Cumulative processes and fractal structure of nuclei
Кумулятивные процессы и фрактальная размерность ядер

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Аннотация на русском языке.

Предполагается, что кумулятивные события обогатят набор данных новым типом взаимодействий, характеризующихся высокой плотностью энергии сжатой ядерной материи. Ожидается, что такой отбор событий позволит обнаружить чёткие признаки фазового перехода, положение критической точки и изучить экстремальные условия в столкновениях тяжёлых ионов. В работе представлен краткий обзор концепции z-скейлинга, основанной на принципах самоподобия, локальности и фрактальности, отражающих общие особенности взаимодействия частиц. Эта закономерность используется для описания рождения частиц в широком диапазоне кинематических переменных, включая кумулятивную область, в p–A и A–A столкновениях. Структура сталкивающихся объектов и процессы фрагментации характеризуются фрактальными размерностями. Образующаяся ядерная среда описывается «удельной теплоёмкостью». Обсуждаются результаты анализа данных по кумулятивному рождению адронов в p–A и A–A столкновениях. Изменение фрактальной размерности ядра, фрагментационной фрактальной размерности и удельной теплоёмкости вблизи критической точки рассматривается как признак новой физики.

This is English translation of the abstract

Selection of cumulative events is assumed to enrich data samples by a new type of collisions characterized by higher energy density and more compressed matter. We expect that this will allow finding more clear signatures of phase transition, location of a critical point and studying extreme conditions in heavy ion collisions. The brief review of z-scaling concept based on the principles of self-similarity, locality and fractality reflecting general features of particle interactions is given. The z-scaling was used to describe of particle production in cumulative and non-cumulative regions in p–A and A–A collisions. Structure of the colliding objects and fragmentation processes is characterized by the structural and fragmentation fractal dimensions. The produced medium is described by a "specific heat". The results of data analysis of cumulative production in p–A and A–A collisions are discussed. The change of nucleus fractal dimension, fragmentation fractal dimension and specific heat near the critical point is considered as a signature of new physics.

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Introduction

The concepts of "scaling" and "universality" have been widely developed to understand critical phenomena [1–3]. Scaling implies that systems near the critical point exhibit self-similarity and are invariant with respect to scale transformations. The universality of their behavior is manifested in the fact that completely different systems behave similarly (are described by the same power-law dependence) near the critical point. Critical exponents in power laws are determined only by the symmetry of the interaction and the dimensionality of space.

New regularity of hadron production in high-energy proton-proton(antiproton) collisions, now known as z-scaling, was established in [4–9]. It manifests itself in the fact that the inclusive spectra of particles of different types are described by a universal scaling function $\psi(z)$, which depends on the self-similarity parameter z over a wide range of transverse momenta, particle emission angles, collision energies, and secondary particle multiplicities.

In collisions of heavy nuclei, phase transitions and other collective effects are assumed to should manifest themselves over a larger spatial volume of nuclear matter than in proton-proton interactions. They are expected to lead to changes in the properties of the particles themselves, the mechanisms of interaction between their constituents, and the fragmentation process. The modification of the latter is determined by the specific properties of the nuclear environment (the high density and temperature of the nuclear matter, the chemical potential and others).

High density compresseed nucleare matter is assumed to be created in the cumulative processes. Cumulative effect was discovered in [10–12]. It means that particles can be produced in the kinematic range forbidden for free nucleon-nucleon interactions. The features of cumulative particle production in p–A and A–A collisions were widely investigated (see [13] and references therein). Different mechanisms of cumulative hadron production (fluctons, short range correlations, rescattering, heavy resonances, quark clasters and others) were suggested for description of the phenomenon.

The z-scaling universality found in data analysis [14] can be a good tool to verify the features of hadron production in cumulative region as well. We propose using the concept of z-scaling to search for signatures of phase transitions in nuclear matter and the critical point in cumulative processes at high transverse momenta of hadrons.

z-Scaling

Here we briefly remind the basic ideas of z-scaling concept [4–6]. We assume that at high energy, the collisions of A_1 and A_2 nuclei can be expressed by the interactions of their constituents. The momentum distribution of a produced inclusive particle h in the process $A_1 + A_2 \to h + X$ is described in terms of a binary collision of the constituents with masses $x_1 M_1$ and $x_2 M_2$.

The quantities x_1 and x_2 are the momentum fractions carried by the interacting constituents of the incoming nuclei with masses M_1 and M_2 and 4-momenta P_1 and P_2, respectively. It is also assumed that the objects with masses m_a/y_a and $(x_1M_1 + x_2M_2 + m_b/y_b)$ are produced in the scattered and recoil directions in such a collision. The y_a and y_b represent the corresponding momentum fractions carried by the inclusive particle with mass m_a and its hadron counterpart with mass m_b moving in opposite direction. The momentum conservation law in the binary collision gives the following condition:

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = M_X^2. \tag{1}$$

The recoil mass is written in the form $M_X = x_1M_1 + x_2M_2 + m_b/y_b$. The produced particle with mass m_b ensures the conservation of additive quantum numbers (electric charge, baryon number, flavor content etc.).

Using the scenario of binary constituent interactions, we also assume that the inclusive hadron production has the property of self-similarity. The self-similarity of hadron interactions at a constituent level is studied by the variable z defined as follows

$$z = z_0\Omega^{-1}, \tag{2}$$

where $z_0 = \sqrt{s_\perp}/[(dN_{\rm ch}/d\eta|_{\eta\approx 0})^c\, m_N]$. The variable z_0 is proportional to the transverse kinetic energy, $\sqrt{s_\perp}$, of a selected constituent sub-process consumed on the production of the inclusive particle with mass m_a and its counterpart with mass m_b. The quantity $dN_{\rm ch}/d\eta|_{\eta\approx 0}$ is a corresponding (pseudo-rapidity) multiplicity density of charged particles produced in the reaction central region at the mid-rapidity, $\eta \approx 0$. The multiplicity density is related to a state of the produced medium. The parameter c characterizes the properties of this medium. The constant m_N is fixed at the value of a nucleon mass. The symbol Ω stands for the maximum of the function,

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\epsilon_a}(1-y_b)^{\epsilon_b} \tag{3}$$

under the kinematic constraint (1). This quantity connects the momentum fractions defining kinematics of a selected binary collision at a microscopic level with the structural and fragmentation characteristics of the reaction. Structure of the colliding hadrons/nuclei and fragmentation of the objects produced in the binary collisions of their constituents are described by δ_1, δ_2 and ϵ_a, ϵ_b, respectively. These parameters are interpreted as fractal dimensions. The quantity Ω is proportional to the relative number of all such configurations in the inclusive process, which contain a state defined by the fractions x_1, x_2, y_a, and y_b. For given values of δ_1, δ_2, ϵ_a, and ϵ_b, the selected sub-process is singled out of all constituent sub-processes by the values of momentum fractions x_1, x_2, y_a, and y_b, which maximize the function $\Omega(x_1, x_2, y_a, y_b)$ under condition (1). The quantity Ω^{-1} characterizes a resolution at which the selected sub-process is singled out of the inclusive reaction. The value of z_0 is proportional to $\sqrt{s_\perp}$ in the selected sub-process consumed on the production of an inclusive particle with mass m_a and its counterpart with mass m_b.

The scaling function $\psi(z)$ is expressed in terms of the experimentally measured inclusive invariant cross section, the (pseudo-rapidity) multiplicity density, $dN/d\eta$, and the total inelastic cross section, $\sigma_{\rm in}$, as follows:

$$\psi(z) = \frac{\pi}{(dN/d\eta)\ \sigma_{\rm in}} J^{-1} E \frac{d^3\sigma}{dp^3}, \tag{4}$$

where J is the Jacobian of the transformation from $\{p_T^2, y\}$ to $\{z, \eta\}$ pairs of variables. The multiplicity density concerns particular hadron species. The integral of $\psi(z)$ over the interval $(0, \infty)$ is normalized to unity. It allows the interpretation of the scaling function as a probability density to produce the inclusive particle with the corresponding value of z.

Fractal dimensions in p–p collisions

The model parameters - structural fractal dimension δ, fragmentation fractal dimension ϵ, and "specific capacity" c, are determined from the requirement of scaling behavior of $\psi(z)$ as a function of self-similarity parameter (see [7] and references therein).

The parameter δ characterizes the internal structure of colliding hadrons. The self-similarity parameter z written in the form (1) allows us to interpret δ as the anomalous fractal dimension. The measure z is tending to infinity with resolution increases $\Omega^{-1} \to \infty$. The value of δ for p–p collisions over a wide range of collision energy was found to be 0.5 for particles with different flavor content [6]. The result also implies self-similarity of constituent interactions at different scales. In the case of pp($\bar{\rm p}$p) interactions, we have $m_1 = m_2 = m_N$ and set the nucleon fractal dimension $\delta_1 = \delta_2 = \delta_N$.

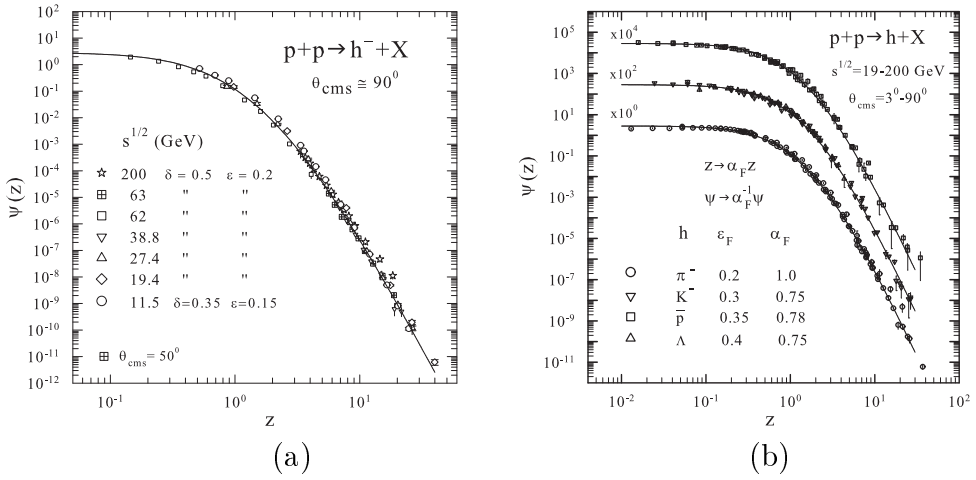


Fig. 1. Scaling function $\psi(z)$ of hadron production in p–p collisions

The parameter ϵ characterizes a fragmentation process. We assume that the fragmentation of the objects moving in the scattered and recoil directions can be described by the same parameter $\epsilon_a = \epsilon_b = \epsilon_F$ which depends on the

type of the inclusive particle. The relation $m_h = m_{\bar{h}}$ is used for each particle species.

Values of the parameters δ and ϵ_F are determined in accordance with the self-similarity requirements of $\psi(z)$ and the experimental data. The properties of data z-presentation for hadron production in p–p collisions can be summarized [7, 15, 16] as follows:
– the energy independence of $\psi(z)$ for $\sqrt{s} > 20$ GeV;
– the angular independence of $\psi(z)$ for $\theta_{cms} = 3^0 - 90^0$;
– the multiplicity independence of $\psi(z)$ for $dN_{ch}/d\eta|_0 = 1.5 - 26$;
– the saturation of $\psi(z)$ at low $z < 0.1$;
– the power law, $\psi(z)\ z^{-\beta}$, at high $z > 4$;
– the flavor independence of $\psi(z)$ for $\pi, K, \phi, \Lambda, .., D, J/\psi, B, \Upsilon, \ldots, top$. The model paraneters were found to have constant values in p–p and p–$\bar{\text{p}}$ collisions at high energies. Deviation from the found values would indicate on new degree of freedom in structure of a particle - nucleon compositeness, and in fragmentation process - transition of constituents into real particles in a scale space.

Figure 1 shows z-presentation of inclusive spectra of hadrons produced in p–p collisions over a wide range of $\sqrt{s}$ and θ_{cms}. The solid line corresponds to the data for π^--meson production. The data is used for comparison with other ones.

Fractal dimensions in p–A collisions

The fractal dimension of nucleus δ_A [17] was found, in agreement with z-scaling, to be described by the additive law $\delta_A = A\delta_N$. These results are valid for high-p_T hadron production in p–A collisions. Atomic weight changes from 2 for deuterium (D) to 207 for lead (Pb). The additive law as it is shown in [18] to be valid for π–A collisions as well. Self-similarity of particle structure, constituent interactions and fragmentation process of hadron production in non-cumulative and cumulative regions in p–A collisions were confirmed [19–21]. Smooth behavior of the scaling function $\psi(z)$ in the overlapping range was found. The discontinuity of fractal dimensions

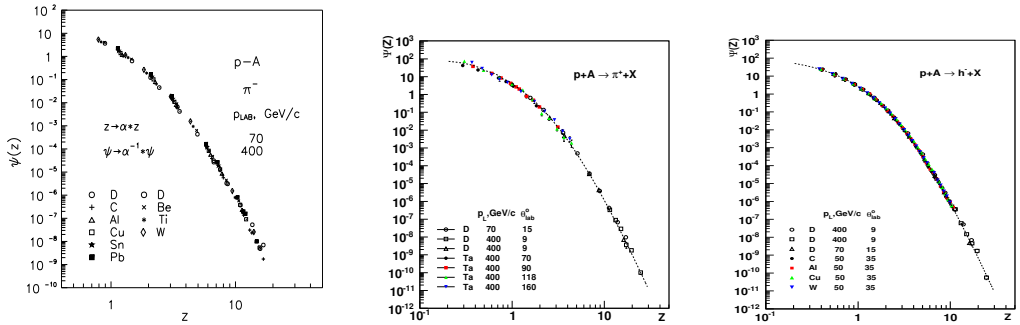


Fig. 2. Scaling function $\psi(z)$ of hadron production in p–A collisions

and "specific heat" was no observed. The general features previously found in p–p collisions were also confirmed for hadron production in p-A interactions: collapse of data points, universal shape of $\psi(z)$, self-similarity of collision structure, constituent interaction and fragmentation process over a wide kinematic range and absent of δ_A-discontinuity. Figure 2 shows that self-similarity covers the non-cumulative (a), cumulative low- (b) and cumulative high-p_T (c) range.

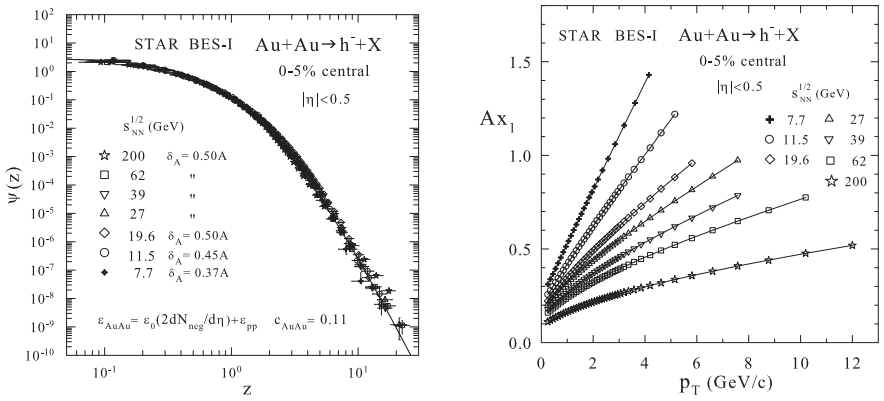


Fig. 3. Scaling function $\psi(z)$ (a) and momentum fraction Ax_1 (b) of negative hadron production in Au–Au collisions

Fractal dimensions in A–A collisions

The STAR BES-I data on negative hadrons and K^0_S-mesons produced in Au–Au collisions cover the cumulative region only at relatively low $\sqrt{s_{NN}}$ =7.7, 11.5 GeV [8, 9]. Spectra are measured over a wide range of energy, centrality and transverse momentum. The smooth behavior of $\psi(z)$ was found. A fractal dimension δ_A of nucleus with atomic weight A determined by the same additive law $\delta_A = A\ \delta_N$. The result is in agreement with the z-scaling found for hadron production in p–A collisions [17]. We see that particle production in Au–Au collisions manifests self-similarity, locality and fractality of hadron interactions at a constituent level in cumulative and non-cumulative range.

Smooth behavior of $\psi(z)$ and constancy of the fractal dimensions indicate that nucleon constituents dominate in nuclei. The enhancement of nucleus collective effects is expected near a kinematic boundary. However, as we found from analysis of the STAR data, it is not possible for heavy nuclei and at high collision energies. Therefore relatively light nuclei and not high energy are preferable to search for violation of the scaling features. Note, that the nucleus fractal dimension decreases with the collision energy. It is equal to zero for point-like particle. Therefore the energy range $\sqrt{s_{NN}} = 10 - 40$ GeV is considered to be optimal for searching discontinuity of fractal dimensions

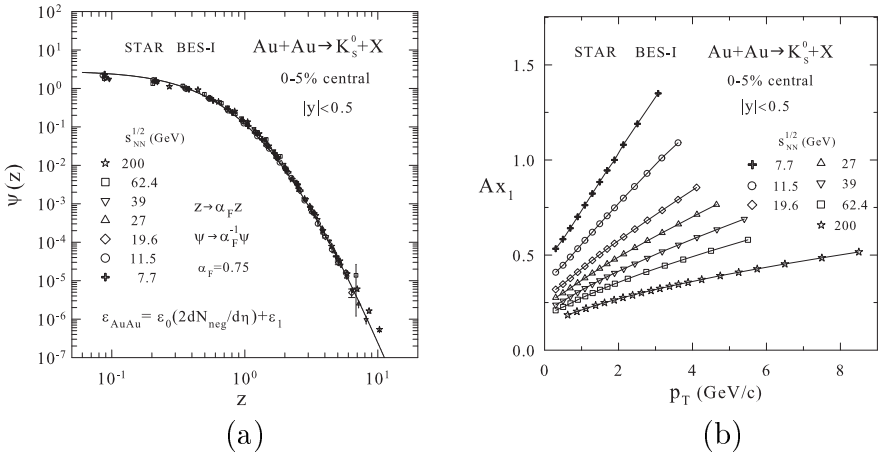


Fig. 4. Scaling function $\psi(z)$ (a) and momentum fraction Ax_1 (b) of K^0_S-meson production in Au–Au collisions

of nuclei and fragmentation process and "specific heat" as a signature of phase transition in cumulative hadron production [14, 22, 23].

Fractal entropy and fractal dimensions

Concept of the fractal entropy $S_{\delta,\epsilon}$ [24, 25] developed in the framework of z-scaling is used to describe a microscopic scenario of hadron production in terms of momentum fractions x_1, x_2, y_a, y_b. The entropy is expressed via the fractal dimensions (3). The maximal entropy principle leads to conservation law of the fractal cumulativity. It is a property of a fractal-like object (or fractal-like process) with a fractal dimension to form a local compact structural aggregate – named as FRACTALON, which carries some fraction of momentum of its parent fractal [26, 27]. Such a way the requirement to present the entropy as a logarithm of the statistical weight of fractal configuration of the internal structures of colliding nuclei and fractal configurations corresponds to the fragmentation processes in the final state, provided the quantization of fractal dimensions $\delta_1 = d{\cdot}n_{\delta_1}, \delta_2 = d{\cdot}n_{\delta_2}, \epsilon_a = d{\cdot}n_{\epsilon_a}, \epsilon_b = d{\cdot}n_{\epsilon_b}$. The quant d of the fractal dimensions and the corresponding quantum numbers $\{n_{\delta_1}, n_{\delta_2}, n_{\epsilon_a}, n_{\epsilon_b}\}$ reflect the fractal structure of particles and their formation at very small scales. Moreover, we consider that every physical particle is a structural one, particle's constituents possess a fractal-like structure, fragmentation is a fractal-like process, compactness of the fractal structures is governed by the Heisenberg uncertainty principle. These properties underlie the nature of particles.

Conclusions

The fractal dimension δ_A of the nucleus is an important ingredient of the z-scaling concept. The constancy of the dimension was found from the analysis of experimental data on cumulative hadron production in p–A collisions at

$\sqrt{s_{NN}} = 11.5 - 27.4$ GeV, obtained at FNAL (Batavia) and IHEP (Protvino) by the groups of G. Leksin, L. Zolin and V. Gapienko. This is consistent with the results obtained within the z-scaling approach for non-cumulative data at high p_T, measured by the groups of J. Cronin, D. Jaffe and R. Sulyaev at FNAL and IHEP. A smooth transition of $\psi(z)$ from the non-cumulative to the cumulative region is observed. The fractal dimension of the nucleus is described by the additive law $\delta_A = A\delta_N$, which is also confirmed by STAR data on the production of negative hadrons and K^0_S-mesons in Au–Au collisions over a wide range of energies $\sqrt{s_{NN}} = 7.7 - 200$ GeV and centrality. We suggest that the violation of the law in cumulative hadron production is associated with a phase transition in compressed nuclear matter and is of interest for searching the location of the critical point.

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