Charge sum rules for quark fragmentation functions as a new manifestation of superconformal symmetry between mesons and baryons.

D. Kotlorz and O.V. Teryaev

XXVI International Baldin Seminar on High Energy Physics Problems "Relativistic Nuclear Physics and Quantum Chromodynamics", Dubna, September 15–20, 2025

Abstract

We have demonstrated that the charge sum rules for the quark fragmentation functions (FFs) hold including simultaneously the contributions of mesons and baryons providing the conservation of the strangeness, electric and baryon charges. We also obtained the expression for the isospin conservation. The results are compatible to Gell-Mann-Nishijima formulas for quarks and hadrons manifesting a new aspect of quark-hadron duality.
Using our results, we formulated the constraints for the quark FFs to mesons π, K and baryons p, n, Λ. The numerical estimates based on some recent parametrizations of FFs confirm these constraints and also are in agreement with the charge and isospin sum rules.

D.K. and O.V. Teryaev, Phys. Rev. C 112, 035204 (2025)

Formalism

The basic concept underlying the theoretical analysis of most high energy interactions is **FACTORIZATION**.

The cross-section for large momentum-transfer reactions may be factorized into long-distance (nonperturbative) pieces that contain the desired information on the structure of the nucleon in terms of its parton densities such as PDFs and FFs, and short-distance (perturbative) parts which describe the hard interactions of the partons e.g. partonic cross section.

PDFs and FFs are universal functions and can be measured with global fit to experiments.

Formalism cont.

Inclusive production of single hadrons is described in terms of

Parton Distribution Functions (PDFs) q_i,

parton-parton interaction Cross Sections $\hat{\sigma}$ calculated in the Standard Model (SM) and Fragmentation Functions (FFs) D_i^h.

$pp \to hX$:

$$E^h \frac{d\sigma_{pp}^h}{d^3P^h} = \frac{1}{\pi} \sum_{ab\to cd} \int_{x_{a,min}}^1 dx_a \int_{x_{b,min}}^1 dx_b \, \frac{1}{z} \times$$
$$\times \left\{ q_a(x_a) q_b(x_b) \left[\frac{d\hat{\sigma}_{ab}^{cd}}{dt} D_c^h(z) + \frac{d\hat{\sigma}_{ab}^{cd}}{du} D_d^h(z) \right] \right.$$
$$\left. + q_a(x_b) q_b(x_a) \left[\frac{d\hat{\sigma}_{ab}^{cd}}{du} D_c^h(z) + \frac{d\hat{\sigma}_{ab}^{cd}}{dt} D_d^h(z) \right] \right\},$$

QCD evolution:

$$q(x) \to q(x, Q^2), \qquad D_q^h(z) \to D_q^h(z, Q^2).$$

Fragmentation Functions

Fragmentation: hadron production from a quark, antiquark, or gluon.

$D_i^h(z, Q^2)$ - fragmentation function of a parton i to hadron h.

It is probability to find the hadron h from a parton i with the energy fraction z.

Energy conservation (momentum sum rule):

$$\sum_h \int_0^1 dz\, z D_i^h(z, Q^2) = 1$$

$$h = \pi^+,\ \pi^0,\ \pi^-,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ p,\ \bar{p},\ \ldots$$

The momentum sum rule is true at all scales Q^2 and is a rigorous assumption used in most phenomenological extractions of FFs.

Charge sum rule

The charge sum rule for FFs expresses charge conservation:

$$Q_i = \sum_h Q_h \int_0^1 dz\, D_i^h(z, Q^2)$$

Q_i – the conserved electric charge of the parent quark of flavor i and the sum runs over all produced hadrons of charge Q_h.

Unlikely to the case of the momentum sum rule, where the suppression of small-z contributions occurs, the charge sum rule can be invalidated due to lack of the experimental data and also an adequate theoretical interpretation of the fragmentation functions in the range of $0 < z < z_{min} \sim 1/Q$.

Therefore, it is reasonable to consider the truncated contribution:

$$Q_i(Q^2) = \sum_h Q_h \int_{z_{min}(Q^2)}^1 dz\, D_i^h(z, Q^2)$$

Procedure

In much of the data for the charged hadron production, the observed hadrons are identified as one of the three lightest ones: the pions ($\pi^{\pm}$), kaons ($K^{\pm}$) and protons ($p/\bar{p}$).

Here, in a simple approach to the charge sum rule, we consider these particles by adding step by step the subsequent components.

Using only isospin SU(2) symmetry for the favored and unfavored fragmentation functions, and also the charge conjugation invariance of the strong interactions, we arrive at the generalized conservation law including charge, strangeness and baryon number.

We use the hadron multiplicity in the form

$$\langle D_q^h \rangle \equiv \int_0^1 dz\, D_q^h(z)$$

$$D_{qval}^{h^+}(z) = D_q^{h^+}(z) - D_{\bar{q}}^{h^+}(z) = D_q^{h^+}(z) - D_q^{h^-}(z)$$

Procedure cont.

The charged hadron production is dominated by charged pions, and this approximation was used long ago by A.V. Efremov and A.V. Radyushkin to perform the pioneering check of charge sum rule and get the u-quark charge from semi-inclusive deep inelastic neutrino data.

Therefore, at a first step, we consider only the pion contributions to the sums over h:

$$Q_u = \frac{2}{3} = \sum_{h=\pi^\pm} Q_h \int_0^1 dz\, D_u^h(z) = \int_0^1 dz\, D_{uval}^{\pi^+}(z)$$

$$Q_d = -\frac{1}{3} = \sum_{h=\pi^\pm} Q_h \int_0^1 dz\, D_d^h(z) = \int_0^1 dz\, D_{dval}^{\pi^+}(z).$$

Discrepancy

Assuming the isospin SU(2) symmetry for the favored and unfavored pion fragmentation functions ($\pi^+ = (u\bar{d})$, $\pi^- = (d\bar{u})$), we have

$$D^{\pi^+}_{dval}(z) = -D^{\pi^+}_{uval}(z)\,,$$

and hence we arrive at the incompatible relation for $Q_u + Q_d$:

$$\frac{1}{3} = 0$$

Adding to the charge sum rules for Q_u and Q_d also the kaon contributions does not remove this discrepancy.

Motivated to find some general law in terms of the fragmentation functions, we proceed our analysis by including also *baryons*.

In this way, we find that current sum rule simultaneously provide conservation of strangeness, electric and baryon charges.

Main result

The charge sum rule $Q_u + Q_d$:

$$\frac{1}{3} = \overbrace{\langle D_u^{K^+}\rangle - \langle D_u^{K^-}\rangle + \langle D_u^{K^0}\rangle - \langle D_u^{\bar{K}^0}\rangle + \langle D_u^{\bar{\Lambda}}\rangle - \langle D_u^{\Lambda}\rangle}^{S_u = 0}$$
$$+ \underbrace{\langle D_u^{p}\rangle - \langle D_u^{\bar{p}}\rangle + \langle D_u^{n}\rangle - \langle D_u^{\bar{n}}\rangle + \langle D_u^{\Lambda}\rangle - \langle D_u^{\bar{\Lambda}}\rangle}_{B_u = 1/3}$$

The isospin conservation $\frac{Q_u - Q_d}{2}$:

$$\begin{aligned}\frac{1}{2} &= \langle D_u^{\pi^+}\rangle - \langle D_u^{\pi^-}\rangle \\ &+ \frac{1}{2}\Big(\langle D_u^{K^+}\rangle - \langle D_u^{K^-}\rangle - \langle D_u^{K^0}\rangle + \langle D_u^{\bar{K}^0}\rangle \\ &+ \langle D_u^{p}\rangle - \langle D_u^{\bar{p}}\rangle - \langle D_u^{n}\rangle + \langle D_u^{\bar{n}}\rangle\Big)\end{aligned}$$

The obtained expressions are compatible to Gell-Mann–Nishijima formulas for quarks and hadrons. Crucial point: fragmentation to both mesons and baryons must be considered.

Constraints for $\langle D^h_{uval} \rangle$

Based on the concept of a common function for favored fragmentation functions from up and down quarks and on a flavor symmetry, we obtain constraints on $\langle D^h_{uval} \rangle$:

$$\langle D^{\pi^+}_{uval} \rangle = \frac{5}{12}\,, \qquad \langle D^{p}_{uval} \rangle = \frac{1}{6}\,,$$
$$\langle D^{K^+}_{uval} \rangle = \langle D^{n}_{uval} \rangle = \langle D^{\Lambda}_{uval} \rangle = \frac{1}{12}$$

To test consistency of our results with a phenomenological model of the fragmentation functions we compare the theoretical predictions on the charge and isospin sum rules, and also the constraints on $\langle D^h_{uval} \rangle$ with the numerical estimates of their truncated at z contributions based on some recent parametrizations of FFs.

Comparison to data

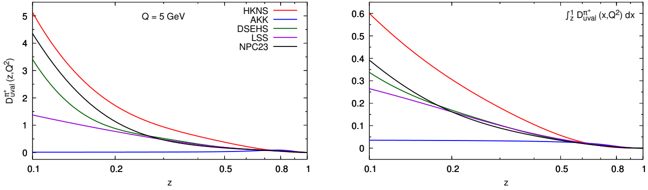

Figure: The valence fragmentation function $D^{\pi^+}_{uval}(z, Q^2)$ (left) and its contribution to the charge of the u-quark, $\int_z^1 D^{\pi^+}_{uval}(x, Q^2)\, dx$ (right), for different parametrization sets at $Q^2 = 25\,\mathrm{GeV}^2$.

HKNS-07, AKK-08, DSS-07, DSEHS-14, DEHSS-17, LSS-15, NPC-23, BS-03, DSV-97, SAK-20 FFs:

M. Hirai, S. Kumano, T. H. Nagai, and K. Sudoh, Phys. Rev. D 75, 094009 (2007)
S. Albino, B. A. Kniehl, and G. Kramer, Nucl. Phys. B 803, 42 (2008)
D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D 76, 074033 (2007)
D. de Florian, R. Sassot, M. Epele, R. J. Hernandez-Pinto, and M. Stratmann, Phys. Rev. D 91, 014035 (2015)
D. de Florian, M. Epele, R. J. Hernandez-Pinto, R. Sassot, and M. Stratmann, Phys. Rev. D 95, 094019 (2017)
E. Leader, A. V. Sidorov, and D. B. Stamenov, Phys. Rev. D 93, 074026 (2016)
J. Gao, C. Liu, X. Shen, H. Xing, and Y. Zhao, Phys. Rev. D 110, 114019 (2024)
C. Bourrely and J. Soffer, Phys. Rev. D 68, 014003 (2003)
D. de Florian, M. Stratmann, and W. Vogelsang, Phys. Rev. D 57, 5811 (1998)
M. Soleymaninia, H. Abdolmaleki, and H. Khanpour, Phys. Rev. D 102, 114029 (2020)

Comparison cont.

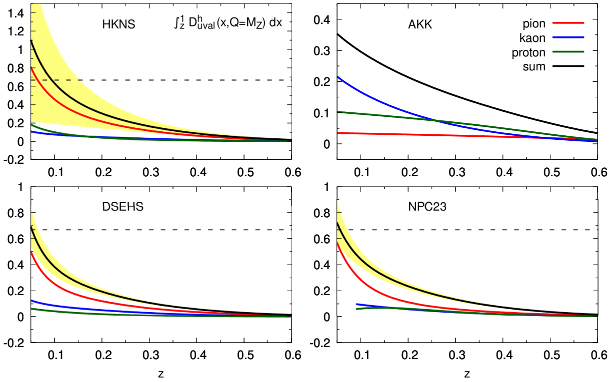

Figure: Contributions to the charge of the u-quark coming from the pion, kaon, proton, $\int_z^1 D^h_{uval}(x, Q^2)\,dx$, and their sum, for different FFs fits at $Q = M_Z$. The uncertainty band for the sum is shown.

Comparison cont.

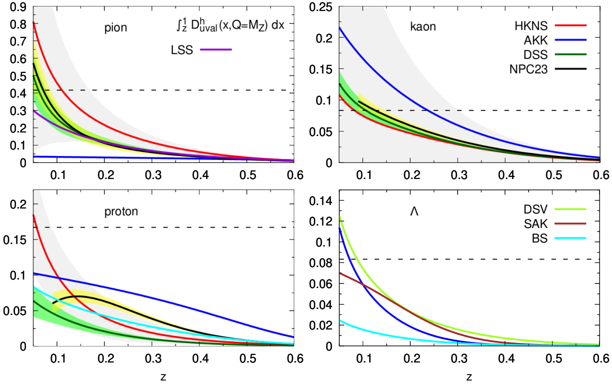

Figure: The truncated moments, $\int_z^1 D^h_{uval}(x, Q^2)\,dx$, for mesons (the pion and kaon) and baryons (the proton and Λ), calculated for different FFs sets, compared to the constraints for $\langle D^h_{uval} \rangle$. The uncertainty bands for HKNS, DSEHS and NPC23 are shown.

Comparison cont.

The isospin conservation:

$$I_3(u) \approx \langle D_{uval}^{\pi^+} \rangle + \frac{1}{2} \langle D_{uval}^{K^+} \rangle + \frac{1}{4} \langle D_{uval}^{p} \rangle$$

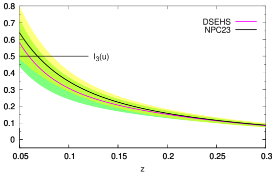

Figure: Truncated contributions to the isospin sum rule for the u-quark.

The experimental data based on some recent parametrizations of FFs are in agreement with the charge and isospin sum rules, and also confirm the constraints on $D_{uval}^{h}(z, Q^2)$.

Conclusions

- We demonstrated that the charge sum rules for the quark fragmentation functions hold including simultaneously the contributions of mesons and baryons providing the conservation of the strangeness, electric and baryon charges. We also obtained the expression for the isospin conservation. The results are compatible to Gell-Mann–Nishijima formulas for quarks and hadrons manifesting a new aspect of quark-hadron duality.
- Using our results, we formulated the constraints for $D^h_{uval}(z, Q^2)$, where h denotes mesons π, K and baryons p, n, Λ. The numerical estimates of the truncated moments based on some recent parametrizations of FFs confirm these constraints and also are in agreement with the charge and isospin sum rules.

"Be careful of success;
it has a dark side."

Robert Redford