Ta}^{1/3}+A_{Be}^{1/3})^2}{(A_{O}^{1/3}+A_{Be}^{1/3})^2} = 3.1$$

$$R(A) = \frac{A_{Ta}^{1/3}+A_{Be}^{1/3}}{A_{O}^{1/3}+A_{Be}^{1/3}} = 1.76$$

Fig. 3. A comparison of cross sections in production of neutron-rich isotopes for elements He, Li, Be, B and C induced in the reactions of ^{18}O (35 *A* MeV) with a heavy target ^{181}Ta and a light target ^{9}Be.

$$R^J(A_s) = \sigma^J(A_s)_{Ta}/\sigma^J(A_s)_{Be},$$

Target Ratio σ(Kr+Ta) / σ(Kr+Be), 64 *A* MeV

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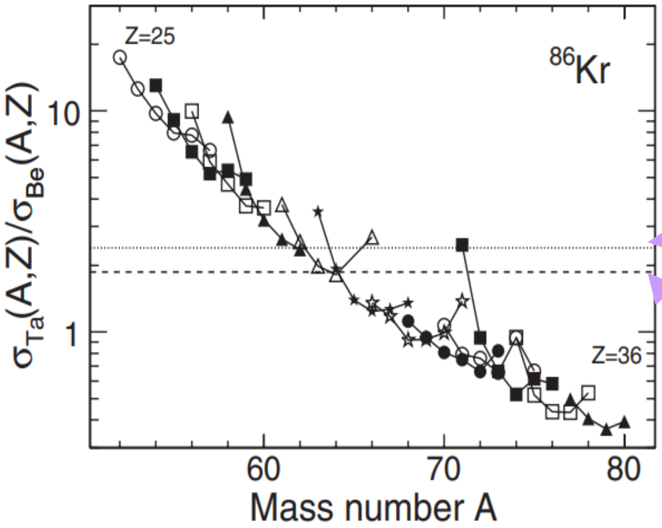


FIG. 10. Ratios of the fragmentation cross sections on Ta and Be targets, $\sigma_{\text{Ta}}(A, Z)/\sigma_{\text{Be}}(A, Z)$, for fragments with $25 \leqslant Z \leqslant 36$ for the ^{86}Kr beam. Only ratios with relative errors smaller than 25% are shown. Open and solid symbols represent odd and even elements starting with $Z = 25$. The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Eq. (4), respectively.

$$\frac{\sigma_{\text{Ta}}(A, Z)}{\sigma_{\text{Be}}(A, Z)} = \frac{\left(A_{\text{Kr}}^{1/3} + A_{\text{Ta}}^{1/3}\right)^2}{\left(A_{\text{Kr}}^{1/3} + A_{\text{Be}}^{1/3}\right)^2} = 2.4,$$

$$\frac{\sigma_{\text{Ta}}(A, Z)}{\sigma_{\text{Be}}(A, Z)} = \frac{\left(A_{\text{Kr}}^{1/3} + A_{\text{Ta}}^{1/3} - 2.38\right)}{\left(A_{\text{Kr}}^{1/3} + A_{\text{Be}}^{1/3} - 2.38\right)} = 1.9.$$

RARE ISOTOPE PRODUCTION
By

Michal Mocko

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics and Astronomy

2006

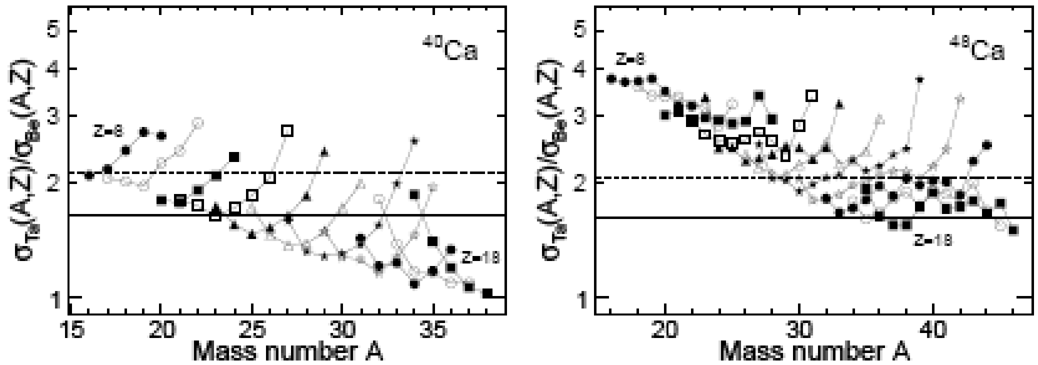


Figure 4.23: Target ratios of the fragmentation cross sections $\sigma_{Ta}(A,Z)/\sigma_{Be}(A,Z)$, of fragments $8 \leq Z \leq 18$ for two projectiles ^{40}Ca (left panel) and ^{48}Ca (right panel). The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Equation (4.22), respectively.

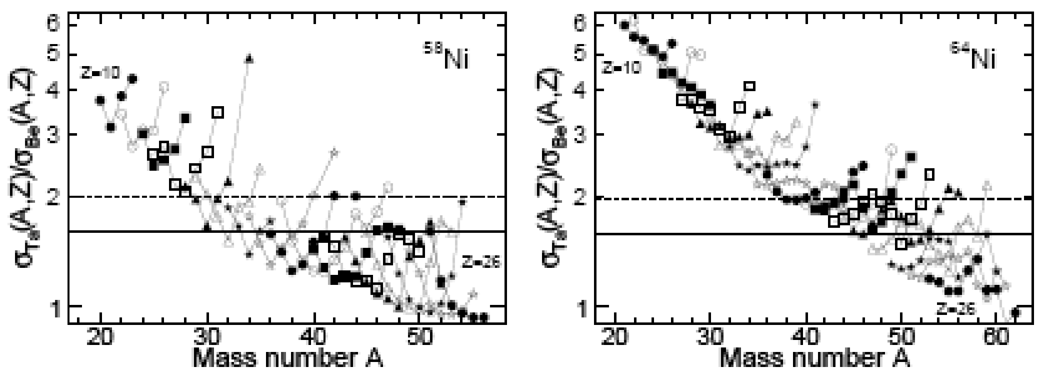
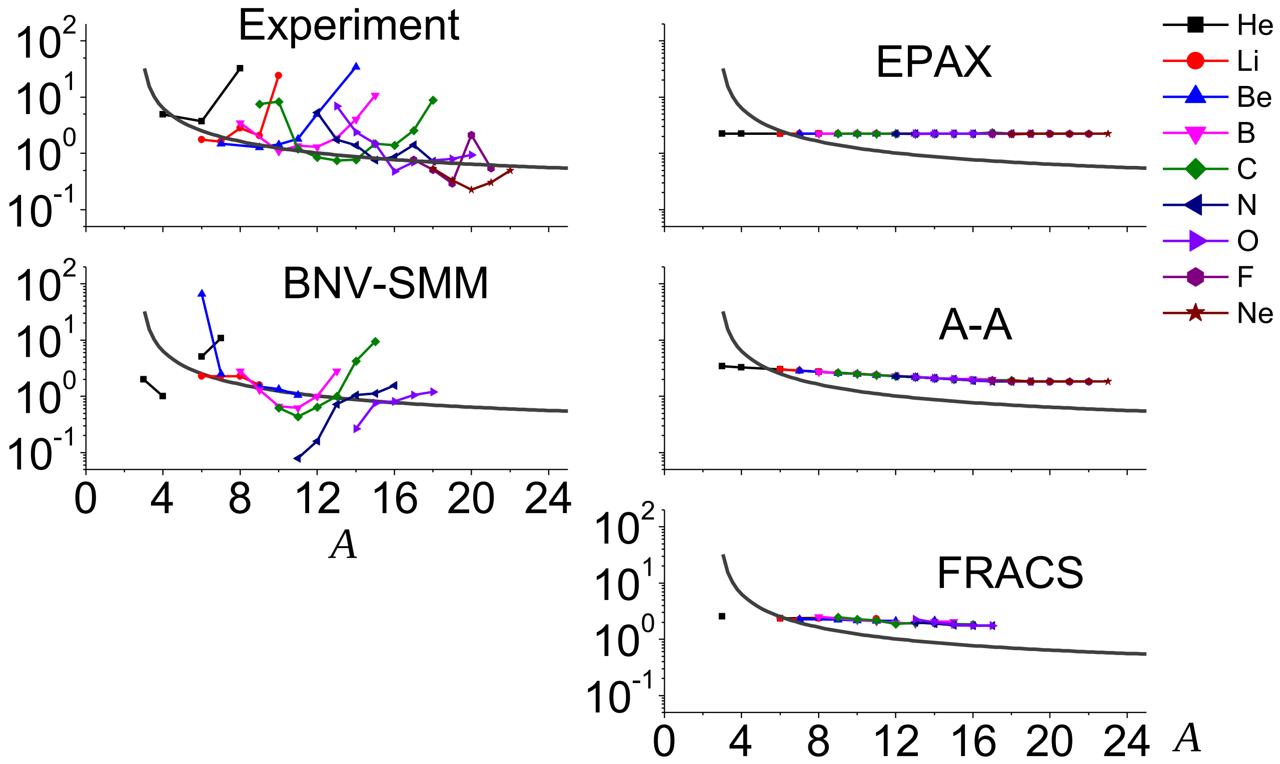


Figure 4.24: Target ratios of the fragmentation cross sections $\sigma_{Ta}(A,Z)/\sigma_{Be}(A,Z)$ of fragments $10 \leq Z \leq 26$ for two projectiles ^{58}Ni (left panel) and ^{64}Ni (right panel). The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Equation (4.22), respectively.

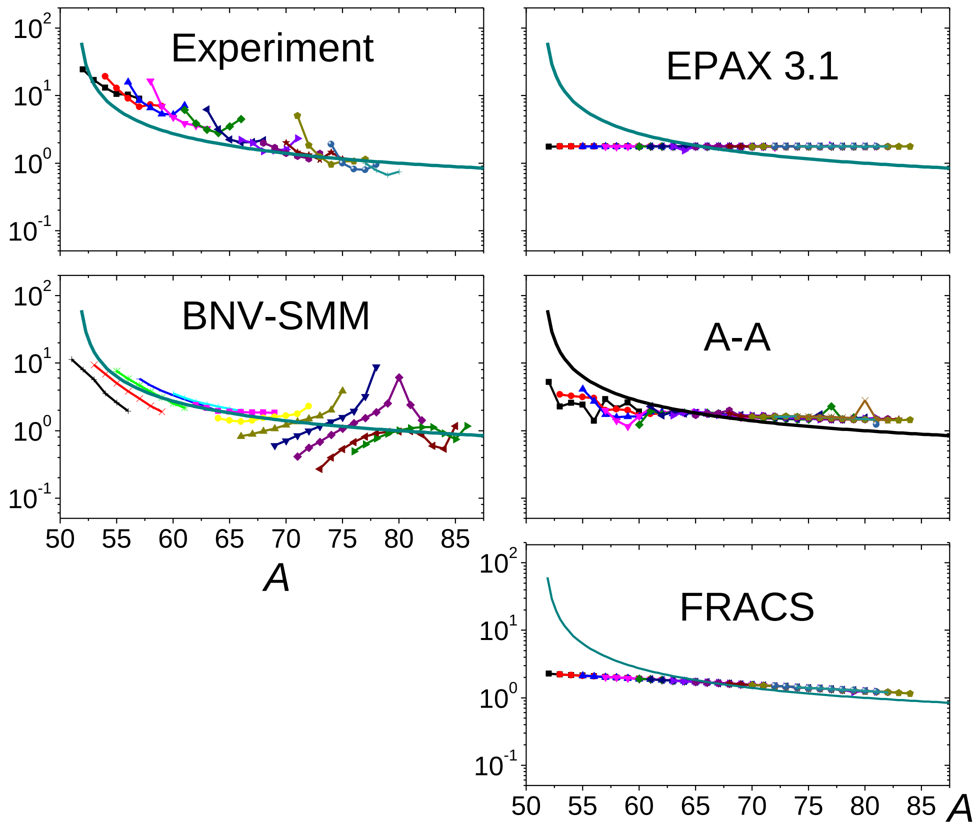
$$R(A_s) = \frac{A_{pr}^{1/3} + A_{Ta}^{1/3}}{A_{pr}^{1/3} + A_{Be}^{1/3}}$$

Target Ratio σ(^{18}O+^{181}Ta) /σ(^{18}O+^{9}Be), 35 *A* MeV, Experiment and model calculations



Experiment: Combas set-up, FLNR, JINR
Artukh, A.G, et al. Nucl. Phys.A701, p96. (2001).

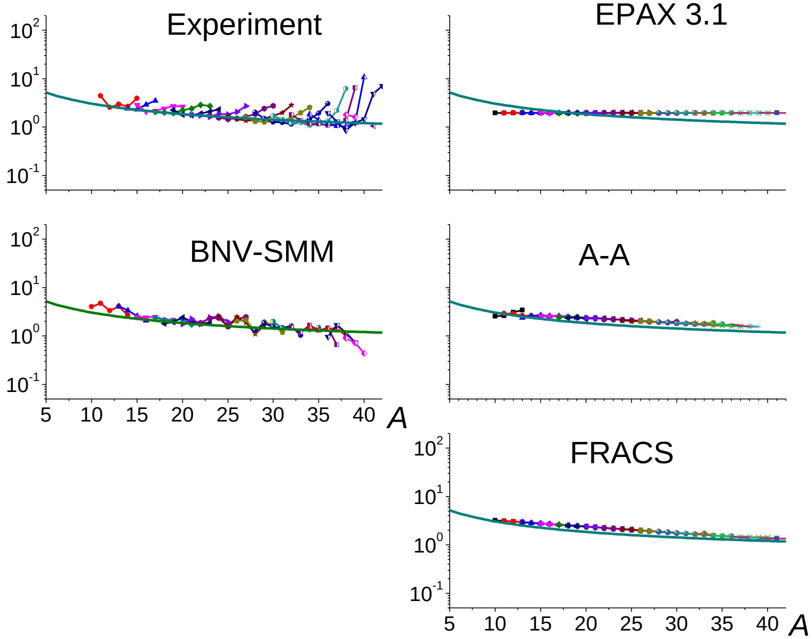
Target ratio σ(^{86}Kr+^{181}Ta) /σ(^{86}Kr+^{9}Be), 64 *A* MeV, Experiment and model calculations



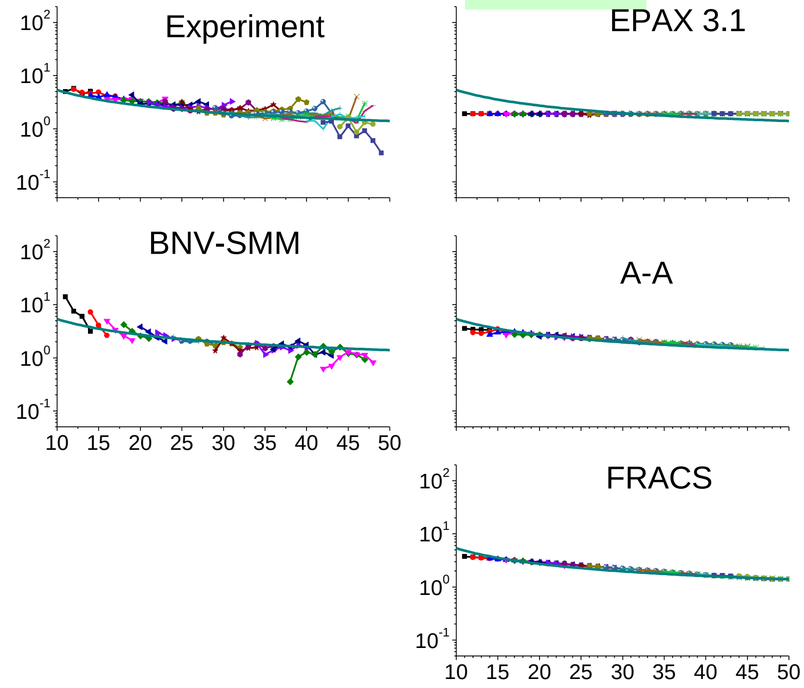
Experiment:
M. Mocko et.al Phys. Rev. C76, 014609(2007)

Target ratio $\sigma(^{40/48}Ca+^{181}Ta)/\sigma(^{40/48}Ca+^{9}Be)$, 140 *A*MeV, Experiment and model calculations

^{40}Ca



^{48}Ca

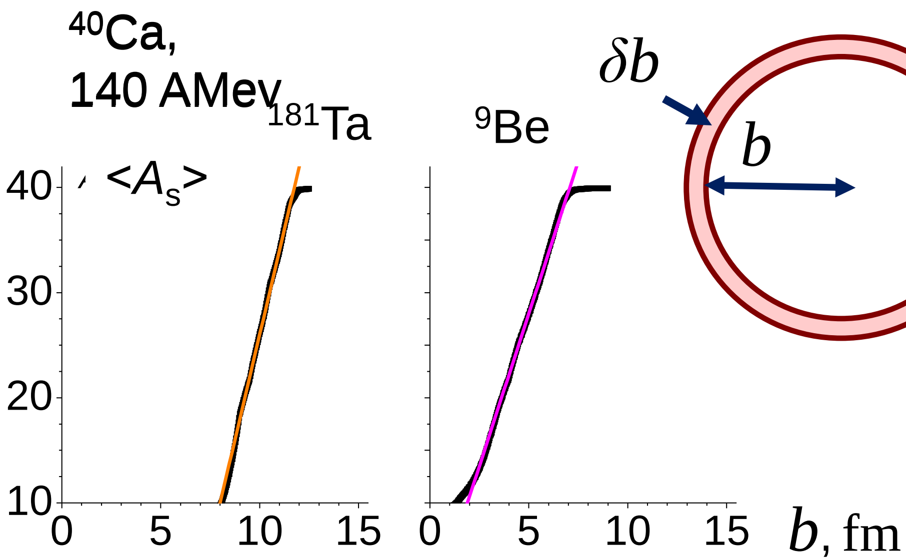


Experiment: Rare isotope production, Dissertation , M. Mocko, 2006

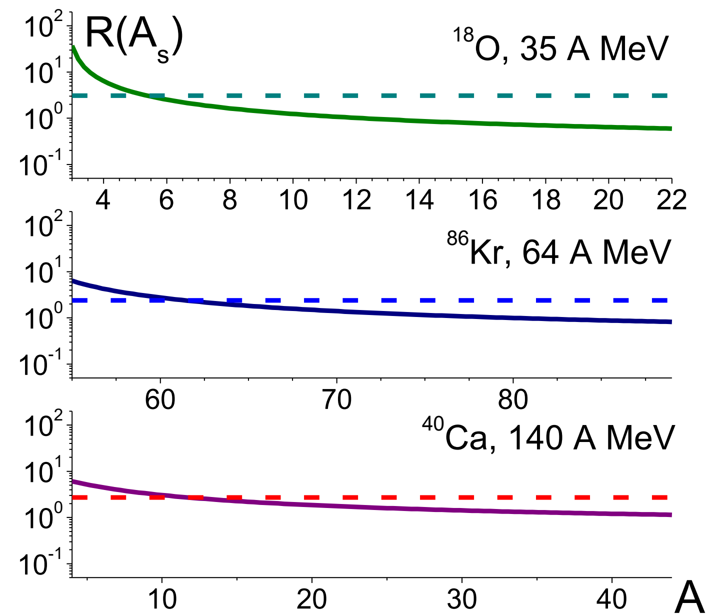
Calculation of the target ratio

$R_J(A_s) = \sigma_J(A_s)_{Ta} / \sigma_J(A_s)_{Be}$,

for projectiles ^{40}Ca and ^{18}O



$\langle A_s \rangle = 5.8 * b - 1$ $\langle A_s \rangle = 8 * b - 54$



$$R(A) = \frac{A_{Pr}^{1/3} + A_{Ta}^{1/3}}{A_{Pr}^{1/3} + A_{Be}^{1/3}} = 1.76$$

$$R(A_s) = b(A_s)_{Ta} \left(\frac{\partial b}{\partial As}\right)_{Ta} \Big/ b(A_s)_{Be} \left(\frac{\partial b}{\partial As}\right)_{Be}$$

Conclusions

There is a variety of models that describe very well the yields of projectile like fragments near stability line . The most accurate predictions gives the EPAX model.

Transport BNV (+SMM) model makes it possible to follow the evolution of the reaction and analyze isotope and energy distributions of the projectile-like fragments. From this calculations it follows that:

1) The hyperbolic envelope of target ratios plot is manifestation of the fragment mass dependence on the impact parameter of the collision;

2) In case of the collision of two non identical nuclei, the lighter nucleus undergoes more pronounced changes than the heavier one.

The increase of the yields of the neutron-rich isotopes in the reactions on heavy targets in comparison with the light ones could be due to the presence of the pick-up reactions.


Thank you for your attention