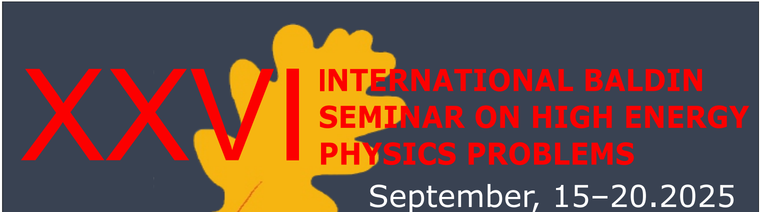
Phase diagram of QCD and two color QCD: dualities and inhomogeneous phases

Roman N. Zhokhov

IZMIRAN, IHEP

The XXVIth International Baldin Seminar on High Energy Physics Problems "Relativistic Nuclear Physics and Quantum Chromodynamics"



Group and Acknowlegements

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RSF | Russian Science Foundation

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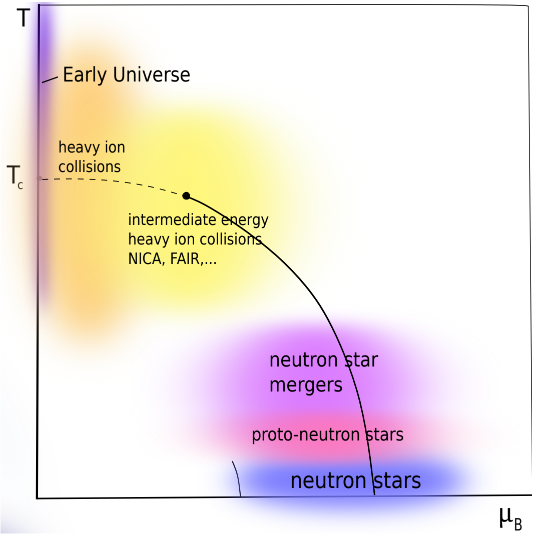
БАЗИС
Фонд развития теоретической физики и математики

QCD Dhase Diagram

QCD at T and μ

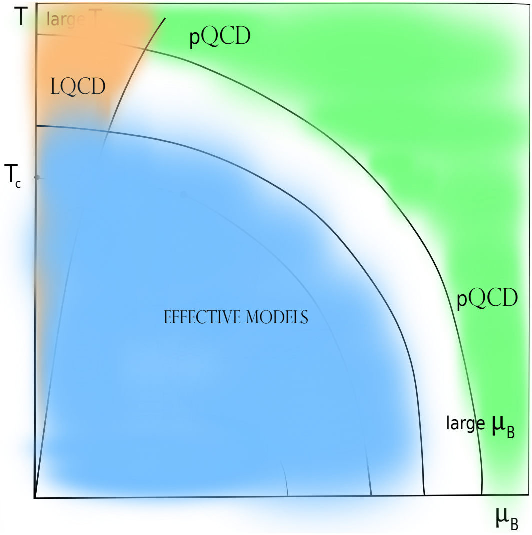
(QCD at extreme conditions)

- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers

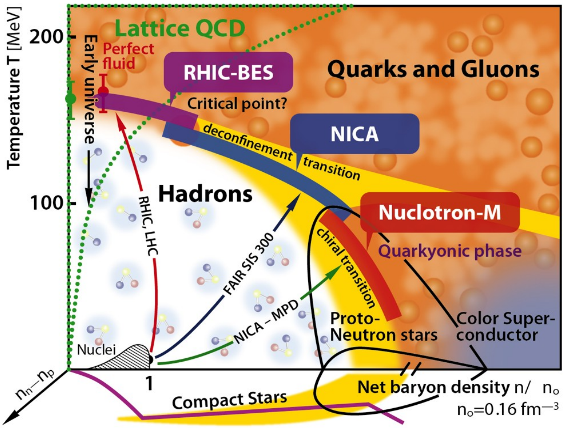


Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- Gauge/Gravity duality
- .....



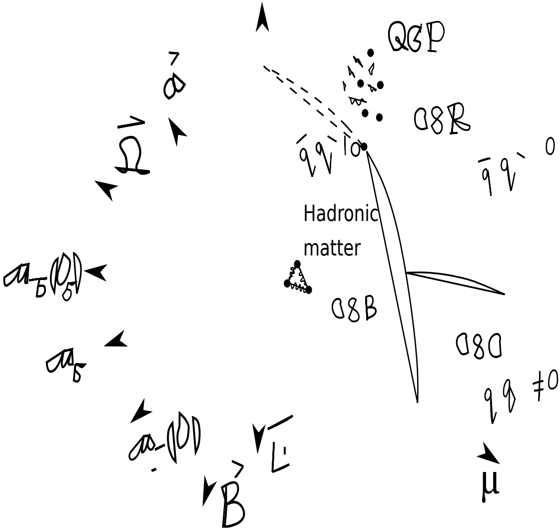
QCD Phase Diagram



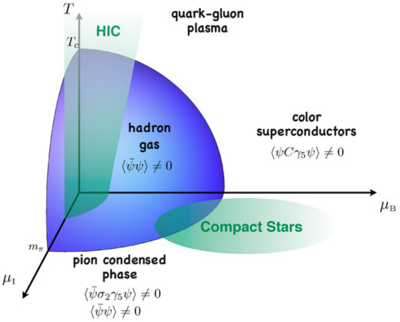
More external conditions to QCD

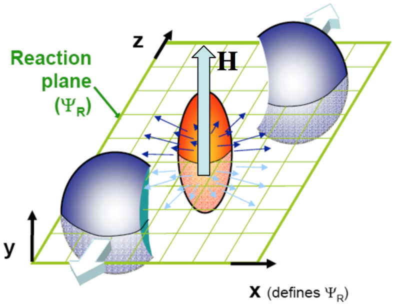
More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



Chemical potentials





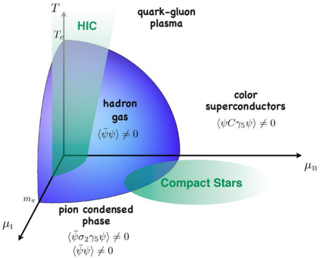
$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

Chemical potentials

- **Isotopic chemical potential** μ_I

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\,(\bar{q}\gamma^0\tau_3 q), \qquad n_I = n_u - n_d$$

Neutron stars, intermediate energy heavy-ion collisions, neutron star mergers



- **Chiral (axial) chemical potential** μ_5

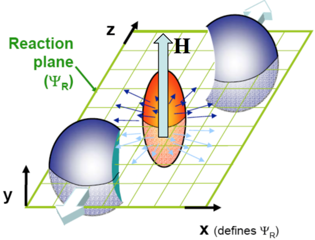
$$\mu_5\,\bar{q}\gamma^0\gamma^5 q, \qquad n_5 = n_R - n_L, \;\; \mu_5 = \mu_R - \mu_L$$

- **Chiral isospin chemical potential** μ_{I5}

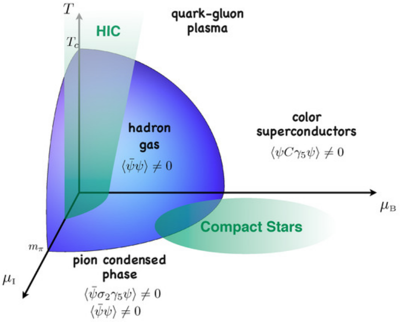
$$\mu_5^u \neq \mu_5^d \qquad\qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

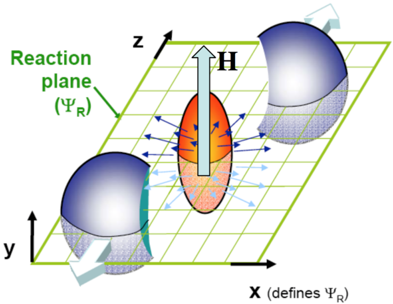
$$\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$$

$$n_{I5} = n_{u5} - n_{d5}, \qquad\qquad n_{I5} \quad \longleftrightarrow \quad \nu_5$$



Chemical potentials





$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

- **Recalling the dualities of phase diagram**
- **Dualities in QCD and QC_2D from first principles**
- **Wide swathes of application of dual**
  - **Speed of sound in quark matter with different properties**
  - **Inhomogeneous phases**

Recall that in NJL model **in** $1/N_c$ **approximation or in the mean field** there have been found **dualities**

(*It is not related to holography or gauge/gravity duality*)

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

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The TDP

$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...)$ $\Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

- A lot of densities and imbalances
  baryon, isospin, chiral, chiral isospin imbalances

- Finite temperature $T \neq 0$

- Physical pion mass $m_\pi \approx 140$ MeV

- Inhomogeneous phases (case)

$$\langle \sigma(x) \rangle = M(x), \quad \langle \pi_\pm(x) \rangle = \pi(x), \quad \langle \pi_3(x) \rangle = 0.$$

- Inclusion of color superconductivity phenomenon

Dualities in QC_2D

Similarity of SU(2) and SU(3)

- similar phase transitions:
  confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of physical quantities coincide with some accuracy
  Critical temperature, shear viscosity etc.
- There is **no sign problem** in SU(2) case and lattice simulations at non-zero baryon density are possible — $(Det(D(\mu)))^\dagger = Det(D(\mu))$

It is a great playground for studying dense matter

$$\sigma(x) = -2H(\bar{q}q), \qquad \Delta(x) = -2H\left[\overline{q^c} i\gamma^5 \sigma_2 \tau_2 q\right]$$

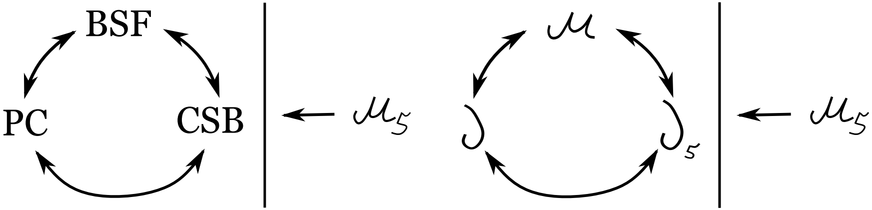
$$\vec{\pi}(x) = -2H(\bar{q}i\gamma^5\vec{\tau}q), \qquad \Delta^*(x) = -2H\left[\bar{q}i\gamma^5\sigma_2\tau_2 q^c\right]$$

Condensates and phases

$M = \langle\sigma(x)\rangle \sim \langle\bar{q}q\rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle\pi_1(x)\rangle = \langle\bar{q}\gamma^5\tau_1 q\rangle,$ PC phase: $\pi_1 \neq 0,$

$\Delta = \langle\Delta(x)\rangle = \langle qq\rangle = \langle q^T C\gamma^5\sigma_2\tau_2 q\rangle,$ BSF phase: $\Delta \neq 0.$



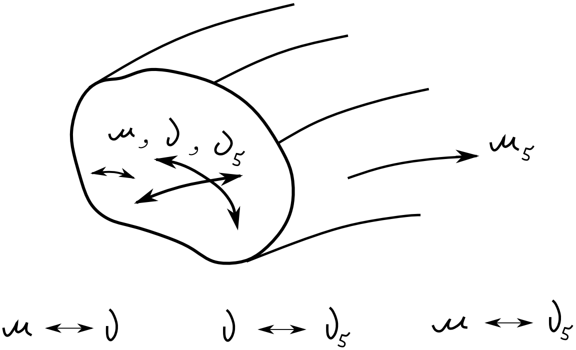
(I) $\mathcal{D}_1 :$ $\mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow \Delta,$ PC $\longleftrightarrow$ BSF

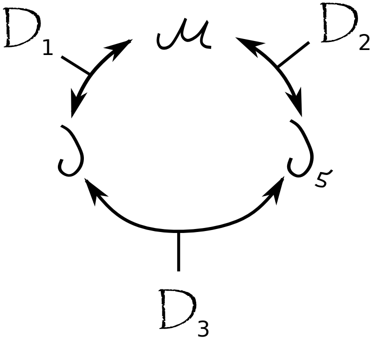
(II) $\mathcal{D}_3 :$ $\nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1,$ PC $\longleftrightarrow$ CSB

(III) $\mathcal{D}_2 :$ $\mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \Delta,$ CSB $\longleftrightarrow$ BSF

The phase diagram of (μ, ν, μ_5, ν_5)

The phase diagram is foliation of dually connected cross-section of (μ, ν, ν_5) along the μ_5 direction





$D_1: \ \mu \longleftrightarrow \nu$

$D_2: \ \mu \longleftrightarrow \nu_5$

$D_3: \ \nu \longleftrightarrow \nu_5$

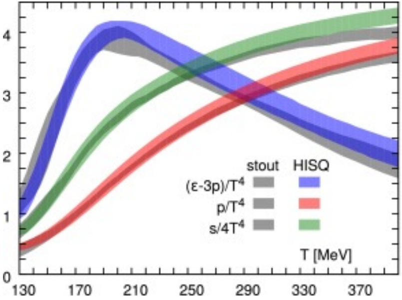
From first principles in QC_2D

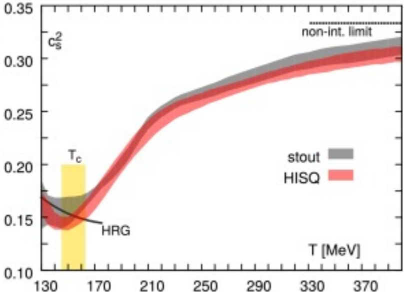
$$\mathcal{D}_{\mathrm{II}}: \qquad \langle\bar{\psi}\psi\rangle \longleftrightarrow \langle i\bar{\psi}\gamma^5\tau_1\psi\rangle, \quad M \longleftrightarrow \pi, \quad \nu \leftrightarrow \nu_5$$

From first principles

Speed of sound c_s^2

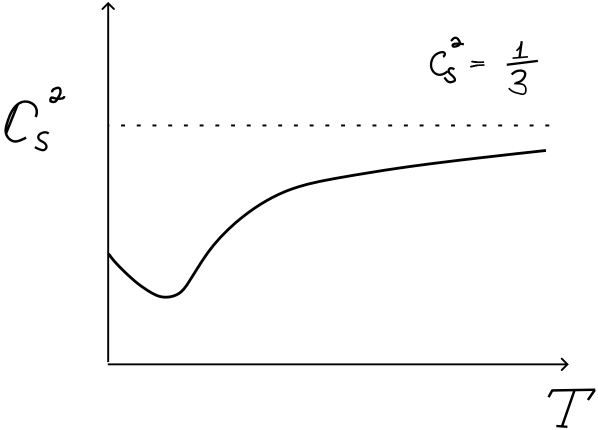
Thermodynamic properties could be calculated in lattice QCD





A. Bazavov et al. [HotQCD], Phys. Rev. D ***90*** *(2014), 094503*

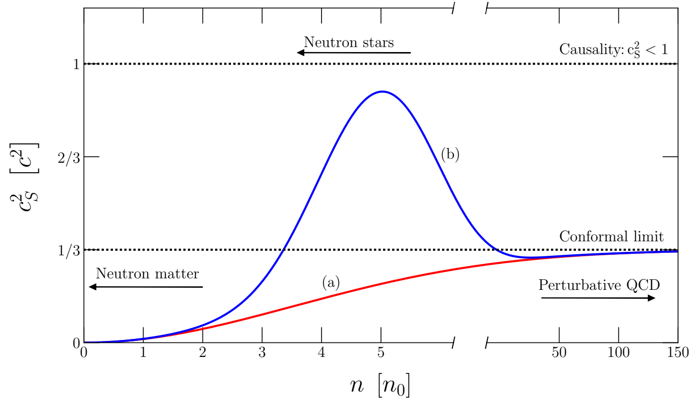
Sound speed in QCD at finite T: skematic



There was discussed bound from holography

A. Cherman, T. D. Cohen and A. Nellore, Phys. Rev. D 80 (2009), 066003

Two possible scenario of speed of sound at non-zero baryon density



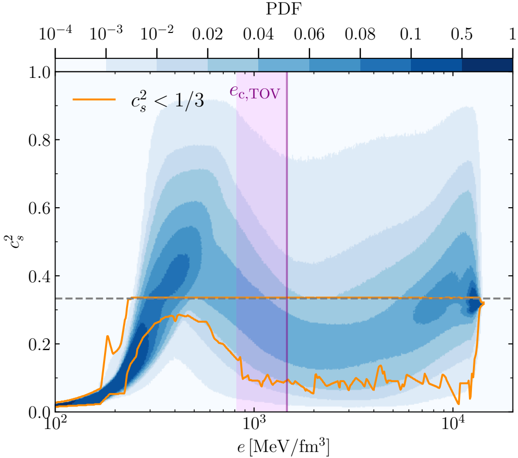
*taken from S. Reddy et al, Astrophys. J. **860** (2018) no.2, 149*

Sound speed in QCD with non-zero baryon density

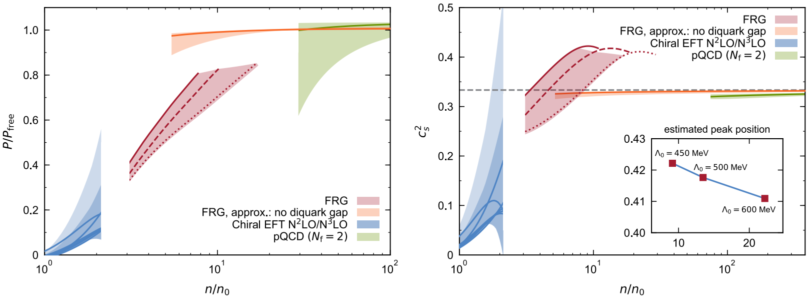
EOS with continuous c_s^2 consistent not only with nuclear theory and perturbative QCD, but also with astrophysical observations.

EOS with sub-conformal sound speeds, i.e.,$c_s^2 < 1/3$ are **possible in principle but very unlikely in practice**

L. Rezzolla et al, Astrophys.J.Lett. 939 (2022) 2, L34



Sound speed in FRG approach



- **Sound speed** squared has been obtained from **FRG approach**

Phys.Rev.Lett. 125 (2020) 14, 142502

$$Z = \int D[\text{gluons}]\, D[\text{quarks}]\, e^{-S^E_{QCD}}$$

$$Z = \int D[\text{gluons}]\, Det\, \mathbb{D}(\mu)\, e^{-S^E_{\text{gluons}}}$$

It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**
complex determinant

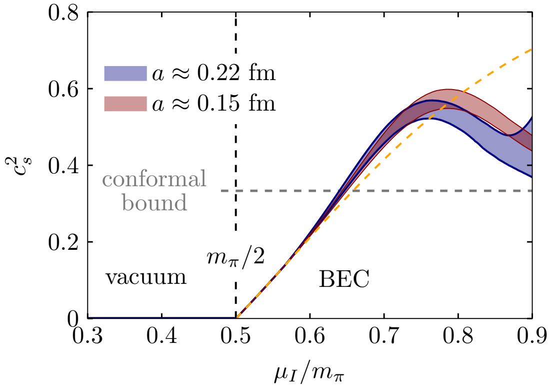
$$Det(D(\mu))^\dagger = Det(D(-\mu))$$

For isospin chemical potential μ_I

$$Det(D(\mu_I))^\dagger = Det(D(\mu_I))$$

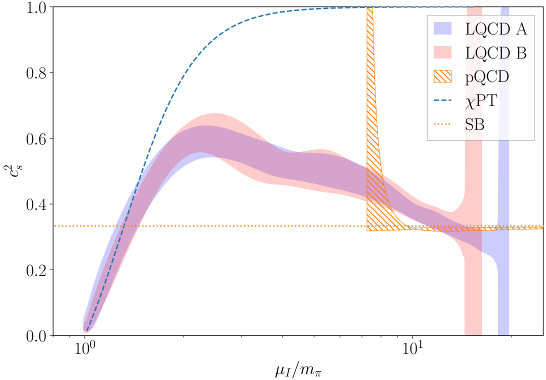
- **Sound speed** squared has been obtained from **lattice QCD simulations** for **QCD with non-zero isospin** μ_I

  B. B. Brandt, F. Cuteri and G. Endrodi, JHEP 07, 055 (2023)



- **Sound speed** squared has been obtained from **lattice QCD simulations** for **QCD with non-zero isospin** μ_I for values of μ_I up to $10 m_\pi$

R. Abbott et al. [NPLQCD], Phys. Rev. D 108, no.11, 114506 (2023)



Duality between chiral symmetry breaking and pion condensation

$$\mathcal{D}: \ M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

The TDP of the quark matter

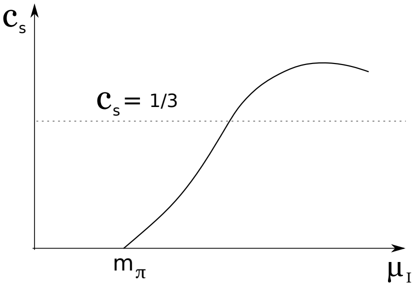
$$\Omega(T,\, \mu,\, \nu,\, \nu_5,\, \mu_5,\, |\, M,\, \pi) = \mathrm{inv}$$

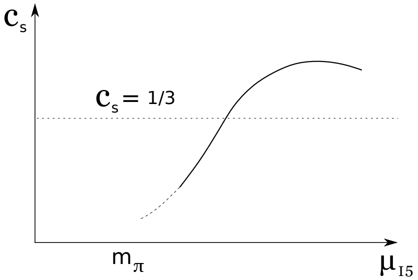
The speed of sound $\quad c_s^2 = \dfrac{dp}{d\epsilon}$

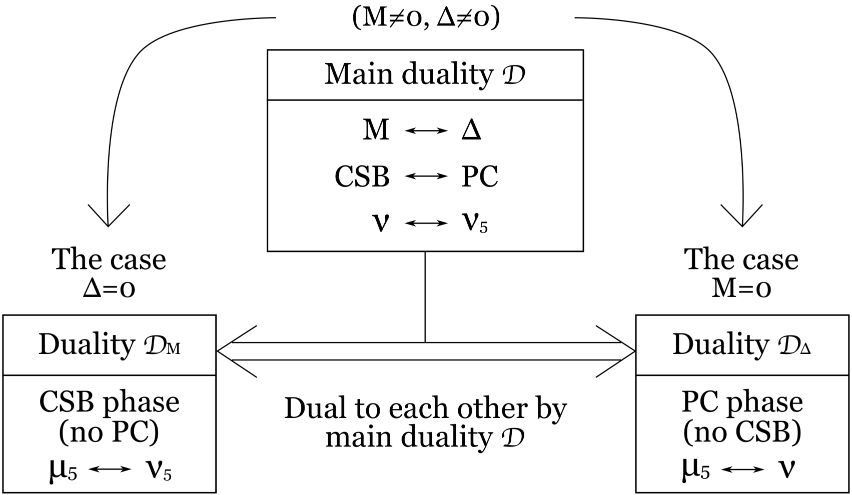
$$\Omega(T, ...) \Longrightarrow c_s^2(T, ...)$$

The speed of sound $c_s^2 = \dfrac{dp}{d\epsilon}$, $\Omega(T, ...) \Longrightarrow c_s^2(T, ...)$

$$\Omega(T, ..., \nu) = \Omega(T, ..., \nu_5) \Longrightarrow c_s^2(T, ..., \nu) = c_s^2(T, ..., \nu_5)$$



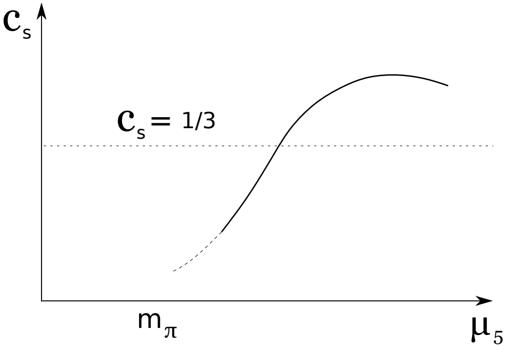



(M≠0, Δ≠0)
Main duality $\mathcal{D}$
M ⟷ Δ
CSB ⟷ PC
$\nu \longleftrightarrow \nu_5$
The case Δ=0
The case M=0
Duality $\mathcal{D}_M$
CSB phase (no PC)
$\mu_5 \longleftrightarrow \nu_5$
Dual to each other by main duality $\mathcal{D}$
Duality $\mathcal{D}_\Delta$
PC phase (no CSB)
$\mu_5 \longleftrightarrow \nu$

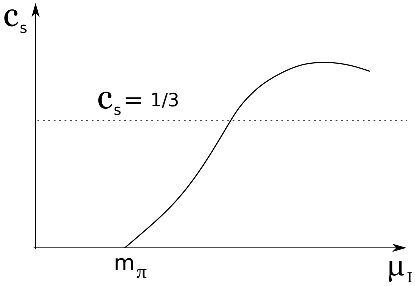
Duality

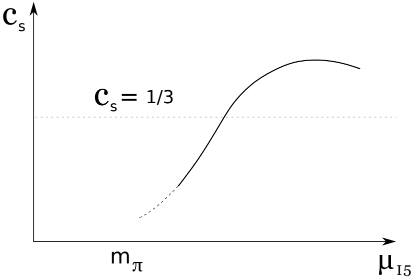
$$\nu_5 \longleftrightarrow \mu_5, \qquad M \neq 0, \quad \langle \pi \rangle = \langle \Delta \rangle = 0$$

- **Sound speed** squared for **QCD with non-zero chiral imbalance** μ_5 **only in the framework of effective model**



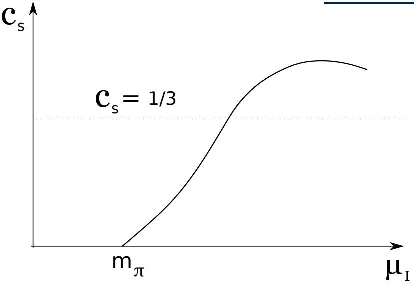

C_s^2
$C_s=\frac{1}{3}$
μ

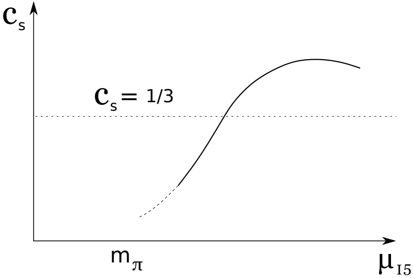

c_s
$c_s = 1/3$
m_π
μ_I

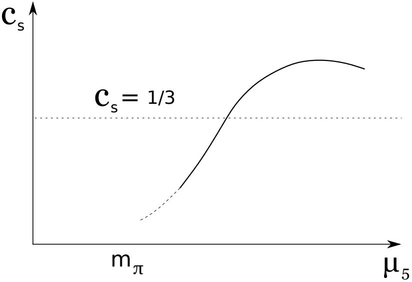

c_s
$c_s = 1/3$
m_π
μ_{I5}


C_s^2
$C_s=\frac{1}{3}$

Speed of sound in QCD: Effective models





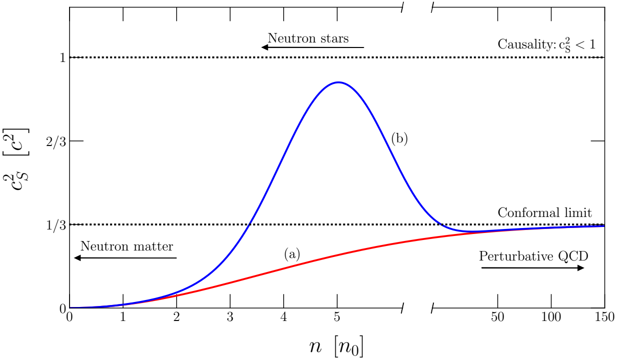


Even more results

for sound speed at various densities

for two color QCD


c_s^2
$c_s = \frac{1}{3}$
μ


Neutron stars
Causality: $c_S^2 < 1$
(b)
Conformal limit
Neutron matter
(a)
Perturbative QCD
$c_S^2\ [c^2]$
$n\ [n_0]$
0
1/3
2/3
1
0
1
2
3
4
5
50
100
150

Inhomogeneous phases in QCD and QC_2D

It is open question if there is inhomogeneous chiral symmetry breaking phase at $\mu_B \neq 0$

As usual condensates i. e. order parameters, $\langle \bar{q}q \rangle$ or ($\langle \bar{q}\gamma_5\vec{\tau}q \rangle$ and $\langle qq \rangle$) are assumed to be spatially constant, e. g.

$$\langle \bar{q}q \rangle = M = \mathbf{const}$$

But at $\mu_B \neq 0$ it was found that inhomogeneous phase $\langle \bar{q}q \rangle$ could take place

$$\langle \bar{q}q \rangle = M(x)$$

Inhomogeneous phases in QCD

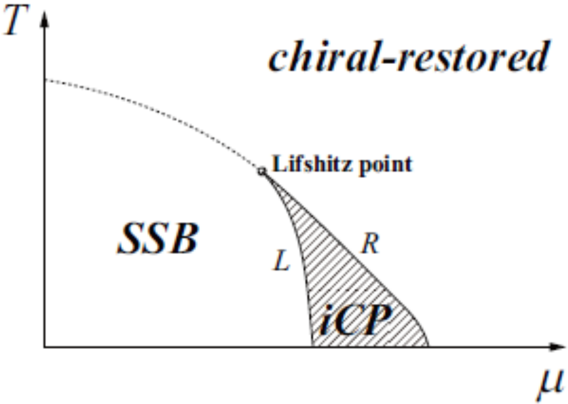
- 1960-s:
  - spin-density waves in nuclear matter (*Overhauser)*)
  - Crystalline superconductors (*Fulde, Ferrell, Larkin, Ovchinnikov*)
- 1970 – 1990-s:
  - p-wave pion condensation (*Migdal*)
  - chiral density wave (*Dautry, Nyman*)
- 2000-s:
  - (1+1)-dimensional Gross-Neveu model (*M. Thies et al.*)
  - Crystalline color superconductors (*M. Alford, J. Bowers, K. Rajagopal*) *arXiv:hep-ph/0008208*
  - Lattice studies on (1+1)-, (2+1)- and (3+1)-dimensional (*M. Winstel, M. Wagner, A. Wipf et al.*)
  - First indication of inhomogeneous phases in framework of effective models (*D. Nickel, 2009*)

Chiral density wave ansatz

$$\langle \bar{q}q \rangle = M \cos(\vec{q}\,\vec{x}), \qquad \langle \bar{q}\gamma_5\tau_3 q \rangle = M \cos(\vec{q}\,\vec{x})$$

At $\mu_B \neq 0$ there is a phase with non-zero $M \neq 0$ and $q \neq 0$, i. e. inhomogeneous phase

$$\langle \bar{q}q \rangle \sim \cos(\vec{q}\,\vec{x})$$



$$\langle \bar{q}q \rangle \sim M(x)$$

- **Inhomogeneous phase was predicted in: (1+1)-dimensional Gross-Neveu (GN) model**

  M. Thies, ...

  A. Wipf, M. Wagner, M. Winstel, L. Pannullo etc.

- **Inhomogeneous phase in (3+1)-dimensional effective models**

- **Inhomogeneous phase in effective models: dependence on the chosen regularization scheme**

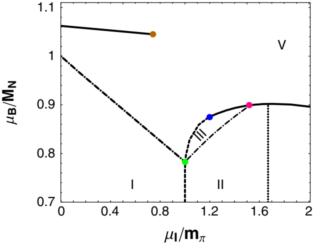
  M. Wagner et al, Phys. Rev. D 110 (2024) 7, 076006

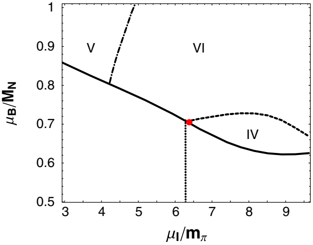
- **Inhomogeneous phase shown in functional approach**

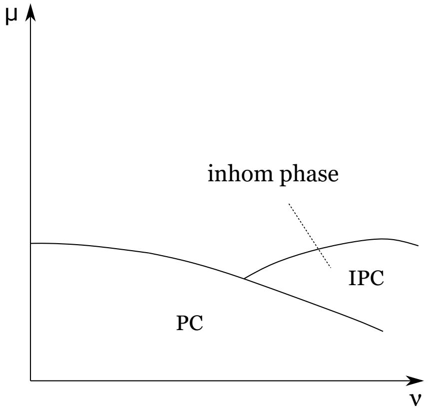
  C. Fischer et al, Phys. Rev. D 108 (2023) 11, 114019, Phys.Rev.D 110 (2024) 7, 074014

Almost no results on inhomogeneous phases in two color case, in QC_2D

Inhomogeneous PC at μ_I: skematic

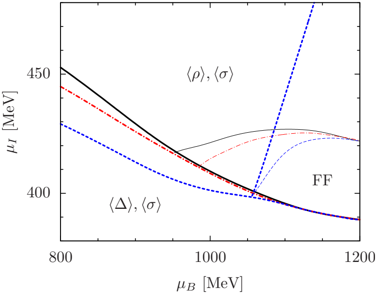


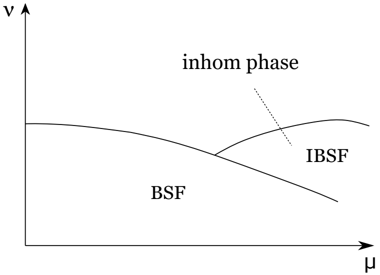




Lianyi He et al, Phys.Rev.D 82 (2010) 056006

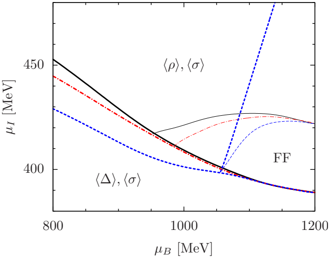
Inhomogeneous diquark condensation found in two color case in the framework of effective models

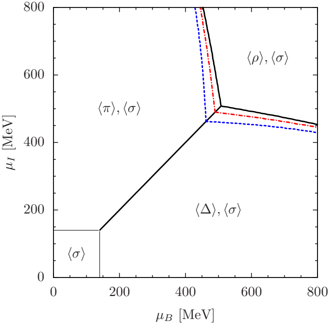


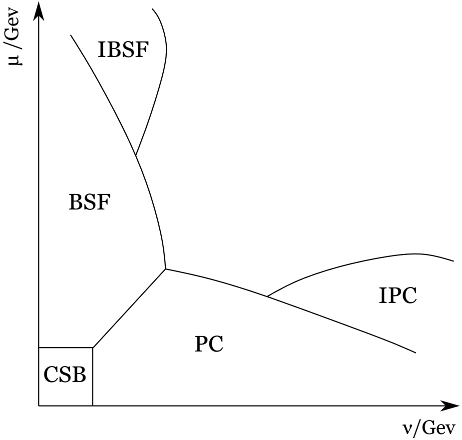


J. Andersen et al Phys.Rev.D 81 (2010) 096004

Inhomogeneous phase at μ_I: skematic





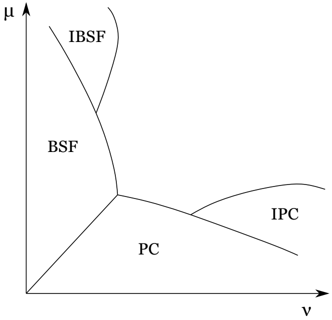


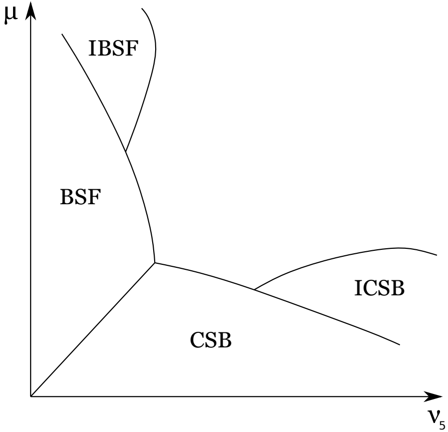
$$\mu \longleftrightarrow \nu, \qquad \pi_1 \longleftrightarrow \Delta$$

$$\text{PC} \longleftrightarrow \text{BSF}, \qquad \text{IPC} \longleftrightarrow \text{IBSF}$$

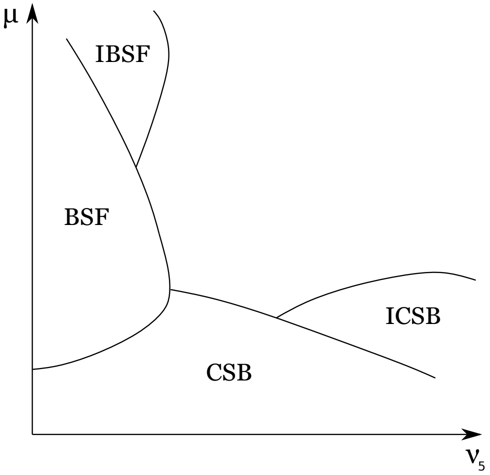
Inhomogeneous phase: skematic

$$\nu \longleftrightarrow \nu_5, \qquad M \longleftrightarrow \pi_1, \qquad \text{CSB} \longleftrightarrow \text{PC}, \qquad \text{ICSB} \longleftrightarrow \text{IPC}$$





Inhomogeneous phase: skematic



$$\mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \Delta, \qquad \text{CSB} \longleftrightarrow \text{BSF}, \qquad \text{ICSB} \longleftrightarrow \text{IBSF}$$

Inhomogeneous phases

Homogeneous case

$$\langle \sigma(x) \rangle = M, \quad \langle \pi_{\pm}(x) \rangle = \pi, \quad \langle \pi_3(x) \rangle = 0.$$

Inhomogeneous phases (three color case)

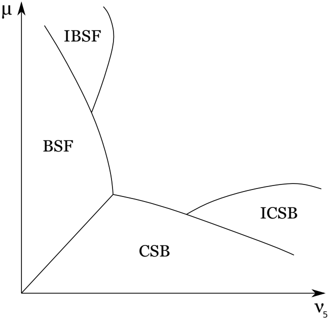
$$\langle \sigma(x) \rangle = M(x), \quad \langle \pi_{\pm}(x) \rangle = \pi(x), \quad \langle \pi_3(x) \rangle = 0.$$

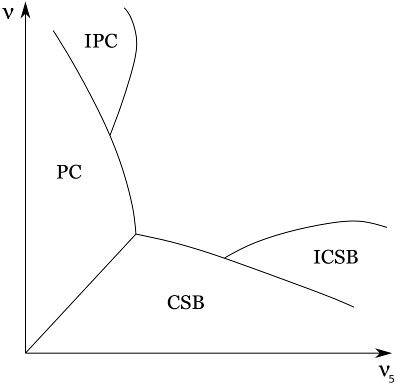
$$\mathcal{D}: \; M(x) \longleftrightarrow \pi(x), \quad \nu \longleftrightarrow \nu_5$$

$$\text{ICSB} \longleftrightarrow \text{IPC} \quad \nu \longleftrightarrow \nu_5$$

Inhomogeneous phase: skematic

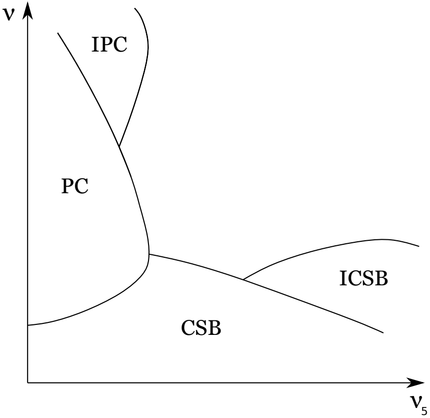
$$\mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow \Delta, \quad \text{PC} \longleftrightarrow \text{BSF}, \quad \text{IPC} \longleftrightarrow \text{IBSF}$$





Inhomogeneous phases exist usually at $\mu_B \neq 0$

Inhomogeneous phase in two color case exist at $\mu_B = 0$



$$\nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \quad \text{CSB} \longleftrightarrow \text{PC}, \quad \text{ICSB} \longleftrightarrow \text{IPC}$$

Dualities has been proven from first principles

Speed of sound exceeding the conformal limit is rather **natural** and taking place in a lot of systems, **with various chemical potentials**

And it is natural if it has similar structure in QCD at non-zero baryon density, the most interesting case

Inhomogeneous phases in two and three color case have been studied, in two color case exist at $\mu_B = 0$