Collective flow harmonics correlations analysis for model data at Nuclotron-NICA energies

N. Bikmetov, P. Parfenov, A. Taranenko
(JINR LHEP, NRNU MEPhI)

The XXVI International Baldin Seminar on High Energy Physics Problems "Relativistic Nuclear Physics and Quantum Chromodynamics"
15-20 September, 2025

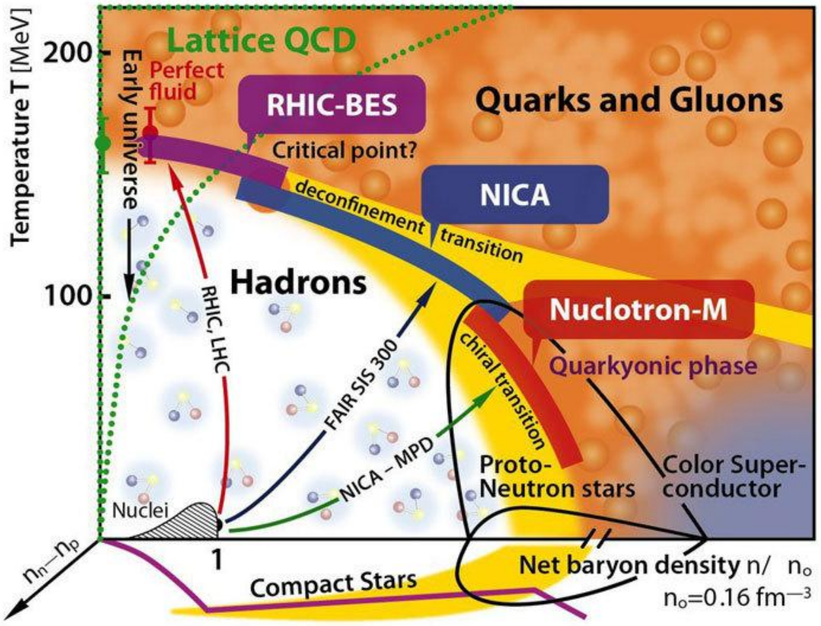
The work was funded by the Ministry of Science and Higher Education of the Russian Federation, Project “Fundamental and applied research at the NICA (JINR) megascience experimental complex” FSWU-2025-0014

16.09.2025

Relativistic heavy-ion collisions

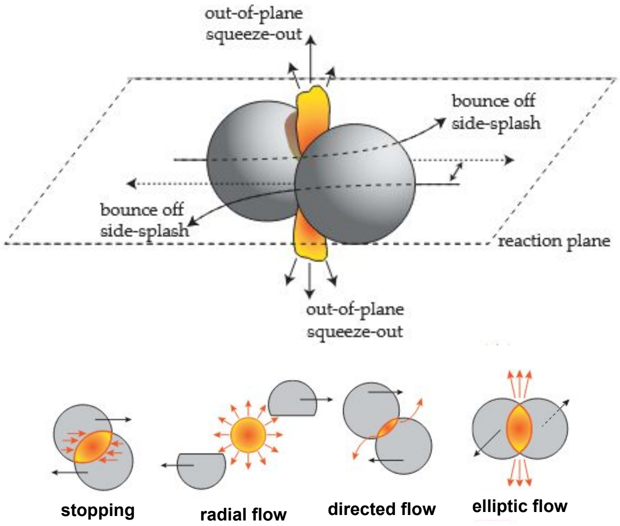
- **The Goal:** Explore the high baryon density region of the QCD phase diagram to search for **first-order phase transition** and the **Critical Point (CEP)**.
- MPD experiments at NICA will collide heavy ions at center-of-mass energies $\sqrt{s_{NN}}$ = **2.4 – 11.5 GeV**.
  - This fills the gap between beam energy scans.

- **Low beam energies:**
  - Intermediate temperature (T);
  - **High net-baryon density**;
  - Analogous to the conditions found in the inner structure of neutron stars and neutron star mergers.



Anisotropic flow

- Flow describes **anisotropy** in particle emission;
- Sensitive to early **pressure gradients** and **Equation of State (EoS)**



The anisotropic flow is **quantified as**:

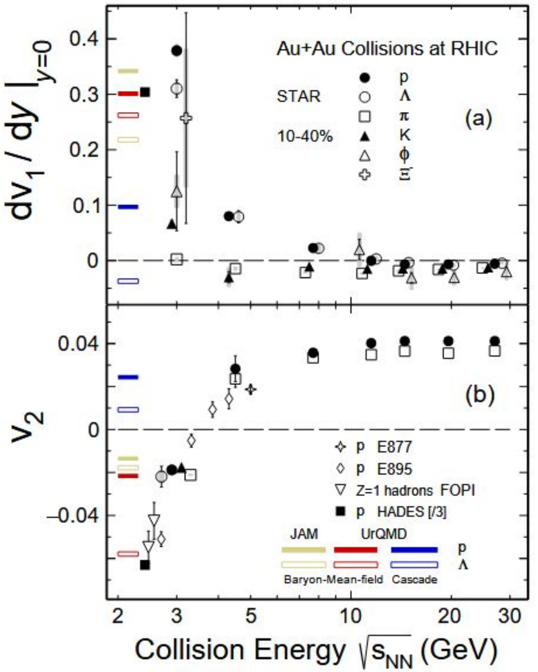
$$\frac{dN}{d\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_n)]$$

Extraction of **azimuthal moments v_n**:

$$v_n = \langle \cos[n(\phi - \Psi_n)] \rangle$$

Poskanzer & Voloshin, Phys. Rev. C 58, 1671 (1998)

STAR Collaboration, Phys. Lett. B 827 (2022) 137003.

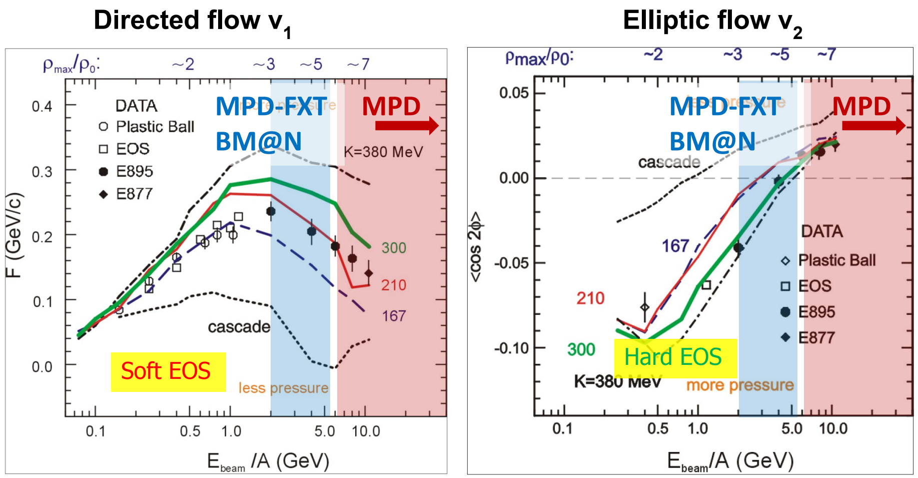


At NICA energies (**2 ÷ 11 GeV**), both **v_1** and **v_2 change strongly** with √sNN

Sensitivity of anisotropic flow to the Equation of State (EoS)

- **Anisotropic flow** is a **sensitive probe** of the pressure gradient built up in the **early, high-density** stage of the collision.
  - Stronger flow = Stiffer EoS (higher pressure)
  - Weaker flow = Softer EoS (lower pressure)
- The discrepancy in the interpretation:
  - Directed flow v_1 **suggests a soft EoS** ($K_0 \approx 210$ MeV).
  - Elliptic flow v_2 **suggests a stiff EoS** ($K_0 \approx 380$ MeV).

P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002)



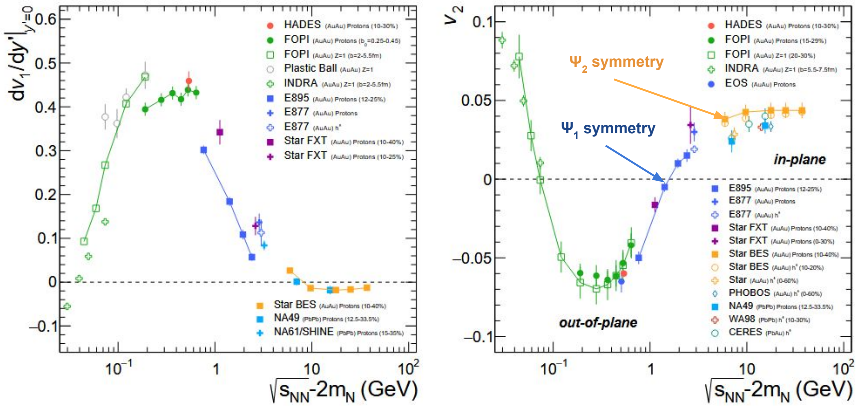
$$F = \frac{d\langle p_x/A\rangle}{d(y/y_{cm})}\bigg|_{y/y_{cm}=1}$$

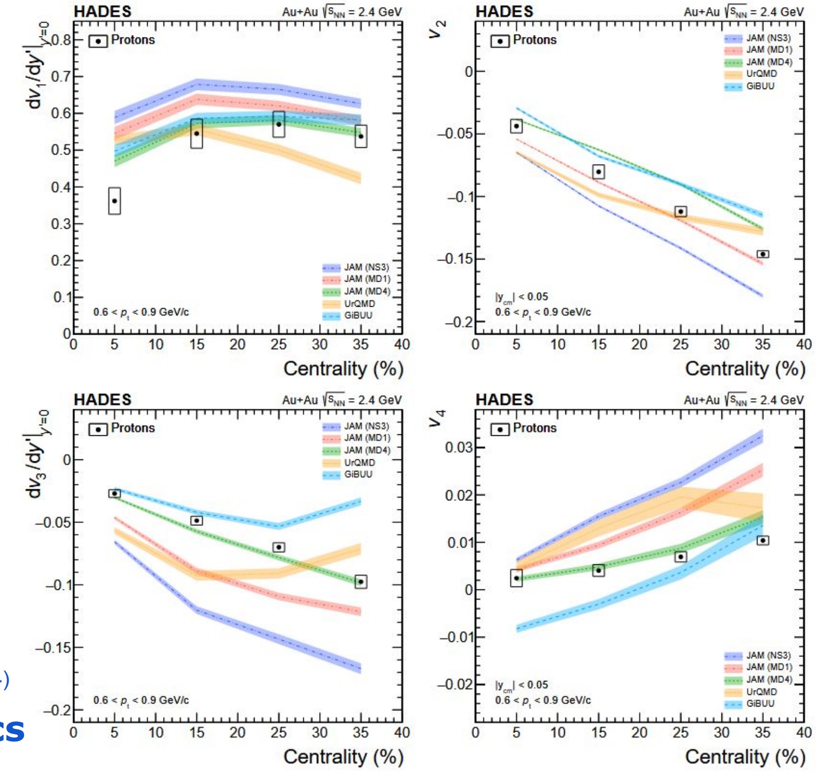
$$v_2 \equiv \langle \cos(2(\varphi - \Psi_{RP}))\rangle$$

Nuclear incompressibility:

$$K_0 = 9\rho_0^2 \frac{\partial^2 (E/A)}{\partial \rho^2}\bigg|_{\rho=\rho_0}$$

HADES results on anisotropic flow correlations





HADES results:

- Show high sensitivity to EoS of **higher flow harmonics**;
- Provide an insight on flow harmonics being **originated from v_2** (?) Reichert, T., & Aichelin, J. arXiv:2411.12908 (2024)

↳ It is interesting to investigate **the flow harmonics correlations** for **stricter EoS** constraints

HADES, Eur. Phys. J. A 59, 80 (2023).

Dataset and applied cuts

Dataset:

- Model: JAM v1.9
- Equation of state: MD2
- Collision system: Au+Au
- Energy: $\sqrt{s_{NN}}$ = 2.0 ÷ 4.5 GeV
- Statistics per energy: ~20÷40M

Event selection:

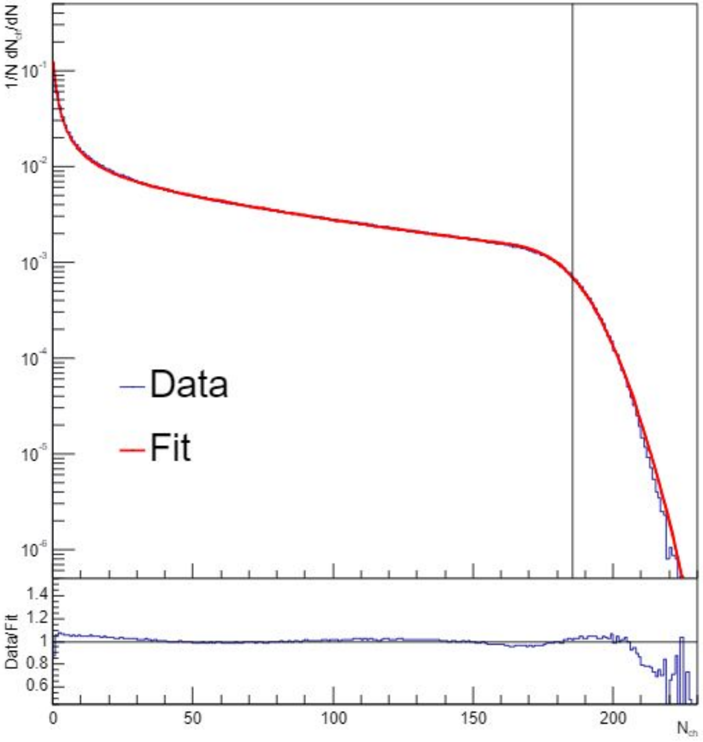
- Multiplicity-based centrality

Particle selection:

- Protons (pdg cut)
- $|y| < 0.5$
- $p_T > 0.5$ GeV/c

Centrality determination

- **Centrality determination** method: Bayesian inversion method (**Γ-fit**)
- Centrality was determined for **charged particle multiplicity (Nch)**;
- Further centrality classes cuts are applied by **selecting events in range of Nch**;

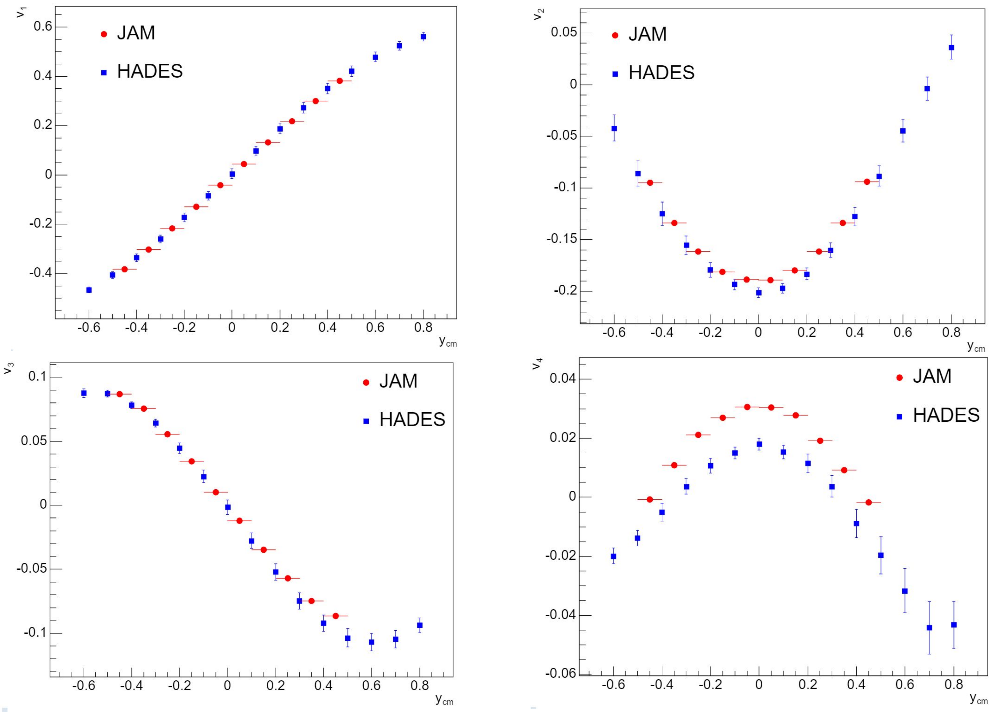


Γ-fit results for √sNN = 2.5GeV

Parfenov et al., Particles 4, 275 (2021). (URL)

v_n dependence on y_{cm} comparison

- For correct comparison, assuming that **20-30%** centrality is **equal** to **6 < b < 9 fm**.
- Results for v_1, v_2 and v_3 are in **good agreement** with HADES data;
- JAM predicts **higher v_4** signal than HADES data.

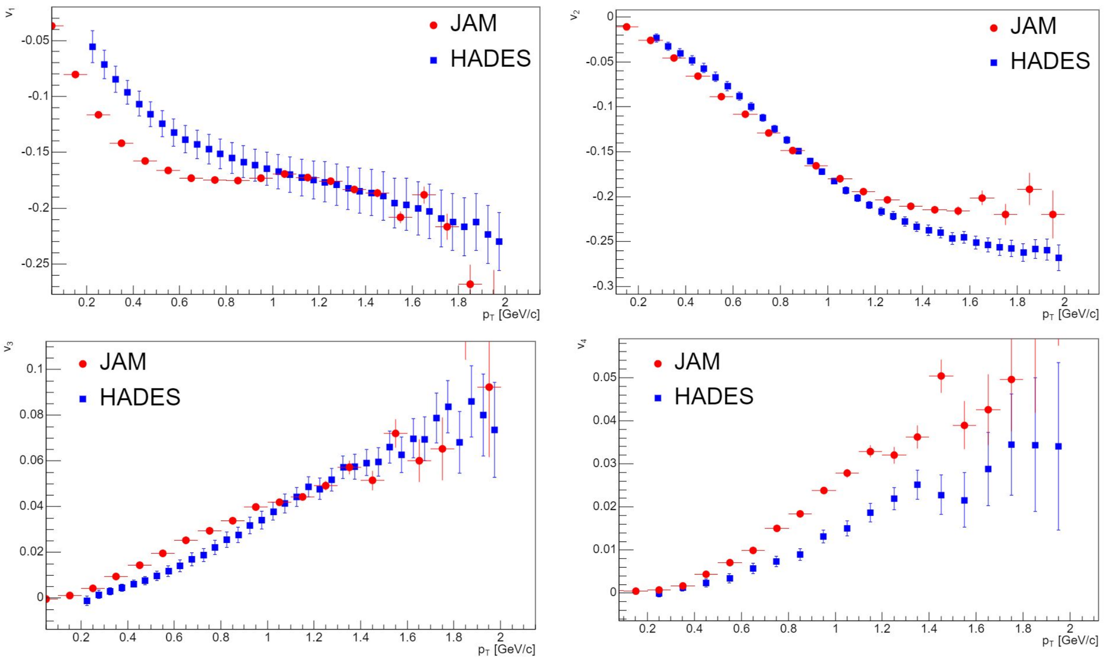


v_1, v_2, v_3 and v_4 distributions over y comparison with HADES data

HADES, Phys. Rev. Lett. 125, 262301 (2020)

v_n dependence on p_T comparison

- For correct comparison, assuming that **20-30%** centrality is **equal** to **6 < b < 9 fm**;
- Results for v_1, v_2 and v_3 are in **agreement at different pt ranges** with HADES data;

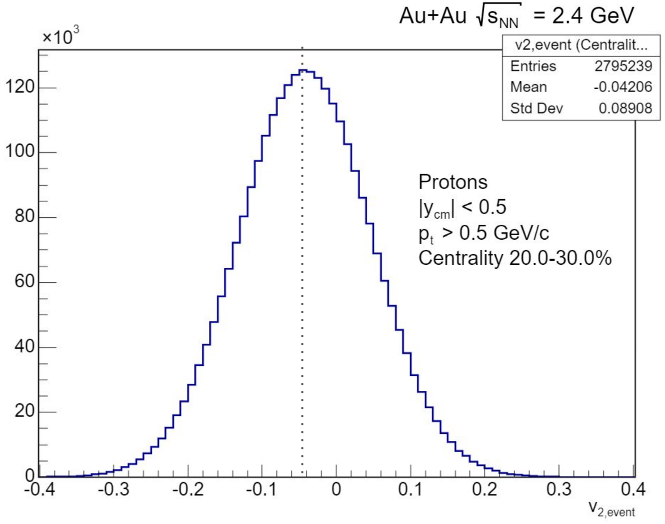


v_1, v_2, v_3 and v_4 distributions over p_T comparison with HADES data

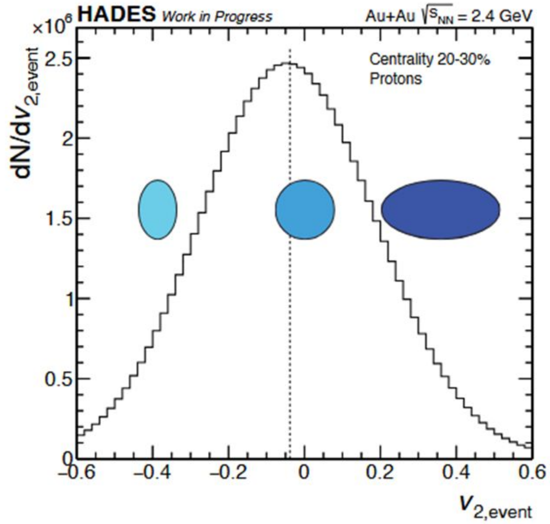
HADES Collaboration, Phys. Rev. Lett. 125, 262301 (2020)

$v_{2,event}$ distribution comparison

- v_2 terms were averaged in one collision event;
- JAM $v_{2,event}$ distribution is **narrower**;
- Mean values are **approximately equal**.



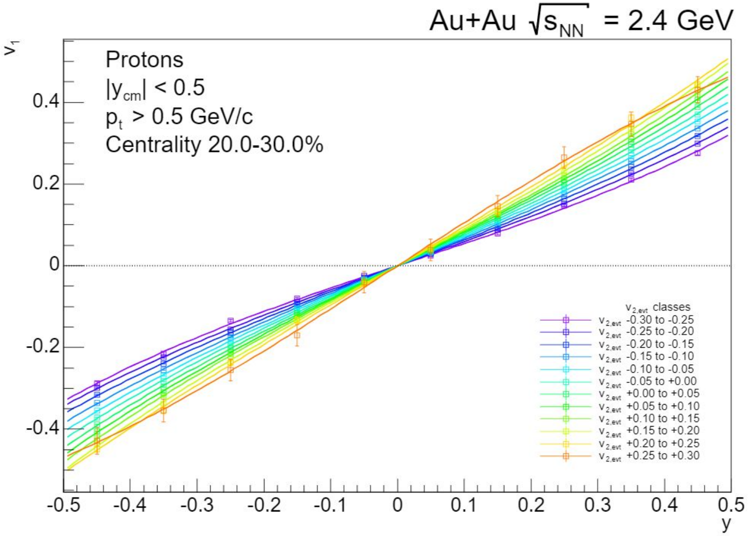
JAM $v_{2,event}$ distribution



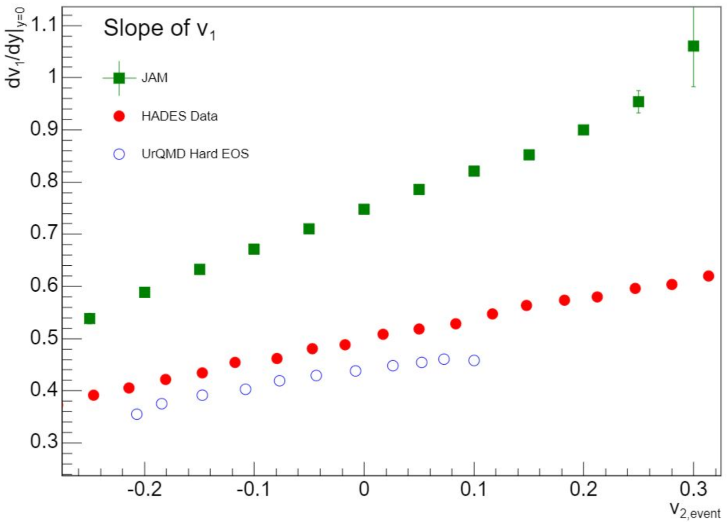
HADES $v_{2,event}$ distribution

B. Kardan, EMMI EOS Workshop II (2024).(URL)

$v_1/dy|_{y=0}$ vs $v_{2,event}$ classes



v_1 distribution for $v_{2,event}$ classes for JAM model

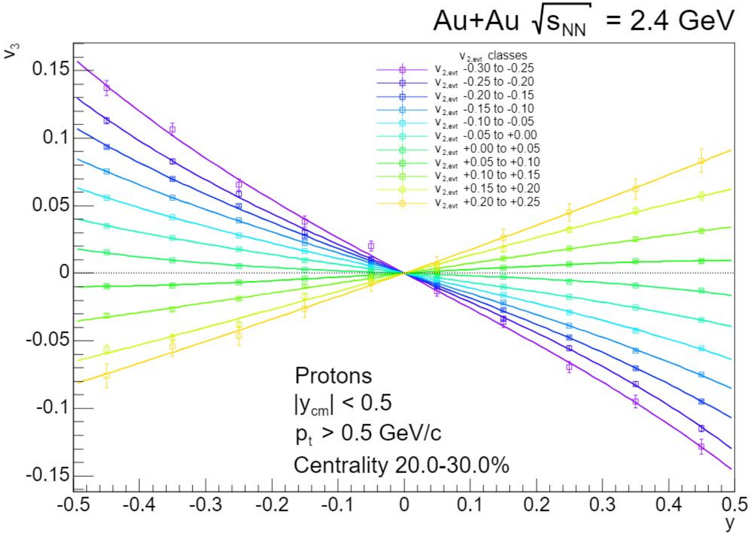


Slopes comparison for v_1 in $v_{2,event}$ classes

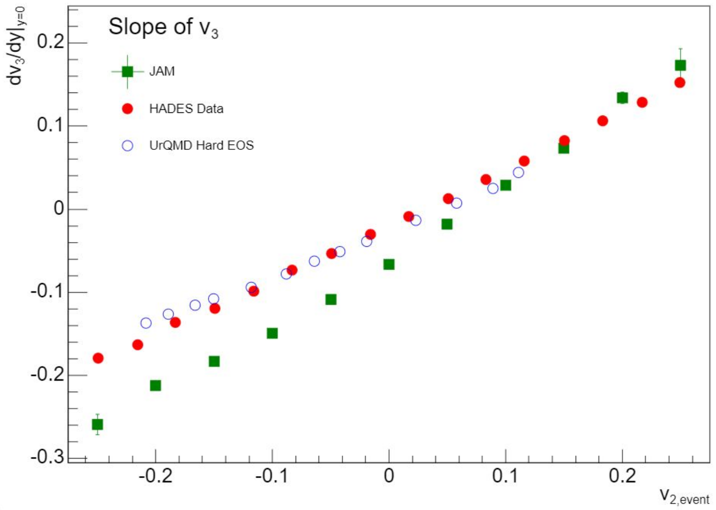
- For JAM the slope dependency of v_1 on $v_{2,event}$ is much **steeper**

B. Kardan, EMMI EOS Workshop II (2024).(URL)

$v_3/dy|_{y=0}$ vs $v_{2,event}$ classes



v_3 distribution for $v_{2,event}$ classes for JAM model

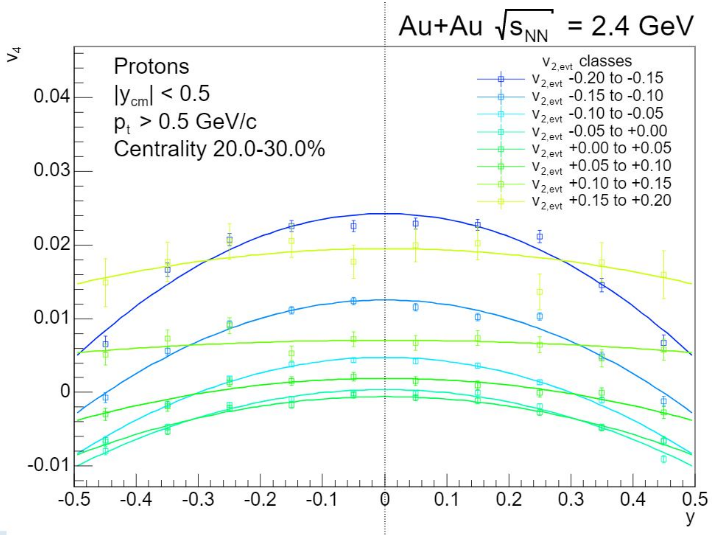


Slopes comparison for v_3 in $v_{2,event}$ classes

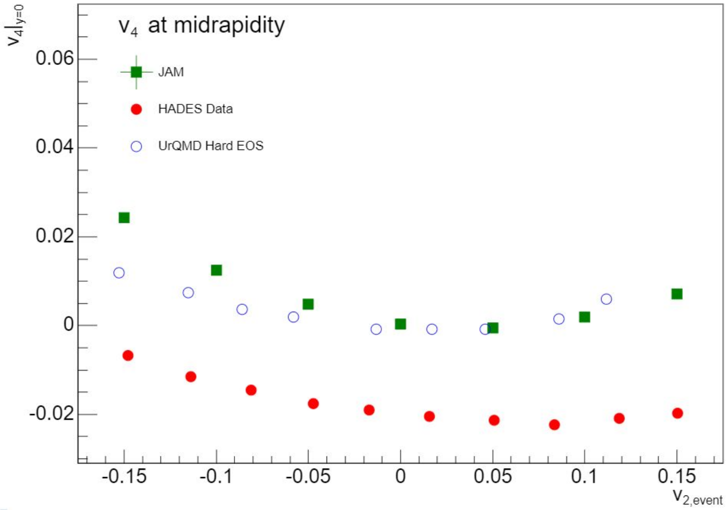
- $v_3/dy|_{y=0}$ dependency on v2,event is in **good agreement** with **UrQMD Hard EoS**

B. Kardan, EMMI EOS Workshop II (2024).(URL)

$v_4|_{y=0}$ vs $v_{2,event}$ classes



$v_4|_{y=0}$ distribution for $v_{2,event}$ classes for JAM model

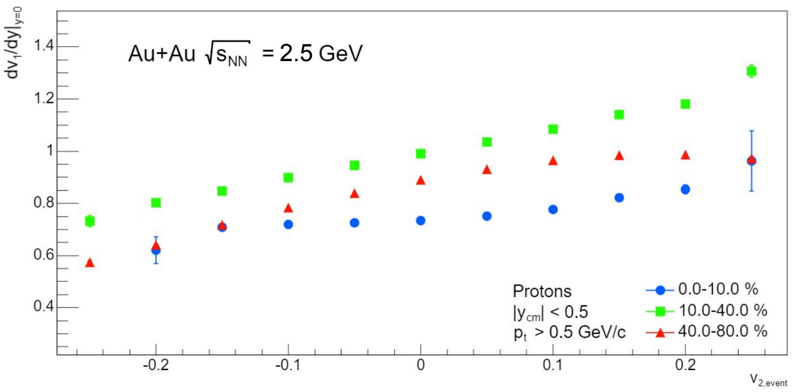


$v_4|_{y=0}$ comparison in $v_{2,event}$ classes

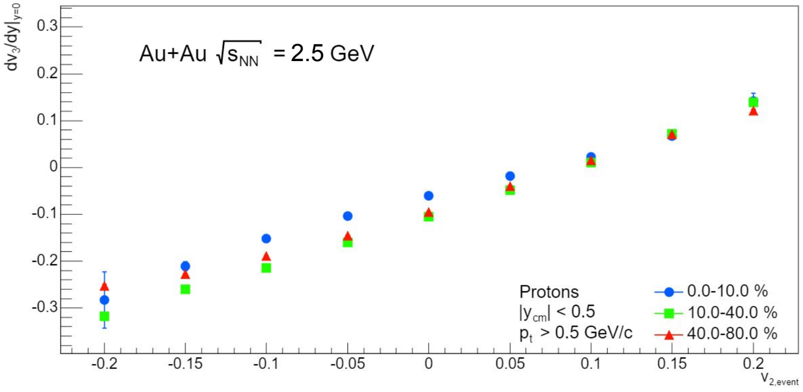
- $v_4|_{y=0}$ dependency of $v_{2,event}$ from JAM is **in agreement** for $v_{2,event}$ > -0.05 with **UrQMD Hard EoS**, however is **stronger than HADES**

B. Kardan, EMMI EOS Workshop II (2024).(URL)

Flow harmonics correlations in centrality classes



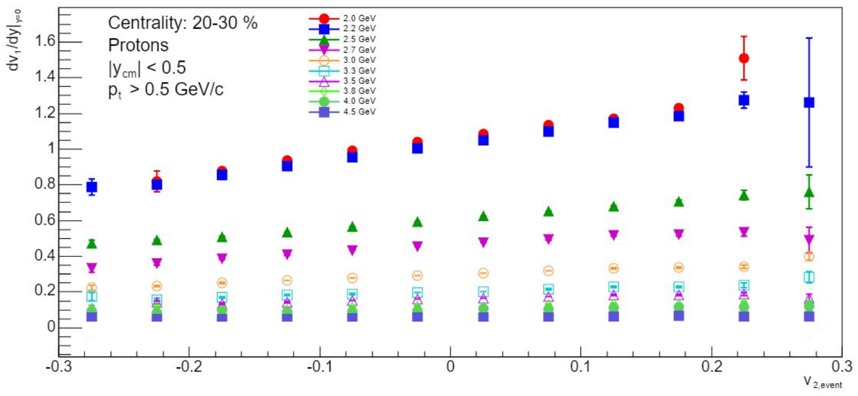
Slopes comparison for v_1 in $v_{2,event}$ classes in different centrality classes



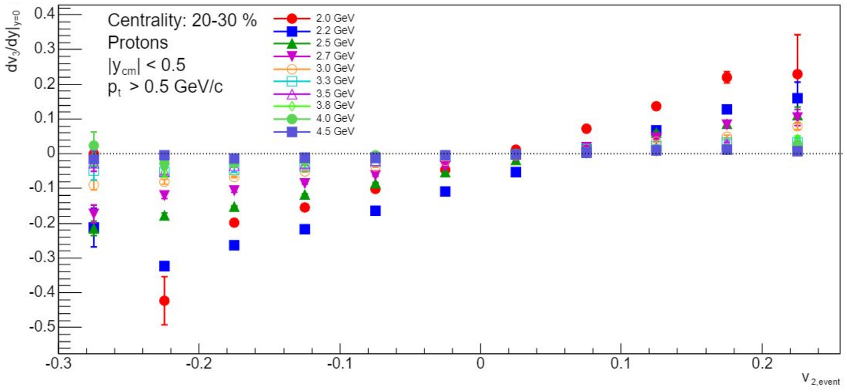
Slopes comparison for v_3 in $v_{2,event}$ classes in different centrality classes

- $dv_1/dy|_{y=0}$ has **stronger dependency** on **centrality** than the $dv_3/dy|_{y=0}$;
- $dv_1/dy|_{y=0}$ vs $v_{2,event}$ has the **strongest** correlation at **midcentral** (10-40%)

Flow harmonics correlations at different energies



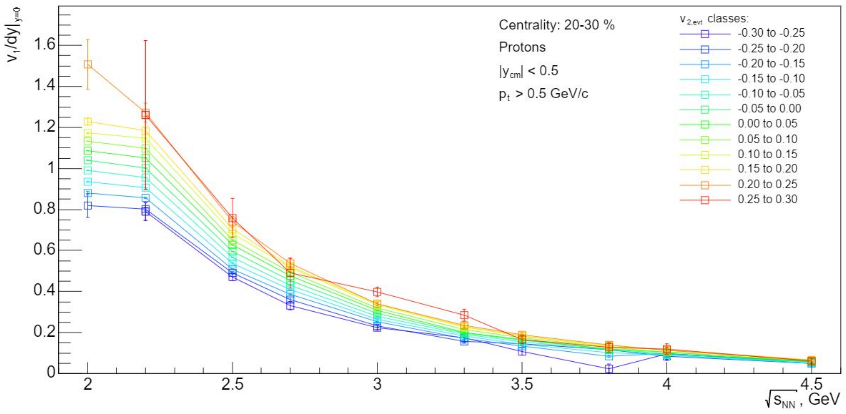
Slopes comparison for v_1 in $v_{2,event}$ classes at $\sqrt{s_{NN}}$ = 2.0 ÷ 4.5 GeV



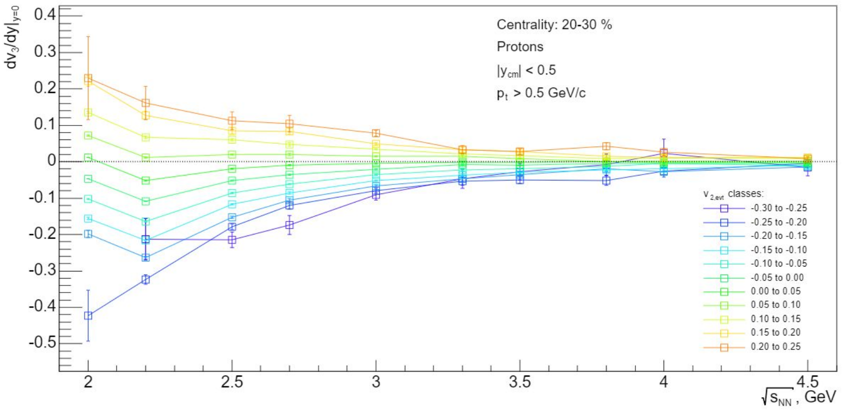
Slopes comparison for v_3 in $v_{2,event}$ classes at $\sqrt{s_{NN}}$ = 2.0 ÷ 4.5 GeV

- $dv_1/dy|_{y=0}$ **decreases** with the beam energy;
- The slope of v_1 and v_3 correlation with $v_{2,event}$ **strongly depends** on the **beam energy**;

Flow harmonics at different energies



Slopes comparison for v1 in v2,event classes
at $\sqrt{s_{NN}}$ = 2.0 ÷ 4.5 GeV



Slopes comparison for v3 in v2,event classes
at $\sqrt{s_{NN}}$ = 2.0 ÷ 4.5 GeV

- $dv_1/dy|_{y=0}$ seems to hit a **plateau** around $\sqrt{s_{NN}} \approx 2.2$ GeV;
- $dv_3/dy|_{y=0}$ is asymptotically **approaches 0** with **energy increase**.

Summary

- **Correlations of v_1, v_3, v_4 with $v_{2,event}$ were studied using JAM for Au+Au collisions at $\sqrt{s_{NN}}$ = 2.0 ÷ 4.5 GeV for hard EoS (MD2):**
  - Comparison with similar data from UrQMD and HADES shows that v_3 slope and v_4 at midrapidity in agreement with UrQMD hard EoS;
  - v_1 slope is steeper than in HADES and UrQMD results.
  - Stronger centrality dependence observed for v1 slope than for v3 slope.
- **Energy dependence:**
  - v_1 slope decreases with energy, v_3 slope approaches zero.
  - v_1 slope hits a plateau around $\sqrt{s_{NN}} \approx 2.2$ GeV (**coincidence?**)

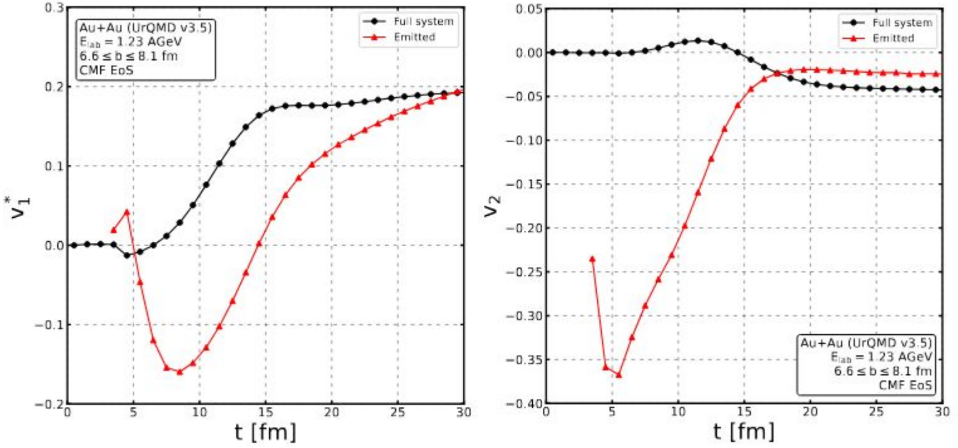
To do:

- Investigate event-wise correlation dependencies for different EoS and different models.
- Implementation of realistic v_n correlation measurements using reconstructed data at the MPD (feasibility study)

Backup Slides

Harmonic flow correlations

- v_1 and v_2 evolve rapidly with time → complex behavior
- Correlations (v_n–v_2) formed at early stage → more robust to late dynamics
- Harmonic flow correlations allows for more precise EoS extraction from data
- Such correlations are largely unexplored → motivation for model studies



The directed and elliptic flow coefficient flow coefficient in the whole phase space at time t and of the nucleons emitted at time t + Δt. Taken from [3]

[3] - Reichert, T., et al., Phys. Lett. B 841 (2023) 137947.

Bayesian inversion method (Gamma-fit)

- Charged particle multiplicity and impact parameter are related by probability distribution as:

$$P(N_{ch}|b) = \frac{1}{\Gamma(k)\theta^k} N_{ch}^{k-1} e^{-N_{ch}/\theta} \qquad [1,3]$$

- c_b – cumulative probability distribution written as: $c_b = \int_0^{\infty} P(b')db'$.
- Mean multiplicity for centrality class based on impact parameter:

$$\langle N_{ch} \rangle = N_{knee} \exp\left(\sum_{i=1}^{3} a_i (c_b)^i\right)$$

- 5 parameters: $N_{knee}, \theta, a_1, a_2, a_3$;
- Fit function for multiplicity distribution:

$$P(N_{ch}) = \int_0^1 P(N_{ch}|c_b)dc_b$$

- Impact parameter for given multiplicity range at certain centrality class:

$$P\left(b|N_{ch}^{low} < N_{ch} < N_{ch}^{high}\right) = p(b)\frac{\int_{N_{ch}^{low}}^{N_{ch}^{high}} P(N'_{ch}|b)dN'_{ch};}{\int_{N_{ch}^{low}}^{N_{ch}^{high}} P(N'_{ch})dN'_{ch};}$$

2 main steps of gamma-fit:

1) Fit multiplicity distribution from data with $P(N_{ch})$;
2) Construct impact parameter distribution using Bayes theorem.

Parfenov et al., Particles 4, 275 (2021). https://doi.org/10.3390/particles4020024