Contribution of linearly polarised gluons in charmonium production within the Soft Gluon Resummation approach

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Outline

- Motivation & General remarks
- TMD factorisation and Soft Gluon Resummation approach
- Nonrelativistic QCD (NRQCD)
- J/ψ and η_c production at small-p_T within current data
- Estimations of J/ψ and η_c production at SPD NICA energy

Motivation

- Previous task:
  only leading contributions in TMD factorisation — unpolarised partons (PDFs) within NRQCD and ICEM for unpolarised and polarised J/ψ and η_c production
  - agreement of NRQCD and ICEM predictions for SPD NICA [V. Saleev, K. Shilyaev (2025)]

- Current work:
  estimation of contribution of Boer-Mulders PDFs — BM PDFs, i.e. linearly polarised partons within protons, for J/ψ and η_c production

- Forthcoming study:
  impact of Boer-Mulders PDFs on the polarisation observables for heavy quarkonium production

$$\frac{d\sigma}{d\Omega} \sim 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \underline{\nu \sin^2\theta \cos 2\phi}, \qquad \nu \sim \text{Re } d\sigma_{-11} \text{ — double spin-flip}$$

General remarks on our approach

- Preliminary estimation for contribution of Boer-Mulders PDFs in pp collisions
  - uncertainties in input for TMD parton distributions
  - considering only leading subprocesses

- Study of gluon-gluon fusion subprocesses only
  - quark-antiquark annihilation contributes less than $10\,\%$ in NRQCD calculations
  - LDMEs for states produced in gg subprocesses are an order greater than in $q\bar{q}$ subprocesses

- Direct production only: $g + g \to J/\psi$
  - neglected $\chi_{cJ} \to J/\psi + \gamma$ decays are estimated to be about $5 - 7\,\%$ in NRQCD calculations

TMD factorisation and TMD PDFs

- **Transverse Momentum Dependent (TMD) factorisation**: $q_T, k_T \ll \mu_F \sim M$
- TMD parton distributions (PDFs & Boer-Mulders PDFs) $F(x, \boldsymbol{q_T}, \mu_F, \zeta) \Rightarrow$ two-scale **CS** equations:

$$\begin{cases} \dfrac{\partial \ln \hat{F}(x, \boldsymbol{b_T}, \mu_F, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu_F) & \text{with CS kernel } \tilde{K}(b_T, \mu_F) \\ \dfrac{\partial \tilde{K}(b_T, \mu)}{\partial \ln \mu} = -\gamma_K[\alpha_s(\mu_F)] & \text{with anomalous dimension } \gamma_K[\alpha_s(\mu_F)] \end{cases}$$

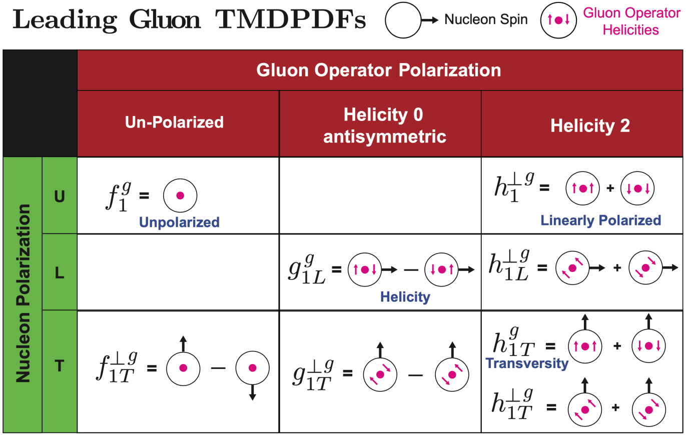
- Partons' momenta decomposition:

$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu$$

  - preserving $\mathcal{O}(q_T/M)$ terms, neglecting $\mathcal{O}(q_T^2/M^2)$ terms and, therefore, assuming $y_{1,2} \to 0$:

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2} \right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2} \right), \qquad x_{1,2} \approx \frac{M}{\sqrt{s}} e^{\pm y}$$

- Gluon parton distributions within unpolarised and polarised nucleons [TMD Handbook, arXiv:2304.03302]



TMD factorisation and TMD PDFs

- General formula of TMD factorisation:

$$d\sigma = \sigma_0 \int dx_1\, dx_2\, d\boldsymbol{q_{1T}}\, d\boldsymbol{q_{2T}}\, \delta(\boldsymbol{q}_{1T} + \boldsymbol{q}_{2T} - \boldsymbol{p}_T)\,(2x_1x_2)\,\Phi_{g\,\mu\nu}(x_1, \mathbf{q}_{1T})\Phi_g^{\mu\nu}(x_2, \mathbf{q}_{2T})$$

with $\Phi_g^{\mu\nu}(x, \mathbf{q}_T)$ being the correlator of gluon field strengths:

$$\Phi_g^{\mu\nu}(x, \mathbf{q}_T) = -\frac{1}{2x}\left[g_T^{\mu\nu} f_1^g(x, \mathbf{q}_T) - \left(\frac{q_T^\mu q_T^\nu}{M_h^2} + g_T^{\mu\nu}\frac{\mathbf{q}_T^2}{2M_h^2}\right) h_1^{\perp g}(x, \mathbf{q}_T)\right]$$

- The cross section as convolutions of TMD PDFs in transverse momentum:

$$d\sigma = \sigma_0\left(\mathcal{C}[f_1^g(x_1, \boldsymbol{q}_{1T}) f_1^g(x_2, \boldsymbol{q}_{2T})] + \mathcal{C}[w_H(\boldsymbol{q}_{1T}, \boldsymbol{q}_{2T}) h_1^{\perp g}(x_1, \boldsymbol{q}_{1T}) h_1^{\perp g}(x_2, \boldsymbol{q}_{2T})]\right)$$

$$w_H(\boldsymbol{q}_{1T}, \boldsymbol{q}_{2T}) = \frac{(\boldsymbol{q}_{1T}\cdot\boldsymbol{q}_{2T})^2 - \frac{1}{2}\boldsymbol{q}_{1T}^2\boldsymbol{q}_{2T}^2}{2M_h^2}$$

TMD factorisation and TMD PDFs

- To implement **Collins-Soper** evolution, the transfer to impact parameter $\boldsymbol{b_T}$ space by 2D Fourier transform is done:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int \frac{d\boldsymbol{b_T}}{(2\pi)^2}\, e^{i\boldsymbol{p}_T \boldsymbol{b}_T} \Big(\hat{f}_1^g(x_1, \boldsymbol{b}_T, \mu_F, \zeta_1)\, \hat{f}_1^g(x_2, \boldsymbol{b}_T, \mu_F, \zeta_2) + \\ + \hat{h}_1^{\perp g}(x_1, \boldsymbol{b}_T, \mu_F, \zeta_1)\, \hat{h}_1^{\perp g}(x_2, \boldsymbol{b}_T, \mu_F, \zeta_2)\Big)$$

- Definition of Fourier transformed functions:

$$\hat{f}_1^g(x, \boldsymbol{b_T}, \mu_F, \zeta) = \int d\boldsymbol{q}_T e^{-i\boldsymbol{q}_T \boldsymbol{b}_T} f_1^g(x, \boldsymbol{q}_T, \mu_F, \zeta)$$

$$\hat{h}_1^{\perp g}(x, \boldsymbol{b_T}, \mu_F, \zeta) = \int d\boldsymbol{q}_T \frac{(\boldsymbol{b}_T \cdot \boldsymbol{q}_T)^2 - \frac{1}{2}\boldsymbol{b}_T^2 \boldsymbol{q}_T^2}{\boldsymbol{b}_T^2 M_h^2} e^{-i\boldsymbol{q}_T \boldsymbol{b}_T} h_1^{\perp g}(x, \boldsymbol{q}_T, \mu_F, \zeta)$$

Soft Gluon Resummation approach, perturbative evolution

- Soft and collinear gluon resummation approach by [J. Collins, D. Soper (1981)]:
  - with $\hat{F}(x,\mu'_{b*},b_T^*) \equiv \hat{f}^g(x,\mu'_{b*},b_T^*)$ or $\hat{h}^{\perp g}(x,\mu'_{b*},b_T^*)$

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T^*,\mu'_{b*},Q)}\; e^{-S_{NP}(b_T,Q)}\; \hat{F}(x_1,\mu'_{b*},b_T^*)\, \hat{F}(x_2,\mu'_{b*},b_T^*)$$

- Sudakov factor in LL–LO perturbative calculations for color-octet final state [J. Collins, D. Soper (1982); P. Sun, C. P. Yuan, F. Yuan (2013)]:
  - it is spin-independent, therefore, the same for any convolution [P. Sun, B.-W. Xiao, F. Yuan (2011)]

$$S_P(b_T,\mu_F,Q) = \frac{C_A}{\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \alpha_s(\mu') \left[\ln\frac{Q^2}{\mu'^2} - \left(\frac{11-2N_f/C_A}{6} + \frac{1}{2}\right)\right] + \mathcal{O}(\alpha_s)$$

- Sudakov factor expression is valid only on region $b_0/Q \leqslant b_T \leqslant b_{T,\,\mathsf{max}}$ which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \to \mu'_b = \frac{Qb_0}{Qb_T + b_0} \quad \text{and} \quad b_T^*(b_T) = \frac{b_T}{\sqrt{1+(b_T/b_{T,\,\mathsf{max}})^2}}$$

Soft Gluon Resummation approach, nonperturbative content

- Master formula for soft gluon resummation:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int\limits_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T^*,\mu'_{b^*},Q)}\; e^{-S_{NP}(b_T,Q)}\; \hat{F}(x_1,\mu'_{b^*},b_T^*)\, \hat{F}(x_2,\mu'_{b^*},b_T^*)$$

- **Nonperturbative** quark factor obtained in SIDIS data fitting [S. Aybat, T. Rogers (2011)]:
  - it is phenomenological and should be Casimir-scaled by C_A/C_F for initial gluons

$$S_{NP}(b_T,Q) = \left[g_1 \ln\frac{Q}{2Q_{NP}} + g_2\left(1+2g_3\ln\frac{10xx_0}{x_0+x}\right)\right] b_T^2$$

- In the leading order of α_s, the perturbative tails of TMD distributions are expressed with collinear PDFs [P. Sun, B.-W. Xiao, F. Yuan (2011)]:

$$\hat{f}^g(x,\mu'_{b^*},b_T^*) = f(x,\mu'_{b^*}) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T\Lambda_{\mathsf{QCD}})$$

$$\hat{h}^{\perp g}(x,\mu'_{b^*},b_T^*) = -\frac{C_A\alpha_s(\mu'_{b^*})}{\pi}\int\frac{dx'}{x'}\left(\frac{x'}{x}-1\right) f(x',\mu'_{b^*}) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T\Lambda_{\mathsf{QCD}})$$

Hadronisation model: NRQCD

- J/ψ wave function as a series with respect to relative velocity υ of constituent quarks:

$$|J/\psi\rangle = \mathcal{O}(\upsilon^0)\,|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(\upsilon^1)\,|c\bar{c}[^3P_J^{(8)}]g\rangle + \mathcal{O}(\upsilon^2)\,|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \\ + \mathcal{O}(\upsilon^2)\,|c\bar{c}[^1S_0^{(8)}]g\rangle + \mathcal{O}(\upsilon^2)\,|c\bar{c}[^1D_J^{(1,8)}]gg\rangle + \ldots$$

- Factorisation of hard cross section:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_n d\hat{\sigma}(ab \to c\bar{c}[n]X)\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$$

- Nonperturbative (hadronisation) factors:

$\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$ – long-distance matrix elements (LDME):

color singlet LDMEs — potential models, data for leptonic decay

color octet LDMEs — lattice QCD calculation or experimental data fitting

Hadronisation model: NRQCD

- Relevant $2 \to 1$ subprocesses for J/ψ production:

$$g + g \to \mathcal{C}[^1S_0^{(8)}], \qquad g + g \to \mathcal{C}[^3P_{0,2}^{(8)}],$$

- fitted LDME values in the J/ψ calculations:
  - heavy quark spin symmetry
  - identical p_T-dependence

$$\langle \mathcal{O}^{J/\psi}[^1S_0^{(8)}]\rangle + \frac{7}{m_c^2}\langle \mathcal{O}^{J/\psi}[^3P_0^{(8)}]\rangle \simeq 0.064 \text{ GeV}^3$$

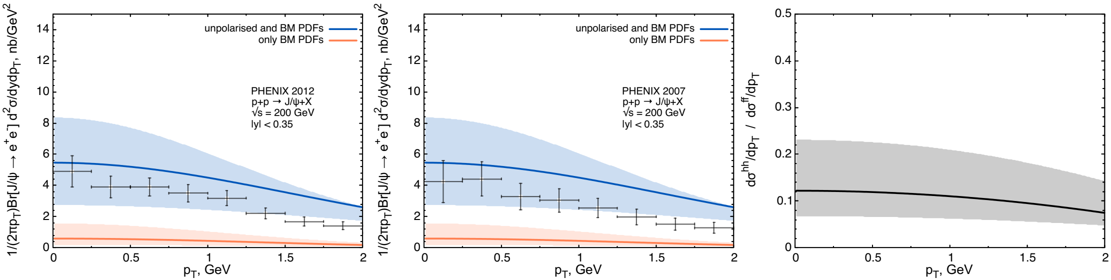
- Relevant $2 \to 1$ subprocess for η_c production:

$$g + g \to \mathcal{C}[^1S_0^{(1)}]$$

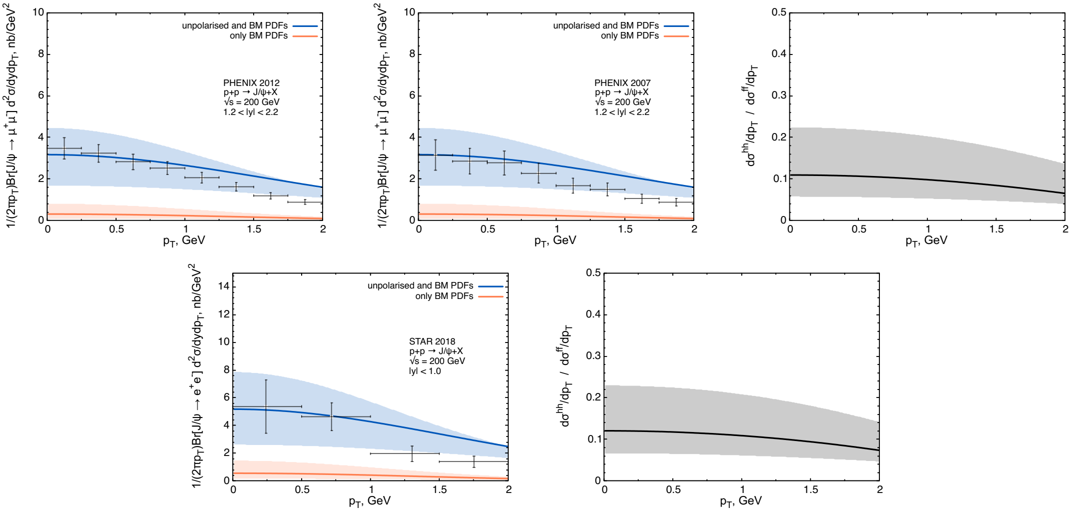
- LDME value in the η_c calculations:
  - heavy quark spin-symmetry

$$\langle \mathcal{O}^{\eta_c}[^1S_0^{(1)}]\rangle = \frac{1}{3}\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}]\rangle \simeq 0.44 \text{ GeV}^3$$

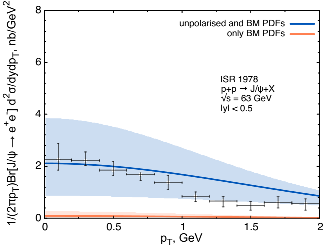
Unpolarised J/ψ production, BM PDFs contribution at $\sqrt{s} = 200$ GeV

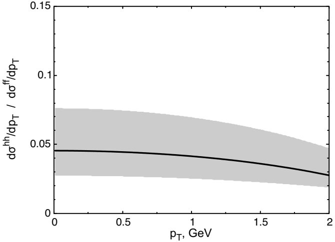


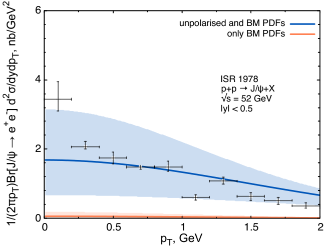
Unpolarised J/ψ production, BM PDFs contribution at $\sqrt{s} = 200$ GeV

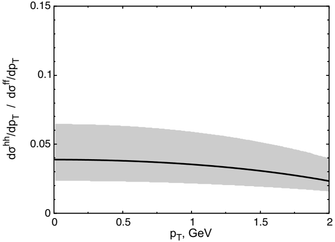


Unpolarised J/ψ production, BM PDFs contribution at $\sqrt{s} = 52 - 63$ GeV

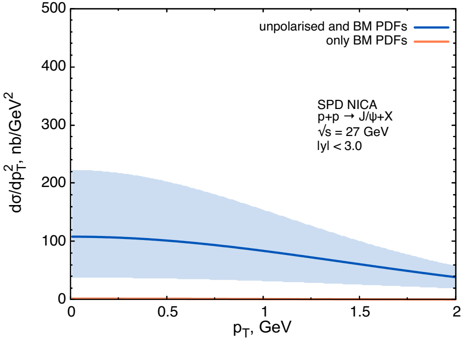


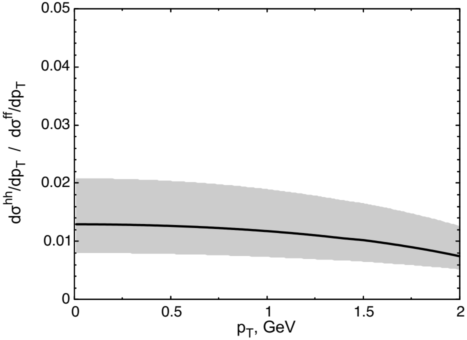




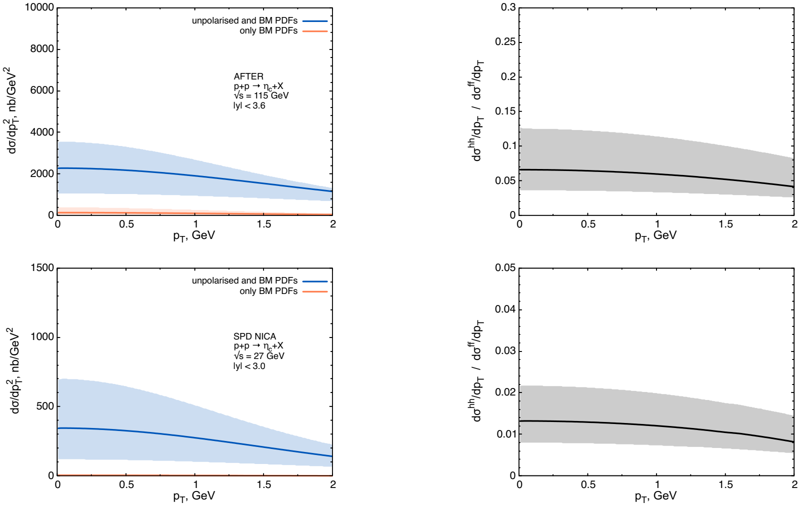


Unpolarised J/ψ production, BM PDFs contribution at $\sqrt{s} = 27$ GeV





η_c production, BM PDFs contribution at $\sqrt{s} = 115$ and $\sqrt{s} = 27$ GeV



Summary

- We have used the Soft Gluon Resummation approach to calculate small-p_T J/ψ and η_c production in the TMD factorisation, including contribution of BM PDFs
- Soft Gluon Resummation approach for gluon TMD PDF and BM PDFs allows to describe satisfyingly experimental data for unpolarised J/ψ production at $\sqrt{s} = 52 - 200$ GeV in the TMD domain of $p_T < 1$ GeV
- We have preliminary estimated the relative contribution the BM PDFs functions for different center-of-mass energies $\sqrt{s}$
- Contribution of BM PDFs decreases when $\sqrt{s}$ decreases, i.e. momentum fraction x increases

THANK YOU FOR ATTENTION!