Isospin symmetry violation in the strong decays of the spin-2 exotic state $X_2(4014)$

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The isospin violation in the strong decays of the spin-2 exotic charmonium-like state $X_2(4014)$ has been studied within the framework of the covariant confined quark model. We have interpreted the exotic hadron X_2 as a four-quark state with a $(D^*\bar{D}^*)$ molecular-type interpolating quark current and computed the leading-order strong decay widths at the level of two-petal quark-loop diagrams. The partial widths of the decay modes into channels $\omega\ J/\Psi$ and $\rho^0\ J/\Psi$, and their branching ratio recently discussed in literature have been calculated and analyzed. In the comparison of our approach to the recent $(D^*\bar{D}^*)$ molecular scenarios, we have shown the explicit appearance of the threshold effect in the latter models. The calculated partial strong decay widths and their branching ratio are in reasonable agreement with the latest experimental data.

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1. Introduction

Recent measurements at the LHCb and Belle collaborations show a significantly larger than expected violation of isospin symmetry in high-energy collisions, which cannot be explained by known effects and requires new theoretical and experimental investigations.

Throughout the past two decades, a number of exotic XYZ states has been identified during the experimental establishment of the heavy hadron spectrum. The first member of the exotic family, the $X(3872)$ state was discovered in the decays through the $\pi^+\pi^-J/\Psi$ channel by Belle collaboration [1]. Since the mass of X is under $\rho(\omega)J/\Psi$ thresholds, the information on those decays may be obtained from an analysis of the $X \to \pi^+\pi^-J/\Psi$ and $X \to \pi^+\pi^-\pi^0J/\Psi$ decays. The branching fractions of the decays were given in [2] as $\mathcal{B}(\rho^0J/\Psi) = (2.8 \pm 0.7)\%$ and $\mathcal{B}(\omega J/\Psi) = (4.1 \pm 1.4)\%$ by using the relevant data from the recent LHCb experiment [3] and previous measurements. These data indicate a severe isospin violation.

The $X(3872)$ state is located very close to the $(D\bar{D}^*)$ threshold, and it is considered as a shallow bound mesonic $J^{PC} = 1^{++}$ molecule $(D\bar{D}^*)$ [4].

A possible existence of the heavier partner of $X(3872)$ with a similar value for the binding energy and mass $M = 4015$ MeV was first predicted in [5]. Then, such a state with $(D^*\bar{D}^*)$ molecular structure has been guessed in [6]. Later, the existence of an isoscalar 2^{++} has been predicted in a number of

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phenomenological models (see, e.g. [7]). A recent observation of a structure in the invariant mass distribution of the $\gamma\psi(2S)$ with a mass of $4014.3\pm4.0\pm 1.5$ MeV and a width of $4\pm11\pm6$ MeV by the Belle collaboration [8] has became a stimulating motive in the investigation of the exotic XYZ states.

This new structure is located near the $(D^*\bar{D}^*)$ threshold, so one may conclude that it is a promising candidate for the corresponding loosely bound state. In particular, this structure was assumed to be a $(D^*\bar{D}^*)$ molecule with $J^{PC}=0^{++}$ in [9]. Furthermore, its measured width has the same order of magnitude as the prediction in [10,11]. Nowadays, this narrow state is indeed a potential candidate for a $(D^*\bar{D}^*)$ molecule with $J^{PC}=2^{++}$.

Alternatively, a 2^{++} tensor state with a similar mass could also be a conventional charmonium state in the first radial excitation $\chi_{c2}(3930)$ [12].

Also, a compact tetraquark model may be used to study the 2^{++} state [13].

The reliable way to disentangle these different multiquark configurations may be just to investigate the decay properties of the $X_2(4014)$ state. A quark model in [14] provides estimates for the X_2 decay width to charmed mesons around tens of MeV by considering the first radial excitation of the P-wave $\chi_{c2}(2^3P_2)$ charmonium. Then, the hadronic decays of the S-wave $(D^*\bar{D}^*)$ hadronic molecule, into $(D\bar{D})$ and $(D\bar{D}^*)$ meson pairs were estimated to be small of the order of a few MeV [10] and, vice versa, as large as 50 MeV [11].

In our previous papers [15, 16] we have studied some of the XYZ-states in the framework of the covariant confined quark model (CCQM) developed by us [17,18]. The CCQM represents an effective, universal, relativistic and a Lorentz covariant approach and allows one to study bound states with an arbitrary number of constituents and any arbitrary quantum numbers.

We below consider the $X_2(4014)$ as a four-quark state of the molecular-type and investigate its hidden-charm strong decays to $\omega\ J/\Psi$ and $\rho^0\ J/\Psi$.

2. Theoretical model framework

The model Lagrangian describes the interaction of a hadron field $H = X_2(4014)$ satisfying the corresponding equation of motion with an interpolating quark current $J_H(x)$ possessing corresponding quantum numbers $I^G(J^{PC}) = 0^+(2^{++})$ for the hadron. It reads as follows in the case of the exotic hadron state X_2:

$$\mathcal{L}_{\rm int} = g_H\, H_{\mu\nu}(x)\cdot J_H^{\mu\nu}(x) + {\rm H.c.}\,, \tag{1}$$

$$J_H^{\mu\nu}(x) = \int dx_1 \ldots \int dx_4 \delta\left(x - \sum_{i=1}^4 w_i x_i\right) \Phi_H\Big(\sum_{i<j}(x_i-x_j)^2\Big) \cdot \tfrac{1}{\sqrt{2}}\Big\{(\bar{q}(x_3)\gamma^\mu c(x_1))\cdot(\bar{c}(x_2)\gamma^\nu q(x_4)) + (\gamma^\mu \leftrightarrow \gamma^\nu)\Big\}, \quad (q=u,d)\,.$$

where $m_1=m_2=m_c$, $m_3=m_4=m_q$, $w_i = m_i/\left(\sum_{j=1}^4 m_j\right)$ and the translationally invariant 4-quark non-local vertex function Φ_H effectively describes the quark distribution inside the hadron.

The ultraviolet convergence of the loop integrals is ensured by the Fourier transform of Φ_H, which falls off in momentum space in the Euclidean region by the Gaussian law as follows:

$$\widetilde{\Phi}_H(-Q^2) = \exp(Q^2/\Lambda_H^2)\,, \tag{2}$$

where we introduce an adjustable size-related parameter Λ_H.

According to our model, the hadron renormalization coupling g_H in Eq. (2) is strictly fixed by the "compositeness condition" [19,20] which imposes that the renormalization constant of the hadron wave function Z_H has to be equal to zero as follows:

$$Z_H = 1 - g_H^2 \frac{d}{dp^2}\,\tilde{\Pi}_H(p^2) = 0\,, \qquad p^2 = M_H^2\,, \tag{3}$$

and does not constitute further free parameters. Therefore, any bare states are removed totally from consideration, the mass and wave function of the hadron are renormalized, and the physical state is dressed. Note that M_H is the hadron mass and $\tilde{\Pi}_H^{(1)}(p^2)$ is the diagonal part of hadron self-energy.

We use the Fock-Schwinger representation for the quark propagator:

$$\widetilde{S}_j(\not{k}) = (m_j + \not{k}) \int\limits_0^\infty d\alpha\, \exp\big(- \alpha\, \big(m_j^2 - k^2\big)\,\big)\,. \tag{4}$$

Many matrix elements describing radiative transitions of hadrons, their mass operators and other decay processes by using Feynman quark-loop diagrams may be represented in terms of convolutions of vertex functions and quark propagators as follows:

$$\Pi^0 = N_c \int_0^\infty dt\, t^{n-1} \int_0^1 d^n\alpha\, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) f(t\alpha_1, \ldots, t\alpha_n). \tag{5}$$

However, at certain relation of kinematic variables, there may appear possible branch points connected with the creation of free quarks, and then, the integral in Eq. (5) diverges. These threshold singularities can be removed by introducing a universal infrared cutoff parameter, λ as follows:

$$\Pi^0 \to \Pi^\lambda = N_c \int_0^{1/\lambda^2} dt\, t^{n-1} \ldots \tag{6}$$

The model free parameters λ, m_q, Λ_H are fixed by fitting the latest experimental data and, if necessary, some lattice results.

The resulting integrals are computed numerically.

3. Strong decays of $X_2(4014)$ into $\omega J/\psi$ and $\rho^0 J/\psi$

The strong decay under consideration $X_2 \to \omega\, J/\psi$ conserves isospin, while the other process $X_2 \to \rho^0\, J/\psi$ breaks isospin symmetry. Sizeable isospin violating decays of X_2 may be expected.

The ratio of the branching fractions

$$\mathrm{BR_{X_2}} \doteq \frac{\Gamma(X_2 \to \omega J/\psi)}{\Gamma(X_2 \to \rho^0 J/\psi)} \tag{7}$$

has recently been investigated using the effective Lagrangian approach by assuming the X_2 as a molecular state of $(D^*\bar{D}^*)$ [21]. The only contributions from the triangle hadron loops made of the charmed mesons D^* and $\bar{D}^*$ have been considered. The decay widths were quite sensitive to the X_2 mass. At the center mass $M_{X_2} = 4.0143$ GeV, the width for the $X_2 \to \rho^0\ J/\psi$ was tens of keV, while it is on the order of $10^2 - 10^3$ keV for the $X_2 \to \omega\ J/\psi$. The corresponding width ratio at the center mass value is $\mathrm{BR_{X_2}} \approx 15$, i.e. one order of magnitude larger than that for the case of $X(3872)$.

Below, we consider the strong decays of $X_2(4014)$ into $\omega\, J/\psi$ and $\rho^0\, J/\psi$ in the framework of the CCQM. We limit ourselves by considering only the leading order (LO) contributions corresponding to the Feynman diagrams with two-petal quark loops. We calculate the partial widths of the related strong decays and estimate the branching ratio $\mathrm{BR_{X_2}}$.

The invariant matrix element for the strong decay $X_2 \to J/\psi + V$ reads

$$\mathcal{M}_{X_2JV} \equiv i\,(2\pi)^4\delta^{(4)}(p-p_1-p_2)\,\varepsilon_{\mu\nu}(p)\,\varepsilon^*_\rho(p_1)\,\varepsilon^*_\sigma(p_2)\; T^{\mu\nu\rho\sigma}_{X_2JV}(p_1,p_2)\,, \tag{8}$$

where $\{p,p_1,p_2\}$ and $\varepsilon_{\mu\nu}(p)$, $\varepsilon^*_\rho(p_1)$, $\varepsilon^*_\sigma(p_2)$ are the momenta and polarization vectors of the X_2, J/ψ and the vector meson $V=\{\omega,\rho^0\}$, correspondingly.

The LO decay amplitude in Eq. (8) reads

$$\begin{aligned}
&T^{\mu\nu\rho\sigma}_{X_2JV}(p_1,p_2) = g_{X_2}\, g_{J/\psi}\, g_V \\
&\times \frac{N_c}{2}\int\frac{d^4k_1}{(2\pi)^4 i}\int\frac{d^4k_2}{(2\pi)^4 i}\,\widetilde{\Phi}_{X_2}\left(-Q^2\right)\widetilde{\Phi}_{J/\psi}\left(-(\ell_1+\ell_2)^2/4\right)\widetilde{\Phi}_V\left(-(\ell_3+\ell_4)^2/4\right) \\
&\times\Big\{\mathrm{tr}\left[\gamma^\mu S_1(\ell_1)\gamma^\rho S_2(\ell_2)\gamma^\nu S_4(\ell_4)\gamma^\sigma S_3(\ell_3)\right] + (\mu\leftrightarrow\nu)\Big\}\,,
\end{aligned} \tag{9}$$

with the following notations introduced:

$$\begin{aligned}
&\widetilde{\Phi}_H(-k^2) = \exp(k^2/\Lambda_H^2), \qquad H=\{X_2, J/\psi, V\}\,,\\
&\qquad Q^2 = \left[(\ell_1+pw_1)^2+(\ell_2-pw_2)^2+(\ell_3+pw_3)^2+(\ell_4-pw_4)^2\right]/2,\\
&\qquad \ell_1 = k_1-pw_1,\ \ell_2 = k_1-pw_1-p_1,\ \ell_3 = k_2-pw_4-p_2,\ \ell_4 = k_2-pw_4\,.
\end{aligned}$$

By substituting the vertices functions and quark propagators into Eq. (9), and performing an explicit $k_{1,2}$-integrations while turning the set of Fock-Schwinger parameters into a simplex, we rewrite the LO amplitude of the strong decay $X_2 \to J/\psi + V$ as follows:

$$\begin{aligned}
T^{\mu\nu\rho\sigma}_{X_2JV}(p_1,p_2) = {}& A_V\cdot\Big(g^{\mu\rho}\Big[g^{\sigma\nu}(p_1\cdot p_2) - p_1^\sigma\, p_2^\nu\Big] + g^{\nu\rho}\Big[g^{\sigma\mu}(p_1\cdot p_2) - p_1^\sigma\, p_2^\mu\Big]\Big)\\
&+ B_V\cdot\Big(g^{\sigma\rho}\Big[p_1^\mu\, p_2^\nu + p_1^\nu\, p_2^\mu\Big] - g^{\mu\sigma}p_1^\nu\, p_2^\rho - g^{\nu\sigma}p_1^\mu\, p_2^\rho\Big)\,,
\end{aligned} \tag{10}$$

where the two independent form factors $A_V(g_X, g_{J/\psi}, g_V, p^2, p_1^2, p_2^2)$ and $B_V(g_X, g_{J/\psi}, g_V, p^2, p_1^2, p_2^2)$ are determined according to Eq. (9).

We calculate

$$|\mathcal{M}_{X_2JV}|^2 \sim |\varepsilon_{\mu\nu}(p)\varepsilon^*_\rho(p_1)\varepsilon^*_\nu(p_2)T^{\mu\nu\rho\sigma}_{X_2JV}|^2 = M^4_{X_2}\left(C^V_A \cdot A^2_V + C^V_{AB} \cdot A_V \cdot B_V + C^V_B \cdot B^2_V\right),$$

where the coefficients C^V_A, C^V_{AB} and C^V_B are defined via the meson masses.

Then, the sought-for hidden-charm two-body strong decay widths read:

$$\Gamma_{X_2JV} = \frac{1}{2S+1}\frac{|\vec{p}_2|}{8\pi M^2_{X_2}}\sum_{polar}|\mathcal{M}_{X_2JV}|^2\,, \qquad (11)$$

where $|\vec{p}_2| = \lambda^{1/2}(M^2_{X_2}, M^2_{J/\psi}, M^2_V)/(2M_{X_2})$ and $\lambda(x,y,z)$ is the Källén kinematical function while $S = 2$ is the spin value of X_2.

4. Numerical results and conclusion

The model parameters in the CCQM are determined by minimizing χ^2 in fits to the latest available experimental data and some lattice results. We keep the updated central values of the basic CCQM parameters, and introduce only a new and totally adjustable size parameter Λ_{X_2} which describes the quark contribution in the exotic hadron X_2.

i). The data from experiments of BABAR [22], Belle [23], and BESIII [24] for the $X(3872)$ strong decay ratio BR_X^{expr} lies in the interval $[0.8 - 1.4]$. In Ref. [25] the ratio $\mathrm{BR}_X \simeq 1$ was explained by the larger effective phase space for the $\rho^0\, J/\psi$ decay than for $\omega\, J/\psi$, due to the large width of the ρ^0, which could compensate the suppression of the small mass difference between the neutral and charged charmed mesons.

If one assumes that the X_2 decay modes under consideration are similar to those of the $X(3872)$, the width ratio BR_{X_2} also should be around unity.

ii). Let us first make a simple and rough approximation to the branching ratio $\mathrm{BR_{X_2}}$. By neglecting the difference between the corresponding form factors $C^\omega_{\{A,AB,B\}}$ and $C^{\rho^0}_{\{A,AB,B\}}$, one can approximately write down the ratio:

$$\mathrm{BR^{rough}_{X_2}} \approx 0.762. \qquad (12)$$

iii). Now we take into account accurately the real contributions of the matrix elements and calculate the partial decay widths defined in Eq. (11) in dependence on the 'size' parameter Λ_{X_2}. The obtained results are represented in Table 1 for a fixed central mass value $M_{X_2} = 4.014$ GeV.

One can see in Table 1 that both the partial decay widths decrease monotonically as the 'size' parameter Λ_{X_2} increases.

However, the branching ratio of these strong decay widths

$$\mathrm{BR^{CCQM}_{X_2}} \approx 1.140 \sim 1.147 \quad \text{for} \quad \Lambda_{X_2} \in [3.0\,,\, 5.0]\ \text{GeV} \qquad (13)$$

almost cancels or at least weakens the Λ_{X_2}-dependence.

Table 1. The dependencies of the partial decay widths $\Gamma(X_2 \to \omega\, J/\psi)$, $\Gamma(X_2 \to \rho^0\, J/\psi)$ and the branching ratio $\mathrm{BR}^{\mathrm{CCQM}}_{\mathrm{X_2}}$ on the size parameter Λ_{X_2}.

Λ_{X_2} [GeV]	3.0	3.5	4.0	4.5	5.0
$\Gamma(X_2 \to \omega\, J/\psi)$ [keV]	1825.3	430.4	138.4	54.4	24.6
$\Gamma(X_2 \to \rho^0\, J/\psi)$ [keV]	1600.9	376.7	120.9	47.5	21.5
$\mathrm{BR}^{\mathrm{CCQM}}_{\mathrm{X_2}}$	1.140	1.143	1.145	1.146	1.146

iv). Recently, by assuming the X_2 as a pure molecule of the $(D^*\bar{D}^*)$, the partial widths of the strong decays of $X_2 \to \omega J/\psi$ and $X_2 \to \rho^0 J/\psi$ via the intermediate meson loops have been estimated in a framework of the effective field theory [21]. The obtained results depend very strongly on the X_2 mass uncertainty. In particular, at center mass 4014.3 MeV, the width for the $X_2 \to \rho^0 J/\psi$ is predicted to be a few tens of keV, while it is on the order of $10^{2\text{-}3}$ keV for the $X_2 \to \omega J/\psi$ [21].

Due to this, we have also investigated the dependence of the partial decay widths $\Gamma(X_2 \to \omega J/\psi)$ and $\Gamma(X_2 \to \rho^0 J/\psi)$ on the exotic hadron mass deviation within the experimental uncertainty reported in [26]. We revealed that the decay widths increase slowly without any peaks and drops in the mass interval from 4.010 GeV to 4.020 GeV. Hereby, the decay width ranges from 260 to 274 keV for $X_2 \to \omega\, J/\psi$, while for $X_2 \to \rho^0\, J/\psi$ it is between 227 and 240 keV, by keeping the branching ratio $\mathrm{BR}^{\mathrm{CCQM}}_{\mathrm{X_2}} = 1.14$.

This result is not surprisingly. The hidden charm decays of the X_2 in our approach occur via the confined quark loops but not the charmed D^* and $\bar{D}^*$ meson loops, without any threshold effects.

By conclusion, we have interpreted the exotic hadron $X_2(4014)$ as a four-quark state with a $D^*\bar{D}^*$ molecular-type interpolating quark current and computed the hidden-charm strong decay widths into channels $\omega J/\Psi$, $\rho^0 J/\Psi$, and their branching ratio recently discussed in literature [7, 21].

Our findings are:

◇ The strong-decay widths $\Gamma(X_2 \to \omega\, J/\psi)$ and $\Gamma(X_2 \to \rho^0\, J/\psi)$ depend significantly on the model size parameter Λ_{X_2}.

◇ Our numerical results clearly show that the branching ratio $\mathrm{BR}^{\mathrm{CCQM}}_{\mathrm{X_2}} \simeq 1.14$ is almost independent on the model parameter Λ_{X_2}.

◇ Our numerical results indicate that the decay widths monotonically increase with no peaks and drops in the mass interval from 4.010 GeV to 4.020 GeV by keeping the branching ratio almost constant ($\mathrm{BR}^{\mathrm{CCQM}}_{\mathrm{X_2}} = 1.14$).

◇ The estimated hidden-charm strong-decay widths and their branching ratio are in good accordance with the most recent theoretical predictions and may support the four-quark molecular-type structure of X_2.

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CONFLICT OF INTEREST

The author of this work declares that she has no conflict of interest.

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