The light-by-light contribution to the anomalous magnetic moment of muon within the nonlocal chiral quark model with vector and axial-vector meson

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Based on:
A.E.Dorokhov,AER,A.S.Zhevlakov EPJC75(2015)417
AER,A.S.Zhevlakov,A.P.Martynenko,F.A.Martynenko PRD108(2023)014033
AER,A.S.Zhevlakov,arxiv:2504.13588

Plan

1. General suggestion
2. QED at one loop level
3. Anomalous magnetic moment of electron
4. Anomalous magnetic moment of muon
5. Strong - HVP, LbL
6. Nonlocal quark model
7. LbL contribution
8. Conclusions

Cosmology tell us that 95% of matter is not described in text-books yet. Dark Matter surrounds us! Where it is ?

Two search strategies

1. High energy physics to excite heavy degrees of freedom. No any evidence till now.
2. Low energy physics to produce Rare processes in view of huge statistics.

There are some rough edges of SM.
Anomalous magnetic moment of the muon $(g-2)_\mu$ is most famous and stable example

Point-like particle.

Dirac Equation Predicts for free point-like spin $\frac{1}{2}$ charged particle:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{p^2}{2m} - \frac{e}{2m}\left(\overrightarrow{L} + 2\,\overrightarrow{S}\right)\cdot\overrightarrow{B}\right]\psi$$

$g = 2,\ a = (g-2)/2 = 0$ (no anomaly at tree level)
a becomes nonzero due to interactions resulting in fermion substructure

Lagrangian of interaction

The general form of element of interaction of lepton with external electromagnetic fields is

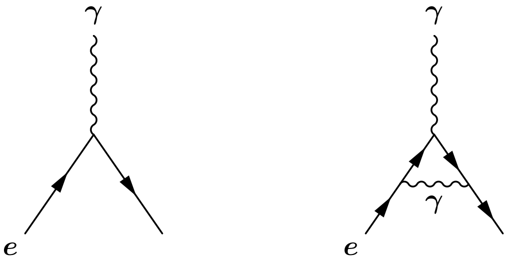
$$-ie\bar{u}(p')\left\{\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q_\nu}{2m}F_2(q^2) + \gamma_5\sigma_{\mu\nu}\frac{q_\nu}{2m}F_3(q^2)\right\}u(p)e_\mu(q)$$

F_1 is the electric charge distribution $e_l = eF_1(0)$

F_2 corresponds to anomalous magnetic moment (AMM) $a_l = (g_l - 2)/2 = F_2(0)$

F_3 corresponds to anomalous electric dipole moment

One loop QED radiative correction



$$\Gamma_\mu = e\gamma_\mu + a\frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

$$a_e = \frac{\alpha}{2\pi} = 0.001162,$$ [1]

$$a_e^{exp} = 0.001145 \pm 0.00004,$$ [2]

[1] J. S. Schwinger, Phys. Rev. **73** (1948) 416.

[2] H. M. Foley and P. Kusch, Phys. Rev. **72**, 1256 (1947).

Anomalous magnetic momentum of electron

1. The best measurement of a_e was performed at one-electron quantum cyclotron at Northwestern University[3]

$$a_e^{\mathrm{NW}} = 1\ 159\ 652\ 180.59(13) \times 10^{-12} \quad [0.11\ \mathrm{ppb}]$$

2. In standard model

$$a_e^{\mathrm{SM}} = \left\{ a_e^{\mathrm{QED}} + a_e^{\mathrm{weak}} + a_e^{\mathrm{hadr}} \right\}^{\mathrm{SM}}, \quad a_e^{\mathrm{QED}} = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi} \right)^n a_e^{(2n)},$$

3. QED itself cannot predict value of α:

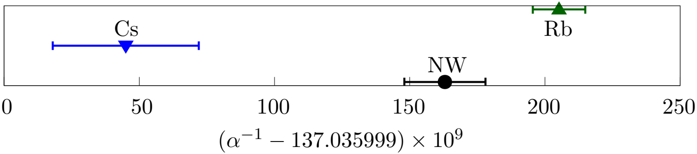
- Equating a_e^{NW} to a_e^{SM} one can obtain the fine structure constant α
- One can use α from other experiment and then compare a_e^{NW} with theoretical prediction

[3] X. Fan, T. G. Myers, B. A. Sukra and G. Gabrielse, PRL 2023 **130** 071801

Anomalous magnetic momentum of electron

3. QED itself cannot predict value of α:

- Equating a_e^{NW} to a_e^{SM} one can obtain the fine structure constant[4] α that is slightly inconsistent with measurements from ^{87}Rb and ^{133}Cs atoms[5]



- One can use α from other experiment and then compare a_e^{NW} with theoretical prediction

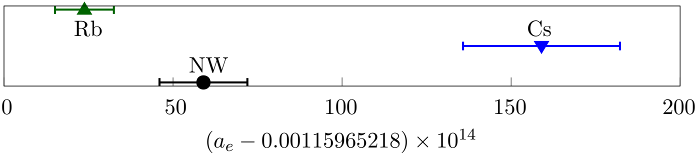
$$a_e^{\mathrm{SM}} = \left\{a_e^{\mathrm{QED}} + a_e^{\mathrm{weak}} + a_e^{\mathrm{hadr}}\right\}^{\mathrm{SM}}, \quad a_e^{\mathrm{QED}} = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)}.$$

4 T. Aoyama, T. Kinoshita, M. Nio, Atoms 2019 **7** 28.

5 R.H. Parker et.al., Science 2018 **360** 191; L.Morel, Z.Yao, P.Clade, S.Guellati-Khelifa, Nature,588(2020)61; WP 2025 arxiv:2505.21476

Anomalous magnetic momentum of electron

There is a discrepancy between the anomalous magnetic moment of the electron measured at Northwestern and the value inferred from ^{87}Rb and ^{133}Cs atoms[6]



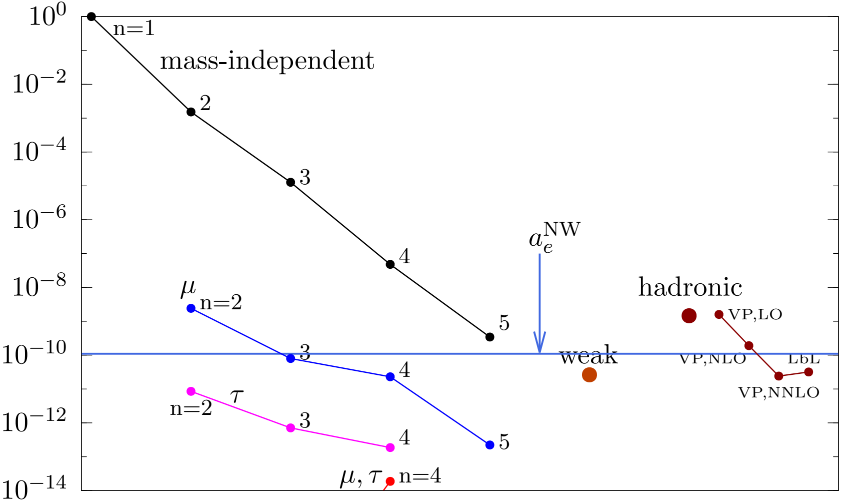
The differences between theory and measurement

$$a_e^{\mathrm{NW}} - a_e^{\mathrm{SM}}[\alpha_{\mathrm{Rb}}] = -1.00(26) \times 10^{-12}$$
$$a_e^{\mathrm{NW}} - a_e^{\mathrm{SM}}[\alpha_{\mathrm{Cs}}] = +0.35(16) \times 10^{-12}$$

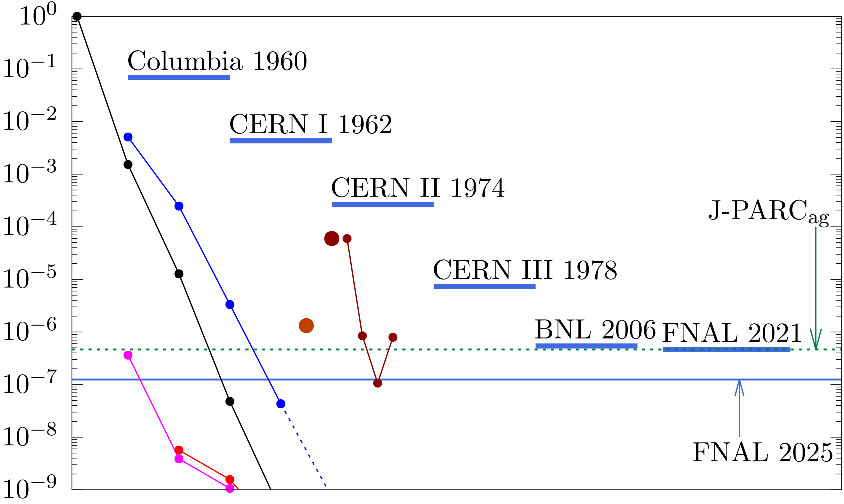
discrepancies: -3.8σ, $+2.2\sigma$ for α_{Rb} and α_{Cs}.

[6] WP 2025 arxiv:2505.21476

Anomalous magnetic momentum of electron. Precision.



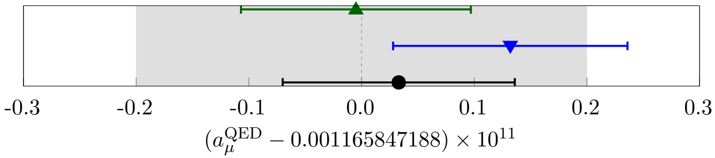
Anomalous magnetic momentum of muon. Precision.



Anomalous magnetic momentum of muon. QED.

3. Tenth-order QED contribution[7] to a_μ

$$a_\mu^{\mathrm{QED}} = 116\ 584\ 718.8(2) \times 10^{-11}$$

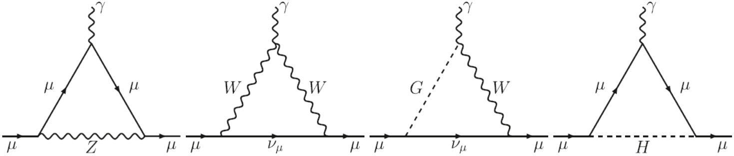


[7] WP 2025 arxiv:2505.21476

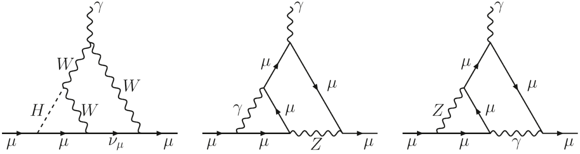
Anomalous magnetic momentum of muon. Electroweak.

4. Electroweak contribution[8] suppressed as $a_\mu^{\mathrm{EW},(1)} \propto \frac{\alpha}{4\pi s_W^2} \cdot \frac{m_\mu^2}{M_W^2}$

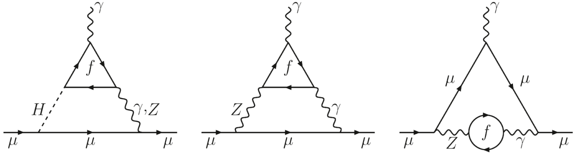
$a_\mu^{\mathrm{EW},(1)}$



$a_{\mu;\mathrm{boson}}^{\mathrm{EW},(2)}$



$a_{\mu;\mathrm{fermion}}^{\mathrm{EW},(2)}$



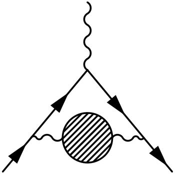
$$a_\mu^{\mathrm{EW}} = a_\mu^{\mathrm{EW},(1)} + a_\mu^{\mathrm{EW},(2)} + a_\mu^{\mathrm{EW},(\geq 3)} = 154.4(4) \times 10^{-11}$$

Strong. HVP.

5. Strong contribution separated into three terms

$$a_\mu^{\text{hadr}} = a_\mu^{\text{HVP,LO}} + (a_\mu^{\text{HVP,NLO}} + a_\mu^{\text{HVP,NNLO}} + ..) + a_\mu^{\text{HLbL}}$$

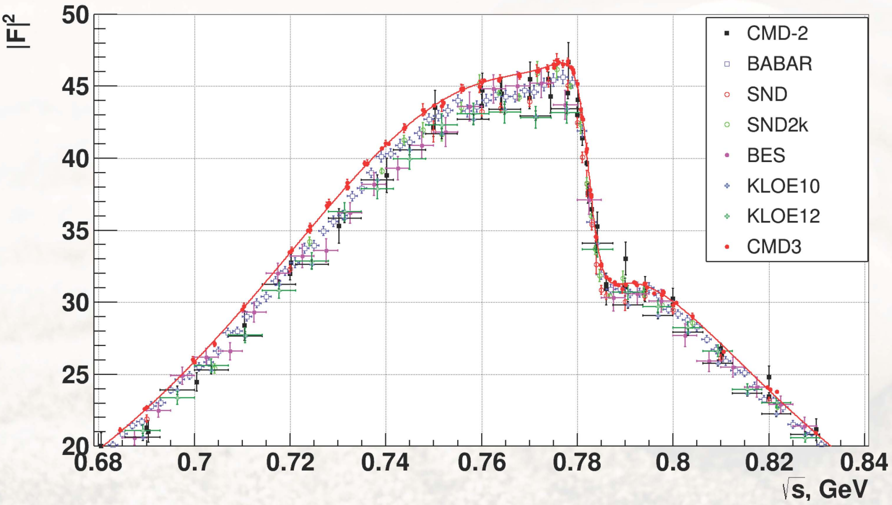
6. Contribution of hadron vacuum polarization can be:



- extracted from experimental data for process $e^+e^- \to$ in hadrons or hadronic τ-lepton decays (tensions between CMD-3 experiment versus previous data)

  HVP LO (e^+e^-, τ) Estimates not provided at this point
- calculated in Lattice QCD

Other experiments



9. Lattice QCD+QED

$$a_\mu^{\text{HVP,LO}} = \alpha^2 \int_0^\infty dt\ K(t)\ G_{1\gamma\text{I}}(t)\ ,$$

$$G(t) = \frac{1}{3e^2} \sum_{i=1,2,3} \int d^3x \langle J_i(\vec{x},t) J_i(0)\rangle\ ,$$

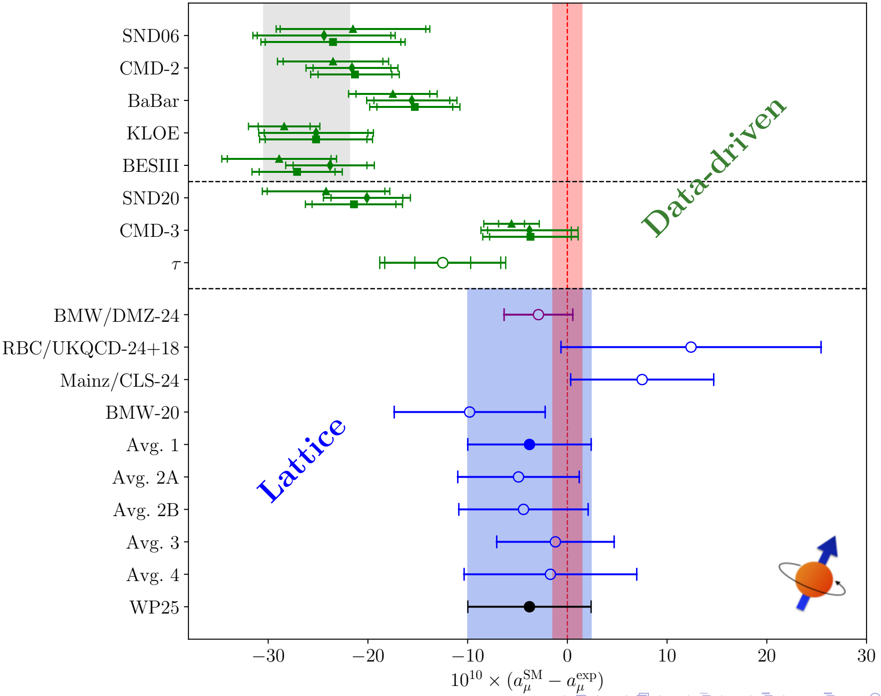
$$K(t) = \int_0^\infty \frac{dQ^2}{m_\mu^2}\, \omega\left(\frac{Q^2}{m_\mu^2}\right) \left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]\ ,$$

where $\omega(r) = [r + 2 - \sqrt{r(r+4)}]^2/\sqrt{r(r+4)}$.
The result from lattice[9]

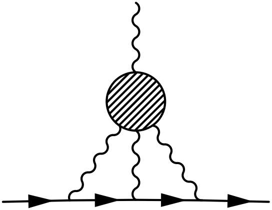
$$a_\mu^{\text{HVP,LO}} = 7075(55) \times 10^{-11}$$

is in tension with data-driven approach for pre-CMD-3 data.

[9] S. Borsanyi *et al.* Nature593(2021)7857


SND06
CMD-2
BaBar
KLOE
BESIII
SND20
CMD-3
τ
BMW/DMZ-24
RBC/UKQCD-24+18
Mainz/CLS-24
BMW-20
Avg. 1
Avg. 2A
Avg. 2B
Avg. 3
Avg. 4
WP25
Data-driven
Lattice
−30
−20
−10
0
10
20
30
$10^{10} \times (a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp}})$

10. LbL scattering amplitude is very complicated object. There is hierarchy in diagrams connected to existence of two small parameters: the inverse number of colors $1/N_c$ and the ratio of the characteristic internal momentum to the chiral symmetry parameter $m_\mu/(4\pi f_\pi) \sim 0.1$.



Contribution of LbL can be:

- Calculated in data-driven approach + effective models
- Calculated in Lattice QCD

LbL, data-driven + phenomenology - Lattice

11. Data-driven + phenomenology WP25 value

$$a_\mu^{\mathrm{LbL}} = 103.3(8.8) \times 10^{-11}$$

is compatible with Lattice QCD results[10]

$$a_\mu^{\mathrm{LbL}} = 109.6(15.9) \times 10^{-11} [\mathrm{Mainz}]$$
$$a_\mu^{\mathrm{LbL}} = 125.5(11.7) \times 10^{-11} [\mathrm{BWMc}]$$
$$a_\mu^{\mathrm{LbL}} = 124.7(14.9) \times 10^{-11} [\mathrm{RBC/UKQCD}]$$

[10] E. H. Chao et. al. Eur. Phys. J. C **82** (2022) 664; T. Blum et. al. PRD 111(2025) 014501; Z. Fodor et. al. PRD 111(2025) 114509.

All SM contributions

12. Combining all SM contributions one obtains [11]

QED	116 584 718.8	(2)
Weak	154.4	(4)
HVP LO(lattice)	7045	(61)
HVP NLO(e^+e^-)	-99.6	(1.3)
HVP NNLO(e^+e^-)	12.4	(1)
HLbL	115.5	(9.9)
SM	116 592 033	(62)

[11] WP 2025 arxiv:2505.21476

Comparison

13. Experimental result[12]

$$a_\mu^{\rm BNL} = 116\ 592\ 089(63) \times 10^{-11}$$
$$a_\mu^{\rm FNAL} = 116\ 592\ 070.5(14.8) \times 10^{-11}$$
$$a_\mu^{\rm Exp} = 116\ 592\ 071.5(14.5) \times 10^{-11}$$

14. Combining all SM contributions one obtains[13]

$$a_\mu^{\rm SM} = 116\ 592\ 033(62) \times 10^{-11}$$

15. The resulting difference between the experimental result and the full SM prediction is

$$a_\mu^{\rm Exp} - a_\mu^{\rm SM} = 38\ \ (63) \times 10^{-11},$$

there is no discrepancy between the SM and experiment !

[12] G.W.Bennett,et al. PRD73(2006)072003;B. Abi *et al.*, PRL135(2025)101802
[13] WP2025 arxiv:2505.21476

Nonlocal quark model. Lagrangian

The Lagrangian of the model with the pseudoscalar–scalar and vector–axial-vector has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{P,S} + \mathcal{L}_{V,A}, \quad \mathcal{L}_{free} = \bar{q}(x)(i\hat{\partial} - M_c)q(x),$$
$$\mathcal{L}_{P,S} = \frac{G_1}{2}\left(\left(J_S^a(x)\right)^2 + \left(J_P^a(x)\right)^2\right),$$
$$\mathcal{L}_{V,A} = \frac{G_2}{2}\left(\left(J_V^{a,\mu}(x)\right)^2 + \left(J_A^{a,\mu}(x)\right)^2\right),$$

M_c is the current quark mass matrix with diagonal elements m_c, G_1 and G_2 are the coupling constants in pseudoscalar–scalar (P,S) and vector–axial-vector sectors (V,A).

Nonlocal quark model. Lagrangian

The Lagrangian of the model with the four-quark and six-quark interactions has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4q} + \mathcal{L}_{tH}, \qquad \mathcal{L}_{free} = \bar{q}(x)(i\hat{\partial} - m_c)q(x)$$

m_c – current quark mass matrix with diagonal elements $m_c^u = m_c^d$, m_c^s

$$\begin{aligned} \mathcal{L}_{4q} &= \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)] \\ \mathcal{L}_{tH} &= -\frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_P^a(x)J_P^b(x)J_P^c(x)] \end{aligned}$$

M_c is the current quark mass matrix with diagonal elements m_c, G and H are the four- and six-quark coupling constants .

Currents

The nonlocal quark currents are given by

$$J_M^{a\{,\mu\}}(x) = \int d^4x_1 d^4x_2 \, f(x_1) f(x_2) \, \bar{q}(x - x_1) \, \Gamma_M^{a\{,\mu\}} q(x + x_2),$$

with $M = S, P, V, A$. The spin-flavour matrices are $\Gamma_S^a = \lambda^a$, $\Gamma_P^a = i\gamma^5\lambda^a$, $\Gamma_V^{a,\mu} = \gamma^\mu\lambda^a$, $\Gamma_A^{a,\mu} = \gamma^5\gamma^\mu\lambda^a$. For the $SU(2)$ model, the flavour matrices are: $\lambda^a \equiv \tau^a$, $a = 0, .., 3$ with $\tau^0 = 1$. Such structure of interaction can be motivated by instanton liquid model[14]. $f(x)$ is the form factor encoding the nonlocality of the QCD vacuum. Since only four-quark interaction is considered, the action of the model can be bosonized by the usual Hubbard-Stratonovich trick with the introduction of auxiliary mesonic fields for each quark current, i.e. P, S, V, A.

[14] I.V. Anikin, A.E. Dorokhov, L. Tomio, PPN(2000)31,509.

Currents

The scalar isoscalar field has a non-zero vacuum expectation value $\langle S^0\rangle_0 = \sigma_0 \neq 0$. The shift of the scalar isoscalar field $S^0 = \tilde{S}^0 + \sigma^0$, which is necessary to obtain a physical scalar field with zero vacuum expectation value, leads to the appearance of the momentum dependent quark mass ($m_d = -\sigma^0$). The separable structure of quark current leads to a solution where momentum dependence is factorized and "gap" equation takes the simple form

$$m(p) = m_c + m_d f^2(p), \quad m_d = G_1 \frac{8N_c}{(2\pi)^4} \int d_E^4 k \frac{f^2(k^2) m(k^2)}{k^2 + m^2(k^2)}.$$

This equation for scalar coefficient m_d can be easily solved numerically. The corresponding quark propagator is

$$\mathrm{S}(p) = (\hat{p} - m(p))^{-1}.$$

Mesons

Meson propagators can be obtained by taking quadratic terms over the meson field from the Lagrangian at one loop level. Quark loops and propagators of vector and axial-vector mesons should be split into longitudinal and transverse parts

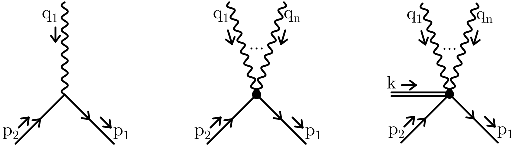
$$\mathrm{D}_M^{\alpha\beta}(p^2) = \mathrm{D}_M^{\mathrm{T}}(p^2)\mathrm{P}_p^{\mathrm{T};\alpha\beta} + \mathrm{D}_M^{\mathrm{L}}(p^2)\mathrm{P}_p^{\mathrm{L};\alpha\beta},$$

with the help of appropriate projectors

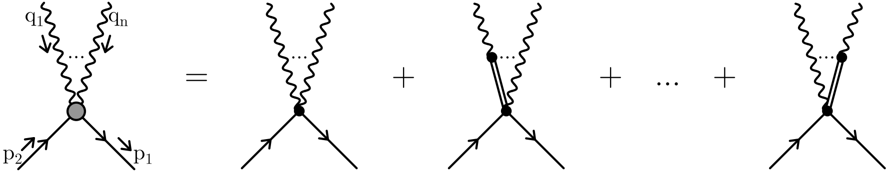
$$\mathrm{P}_p^{\mathrm{T};\alpha\beta} = g^{\alpha\beta} - \frac{p^\alpha p^\beta}{p^2},\ \mathrm{P}_p^{\mathrm{L};\alpha\beta} = \frac{p^\alpha p^\beta}{p^2}.$$

Transverse components correspond to spin-1 states while longitudinal are related to spin-0. In the case of a system of pseudoscalar–axial-vector states, a mixing appears due to a quark polarisation loop with pseudoscalar and axial-vector vertices and physical states can be found as solutions of the matrix equation for $\pi - a_1$ system.

Interactions with the electromagnetic gauge field should be introduced also in the nonlocal quark currents. Photon-quark vertices:

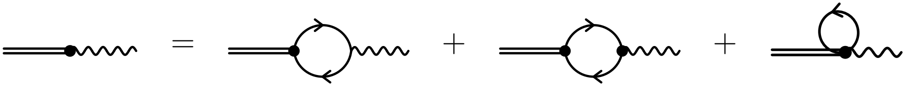


The full quark–antiquark–photon vertices :



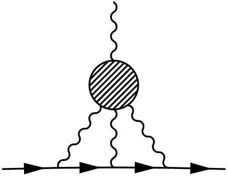
$\rho(\omega) \to \gamma$ transition

In the presence of the vector sector, the photon(s)-quark interaction vertices are additionally dressed by the $\rho(\omega) \to \gamma$ transition



Only one vector meson can be connected with quark-antiquark pair and external EM fields at the point of interaction.

Lbl contribution



The LbL contribution to anomalous magnetic moment of the muon is defined by the projection

$$a_\mu^{\rm HLbL} = \frac{1}{48 m_\mu} {\rm Tr}\left((\hat{p}+m_\mu)[\gamma^\rho,\gamma^\sigma](\hat{p}+m_\mu)\Pi_{\rho\sigma}(p,p)\right),$$

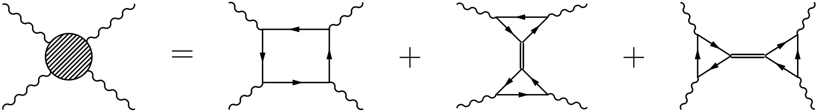
$$\Pi_{\rho\sigma}(p',p) = -ie^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2-k)^2} \times$$

$$\times\, \gamma^\mu \frac{\hat{p}' - \hat{q}_1 + m_\mu}{(p'-q_1)^2 - m_\mu^2} \gamma^\nu \frac{\hat{p} - \hat{q}_1 - \hat{q}_2 + m_\mu}{(p-q_1-q_2)^2 - m_\mu^2} \gamma^\lambda \times$$

$$\times\, \frac{\partial}{\partial k^\rho} \Pi_{\mu\nu\lambda\sigma}(q_1, q_2, k - q_1 - q_2),$$

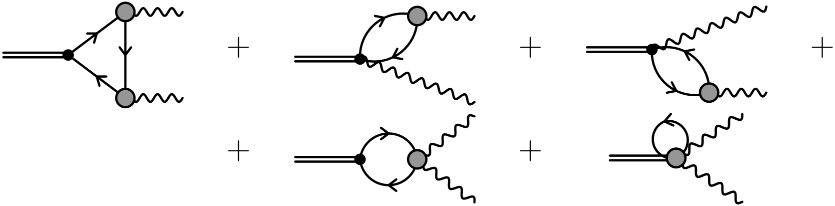
where m_μ is the muon mass, and the static limit $k_\mu \equiv (p'-p)_\mu \to 0$ is implied.

To the leading $1/N_c$ order four-rank polarization tensor $\Pi_{\mu\nu\lambda\sigma}$ can be represented in the form



Contact term is needed for correct asymptotics (LO OPE – axial anomaly[15]) !

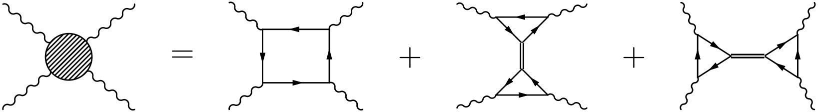
Diagrams for meson transition form-factor in nonlocal model:



[15] K.Melnikov, A. Vainshtein PRD70(2004)113006

Four-rank polarization tensor

To the leading $1/N_c$ order four-rank polarization tensor $\Pi_{\mu\nu\lambda\sigma}$ can be represented in the form



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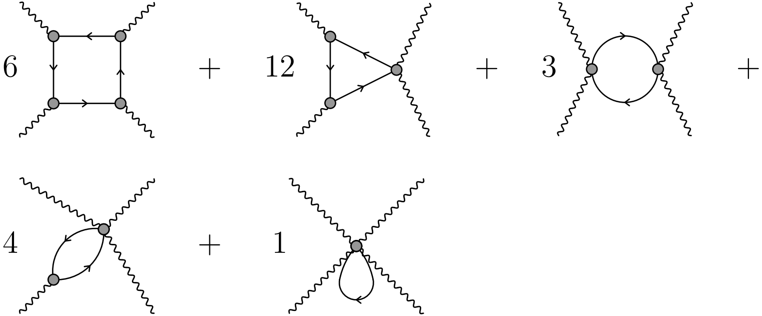
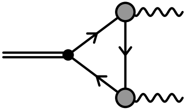


[15] K.Melnikov, A. Vainshtein PRD70(2004)113006

Mesons contribution – off-shell effects

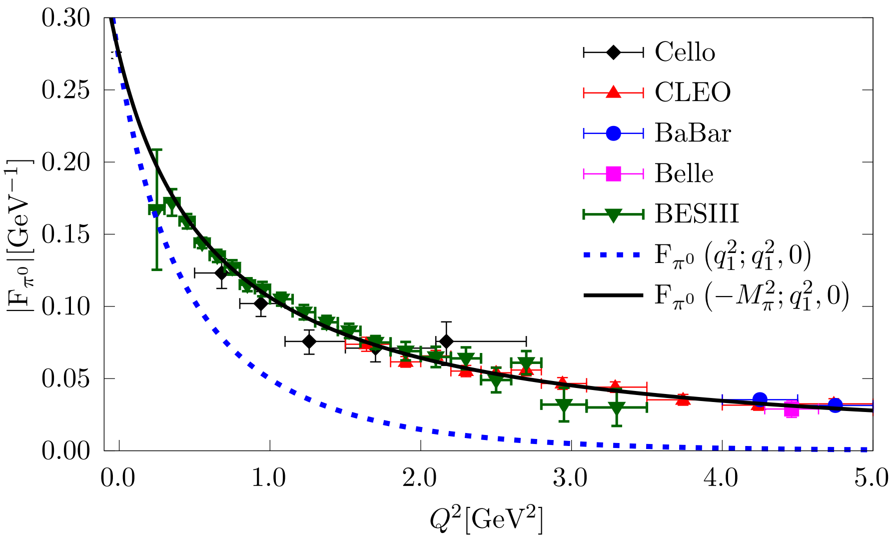
In model, the transition form-factor depends not only on the virtuality of photons but also on that of meson. This dependence results from the fact that mesons are quark-antiquark bound states.



$$\sim \quad F(p^2, q_1^2, q_2^2)$$

It leads to suppression of the LbL contribution due to the virtuality of the meson. Similar situation should occur in the local NJL model or the Dyson-Schwinger approach to QCD.
In approaches based on phenomenological information from experiments the form-factor is taken on meson-mass shell $F(q_1^2, q_2^2)$

Pion transitions form-factors



Lbl contribution.

The LbL contribution in $SU(3)$ nonlocal model without spin-1 fields:

- pseudoscalar(π^0, η, η') mesons
- scalar (σ, $a_0(980)$, $f_0(980)$) mesons
- u, d, s quark loop

The LbL contribution in $SU(2)$ nonlocal model with spin-1 fields:

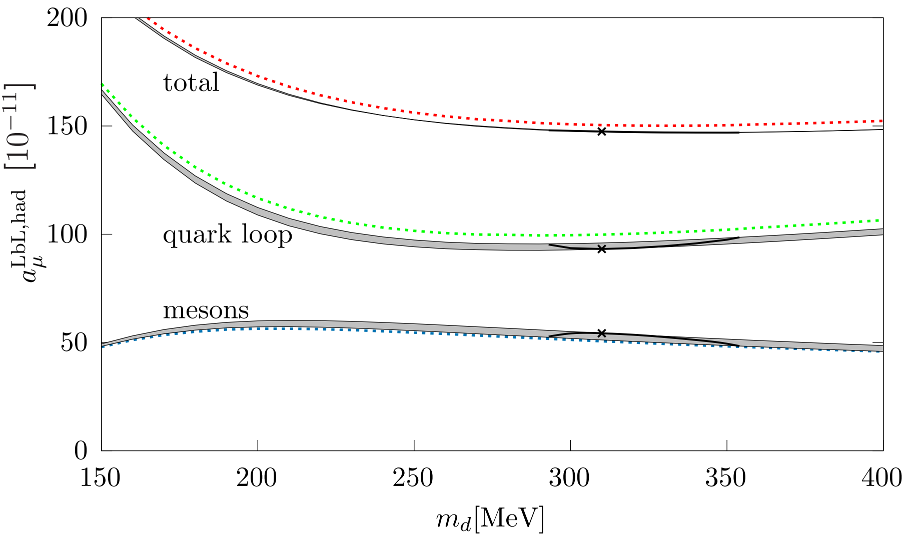
- π^0 meson
- scalar σ meson
- axial-vector a_1, f_1 mesons
- u, d quark loop
- Photons are dressed by intermediate $\rho - \omega$ mesons.

Model Parameters

Form-factor $f(k^2) = \exp(-k^2/\Lambda^2)$ in Euclidean space. Fixing parameters:

- mass and two-photon decay constant of the neutral pion
- strange contribution – estimations w/o spin-1 particles: fitting the K^0 mass and obtaining more or less reasonable values for the η meson mass and the $\eta \to \gamma\gamma$ decay width).
- estimations with spin-1 particles : ρ-meson mass for m_d between 293–354 MeV + calculation in wider region of m_d for fixed G_2/G_1 ratios: -0.08 and -0.14. Central value is taken at $m_d = 310$ MeV.

Quark loop+mesons



Lbl contribution. Total result

The result for LbL contribution in $SU(3)$ nonlocal model without spin-1 fields

$$a_\mu^{\mathrm{HLbL}} = 168(12.5) \times 10^{-11}$$

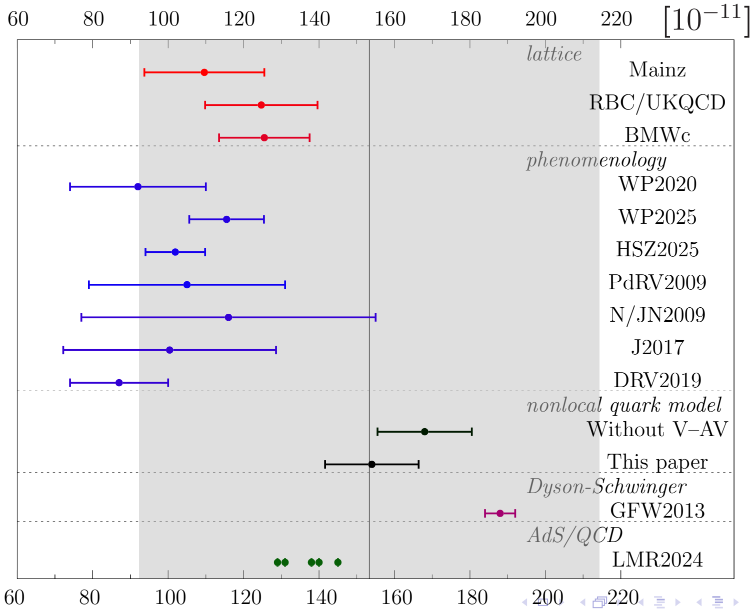
The result for LbL contribution in $SU(2)$ nonlocal model with spin-1 fields

$$a_\mu^{\mathrm{HLbL}} = 147(11.4) \times 10^{-11}$$

The result for LbL contribution in $SU(2)$ nonlocal model with spin-1 fields + strange

$$a_\mu^{\mathrm{HLbL}} = 154(12.4) \times 10^{-11}$$

Lbl contribution. Comparison



Results & Conclusions

- LbL contribution is estimated in $SU(2)$ model with scalar–pseudoscalar+ vector–axial-vector interactions
- model parameters are refitted due to $\pi - a_1$ mixing
- LbL contribution consist of contact term (box) + off–shell resonances
- contact term leads to correct LO OPE asymptotic (off–shell resonances do not contribute)
- dressing of photons by vector-mesons – tiny effect
- for strange contribution the estimation is taken from $SU(3)$ calculations without spin-1 mesons

Conclusions

- Our result for LbL contribution (154) is larger than WP2025 (103.3) or lattice (109.6–125.5) estimates
- With our result the window for something new (New Physics) becomes very small

$$a_\mu^{\text{Exp}} - (a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HVP,(NN)LO}}) = 153.3$$

- $1/N_c$ corrections ?

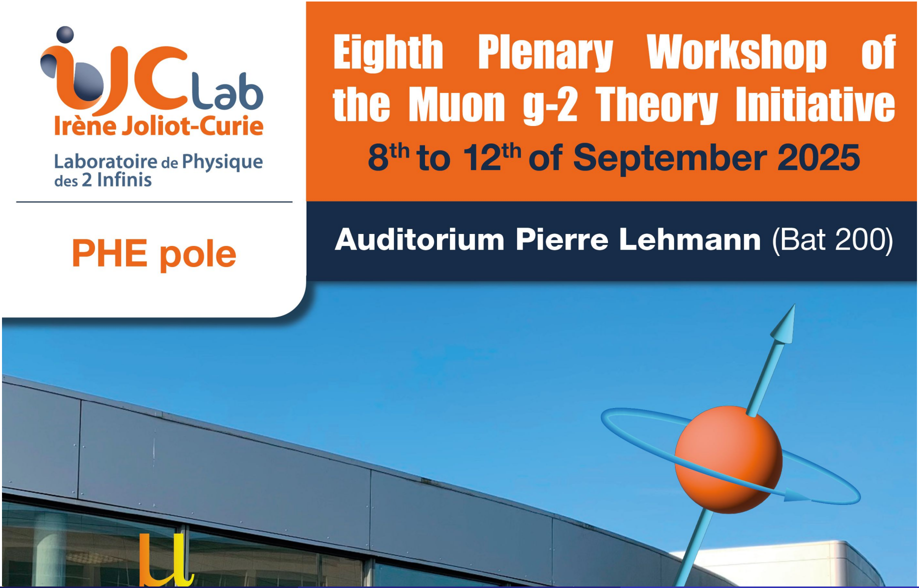
Open questions

- HVP from data-driven approach and lattice
- CMD-3 measurements $e^+e^- \to \pi^+\pi^-$ - larger cross-section than all previous experiments (including the most precise ones from BaBar, KLOE).
- α from electron anomalous magnetic moment and from atoms

Future

- J-PARC g-2/EDM experiment
- MUonE - Measuring the leading hadronic contribution to the muon g-2 via μe scattering
- New lattice calculations: HVP (different windows), LbL
- New data for data-driven HVP expected from BaBar, BES-III and Belle-II, SND

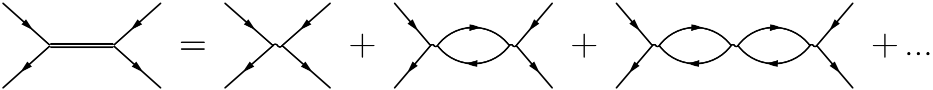
https://indico.ijclab.in2p3.fr/event/11652/



THANKS !

Backup slides

Mesons



On the other hand, the vertex functions and the meson masses can be found from the Bethe–Salpeter equation, which for the pion case is

$$\delta(p_1 + p_2 - p_3 - p_4)\frac{\overline{\mathbf{\Gamma}}^{\pi}_{p_1,p_3} \otimes \mathbf{\Gamma}^{\pi}_{p_2,p_4}}{p^2 - \mathrm{M}_{\pi}^2},$$

where p is the total momentum of the $\bar{q}q$ pair, $\overline{\mathbf{\Gamma}} = \gamma^0 \mathbf{\Gamma}^{\dagger} \gamma^0$ and p_i are quark momenta. The meson vertex functions without mixing in momentum space are

$$\mathbf{\Gamma}^{M\{;\mu\}}_{p_+,p_-} = g_M(p^2)\Gamma^{\{\mu\}}_{M} f(p_-) f(p_+),$$

where $p_{\pm}, k$ are the quark and meson momenta.

External currents

Due to nonlocality, the interactions with the electromagnetic gauge field should be introduced not only in the quark kinetic part but also in the nonlocal quark currents. The part of Lagrangian for meson fields with quark currents in the presence of external gauge fields

$$M^{a\{,\mu\}}(x)J_M^{a\{,\mu\}}(x)$$

where the Schwinger phase factor is attached to each quark field
$Q(x, x + x_2) = E(x, x + x_2)q(x + x_2)$ and
$\bar{Q}(x - x_1, x) = \bar{q}(x - x_1)E(x - x_1, x)$

$$E(x, y) = \mathrm{Pexp}\left\{-ie\mathrm{Q}\int\limits_x^y du_\mu G^\mu(u)\right\},$$

where G^μ is the photon field, e is the elementary charge and Q is the charge matrix of the quark fields.

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$$\int d^4x_1 d^4x_2\, f(x_1)f(x_2)\, \bar{Q}(x-x_1,x)\, M^{a\{,\mu\}}(x)\Gamma_M^{a\{,\mu\}}\, Q(x,x+x_2),$$

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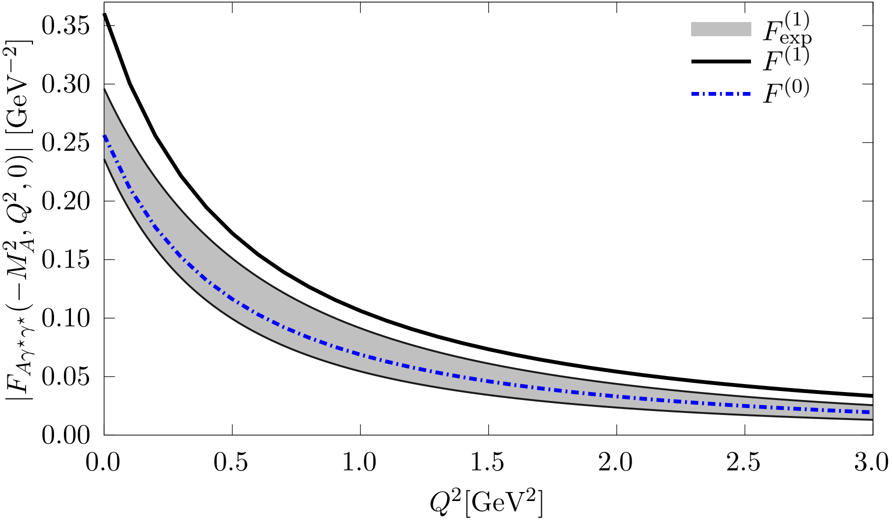
External currents

The Ward identity only fixes the longitudinal part of photon vertices, and to find an expression for the transverse part of vertices one needs to specify rules for the contour integral. Two possible ways:

- the straight-path ansatz $z^\mu = x^\mu + \alpha(y^\mu - x^\mu)$, $0 \leq \alpha \leq 1$.
- Rule for derivative of contour integral: derivative does not depend on the form of the path and the explicit form of the path is not important

$$\frac{\partial}{\partial y^\mu} \int_x^y dz_\nu G^\nu(z) = G_\mu(y), \quad \delta^{(4)}(x-y) \int_x^y dz_\nu G^\nu(z) = 0.$$

AV transitions form-factors. L3 collaboration



$$\tilde{\Gamma}_{\gamma\gamma}(A) = 3.5 \pm 0.6 \pm 0.5\,\text{keV}, \quad \Lambda_{dip} = 1.04 \pm 0.06 \pm 0.05\,\text{GeV}.$$

AV form-factor

Gauge invariance leads to the relations

$$A_2(p^2,q_1^2,q_2^2) = (q_1\cdot q_2)A_6(p^2,q_1^2,q_2^2) + q_1^2 A_5(p^2,q_1^2,q_2^2),$$
$$A_1(p^2,q_1^2,q_2^2) = (q_1\cdot q_2)A_3(p^2,q_1^2,q_2^2) + q_2^2 A_4(p^2,q_1^2,q_2^2),$$

and the Bose symmetry results in:

$$A_1(p^2,q_1^2,q_2^2) = -A_2(p^2,q_2^2,q_1^2), \quad A_3(p^2,q_1^2,q_2^2) = -A_6(p^2,q_2^2,q_1^2),$$
$$A_4(p^2,q_1^2,q_2^2) = -A_5(p^2,q_2^2,q_1^2).$$

The part of the amplitude longitudinal to the meson momentum is

$$\Delta^{\mu\nu}_{A,\alpha;L}(p,q_1,q_2) = i\varepsilon^{\rho\sigma\mu\nu} q_{1\rho} q_{2\sigma} \frac{p_\alpha}{p^2}\Big(A_2(p^2,q_1^2,q_2^2) - A_1(p^2,q_1^2,q_2^2)\Big).$$

AV form-factor.The transverse part of the amplitude[16]

$$\Delta^{\mu\nu}_{A,\alpha}(p,q_1,q_2) = i\varepsilon_{\rho\sigma\tau\alpha}\Big\{$$

$$R^{\mu\rho}_{q_1,q_2} R^{\nu\sigma}_{q_1,q_2} (q_1-q_2)^\tau (q_1\cdot q_2) F^{(0)}_{A\gamma^*\gamma^*}(p^2,q_1^2,q_2^2)$$

$$+ R^{\nu\rho}_{q_1,q_2} Q^\mu_1 q_1^\sigma q_2^\tau \; F^{(1)}_{A\gamma^*\gamma^*}(p^2,q_1^2,q_2^2)$$

$$+ R^{\mu\rho}_{q_1,q_2} Q^\nu_2 q_2^\sigma q_1^\tau \; F^{(1)}_{A\gamma^*\gamma^*}(p^2,q_2^2,q_1^2)\Big\},$$

$$R^{\mu\nu}_{q_1,q_2} = -g^{\mu\nu} + \frac{1}{X}\left\{(q_1\cdot q_2)(q_1^\mu q_2^\nu + q_2^\mu q_1^\nu) - q_1^2 q_2^\mu q_2^\nu - q_2^2 q_1^\mu q_1^\nu\right\},$$

$$Q^\mu_1 = q_1^\mu - \frac{q_2^\mu q_1^2}{(q_1\cdot q_2)},\; Q^\nu_2 = q_2^\nu - \frac{q_1^\nu q_2^2}{(q_1\cdot q_2)},\; X = (q_1\cdot q_2)^2 - q_1^2 q_2^2,$$

where $R^{\mu\nu}_{q_1,q_2}$ is the totally transverse tensor, Q^μ_1 and Q^ν_2 are transverse with respect to q_1 and q_2.

[16] V.Pascalutsa, V.Pauk, M. Vanderhaeghen, PRD85(2012)116001

AV form-factor

The other set of Lorentz structures is suggested for studying the asymptotic behavior of meson transition form factors from a light-cone expansion[17]

$$\mathcal{F}_1^A = M_A^2(A_3 + A_6)/2,$$
$$\mathcal{F}_2^A = -M_A^2(A_3 + A_5),$$
$$\mathcal{F}_3^A = -M_A^2(A_4 + A_6).$$

According to the Landau–Yang theorem, the axial-vector mesons cannot decay into two real photons. However, the coupling of 1^{++} mesons to two photons is allowed if one or both photons are virtual. The two-photon "decay" width for axial-vector mesons is defined for a quasireal longitudinal photon and a real photon as

$$\tilde{\Gamma}_{\gamma\gamma}(A) = \lim_{Q^2\to 0} \frac{1}{2}\frac{M_A^2}{Q^2}\Gamma\left(A \to \gamma_T\gamma_L^*\right) = \frac{\pi\alpha^2 M_A^5}{12}[F^{(1)}_{A\gamma^*\gamma^*}(M_A^2, 0, 0)]^2.$$

[17] M. Hoferichter, P. Stoffer, JHEP05(2020)159.

Model Parameters

Form-factor $f(k^2) = \exp(-k^2/\Lambda^2)$ in Euclidean space. For fixing parameters: mass and two-photon decay constant of the neutral pion + ρ-meson mass $M_\rho = 775.26$ MeV

m_d, MeV	m_c, MeV	Λ, MeV	$G_1\Lambda^2$	$G_2\Lambda^2$	m^2_{pole}, GeV^2	M_{thr}
293	7.12	1066.5	34.808	−3.00	0.152; 0.489	780
300	7.28	1045.5	35.357	−4.36	0.180; 0.404	848
310	7.72	1004.8	36.297	−5.02	0.245 ± 0.091	1006
320	8.24	963.1	37.333	−5.20	0.202 ± 0.152	954
330	8.82	922.3	38.453	−5.09	0.164 ± 0.180	902
340	9.48	882.5	39.665	−4.72	0.131 ± 0.194	854
350	10.24	842.5	40.999	−3.98	0.101 ± 0.200	806
354	10.64	824.2	41.615	−3.33	0.088 ± 0.201	784

$$k^2 + m^2(k^2) = 0, \quad k^2 = -m^2_{\text{pole}}$$

$$\mathrm{M}^2_{\text{thr}} = 2\mathrm{Re}(m^2_{\text{pole}}) + 2\sqrt{\mathrm{Re}(m^2_{\text{pole}})^2 + \mathrm{Im}(m^2_{\text{pole}})^2}$$

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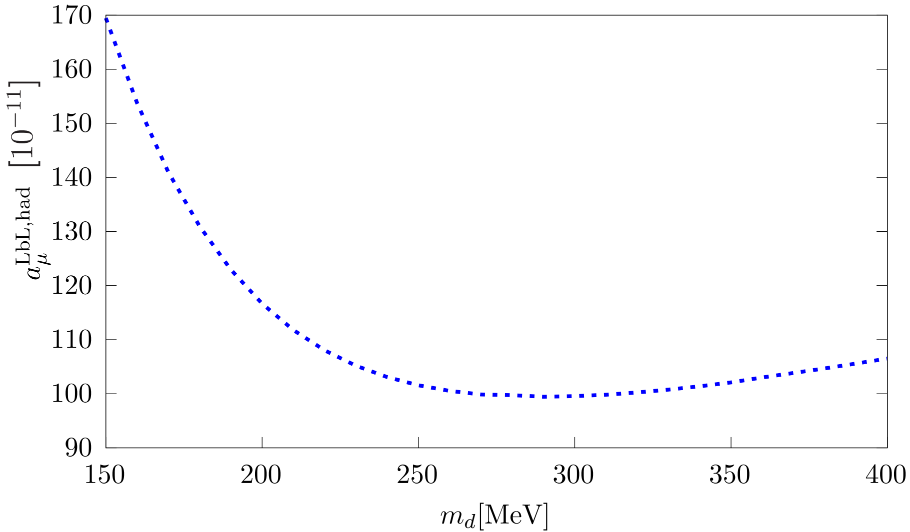
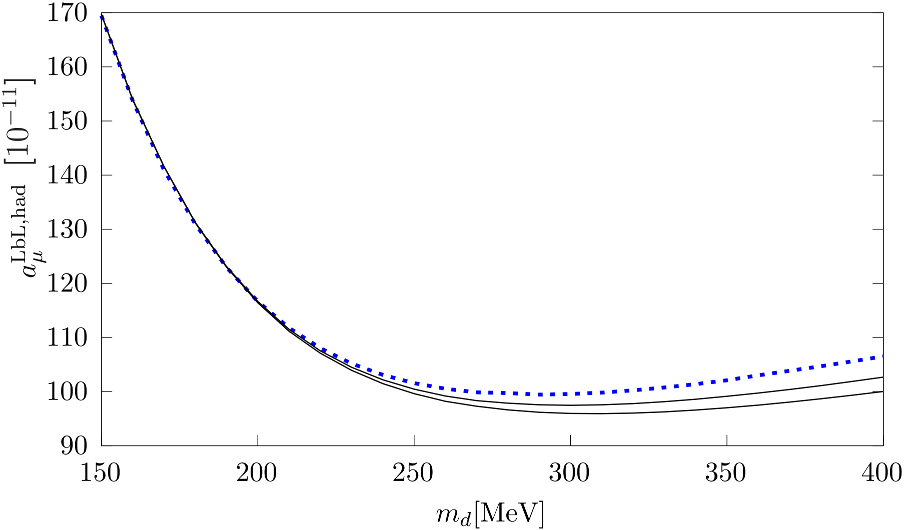
central value is taken at $m_d = 310$ MeV as it has the maximal $\mathrm{M_{thr}}$. The axial-vector meson mass in the model for this parameter set is found to be $M_A = 918$ MeV.

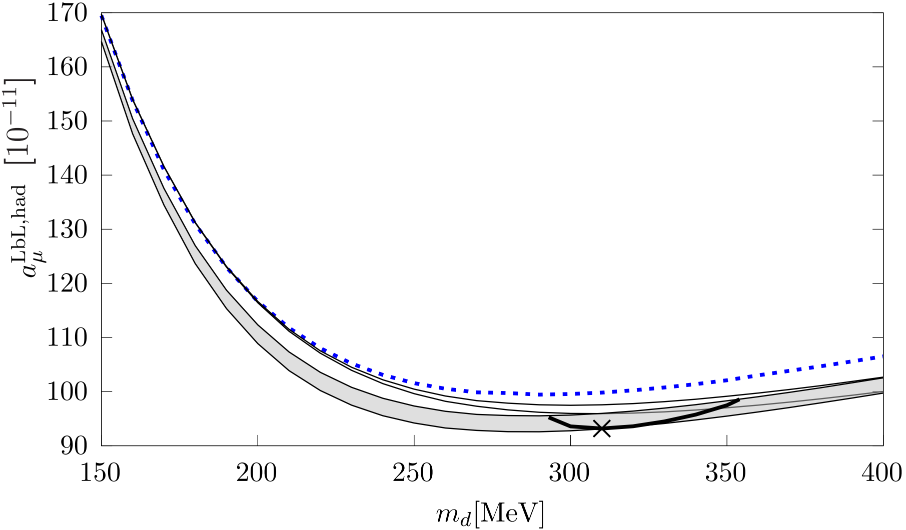
How big is vector meson dressing ? u, d loop w/o spin-1



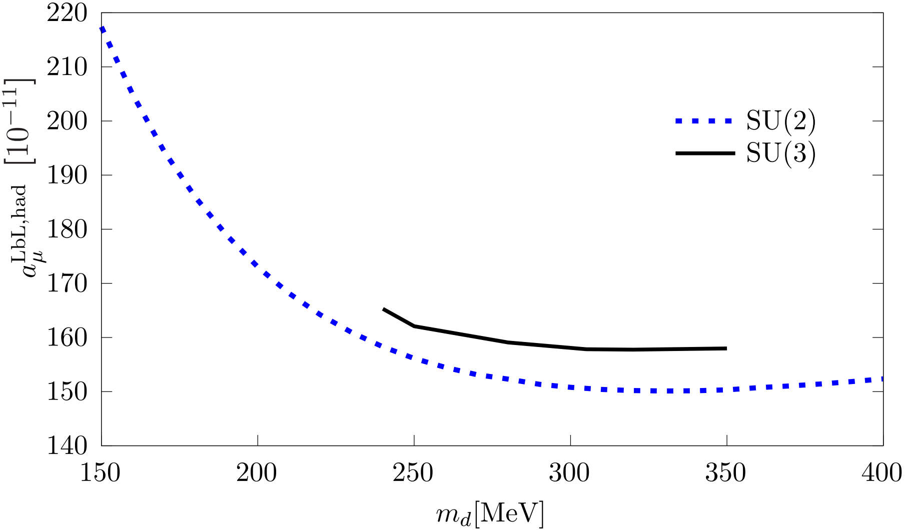
How big is vector meson dressing ? u, d loop $\pi - a_1$



How big is vector meson dressing ? u, d loop with spin-1



Strange contribution w/o spin-1



Mesons

Meson propagators can be obtained by taking quadratic terms over the meson field from the Lagrangian at one loop level. For spin-0 mesons, the unrenormalized propagators are

$$\mathrm{D}_M(p^2) = \frac{1}{-G_1^{-1} + \Pi_{MM}(p^2)} = \frac{g_M^2(p^2)}{p^2 - M_M^2}$$

After redefinition of the meson fields, the spin-0 propagator has the usual form

$$\mathrm{D}_M^R(p^2) = \mathrm{D}_M(p^2)/g_M^2(p^2) = (p^2 - M_M^2)^{-1}.$$

The quark polarization loops are

$$\Pi_{M_1M_2}(p^2) = i\int \frac{d^4k}{(2\pi)^4} f^2(k_+^2) f^2(k_-^2)\,\mathrm{Tr}_{c,d,f}\left[\mathrm{S}(k_-)\Gamma^a_{M_1}\mathrm{S}(k_+)\Gamma^b_{M_2}\right],$$

where $k_\pm = k \pm p/2$ and the trace is taken over color, Dirac and flavour matrices.

Mesons

Quark loops and propagators of vector and axial-vector mesons should be split into longitudinal and transverse parts

$$\mathrm{D}_M^{\alpha\beta}(p^2) = \mathrm{D}_M^{\mathrm{T}}(p^2)\mathrm{P}_p^{\mathrm{T};\alpha\beta} + \mathrm{D}_M^{\mathrm{L}}(p^2)\mathrm{P}_p^{\mathrm{L};\alpha\beta},$$

with the help of appropriate projectors

$$\mathrm{P}_p^{\mathrm{T};\alpha\beta} = g^{\alpha\beta} - \frac{p^\alpha p^\beta}{p^2},\ \mathrm{P}_p^{\mathrm{L};\alpha\beta} = \frac{p^\alpha p^\beta}{p^2}.$$

Transverse components correspond to spin-1 states and unrenormalised propagators are

$$\mathrm{D}_{V,A}^{\mathrm{T}}(p^2) = \frac{1}{-G_2^{-1} + \Pi_{VV,AA}^{\mathrm{T}}(p^2)} = \frac{g_{V,A}^2(p^2)}{M_{V,A}^2 - p^2}.$$

Renormalized propagators are $\mathrm{D}_M^{\mathrm{T};R}(p^2) = \mathrm{D}_M^{\mathrm{T}}(p^2)/g_M^2(p^2)$

Mesons

Longitudinal components of spin-1 mesons are related to spin-0. In the case of a system of pseudoscalar–axial-vector states, a mixing appears due to a quark polarisation loop with pseudoscalar and axial-vector vertices

$$\Pi^{\mu}_{PA}(p^2) = p^{\mu}\Pi_{\pi a_1}(p^2),$$

and physical states can be found as solutions of the matrix equation for $\pi - a_1$ system. One can represent the mixing as a modification of the pion vertex with the contribution of the longitudinal component of the axial-vector mesons.