Essence of Charge and Spin with General Covariant Dirac Equations

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In the framework of the general covariant field-theoretical approach, new representations of natural Time and its duality give an explanation for the existence of nuclei and open up quite new possibilities to puzzle out their internal structure if it is explained why charge and spin exist. To solve this problem, general covariant Dirac equations are established, which connect the essence of spin with gravity and the essence of charge with natural Time. Thus, the general covariant Dirac equations provide a reliable basis for the theory of matter at the fundamental field-theoretical level and an explanation of the phenomena associated with the existence of nuclei.

В общековариантном теоретико-полевом подходе, новые представления о естественном времени и его дуальности дают объяснение существования ядер и тем самым открывают совершенно новые возможности для выяснения их внутренней структуры, требуя при этом объяснения существования заряда и спина. Чтобы решить эту проблему, устанавливаются общековариантные уравнения Дирака, которые связывают сущность спина с гравитацией, а сущность заряда с естественным временем. Таким образом, на фундаментальном, теоретико-полевом уровне общековариантные уравнения Дирака обеспечивают надежную основу для теории материи и объяснения явлений, связанных с существованием ядер.

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Motivation for our study

First of all, we should like to explain the meaning of a combination of words "general covariant" and, hence, very shortly formulate the central tenets of a general covariant field-theoretical approach to physical reality. In accordance with the seminal work of Einstein and Grossmann [1] and the principle recommending that the simplest explanation is the one with the fewest assumptions and entities known as Occam's razor, general covariance means that a function of multiple variables is an elementary structure object. This means that to formulate laws of Nature, i.e.visible and invisible world, it is necessary to use only systems of functions with the known law of transformation under the transition from the initial system of coordinates to any most general one. The last demand is very important to avoid subdivisions and have a bird's eye point of view. Such systems of functions are called general covariant fields, and in what follows, we will only operate with fields of this kind. In common physics, the term "field" is tied to physical reality that expands all over the intuitive space or its region. We mean a space that cannot be separated from our vision. Hence, we accept that the unity

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of Nature is most exactly disclosed by the real-number continuum and the faultlessly beautiful apparatus of mathematical analysis which is considered as the only basis for more complicated constructions. Thus, the general covariant field-theoretical approach is characterized by the minimum of things that are borrowed from observers. We move away all other achievements of observers to the horizon for the time being.

In the framework of this approach, we suppose that a definite system of fundamental general covariant fields forms one whole and is characterized by a very restricted number of parameters. Each state of this system of fields with known distribution of energy and charges and the corresponding integral characteristics is considered as a field-theoretical model of particles and, in particulary, nuclei. As an example, we can consider the closed and self-consistence Maxwell-Dirac system of equations that is characterized by the dimensionless parameter known as a coupling constant and the parameter of the dimension of length known as Compton's length. The integral characteristics of this system are energy and charge, but it is evidently inappropriate for explaining why charge exists. In addition,we can as well remark that in the framework of the general covariant field-theoretical approach, the well-known Pauli and Dirac matrices cannot be used. In view of this, we consider the original Dirac equation as a good point to serve as a rough guide. We believe that the general covariant field-theoretical approach, like microscopes, telescopes and many other devices, essentially enlarges the possibilities of our vision without special expenditures and artificial assumptions.

Space and time

To explain why charge and spin exist let us remind the definition and some main properties of natural Time presented at the Baldin ISHEPP-25 [2]. The general covariant field-theoretical approach is formalized by the introducing a reference space R^4 in which a point is defined as a 4-tuple of real numbers $x = (x^1, x^2, x^3, x^4), \quad -\infty < x^i < \infty$. Independent variables x^1, x^2, x^3, x^4 are considered on an absolutely equal footing and represent the initial system of coordinates. In accordance with [2], we consider four independent variables.

Definition: a moment of natural Time is a number that we put into correspondence to any point of the reference space R^4. Hence, a moment of natural Time t can be found as follows:

$$t = \chi(x^1, x^2, x^3, x^4) = \chi(x),$$

where $\chi(x)$ is a known function of the variables (x^1, x^2, x^3, x^4), i.e.the scalar field.

All points of the reference space that correspond to the same moment of natural Time t constitute physical space $S(t)$ or isochrone. An isochrone is a three–dimensional surface in R^4 which is defined by the equation

$$\chi(x^1, x^2, x^3, x^4) = \chi(x) = t = constant.$$

The gradient of natural Time is the vector field $\mathbf{t}$ with the components

$$t^i = (\nabla\chi)^i = f^{ij}\partial_j\chi = f^{ij}t_j,$$

where f^{ij} are the contravariant components of the symmetric tensor field $f_{ij}(x)$ that defines a positive definite general covariant scalar product in the linear space of the vector fields

$$(u|v) = f_{ij}u^iv^j, \quad (u|u) = f_{ij}u^iu^j \geq 0, \quad (u|u) = 0$$

if and only if $u^i = 0$. From a physical point of view f_{ij} is the potential of the gravitational field. It can be shown that f_{ij} is the very field that allows one to formulate the true laws of Nature itself in a general covariant form. This demonstrates why gravity exists in general and as something universal.

The gradient of natural Time defines fundamental general covariant bilateral symmetry(discrete internal symmetry of left and right). The connection between the right–hand sided and left–hand sided vector fields is represented as a linear transformation (reflection)

$$\overline{v}^i = R^i_jv^j, \quad R^i_j = 2t^it_j - \delta^i_j, \quad R^i_kR^k_j = \delta^i_j, \quad \mathrm{Det}(R^i_j) = -1.$$

The t^i and f_{ij} are invariant under reflection since $R^i_jt^j = t^i, \quad R^i_kR^j_lf_{ij} = f_{kl}$. We consider bilateral symmetry as fundamental discrete internal symmetry of Nature and a strict method for constructing dynamical theory in the framework of the general covariant field-theoretical approach.

The potential $\chi(x)$ of natural Time is a solution to the equation

$$f^{ij}\frac{\partial\chi}{\partial x^i}\frac{\partial\chi}{\partial x^j} = 1, \tag{1}$$

which can be considered as a condition that singles out the temporal scalar field $\chi(x)$ from the manifold of others [2]. The fundamental observation (from a physical point of view) reads that equation (1) has not only a general solution but also a special solution known as the geodesic distance function. This means that there are two different Times in Nature and, hence, two different kinds of natural dynamical processes. An important example: under the condition $f_{ij} = \delta_{ij}$ equation (1) has the following two solutions:

$$\chi(x) = a_ix^i, \quad a_ia^i = 1, \quad \chi(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}.$$

We consider the first solution as the natural Time of translation motion and the second one as the natural Time of rotational motion. Known nuclei are considered as different states of the system of fundamental fields that form one whole and live in the natural Time of rotational motion. One of these key fields is the main subject of our consideration.

Spin as emergent property

We put forward an idea that charge and spin are emergent properties of the system of irreducible fields and our task here is to represent this system and get this surprise. An emergent property is a characteristic of a complex system that is not present in its individual components but arises from their interactions and relationships. These properties are "more than the sum of their parts," meaning that they cannot be deduced by simply examining the qualities of individual elements. One technical example. An aircraft can fly but all its details can only fall. The idea of emergent properties is not new (see, for example Baldin's and Laughlin's works [3],[4]).

Let us consider an ordered tuple of the following general covariant real fields:

$$\mathbf{A}(\mathbf{x}) = \Big(a(x),\ a_i(x),\ a_{ij}(x),\ a_{ijk}(x),\ a_{ijkl}(x),\Big) \quad i,j,k,l = 1,2,3,4, \quad (2)$$

where a is the scalar field, a_i is the covector field and further antisymmetric tensor fields. We have not sufficient reasons to introduce complex fields. Now we can question about internal relationships of these fields to form a complex system. These relationships exist in the form of general covariant operators T that transform the tuple (2) into the tuple $\overline{\mathbf{A}} = \mathrm{T}\mathbf{A}(\mathbf{x})$, where

$$\overline{\mathbf{A}} = (\overline{a}, \overline{a}_i, \overline{a}_{ij}, \overline{a}_{ijk}, \overline{a}_{ijkl}), \quad (i,j,k,l = 1,2,3,4)$$

and the general element of this tuple $\overline{a}_{j_1\cdots j_q}, \quad (q = 0,1,2,3,4)$ is defined by antisymmetrical tensor fields $T^{i_1\cdots i_p}_{j_1\cdots j_q}, \quad (p,q = 0,1,2,3,4)$, as follows

$$\overline{a}_{j_1\cdots j_q} = T^{i_1\cdots i_p}_{j_1\cdots j_q} a_{i_1\cdots i_p}, \quad (p,q = 0,1,2,3,4).$$

The general covariant identical operator E is defined by the following natural conditions:

$$E^{i_1\cdots i_p}_{j_1\cdots j_q} = 0, \quad \text{if} \quad p \neq q, \quad E^{i_1\cdots i_p}_{j_1\cdots j_p} = \delta^{i_1\cdots i_p}_{j_1\cdots j_p},$$

where $\delta^{i_1\cdots i_p}_{j_1\cdots j_p}$ is the generalized Kronecker delta. These general covariant finite transformations of the internal symmetry form the algebraic structure known as the ring and are inseparable from the field representation since they concern field functions and do not affect the coordinates. As one can see, the set of general covariant fields (2) forms a system but this is not enough to recognize its emergent properties. However, instead of finite transformations we can consider the infinitesimal transformations of this kind since it is well-known that the operators of infinitesimal transformations have fundamental meaning in quantum theory, where these operators are defined by the vector fields. For example, from the theory of Lie groups it follows that the operators of angular momenta may be considered as vector fields associated with the group of rotation.

To this end, let us consider the natural general covariant algebraic operators $\overline{\mathbf{A}} = \mathrm{I}_{\mathbf{v}}\mathbf{A}$ and $\overline{\mathbf{A}} = \mathrm{E}_{\mathbf{v}}\mathbf{A}$ defined by the vector field v^i as follows:

$$\mathrm{I}_{\mathbf{v}}\mathbf{A} = (v^m a_m,\ v^m a_{mi},\ v^m a_{mij},\ v^m a_{mijk},\ 0),$$

$$\mathrm{E}_{\mathbf{v}}\mathbf{A} = (0,\, v_i a,\, v_{[i}a_{j]},\, v_{[i}a_{jk]},\, v_{[i}a_{jkl]}),$$

where the square brackets $[\cdots]$ denote the process of alternation and $v_i = f_{ij}v^j$. To complete, let us introduce the numerical operator Z that is defined as follows $\mathrm{Z}\mathbf{A} = (a, -a_i, a_{ij}, -a_{ijk}, a_{ijkl})$.

Now we can introduce the fundamental operators:

$$Q_{\mathbf{v}} = \mathrm{E}_{\mathbf{v}} - \mathrm{I}_{\mathbf{v}}, \quad \widetilde{Q}_{\mathbf{v}} = (\mathrm{E}_{\mathbf{v}} + \mathrm{I}_{\mathbf{v}})\mathrm{Z}$$

for which the very important relations

$$Q_{\mathbf{v}}Q_{\mathbf{w}} + Q_{\mathbf{w}}Q_{\mathbf{v}} = -2(\mathbf{v}|\mathbf{w}), \quad \widetilde{Q}_{\mathbf{v}}\widetilde{Q}_{\mathbf{w}} + \widetilde{Q}_{\mathbf{w}}\widetilde{Q}_{\mathbf{v}} = -2(\mathbf{v}|\mathbf{w}), \quad \widetilde{Q}_{\mathbf{v}}Q_{\mathbf{w}} = Q_{\mathbf{w}}\widetilde{Q}_{\mathbf{v}}$$

are fulfilled at any vector fields $\mathbf{v}$ and $\mathbf{w}$. The commuting operators $Q_{\mathbf{v}}$ and $\widetilde{Q}_{\mathbf{v}}$ correspond to Dirac matrices in the framework of the general covariant field-theoretical approach. Hence, general covariant representations of spin at the fundamental field-theoretical level can be introduced as follows. Let us consider three orthogonal and unit vector fields $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$, and put

$$S_1 = \frac{i}{2}Q_{\mathbf{v}\wedge\mathbf{w}}, \quad S_2 = \frac{i}{2}Q_{\mathbf{w}\wedge\mathbf{u}}, \quad S_3 = \frac{i}{2}Q_{\mathbf{u}\wedge\mathbf{v}},$$

$$\widetilde{S}_1 = \frac{i}{2}\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}}, \quad \widetilde{S}_2 = \frac{i}{2}\widetilde{Q}_{\mathbf{w}\wedge\mathbf{u}}, \quad \widetilde{S}_3 = \frac{i}{2}\widetilde{Q}_{\mathbf{u}\wedge\mathbf{v}},$$

where $2Q_{\mathbf{v}\wedge\mathbf{w}} = Q_{\mathbf{v}}Q_{\mathbf{w}} - Q_{\mathbf{w}}Q_{\mathbf{v}}$ and the same for $\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}}$. One can verify that for these operators the following relations are fulfilled:

$$[S_j,\, S_k] = i\varepsilon_{jkl}S_l, \quad S_1^2 + S_2^2 + S_3^2 = s(s+1) = \frac{3}{4}, \tag{3}$$

$$[\widetilde{S}_j,\, \widetilde{S}_k] = i\varepsilon_{jkl}\widetilde{S}_l, \quad \widetilde{S}_1^2 + \widetilde{S}_2^2 + \widetilde{S}_3^2 = \frac{3}{4}, \quad [S_j,\, \widetilde{S}_k] = 0. \tag{4}$$

Thus, our tuple (2) provides the general covariant field-theoretical realization of the concept of Top of classical mechanics as an emergent property, which is why it can be called a general covariant spinor field or shortly a S-field. We see that the constructive idea of Kronig, Goudsmit and Uhlenbek of spin as a Top can be naturally realized only in the framework of the general covariant field-theoretical approach since, as mentioned above, the general covariant internal symmetry in question is inseparable from the field representations. Thus, the internal symmetry in question is related with phenomena that cannot be described based on classical concepts. We also pay attention to a deep connection between the gravity and concept of spin since the operators $Q_{\mathbf{v}}$ and $\widetilde{Q}_{\mathbf{v}}$ cannot be defined without the gravitational potential f_{ij}. We can say that spin exists since the gravity exists. This is not a surprise because Nature is one whole from the point of view in question. Now our goal is to show that charge exists because natural Time exists. To this end, let us give a general representation of the general covariant Dirac equations for the S-field.

General Covariant Dirac Equations

As in the case of the electromagnetic field [2], bilateral symmetry provides a natural way to formulate dynamical general covariant equations for the S-field. We define that left and right S-fields $\overline{\mathbf{A}}$ and $\mathbf{A}$ are connected by the transformation

$$\overline{\mathbf{A}} = R\mathbf{A} = -Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}\mathbf{A}, \quad R^2 = 1,$$

which can be derived from the definition of bilateral symmetry given above. Simple algebra shows that since for the imaginary unit $i^2 = -1$, then

$$R = -Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}} = i^2 Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}} = (iQ_{\mathbf{t}})(i\widetilde{Q}_{\mathbf{t}}).$$

We conclude that for the complex S-fields $\mathbf{\Psi} = \mathbf{A} + i\mathbf{B}$, $\overset{*}{\mathbf{\Psi}} = \mathbf{A} - i\mathbf{B}$ bilateral symmetry can be realized as two different transformations of the form

$$\overline{\mathbf{\Psi}} = R\mathbf{\Psi} = iQ_{\mathbf{t}}\mathbf{\Psi}, \quad R^2 = 1; \quad \overline{\mathbf{\Psi}} = R\mathbf{\Psi} = i\widetilde{Q}_{\mathbf{t}}\mathbf{\Psi}, \quad R^2 = 1.$$

In view of this, we at first give a general representation of the dynamical theory of a real S-field and then consider the complex case.

To formulate the Lagrange formalism, we introduce the positive definite bilinear quadratic form $(\mathbf{A}|\mathbf{B})$ for the S-fields as follows:

$$(\mathbf{A}|\mathbf{B}) = \sum\nolimits_{p=0}^{4} \frac{1}{p!} a_{i_1\cdots i_p} b^{i_1\cdots i_p}, \quad b^{i_1\cdots i_p} = f^{i_1 j_1}\cdots f^{i_p j_p} b_{j_1\cdots j_p}.$$

The operators $Q_{\mathbf{v}}$ and $\widetilde{Q}_{\mathbf{v}}$ are anti self-conjugated with respect to this scalar product since $(\mathbf{A}|Q_{\mathbf{v}}\mathbf{B}) = -(Q_{\mathbf{v}}\mathbf{A}|\mathbf{B})$ $(\mathbf{A}|\widetilde{Q}_{\mathbf{v}}\mathbf{B}) = -(\widetilde{Q}_{\mathbf{v}}\mathbf{A}|\mathbf{B})$. We introduce the connection

$$W^i_{jk} = \Gamma^i_{jk} + t^i(\partial_j t_k - \Gamma^l_{jk} t_l) - t_k(\partial_j t^i + \Gamma^i_{jl} t^l)$$

with the following remarkable properties:

$$\nabla_i t_j = 0, \quad \nabla_i f_{jk} = 0, \quad \nabla_i e_{jklm} = 0,$$

where ∇_i is the covariant derivative with respect to the connection W^i_{jk}, Γ^i_{jk} is the Levi-Civita connection of f_{ij} and e_{ijkl} is the Levi-Civita tensor field with the essential component $e_{1234} = \sqrt{f}, \quad f = \mathrm{Det}(f_{ij}) > 0$.

We define the main differential operator Π of the dynamical theory of the S-field as follows:

$$(\Pi\mathbf{A})_{i_1\cdots i_p} = p\nabla_{[i_1} a_{i_2\cdots i_p]} + \nabla^k a_{k i_1\cdots i_p} - 2t^k \nabla_{\mathbf{t}} a_{k i_1\cdots i_p}, \quad p = 0, 1, 2, 3, 4,$$

where $\nabla_{\mathbf{t}} = t^k \nabla_k$. For $p = 2$ we have

$$(\Pi\mathbf{A})_{ij} = \nabla_i a_j - \nabla_j a_i + \nabla^k a_{kij} - 2t^k \nabla_{\mathbf{t}} a_{kij}.$$

We consider the Lagrangian of the real S-field

$$\mathcal{L}_{\mathbf{A}} = \frac{1}{2}(\overline{\mathbf{A}}|\Pi\mathbf{A}) - \frac{m}{2}(\overline{\mathbf{A}}|\mathbf{A}),$$

where $\overline{\mathbf{A}} = -Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}\mathbf{A}$. The Lagrangian $\mathcal{L}_{\mathbf{A}}$ is invariant with respect to the transformations

$$\mathbf{A}' = \frac{\Lambda}{|\Lambda|}\mathbf{A}, \quad \Lambda = \Lambda_1 + \Lambda_2\widetilde{Q}_{\mathbf{t}}, \quad |\Lambda|^2 = {\Lambda_1}^2 + {\Lambda_2}^2.$$

The operator $\widetilde{Q}_{\mathbf{t}}$ defined by the gradient of natural Time is the operator of the electric charge that provides the law of conservation of the electric charge in the case of real S-field.

Further, we consider the operator $\widetilde{H}$ defined by the tensor field e_{ijkl} as follows:

$$\widetilde{H}\mathbf{A} = (\frac{1}{4!}e_{ijkl}a^{ijkl},\ -\frac{1}{3!}e_{ijkl}a^{jkl},\ -\frac{1}{2!}e_{ijkl}a^{kl},\ e_{ijkl}a^{l},\ e_{ijkl}a)$$

and introduce the operator $\widetilde{J} = -\widetilde{H}Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}$. The following relations are fulfilled:

$$\widetilde{J}^2 = -1, \quad \widetilde{J}\widetilde{Q}_{\mathbf{t}} + \widetilde{Q}_{\mathbf{t}}\widetilde{J} = 0, \quad \Pi\widetilde{J} - \widetilde{J}\Pi = 0;$$

however, the Lagrangian $\mathcal{L}_{\mathbf{A}}$ is not invariant under the transformations $\mathbf{A}' = \Lambda/|\Lambda|\mathbf{A}, \quad \Lambda = \Lambda_1 + \Lambda_2\widetilde{J}, \quad |\Lambda|^2 = {\Lambda_1}^2 + {\Lambda_2}^2$, but for the complex S-field this is the case.

The fundamental Lagrangian of the complex S-field takes the form

$$\mathcal{L}_{\mathbf{\Psi}} = \frac{1}{2}(\overline{\mathbf{\Psi}}|\Pi\mathbf{\Psi}) - \frac{1}{2}(\Pi\overline{\mathbf{\Psi}}|\mathbf{\Psi}) - m(\overline{\mathbf{\Psi}}|\mathbf{\Psi}),$$

where $\overline{\mathbf{\Psi}} = -iQ_{\mathbf{t}}\overset{*}{\mathbf{\Psi}}$. This Lagrangian is invariant under the transformations $\mathbf{\Psi}' = \widetilde{\Lambda}/|\widetilde{\Lambda}|\mathbf{\Psi}$, where

$$\widetilde{\Lambda} = \widetilde{\Lambda}_1 + \widetilde{\Lambda}_2\widetilde{I} + \widetilde{\Lambda}_3\widetilde{J} + \widetilde{\Lambda}_4\widetilde{K},\ |\widetilde{\Lambda}|^2 = \widetilde{\Lambda}_1^2 + \widetilde{\Lambda}_2^2 + \widetilde{\Lambda}_3^2 + \widetilde{\Lambda}_4^2,\ \widetilde{I} = \widetilde{Q}_{\mathbf{t}},\ \widetilde{K} = \widetilde{I}\widetilde{J}$$

The general covariant operators E, $\widetilde{I}$, $\widetilde{J}$, $\widetilde{K}$ represent the realization of quaternion algebra. The operator $\widetilde{J}$ is tightly connected with the concept of orientation and, hence, parity non conservation. We will call this operator the operator of dual electrical charge that defines dual positive and negative electromagnetism. The algebra of self-conjugated operators of the electric charge $Q_e = i\widetilde{Q}_{\mathbf{t}}$ and dual electrical charge $Q_\nu = i\widetilde{J}$ is very simple

$${Q_e}^2 = 1, \quad {Q_\nu}^2 = 1, \quad Q_e\,Q_\nu + Q_\nu\,Q_e = 0.$$

With this algebra we can prepare a set of states of the complex S-field. Let us introduce the projection operators:

$$P_+ = \frac{1}{2}(\mathrm{E} + Q_e), \quad P_- = \frac{1}{2}(\mathrm{E} - Q_e), \quad R_+ = \frac{1}{2}(\mathrm{E} + Q_\nu), \quad R_- = \frac{1}{2}(\mathrm{E} - Q_\nu).$$

For the identity operator we have the following expansions:

$$\mathrm{E} = P_+R_+ + P_+R_- + P_-R_+ + P_-R_-, \quad \mathrm{E} = R_+P_+ + R_+P_- + R_-P_+ + R_-P_-.$$

Hence, the complex S-field can be in four different states:

$$\mathbf{\Psi}_{++} = P_+R_+\mathbf{\Psi},\ \mathbf{\Psi}_{+-} = P_+R_-\mathbf{\Psi},\ \mathbf{\Psi}_{-+} = P_-R_+\mathbf{\Psi},\ \mathbf{\Psi}_{--} = P_-R_-\mathbf{\Psi},$$

which are the eigenstates of the operator of the electric charge. In these states, the operator of the dual electrical charge manifests itself in a hidden form.

We evidently also have four states

$$\mathbf{\Phi}_{++} = R_+P_+\mathbf{\Psi},\ \mathbf{\Phi}_{+-} = R_+P_-\mathbf{\Psi},\ \mathbf{\Phi}_{-+} = R_-P_+\mathbf{\Psi},\ \mathbf{\Phi}_{--} = R_-P_-\mathbf{\Psi},$$

which are the eigenstates of the operator of the dual electric charge, where the electric charge manifests itself in a hidden form.

Thus, we need no to pair "electrons" and "neutrinos" since they represent different states of the same entity, i.e. the complex S-field. But since there are two general covariant Dirac equations we must distinguish between "electrons" emitted under definite conditions by atoms and "electrons" emitted by nuclei. Really, the former carry only an electrical charge and correspond to the real S-field, whereas the latter additionally carry a dual electrical charge that is not registered by our known devices.

In the free case, the general covariant Dirac equations for the real and complex S-fields have formally the same form and read

$$\Pi\mathbf{A} + \frac{1}{2}\varphi Q_{\mathbf{t}}\mathbf{A} = m\mathbf{A}, \quad \Pi\mathbf{\Psi} + \frac{1}{2}\varphi Q_{\mathbf{t}}\mathbf{\Psi} = m\mathbf{\Psi},$$

where $\varphi = \nabla_i t^i$. It should be noted that these equations describe the hydrogen atom as well as the original Dirac equation.

Thus, our main goal outlined above is achieved. In conclusion, we put forward an idea that in the framework of the general covariant field-theoretical approach nuclei are described by the system of fields that includes the dual natural Time, the complex S-field, the Yang-Mills field, which corresponds to the quaternion structure E, $\widetilde{I}$, $\widetilde{J}$, $\widetilde{K}$, and the ground state of this field. Each nucleus should be considered as a definite solution of the corresponding closed and self-consistent system of equations. This system cannot be considered in more detail here.

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