Impact of antiparticle degrees of freedom on neutrino flavor oscillations in frames of quantum field theory
Влияние степеней свободы связанных с античастицами на флейворные осцилляции нейтрино в рамках квантовой теории поля

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Изучаются флейворные осцилляции нейтрино с использованием подхода основанного на квантовой теории поля (КТП), в котором нейтрино считаются виртуальными частицами. В данном формализме используются пропагаторы массовых состояний нейтрино. Ранее, применяя этот подход к осцилляциям нейтрино во внешних полях, применялось разложение пропагаторов с последующим использованием только вклада частиц при вычислении матричного элемента. В настоящей работе приведено обоснование справедливости такого рода преобразований на примере осцилляции нейтрино в вакууме. В принципе, полученные результаты могут быть обобщены на случай осцилляциям нейтрино во внешних полях в рамках КТП.

We study neutrino flavor oscillations using the approach based on the quantum field theory (QFT), where neutrinos are taken to be virtual particles. One deals with the propagators of neutrino mass eigenstates in this formalism. Previously, while applying this approach to neutrino oscillations in external fields, we decomposed the propagators and used only the particle contribution in the calculation of the matrix element. In the present work, we carefully justify the validity of this kind of transformation by considering neutrino oscillations in vacuum. In principle, the results obtained can be extended for the QFT applied to neutrino oscillations in external fields.

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In the wake of numerous experiments (see, e.g., Ref. [1,2]), neutrinos were established to be massive particles having nonzero mixing between different flavors. These neutrino properties result in transitions between neutrino types which are called neutrino flavor oscillations. Historically, neutrino oscillations are described using the quantum mechanical approach (see, e.g., Ref. [3]). However, this kind of formalism was shown in Ref. [4] to have certain shortcomings.

To avoid some of the shortcomings, the approach, where neutrino states are represented as noncovariant wave packets, was proposed (see, e.g., Ref. [5]). The covariant generalization of the neutrino wave packets formalism was developed in Ref. [4]. Nevertheless, an approach to scrutinize neutrino oscillations, consistent with

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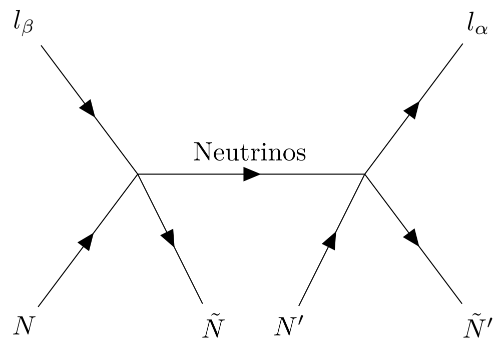

Fig. 1. The schematic illustration of the process which corresponds to neutrino flavor oscillations in frames of QFT.

the quantum fields theory (QFT), has to be developed. This kind of formalisms was developed in Refs. [6–8]. The originally proposed QFT approach was significantly extended in Ref. [4], e.g., by the consideration of the covariant wave packets of in- and out-states of particles involved in a reaction.

As a rule, the QFT approach incorporates both the production, the propagation, and the detection of neutrinos. Neutrinos are supposed in Ref. [6] to be produced in a source, which is a heavy nucleus N, in a reaction with the incoming charged lepton l_β, $l_\beta + N \to \tilde{N} +$ neutrinos. Then, these neutrinos propagate in space and interact with a detector, which again is a heavy nucleus N'. A charged lepton l_α is produced in the wake of this interaction, neutrinos $+ N' \to \tilde{N}' + l_\alpha$. If $l_\alpha \neq l_\beta$, the process in question is interpreted as neutrino oscillations. Note that neutrinos are taken to be virtual particles in this approach. The described process is schematically depicted in Fig. 1.

In Refs. [9, 10], we extended the above picture of neutrino oscillations to include various external fields which neutrinos can interact with. In particular, we considered the cases of the neutrino interaction with background matter and with an external magnetic field. Note that neutrino oscillations in external fields in frames of QFT were also studied in Refs. [11–13].

The main purpose of the present work is to justify one assumption made in Refs. [9, 10]. As we shall see shortly, a process interpreted as neutrino oscillations involves the propagators of virtual neutrinos. In Refs. [9, 10], these propagators were decomposed into particle and antiparticle parts, with only one them being used in the computation of the matrix element. The remaining part was claimed to give a negligible contribution to the matrix element for ultrarelativistic neutrinos. It is an expected but not so obvious assumption. Now, we demonstrate the validity of this statement considering neutrino oscillations in vacuum.

In frames of QFT, the following S-matrix element corresponds to neutrino flavor oscillations,

$$S = -\frac{1}{2}\left(\sqrt{2}G_{\text{int}}\right)^2 \int \mathrm{d}^4x\mathrm{d}^4y \times \left\langle \tilde{N}, \tilde{N}', l_\alpha \left| T\left\{ j_\mu^\dagger(x) J^\mu(x) j^\nu(y) J_\nu^\dagger(y) \right\} \right| N, N', l_\beta \right\rangle, \tag{1}$$

where

$$j_\mu = \sum_\lambda \bar{\nu}_{\lambda\mathrm{L}} \gamma_\mu l_{\lambda\mathrm{L}}, \tag{2}$$

is the operator valued leptonic current, J^μ is the nuclear current operator, and G_{int} is the coupling constant, and $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$ are the Dirac matrices. In Eq. (2), the operators of flavor neutrinos, ν_λ, and of charged leptons, l_λ, stay as the left chiral projections $\propto \frac{1}{2}(1-\gamma^5)$, where $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$.

The flavor neutrinos, which interact with other particles in the standard model, do not have definite masses. To diagonalize the mass term in the neutrino Lagrangian, one introduces the neutrino mass eigenstates ψ_a, with the masses m_a, which are the superposition of the neutrino flavor eigenstates. If we deal with only two neutrinos for simplicity, e.g., ν_e and ν_μ, this matrix transformation reads

$$\nu_\lambda = \sum_{a=1,2} U_{\lambda a}\psi_a, \quad (U_{\lambda a}) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{3}$$

where θ is the vacuum mixing angle.

Assuming that the nuclei in a source and a detector of neutrinos are quite heavy, as well as that charged leptons propagate as plane waves, one rewrites Eq. (1) as

$$S = -2\pi G_{\text{int}}^2 \delta(E_\alpha - E_\beta) e^{-i\mathbf{p}_\alpha\mathbf{x}_2 + i\mathbf{p}_\beta\mathbf{x}_1} \mathrm{i}\mathcal{M}_{\beta\to\alpha}, \tag{4}$$

where

$$\mathcal{M}_{\beta\to\alpha} = \bar{u}_\alpha \gamma_0^{\mathrm{L}} \sum_a U_{\alpha a} U_{\beta a}^* \left(\int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{\mathrm{i}\mathbf{qL}} S_a(E, \mathbf{q}) \right) \gamma_0^{\mathrm{L}} u_\beta(p_\beta), \tag{5}$$

is the matrix element, $u_\beta = u_\beta(p_\beta)$ and $u_\alpha = u_\alpha(p_\alpha)$ are the spinors of incoming and outgoing charged leptons having the momenta $p_\beta^\mu = (E_\beta, \mathbf{p}_\beta)$ and $p_\alpha^\mu = (E_\alpha, \mathbf{p}_\alpha)$, $E = (E_\alpha + E_\beta)/2$, $\mathbf{L}$ is the vector connecting the positions of a source $\mathbf{x}_1$ and a detector $\mathbf{x}_2$, and

$$S_a(q^\mu) = \frac{\gamma_\mu q^\mu + m_a}{q^2 - m_a^2 + \mathrm{i}0}, \tag{6}$$

is the 4D Fourier image of the vacuum propagator of a neutrino mass eigenstate $S_a(x-y) = -\mathrm{i}\left\langle 0 \left| T\left\{ \psi_a(x)\bar{\psi}_a(y) \right\} \right| 0 \right\rangle$.

We cast the propagator to the form, $S_a = S_a^{(\psi)} + S_a^{(\bar{\psi})}$, where

$$S_a^{(\psi)} = \frac{1}{2(q_0 - E_a + \mathrm{i}0)} \left(\gamma^0 - \frac{1}{E_a}\boldsymbol{\gamma}\mathbf{q} + \frac{m_a}{E_a} \right), \tag{7}$$

$$S_a^{(\bar{\psi})} = \frac{1}{2(q_0 + E_a - \mathrm{i}0)} \left(\gamma^0 + \frac{1}{E_a}\boldsymbol{\gamma}\mathbf{q} - \frac{m_a}{E_a} \right). \tag{8}$$

Here $E_a = \sqrt{q^2 + m_a^2}$ is the energy of a massive neutrino. The propagator $S_a^{(\psi)}$ in Eq. (7) has the positive imaginary term $+\mathrm{i}0$ in the denominator. We referred to such a function in Ref. [9] as to the propagator of particles. On the contrary, $S_a^{(\bar{\psi})}$ in Eq. (8) has the negative imaginary term $-\mathrm{i}0$ in the denominator. We can attribute it to the antiparticle propagator.

We calculate the contributions of both neutrinos and antineutrinos to the matrix element in Eq. (5) by replacing $S_a \to S_a^{(\psi)}$ and $S_a \to S_a^{(\bar{\psi})}$. To simplify the calculations we adopt the forward scattering approximation for leptons by taking that $u_{\alpha,\beta}^{\mathrm{L}} = (0,0,0,1)^{\mathrm{T}}$. It means that both leptons are left polarized and propagate along the positive direction of the z-axis. We rely on the Dirac matrices in the chiral representation here. We also suppose that $\mathbf{L} = L\mathbf{e}_z$.

First, we study the particles contributions to the matrix element. Using Eq. (7), one gets that

$$\bar{u}_\alpha \gamma_0^{\mathrm{L}} \left(\gamma^0 - \frac{1}{E_a}\boldsymbol{\gamma}\mathbf{q} + \frac{m_a}{E_a} \right) \gamma_0^{\mathrm{L}} u_\beta(p_\beta) = \frac{1}{2}\left(1 + \frac{q_z}{E_a}\right). \tag{9}$$

Then, we use the cylindrical coordinates for the momentum of the virtual neutrino, $\mathbf{q} = \rho\mathbf{e}_\rho + z\mathbf{e}_z$. Taking into account Eq. (9), the integral in Eq. (5), averaged over leptonic states, becomes

$$\begin{aligned} I_\psi =& \bar{u}_\alpha \gamma_0^{\mathrm{L}} \left(\int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{\mathrm{i}\mathbf{q}\mathbf{L}} S_a^{(\psi)}(E, \mathbf{q}) \right) \gamma_0^{\mathrm{L}} u_\beta(p_\beta) \\ =& \frac{1}{8\pi^2} \int_0^\infty \rho \mathrm{d}\rho \int_{-\infty}^{+\infty} \mathrm{d}z \frac{\left(1 + \frac{z}{E_a}\right) e^{\mathrm{i}zL}}{E - E_a + \mathrm{i}0}. \end{aligned} \tag{10}$$

The integral over z in Eq. (10) is computed in the complex plane. Assuming that $L > 0$, we should close the contour in the upper half plane to provide the integral convergence. The poles of the integrand, which are the solution of the equation $E - E_a + \mathrm{i}0 = 0$, have the following form:

$$z_0 = \begin{cases} \sqrt{\rho_0^2 - \rho^2} + \mathrm{i}0, & \text{if } \rho < \rho_0, \\ \mathrm{i}\sqrt{\rho^2 - \rho_0^2}, & \text{if } \rho > \rho_0, \end{cases} \tag{11}$$

where $\rho_0 = \sqrt{E - m_a^2}$.

Decomposing the denominator of the integrand in Eq. (10) near the pole,

$$E - E_a + \mathrm{i}0 = -(z - z_0) \left.\frac{\mathrm{d}E_a}{\mathrm{d}z}\right|_{z=z_0} + \dots = -(z - z_0)\frac{z_0}{E}, \tag{12}$$

since $E_a(z_0) = E$, one obtains that

$$\begin{aligned} I_\psi =& -\frac{\mathrm{i}}{4\pi} \left[\int_0^{\rho_0} \rho \mathrm{d}\rho e^{\mathrm{i}L\sqrt{\rho_0^2 - \rho^2}} \left(1 + \frac{E}{\sqrt{\rho_0^2 - \rho^2}}\right) \right. \\ & \left. + \int_{\rho_0}^\infty \rho \mathrm{d}\rho e^{-L\sqrt{\rho^2 - \rho_0^2}} \left(1 - \frac{\mathrm{i}E}{\sqrt{\rho^2 - \rho_0^2}}\right) \right]. \end{aligned} \tag{13}$$

The remaining integrals over ρ in Eq. (13) are computed with help of the expressions,

$$\int_0^{\rho_0} \rho \mathrm{d}\rho \frac{e^{\mathrm{i}L\sqrt{\rho_0^2-\rho^2}}}{\sqrt{\rho_0^2-\rho^2}} = -\frac{\mathrm{i}}{L}(e^{\mathrm{i}\rho_0 L}-1), \tag{14}$$

$$\int_0^{\rho_0} \rho \mathrm{d}\rho e^{\mathrm{i}L\sqrt{\rho_0^2-\rho^2}} = -\frac{\mathrm{i}}{L^2}e^{\mathrm{i}\rho_0 L}(L\rho_0+\mathrm{i}) - \frac{1}{L^2}, \tag{15}$$

$$\int_{\rho_0}^{\infty} \rho \mathrm{d}\rho \frac{e^{-L\sqrt{\rho^2-\rho_0^2}}}{\sqrt{\rho^2-\rho_0^2}} = \frac{1}{L}, \tag{16}$$

$$\int_{\rho_0}^{\infty} \rho \mathrm{d}\rho e^{-L\sqrt{\rho^2-\rho_0^2}} = \frac{1}{L^2}. \tag{17}$$

Eventually, one gets that

$$I_\psi = -\frac{E e^{\mathrm{i}\sqrt{E-m_a^2}L}}{4\pi L}\left(1+\frac{\sqrt{E-m_a^2}}{E}+\frac{\mathrm{i}}{LE}\right). \tag{18}$$

As a rule, one considers the situation when the propagation distance is great, $L \gg E^{-1}$. Thus, we can neglect the last term in Eq. (18).

Using Eqs. (3) and (18) and considering the two neutrinos system, one gets the particle contribution to the matrix element in the form,

$$\begin{aligned}\mathcal{M}_{e\to\mu}^{(\psi)} \approx & -\frac{E e^{\mathrm{i}EL}}{4\pi L}\sin 2\theta\left[e^{-\mathrm{i}\frac{m_2^2 L}{2E}}\left(1-\frac{m_2^2}{4E^2}\right)-e^{-\mathrm{i}\frac{m_1^2 L}{2E}}\left(1-\frac{m_1^2}{4E^2}\right)\right]. \\ = & -\frac{E e^{\mathrm{i}EL}}{4\pi L}\sin 2\theta\left[e^{-\mathrm{i}\frac{m_2^2 L}{2E}}-e^{-\mathrm{i}\frac{m_1^2 L}{2E}}+\mathcal{O}\left(\frac{m_a^2}{E^2}\right)\right],\end{aligned} \tag{19}$$

where we assume that neutrinos are ultrarelativistic with $m_a \ll E$.

Now, we compute the antiparticle contribution to the matrix element. For this purpose, we substitute $S_a^{(\bar{\psi})}$ in Eq. (8) to Eq. (5). We just list the main modifications in Eqs. (9)-(18). Equation (9) becomes

$$\bar{u}_\alpha \gamma_0^{\mathrm{L}}\left(\gamma^0+\frac{1}{E_a}\boldsymbol{\gamma}\mathbf{q}-\frac{m_a}{E_a}\right)\gamma_0^{\mathrm{L}}u_\beta(p_\beta) = \frac{1}{2}\left(1-\frac{q_z}{E_a}\right). \tag{20}$$

The integral in the matrix element reads

$$\begin{aligned}I_{\bar{\psi}} = & \bar{u}_\alpha \gamma_0^{\mathrm{L}}\left(\int \frac{\mathrm{d}^3 q}{(2\pi)^3}e^{\mathrm{i}\mathbf{qL}}S_a^{(\bar{\psi})}(E,\mathbf{q})\right)\gamma_0^{\mathrm{L}}u_\beta(p_\beta) \\ = & \frac{1}{8\pi^2}\int_0^\infty \rho \mathrm{d}\rho \int_{-\infty}^{+\infty}\mathrm{d}z\frac{\left(1-\frac{z}{E_a}\right)e^{\mathrm{i}zL}}{E+E_a-\mathrm{i}0}.\end{aligned} \tag{21}$$

Instead of Eq. (11), the poles of the integrand in Eq. (21) are

$$\bar{z}_0 = \begin{cases} -\sqrt{\rho_0^2-\rho^2}+\mathrm{i}0, & \text{if } \rho < \rho_0, \\ \mathrm{i}\sqrt{\rho^2-\rho_0^2}, & \text{if } \rho > \rho_0. \end{cases} \tag{22}$$

Note that we close the contour also in the upper half plane while integrating over z. The decomposition in Eq. (12) takes the form,

$$E + E_a - \mathrm{i}0 = (z - \bar{z}_0) \left. \frac{\mathrm{d}E_a}{\mathrm{d}z} \right|_{z=\bar{z}_0} + \dots = -(z - \bar{z}_0)\frac{\bar{z}_0}{E}, \tag{23}$$

since $E_a(\bar{z}_0) = -E$.

The integral in Eq. (13) is rewritten as

$$\begin{aligned} I_{\bar{\psi}} = & \frac{\mathrm{i}}{4\pi} \left[\int_0^{\rho_0} \rho \mathrm{d}\rho e^{-\mathrm{i}L\sqrt{\rho_0^2-\rho^2}} \left(1 - \frac{E}{\sqrt{\rho_0^2 - \rho^2}} \right) \right. \\ & \left. + \int_{\rho_0}^{\infty} \rho \mathrm{d}\rho e^{-L\sqrt{\rho^2-\rho_0^2}} \left(1 - \frac{\mathrm{i}E}{\sqrt{\rho^2 - \rho_0^2}} \right) \right]. \end{aligned} \tag{24}$$

The integrations in Eq. (24) can be carried out if we replace $L \to -L$ in Eqs. (14) and (15). The final result for $I_{\bar{\psi}}$ reads

$$I_{\bar{\psi}} = \frac{E e^{-\mathrm{i}\sqrt{E-m_a^2}L}}{4\pi L} \left(1 - \frac{\sqrt{E - m_a^2}}{E} + \frac{\mathrm{i}}{LE} \right). \tag{25}$$

Using Eq. (25), one gets the antiparticle contribution to the matrix element,

$$\mathcal{M}_{e\to\mu}^{(\bar{\psi})} \approx \frac{E e^{-\mathrm{i}EL}}{4\pi L} \sin 2\theta \left[\frac{m_2^2}{4E^2} e^{\mathrm{i}\frac{m_2^2 L}{2E}} - \frac{m_1^2}{4E^2} e^{-\mathrm{i}\frac{m_1^2 L}{2E}} \right], \tag{26}$$

where we are based on the same assumptions as for the derivation of Eq. (19).

Comparing Eqs. (19) and (26), one obtains that

$$\frac{|\mathcal{M}_{e\to\mu}^{(\bar{\psi})}|}{|\mathcal{M}_{e\to\mu}^{(\psi)}|} \sim \frac{m_a^2}{E^2} \ll 1. \tag{27}$$

Thus, the antiparticle part of the propagator in Eq. (8) gives a contribution to the matrix element which is negligible for ultrarelativistic neutrinos.

Finally, we demonstrate that the matrix element in Eq. (19) results in the appropriate transition probability expression. Based on the fact that $P_{\nu_e\to\nu_\mu} \propto |\mathcal{M}_{e\to\mu}|^2$, one gets that

$$P_{\nu_e\to\nu_\mu}(L) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4E} L \right), \tag{28}$$

where $\Delta m^2 = m_2^2 - m_1^2$. One can see that Eq. (28) reproduces the known expression for the transition probability of neutrino oscillations in vacuum (see, e.g., Ref. [3]).

In conclusion, we mention that we have analyzed the validity of the decomposition of the propagators of neutrino mass eigenstates which was used in Ref. [9]. After the separation of the particle and antiparticle parts in the propagator, one uses only the particle part in the formalism for the description of neutrino flavor oscillations based on QFT. We have shown that the contribution of the antiparticle part to the matrix element is suppressed for ultrarelativistic neutrinos. This fact has been demonstrated for neutrino oscillations in vacuum since one can carry out all the calculations analytically in this case. Despite the QFT based description of neutrino oscillations in external fields is more complicated technically, one can extrapolate the obtained result to this kind of situations.

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Conflict of interest

The author declares that there is no conflict of interest.

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