Contribution of linearly polarised gluons in charmonium production within the Soft Gluon Resummation approach
Вклад линейно поляризованных глюонов в рождение чармониев в подходе пересуммирования мягких глюонов

V. Saleev[a,b,1], *K. Shilyaev*[b,2]

В.А. Салеев[a,b,1], *К.К. Шиляев*[b,2]

[a] Joint Institute for Nuclear Research, Dubna, 6 Joliot-Curie St, 141980

[a] Объединенный институт ядерных исследований, Дубна, ул. Жолио-Кюри, 6, 141980

[b] Samara National Research University, Samara, 34 Moskovskoe shosse, 443086

[b] Самарский университет, Самара, Московское шоссе, д. 34, 443086

В данной работе используется подход пересуммирования мягких глюонов для описания неколлинеарных партонных распределений. Неколлинеарная факторизация позволяет получить доступ не только к неполяризованным распределениям, но и к функциям Бура-Малдерса, описывающим распределения линейно поляризованных партонов в неполяризованных протонах. В работе сделана оценка отношения вкладов от функций Бура-Малдерса и неполяризованных распределений для рождения чармониев при разных значениях энергии в системе центра масс начальных протонов $\sqrt{s}$. В качестве модели адронизации используется нерелятивистская КХД.

In this article, the Soft Gluon Resummation approach is used for modelling transverse-momentum dependent parton distributions. The Transverse-Momentum Dependent factorisation allows to get access both to unpolarised parton distribution functions and to the Boer-Mulders functions which describe linearly polarised partons in unpolarised protons. The estimation for ratio of contributions of Boer-Mulders and unpolarised parton distributions for charmonium production in a range of center-of-mass energies of initial protons $\sqrt{s}$ is made. The nonrelativistic QCD is used as a hadronisation model.

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Introduction

[1]E-mail: saleev.va@ssau.ru

[2]E-mail: kirill.k.shilyaev@gmail.com

[1]E-mail: saleev.va@ssau.ru

[2]E-mail: kirill.k.shilyaev@gmail.com

The study of J/ψ production in proton-proton collisions is a part of the future experimental program of the SPD NICA Collaboration [1]. Due to the relatively low center-of-mass energy $\sqrt{s} = 27$ GeV, the kinematic domain of small transverse momentum of final state is principal for investigation of internal structure of protons. This domain, $p_T \ll M$ with M as a charmonium mass, is described within the Transverse Momentum Dependent (TMD) factorisation [2]. The TMD factorisation theorem allows to express the cross section as a convolution of hard parton cross section and TMD parton distribution functions (PDFs). The latter demand some other framework to be described and modelled. In our work, we use the Soft Gluon Resummation (SGR) approach for this purpose [3]. The TMD factorisation provides access not only to unpolarised PDFs but to the Boer-Mulders PDFs as well, they describe the distribution of linearly polarised partons inside unpolarised protons [4]. The nonrelativistic QCD is used to describe hadronisation of produced pair of heavy quarks into observable heavy quarkonium [5].

In the previous work [6], we made estimations for p_T-spectra of η_c and J/ψ production considering only unpolarised PDFs. In the current study, we make calculations for the ratios of Boer-Mulders and unpolarised PDFs in the J/ψ production for a range of center-of-mass energies $\sqrt{s} = 27 - 200$ GeV, including energy of the NICA collider.

Soft Gluon Resummation approach

The TMD factorisation is an approach valid for small transverse momentum of the final state. In case of charmonium production, the applicability domain is $p_T \ll M$, where M is a charmonium mass conventionally taken as a hard scale of the production process [2]. The momenta of the initial partons are of the form $q^\mu_{1,2} = x_{1,2}p^\mu_{1,2} + q^\mu_{1,2T}$ with protons momenta $p_{1,2} = \frac{\sqrt{s}}{2}(1, 0, 0, \pm 1)$, momentum fractions $x_{1,2} = Me^{\pm y}/\sqrt{s}$ and transverse momenta $q^\mu_{1,2T}$ such that $q^2_{1,2T} = -\mathbf{q}^2_{1,2T}$. In approximation of small transverse momenta, when only the first power of $\mathbf{q}_{1,2T}$ is preserved, the initial partons are on-mass-shell.

Further, we will consider only gluons as initial partons because the quark-antiquark annihilation subprocesses are suppressed by the gluon-gluon fusion in quarkonium production. The TMD parton distributions are expressed via the gluon correlator [7]:

$$\Phi^{\mu\nu}_g(x, \mathbf{q}_T) = -\frac{1}{2x}\left[g^{\mu\nu}_T f^g_1(x, \mathbf{q}_T) - \left(\frac{q^\mu_T q^\nu_T}{M^2_h} + g^{\mu\nu}_T \frac{\mathbf{q}^2_T}{2M^2_h}\right) h^{\perp g}_1(x, \mathbf{q}_T)\right] \quad (1)$$

where $g^{\mu\nu}_T = g^{\mu\nu} - (P^\mu n^\nu + n^\mu P^\nu)/(P\cdot n)$ with a momentum P of the parent proton and a light-like vector n conjugated to P, f^g_1 is an unpolarised gluon PDF, $h^{\perp g}_1$ is a linearly polarised gluon PDF, or the Boer-Mulders function, M_h is of order of the proton mass. The evolution of TMD PDFs with the factorisation scale μ_F and the rapidity variable ζ is decribed within the Collins-Soper and renormalisation group equations [8]. For simplicity, the following standard choice is made for the scales: $\mu_F = \sqrt{\zeta}$. The equations are solved in Fourier-conjugated space of impact parameter $\mathbf{b}_T$, and the solution is obtained with the perturbative Sudakov factor S_P with the coefficients related to logarithmic approximation and approximation with respect to the coupling constant α_s [9]:

$$S_P(\mu_F, \mu_b, b_T) = \frac{C_A}{\pi} \int\limits_{\mu_b^2}^{\mu_F^2} \frac{d\mu'^2}{\mu'^2} \alpha_s(\mu') \left[\ln \frac{\mu_F^2}{\mu'^2} - \left(\frac{11 - 2N_f/C_A}{6} + \frac{1}{2} \right) \right], \quad (2)$$

where N_f is a number of quark flavors and $C_A = N_c = 3$. The Sudakov factor realises PDF evolution form the initial scale $\mu_{F0} = \mu_b = \sqrt{\zeta_0}$ to the final ones: $\mu_F = \sqrt{\zeta}$. In order to stay in the perturbative regime during the PDF evolution and, therefore, for S_P to be valid, it is necessary to make a prescription $\mu_b' = \mu_F b_0/(\mu_F b_T + b_0)$ for the hard scale (with $b_0 = 2e^{-\gamma}$ where γ is the Euler-Mascheroni constant) and cut-off $b_T^*(b_T) = b_T/\sqrt{1 + (b_T/b_{T,\max})^2}$ for the impact parameter with the value $b_{T,\max} = 1.5$ GeV^{-1} (we use natural units, $\hbar = c = 1$) [10]. In this study, we are working in the leading logarithmic (LL) approximation and leading order (LO) with respect to the coupling constant α_s. The nonperturbative input for initial partons is incorporated within the nonperturbative Sudakov factor S_{NP} of the form [11]:

$$S_{NP}(b_T, \mu_F) = \left[g_1 \ln \frac{\mu_F}{2Q_{NP}} + g_2 \left(1 + 2g_3 \ln \frac{10xx_0}{x_0 + x} \right) \right] b_T^2, \quad (3)$$

where $g_1 = 0.184$ GeV^2, $g_2 = 0.201$ GeV^2, $g_3 = -0.129$, $x_0 = 0.009$, $Q_{NP} = 1.6$ GeV. This expression is valid for initial quarks, in order to apply it to gluons additional color factor change C_A/C_F should be made.

The Fourier transformed PDFs are defined in this way:

$$\hat{f}_1^g(x, \mathbf{b}_T, \mu_F, \zeta) = \int d\mathbf{q}_T e^{-i\mathbf{q}_T \mathbf{b}_T} f_1^g(x, \mathbf{q}_T, \mu_F, \zeta), \quad (4)$$

$$\hat{h}_1^{\perp g}(x, \mathbf{q}_T, \mu_F, \zeta) = \int d\mathbf{q}_T \frac{(\mathbf{b}_T \cdot \mathbf{q}_T)^2 - \frac{1}{2}\mathbf{b}_T^2 \mathbf{q}_T^2}{\mathbf{b}_T^2 M_h^2} e^{-i\mathbf{q}_T \mathbf{b}_T} h_1^{\perp g}(x, \mathbf{q}_T, \mu_F, \zeta), \quad (5)$$

and these Fourier transformed unpolarised and linearly polarised PDFs within the SGR approach are expressed with the collinear PDFs in the following way up to corrections $\mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$ and $\mathcal{O}(\alpha_s)$ for unpolarised PDF, $\mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$ and $\mathcal{O}(\alpha_s^2)$ for linearly polarised PDF [3]:

$$\hat{f}_1^g(x, b_T^*, \mu_{b^*}') = f(x, \mu_{b^*}'), \quad (6)$$

$$\hat{h}_1^{\perp g}(x, b_T^*, \mu_{b^*}') = -\frac{C_A \alpha_s(\mu_{b^*}')}{\pi} \int\limits_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1 \right) f(x', \mu_{b^*}'). \quad (7)$$

Overall, the quarkonium production cross section can be written shortly as follows:

$$\frac{d\sigma}{dy\, d\mathbf{p}_T^2} = \frac{\overline{|\mathcal{M}|^2}}{M^2 s} \left(\mathcal{C}[f_1^g f_1^g] + \mathcal{C}[w h_1^{\perp g} h_1^{\perp g}] \right), \quad (8)$$

with the convolutions of unpolarised, linearly polarised PDFs and weight originated from the gluon correlator (1). The convolutions read

$$\mathcal{C}[wff] = \int d\mathbf{q}_{1T} d\mathbf{q}_{2T} \delta(\mathbf{q}_{1T} + \mathbf{q}_{2T} - \mathbf{p}_T) w(\mathbf{q}_{1T}, \mathbf{q}_{2T}) f(x_1, \mathbf{q}_{1T}) f(x_2, \mathbf{q}_{2T}) \quad (9)$$

or, using the SGR approach for TMD PDFs, they are expressed in the following way:

$$\mathcal{C}[wff] = \int \frac{b_T db_T}{2\pi} J_0(p_T b_T) e^{-S_P} e^{-S_{NP}} \hat{f}(x_1, \mu'_{b^*}, b^*_T) \hat{f}(x_2, \mu'_{b^*}, b^*_T), \quad (10)$$

where $\hat{f}(x, \mu'_{b^*}) \equiv \hat{f}_1^g(x, \mu'_{b^*}, b^*_T)$ or $\hat{h}_1^{\perp g}(x, \mu'_{b^*}, b^*_T)$.

Nonrelativistic QCD

The nonrelativistic QCD (NRQCD) is a conventional framework to describe the hadronisation of the produced heavy quark pair [5]. The small value of relative velocity of constituent quark in charmonium ($v^2 \approx 0.3$) allows to utilise the scaling of kinematic variables in order to expand the charmonium wave function into the series of the Fock states. The calculation of matrix elements within the NRQCD is made with the projections of amplitude onto the states with the required quantum numbers [12].

The hard cross section is factorised into cross section of production of quark pair in some Fock state and corresponding long-distance matrix element (LDME) which is of the nonperturbative nature:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_n d\hat{\sigma}(ab \to c\bar{c}[n]X) \langle \mathcal{O}^{\mathcal{C}}[n] \rangle \quad (11)$$

where the summation over the Fock states is indicated. Contrary to J/ψ, the first color singlet term in the NRQCD expansion is enough approximation in the cases of η_c and $\chi_{c0,2}$ production, i.e. Color Singlet Model (CSM).

Results of calculations

We used the NNPDF2.3 set [13] for collinear PDFs and the charmonium mass $M = 3.096$ GeV [14].

As the sum of convolutions of the PDFs is included in the cross section expression (8), it is convenient to introduce the ratio of PDFs convolutions $R(\mathbf{p}_T^2) = \mathcal{C}[w h_1^{\perp g} h_1^{\perp g}] / \mathcal{C}[f_1^g f_1^g]$ and to estimate the Boer-Mulders contribution to production of different states of charmonium in this way [15]:

$$\frac{d\sigma(\eta_c)}{dy\, d\mathbf{p}_T^2} = \frac{2\pi^2 \alpha_s^2}{9 M^3 s} \langle \mathcal{O}^{\eta_c}[^1S_0^{(1)}] \rangle\, \mathcal{C}[f_1^g f_1^g]\, [1 - R(\mathbf{p}_T^2)], \quad (12)$$

$$\frac{d\sigma(\chi_{c0})}{dy\, d\mathbf{p}_T^2} = \frac{8\pi^2 \alpha_s^2}{3 M^5 s} \langle \mathcal{O}^{\chi_{c0}}[^3P_0^{(1)}] \rangle\, \mathcal{C}[f_1^g f_1^g]\, [1 + R(\mathbf{p}_T^2)], \quad (13)$$

$$\frac{d\sigma(\chi_{c2})}{dy\, d\mathbf{p}_T^2} = \frac{32\pi^2\alpha_s^2}{45M^2 s}\langle\mathcal{O}^{\chi_{c2}}[^3P_2^{(1)}]\rangle\, \mathcal{C}[f_1^g f_1^g]. \tag{14}$$

In the Fig. 1, the ratio $R(p_T^2)$ is shown for three values of $\sqrt{s}$: 200 GeV (RHIC PHENIX), 63 GeV (ISR), 27 GeV (SPD NICA). It is clear that the relative contribution of Boer-Mulders PDFs decreases when the center-of-mass energy $\sqrt{s}$ decreases and, therefore, the momentum fraction x increases, which makes it difficult to observe directly the Boer-Mulders effect in unpolarised production data at low values of $\sqrt{s}$. Some more complex observables are necessary to study in order to observe the impact of Boer-Mulders effect like polarisation in case of a single charmonium production or azymuthal asymmetries in case of pair charmonium production.

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Conflict of interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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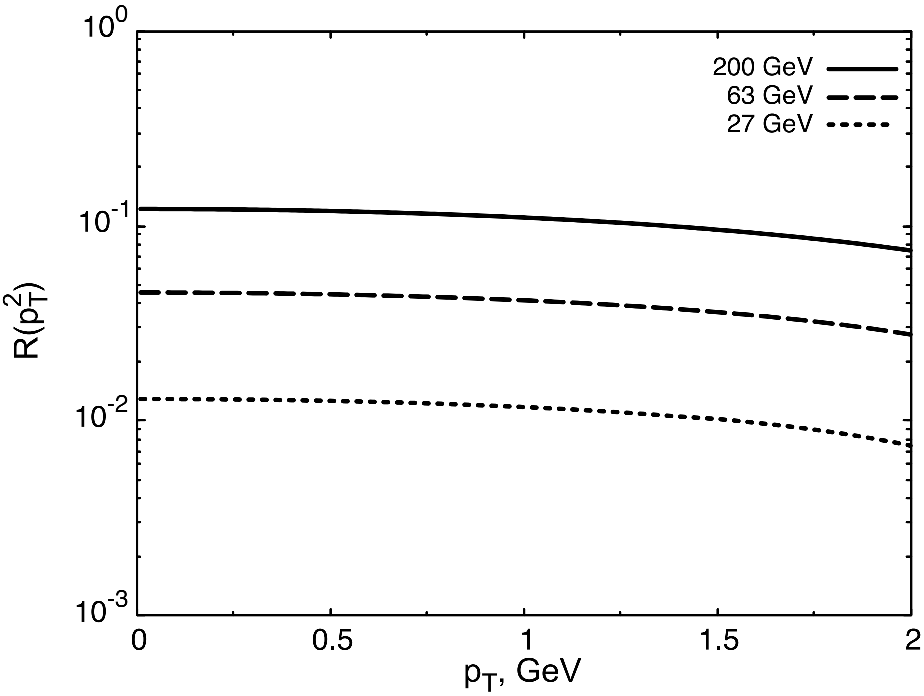

Fig. 1. The ratio $R(p_T^2) = \mathcal{C}[wh_1^{\perp g} h_1^{\perp g}]/\mathcal{C}[f_1^g f_1^g]$ over charmonium transverse momentum for several values of center-of-mass energies $\sqrt{s}$

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