Analysis of the running coupling constant $\alpha_S(q^2)$ of π^- mesons and protons from $\pi^- + C$ interactions at 40 GeV/c and $p + p$ interactions at 205 GeV/c

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Plan of the talk

1. Introduction
2. Experimental method
3. Cumulative number distributions of π^- mesons and protons from $p + p \rightarrow \pi^-, p + X$ at 205 GeV/c and $\pi^- + C \rightarrow \pi^-, p + X$ at 40 GeV/c
4. The square of the transferred momentum and the running coupling constants $\alpha_S(q^2)$
5. $\alpha_S(q^2)$ distributions of π^- mesons and protons from above mentioned reactions and comparison with QCD predictions
6. Conclusion

1. Introduction

Investigations of the multiparticle production processes in hadron-nucleon (hN) hadron-nucleus (hA) and nucleus-nucleus (AA) interaction at high energies and large momentum transfers play a very important role in understanding the strong interaction mechanism and the inner quark-gluon structure of nuclear matter. It is well-known experimental fact that in hA and AA interactions at high energies the secondary particles are produced in the region kinematically forbidden for hN interactions. These particles are the cumulative particles. Thus, study of the cumulative particle production process allows to examine the features of the nuclear matter under extreme conditions.

In multiparticle production processes at high energies the secondary particles are produced at different scattering angles with different values of momenta. Values of the scattering angles, momentum and running coupling constants are mainly described by square of the four-dimensional transferred momentum q^2. In other words, q^2 is one of the variables sensitive to the mechanism of the multiparticle production processes.

2. Experimental method

The experimental material was obtained using the Dubna 2-meter propane (C3H8) bubble chamber exposed to π^- mesons with a momentum of 40 GeV/c from the Serpukhov accelerator. The advantage of the bubble chamber experiment in this paper is that the distributions are obtained under the condition of 4π geometry of secondary particles. The average error of the momentum measurements is $\sim$ 12% and the average error of the angular measurements is $\sim$ 0.6%. The 8791 $\pi^- + C$ events with 30162 π^- mesons and 12441 protons at 40 GeV/c have been used in the experimental analysis.

Data on $p + p$ interactions at 205 GeV/c were obtained with the help of the 30-inch hydrogen bubble chamber at NAL (National Accelerator Laboratory). The 5025 $p + p$ events with 12095 π^- mesons and 2086 protons at 205 GeV/c have been used in the experimental analysis.

Cumulative number n_c

The variable called the cumulative number in the fixed target experiment, is determined by the following formula:

$$n_c = \frac{P_a \cdot P_c}{P_a \cdot P_b} = \frac{E_c - \beta_a P_c^{||}}{m_p}, \qquad n_c m_p = E_c - \beta_a P_c^{||}$$

Where P_a, P_b and P_c are the four-dimensional momenta of the incident, target and secondary particles under consideration. E_c and $P_c^{||}$ are the energy and longitudinal momentum of the secondary particle, respectively, and $\beta_a = P_a/E_a$ is the velocity of the incident particle. At high energy experiments β_a=1 and m_p is the proton mass.

It can be seen from this formula that the variable n_c is a relativistic invariant and dimensionless. Furthermore, this variable provides an opportunity to know which particles in the event under consideration, are produced in the cumulative region (n_c > 1) and vice versa.

3. Cumulative number distributions of π^- mesons and protons from $p + p \rightarrow \pi^-, p + X$ at 205 GeV/c and $\pi^- + C \rightarrow \pi^-, p + X$ at 40 GeV/c

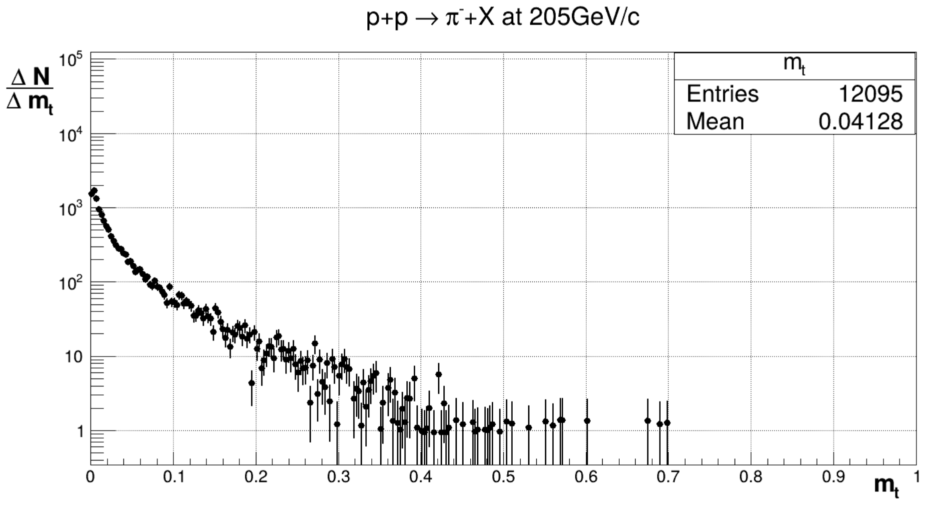


Figure 1 a

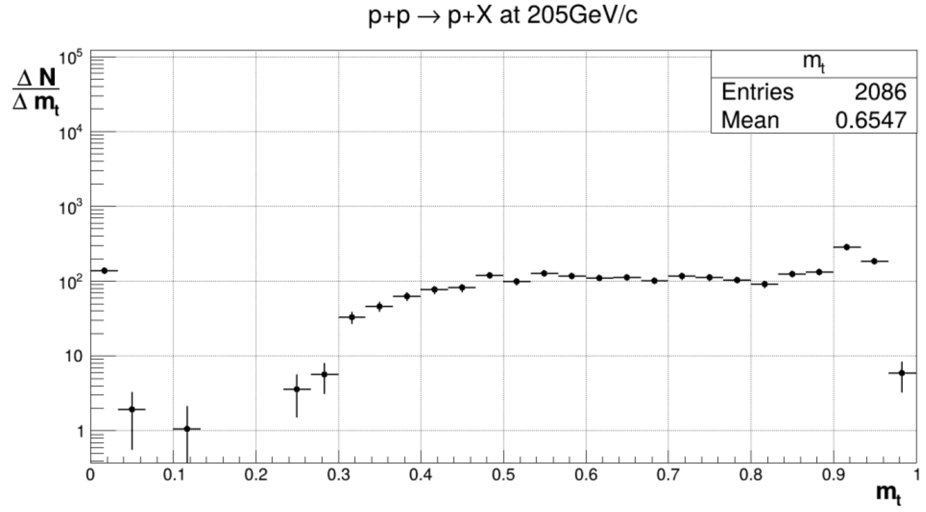


Figure 1 b

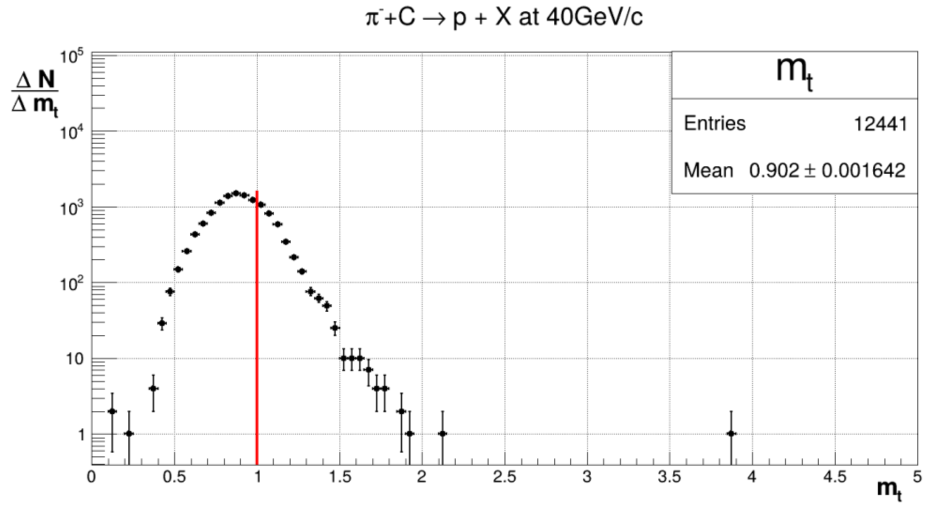


Figure 1 c

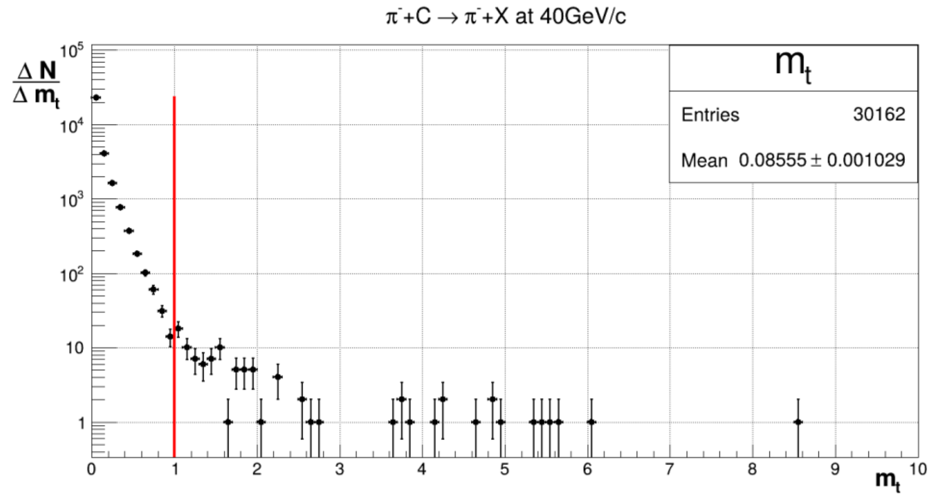


Figure 1 d

The square of 4-momentum transfer q^2 and the running coupling constant

As mentioned in the Introduction, the square of the momentum transfer q^2 plays very important role in the multiparticle production process at high energies. The square of the transferred momentum q^2 is determined by the following formula:

$$q^2 = -(P_a - P_c)^2 = 2E_a\left(E_c - \beta_a P_c^{||}\right) - (m_a^2 + m_c^2) = 2E_a m_p n_c - (m_a^2 + m_c^2)$$

Where E_a and m_a are the energy and mass of the incident particle, respectively m_c is the mass of the secondary particles under consideration.

Previous formula may be rewritten in the following form using cumulative number formula:

$$\frac{q^2}{m_a^2 + m_c^2} = \frac{2E_a m_p n_c - (m_a^2 + m_c^2)}{m_a^2 + m_c^2} = \frac{2E_a m_p n_c}{m_a^2 + m_c^2} - 1$$

Now we can take the natural logarithm from both sides of this formula,

$$\ln\left(\frac{q^2}{m_a^2+m_c^2}\right)=\ln\left(\frac{2E_a m_p n_c-(m_a^2+m_c^2)}{m_a^2+m_c^2}\right)=\ln\left(\frac{2E_a m_p n_c}{m_a^2+m_c^2}-1\right)$$

On the other words, the coupling constant of the strong interaction in the LO approximation as a function of the 4-momentum transfer q^2 is determined by the following formula:

$$\alpha_S(q^2)=\frac{4\pi}{\beta_0\ln\left(\frac{q^2}{\Lambda^2}\right)}=\frac{4\pi}{\beta_0\ln\left(\frac{q^2}{m_a^2+m_c^2}\right)}$$

The experimental dependences of $\alpha_S(q^2)$

The strong coupling constant in the LO approximation is determined by following formula:

$$\alpha_S(q^2) = \frac{4\pi}{\beta_0 \ln(\frac{q^2}{\Lambda^2})}, \qquad \beta_0 = 11 - \frac{2}{3}n_f$$

Where β_0 is the coefficient obtained at calculating the different radiation corrections. n_f is number of flavors. The corresponding values of χ^2 are presented in every figures. Experimental distribution of strong coupling constant for all reactions calculated by previous formula are presented in Figure 2 a, b, c, d. We note that the corresponding values of the cut parameter Λ and $\alpha_S(q^2)$ are obtained using the program “Fumili”. The results of fitting by program “Fumili” are in the agreement within the limit of 2÷3 errors with the results calculated by $\alpha_S(q^2)$ formula.

The experimental dependences of $\alpha_s(q^2)$

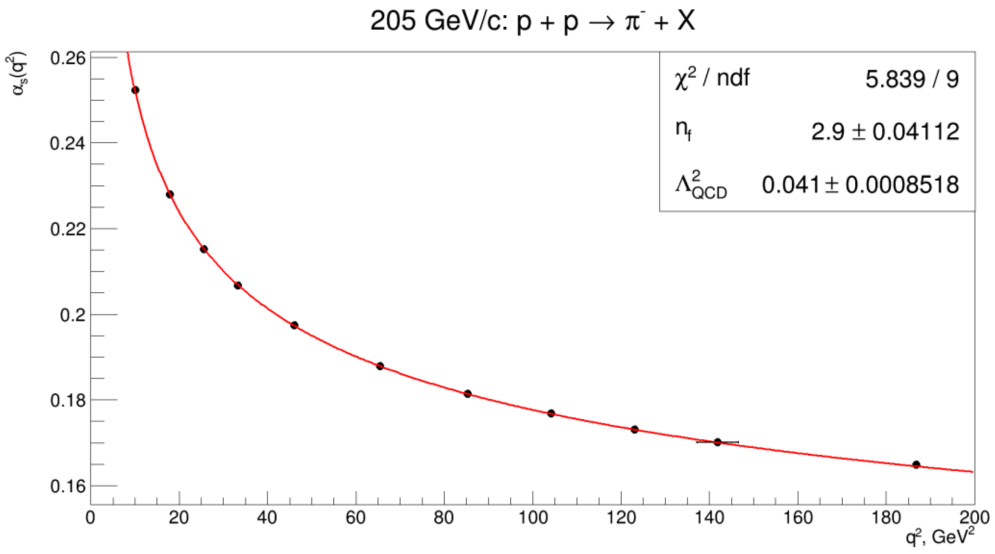


Figure 2 a

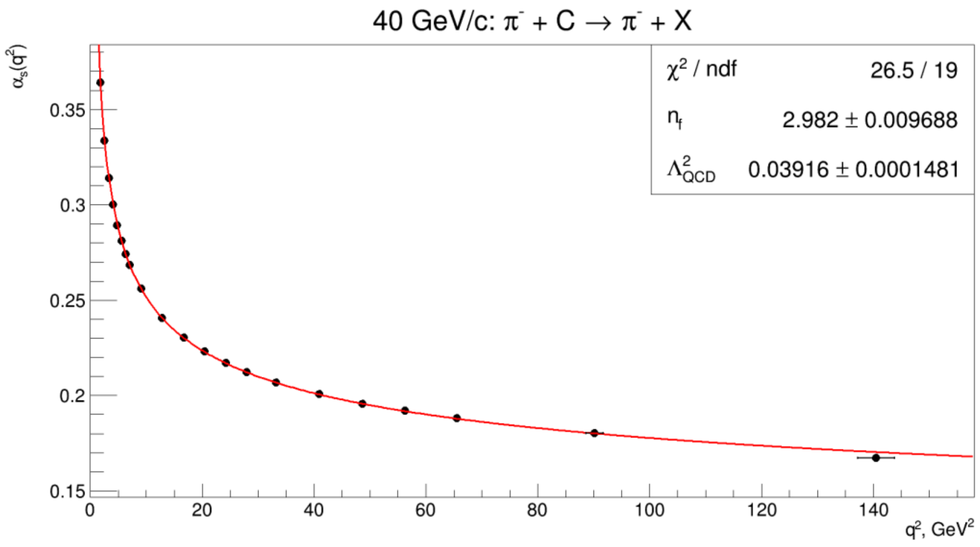


Figure 2 c

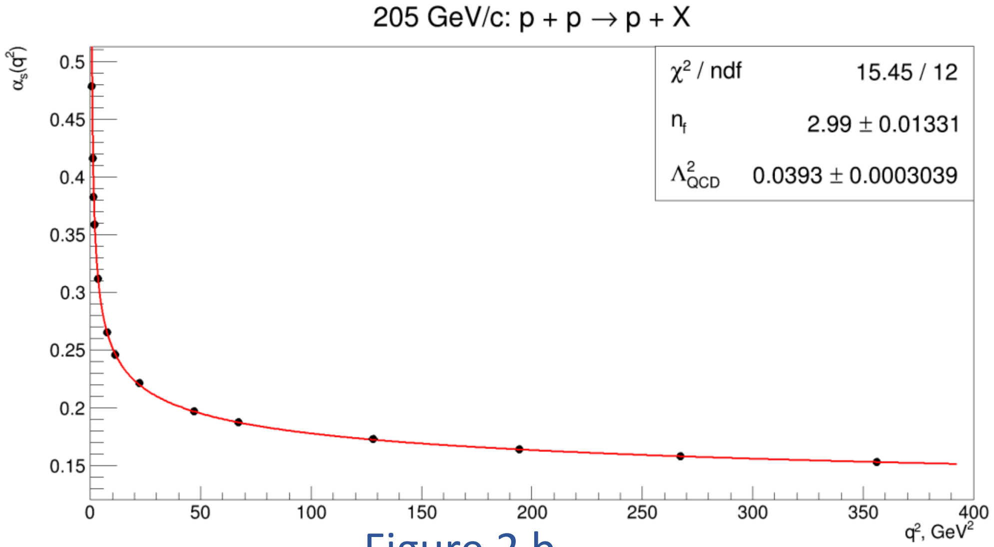


Figure 2 b

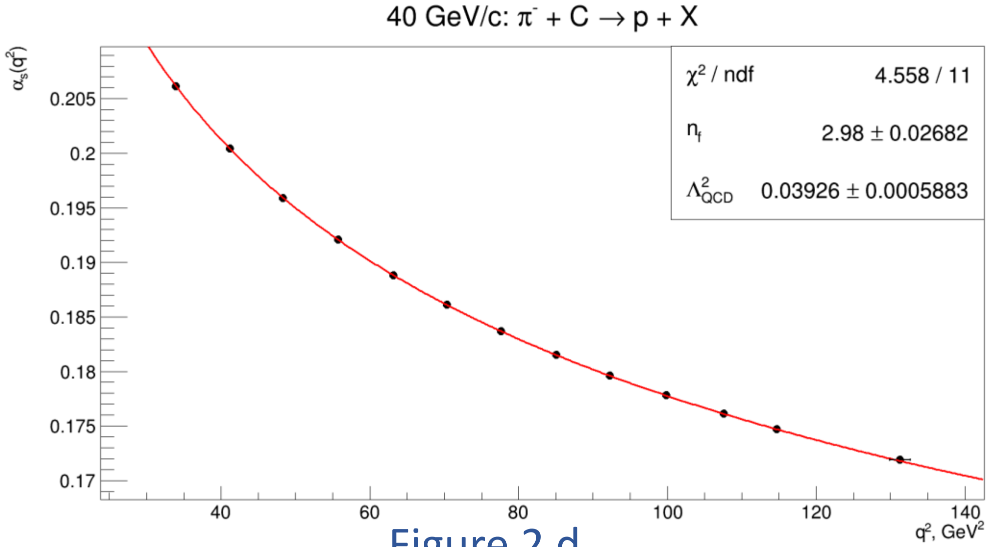


Figure 2 d

$\alpha_S(q^2)$ distributions of π^- mesons and protons from $\pi^- + C$ interactions at 40 GeV/c and comparison with QCD predictions

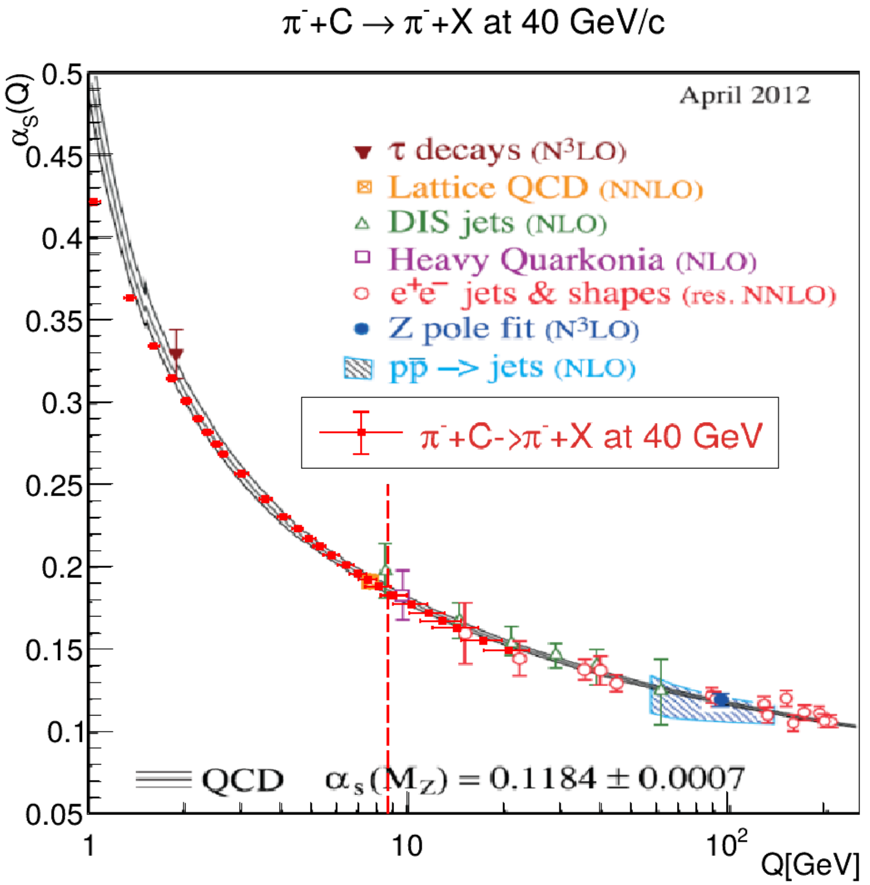


Figure 3 a

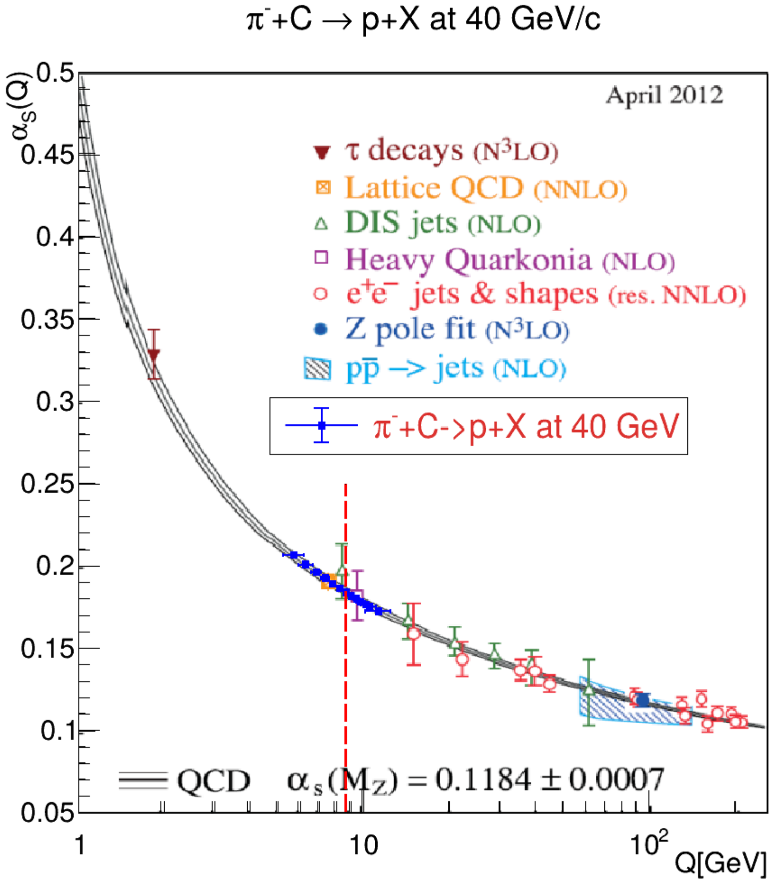


Figure 3 b

$\alpha_S(q^2)$ distributions of π^- mesons and protons from $p+p$ interactions at 205 GeV/c and comparison with QCD predictions

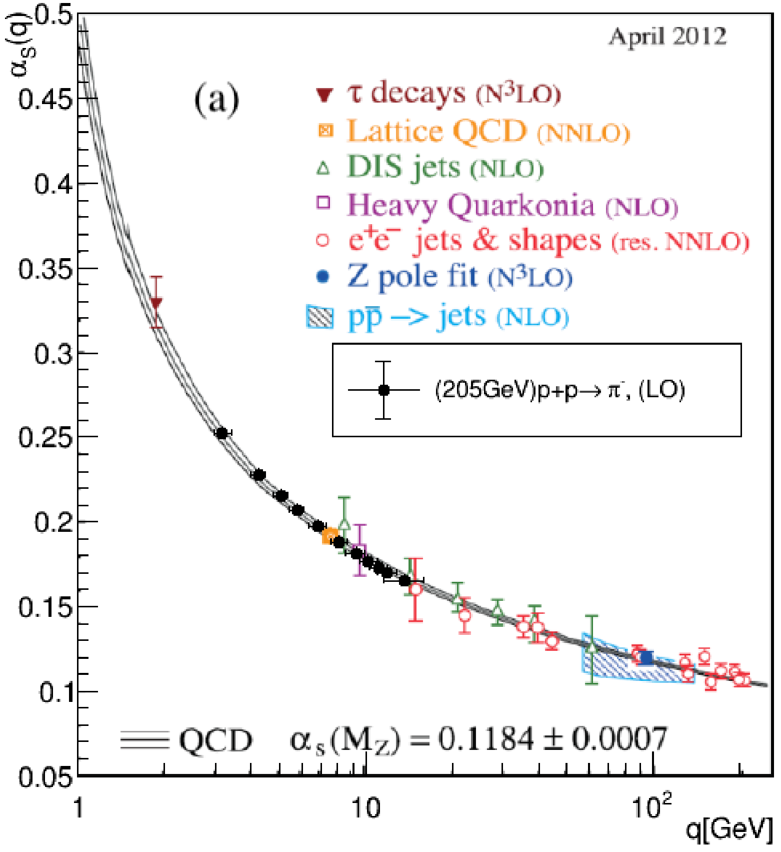


Figure 3 c

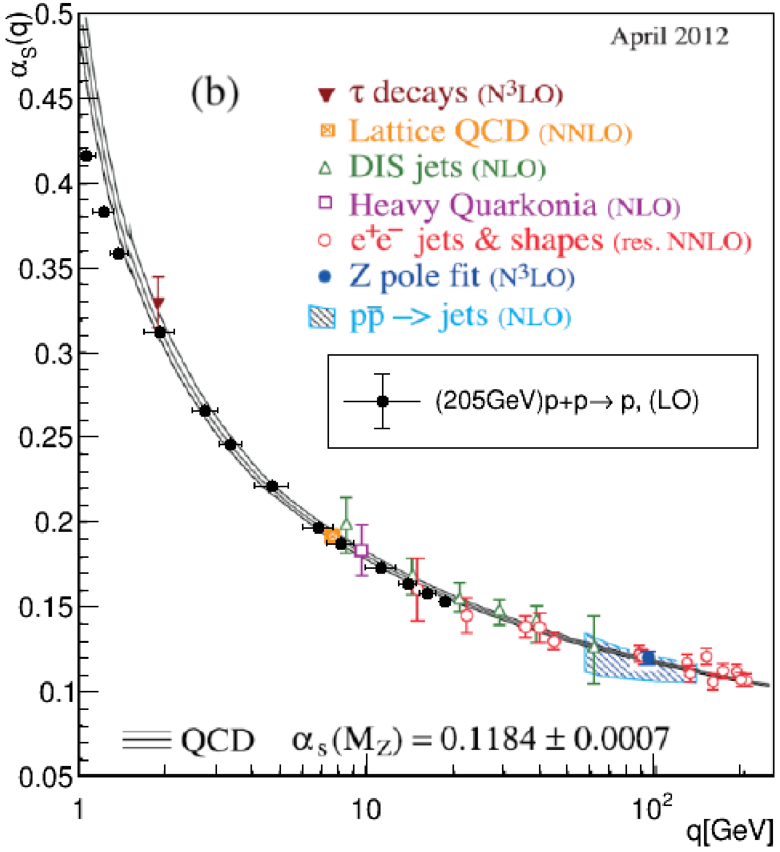
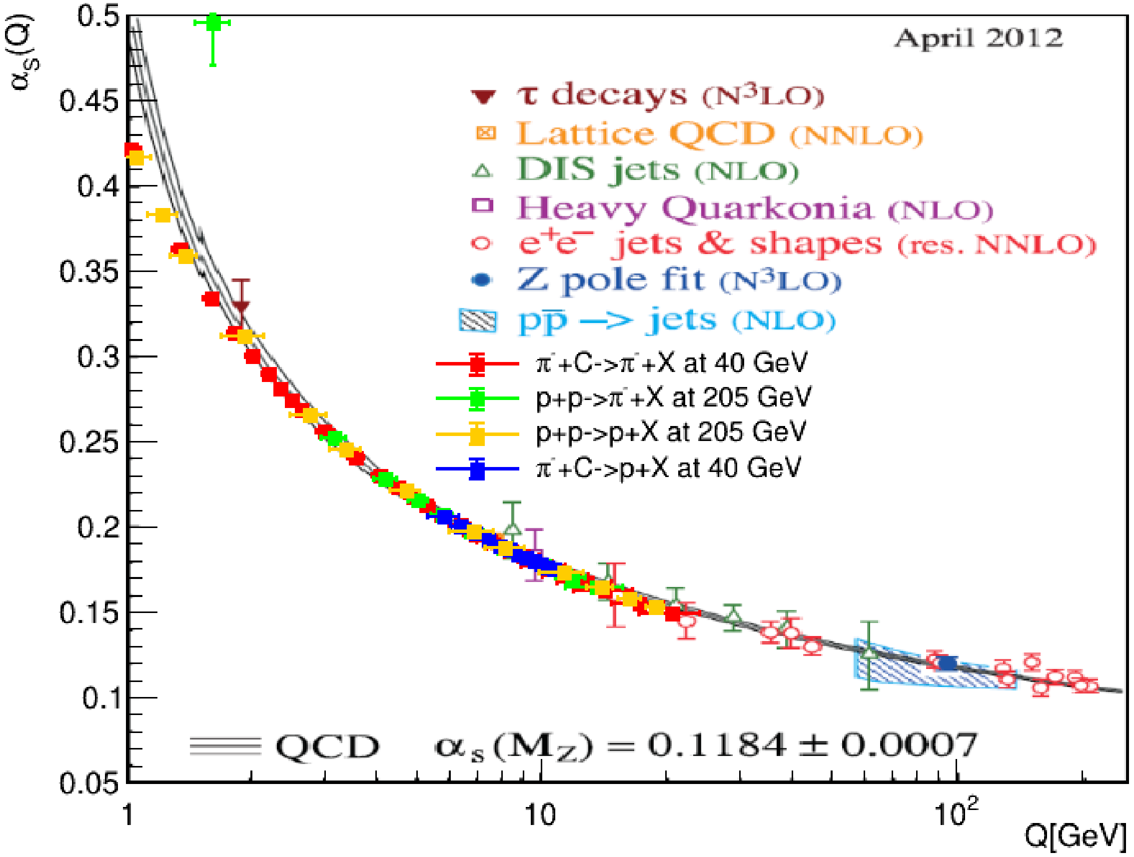


Figure 3 d

Conclusion



The running coupling constant $\alpha_S(q^2)$ of π^- mesons and protons, including the cumulative ones with $n_c > 1$ produced in $\pi^- + C$ interactions at 40 GeV/c are in agreement with $p + p$ interactions data at 205 GeV/c.

These values of the running coupling constants are also in agreement with QCD predictions. In other words, the production of the cumulative and the noncumulative particles, both obey the QCD mechanism.

THANK YOU FOR YOUR ATTENTION.

Conclusion

The running coupling constant $\alpha_S(q^2)$ of π^- mesons and protons, including the cumulative ones with $n_c > 1$ produced in $\pi^- + C$ interactions at 40 GeV/c are in agreement with $p + p$ interactions data at 205 GeV/c.

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Cumulative number distributions